Reg. No. :

21/05/18

Question Paper Code : 40057

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Second Semester

Aeronautical Engineering

MA 8251 – ENGINEERING MATHEMATICS – II

(Common to all branches, except Marine Engineering)

(Regulations 2017)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. If 3 and 5 are two eigenvalues of the matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ then find its third eigenvalue and hence $|A|$.

2. Show that the eigenvalues of a null matrix are zero.

3. If $\vec{F} = x^3\,\vec{i} + y^3\vec{j} + z^3\,\vec{k}$, then find div curl $\vec{F}$.

4. Find the values of a, b, c such that the following vector is irrotational.
$\vec{F} = (x + 2y + az)\,\vec{i} + (bx - 3y - z)\,\vec{j} + (4x + cy + 2z)\vec{k}$.

5. If $f(z) = r^2 (\cos 2\theta + i \sin p\theta)$ is analytic, then find the value of 'p'.

6. Examine whether the function $u = xy^2$ can be a real part of an analytic function.

7. If 'C' is the circle $|z| = 3$ and if $g(z_0) = \int_C \frac{2z^2 - z - 2}{z - z_0}\,dz$ then find g(2).

8. Find the value of $\int_C \frac{3z^2 + 7z + 1}{z + 1}\,dz$ if C is $|z| = \frac{1}{2}$.

9. If $L[f(t)] = F(s)$ then prove that $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$.

10. Find the Laplace transform of $\left[\frac{t}{e^t}\right]$.

PART – B **(5×16 = 80 Marks)**

11. a) i) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ (8)

ii) Using Cayley-Hamilton theorem find the inverse of the given matrix

$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$. (8)

(OR)

b) Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to a canonical form through an orthogonal transformation. Find also its nature. (16)

12. a) Verify the Gauss divergence theorem for $\vec{F} = x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}$ taken over the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0$ and $z = a$. (16)

(OR)

b) Verify Stoke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - (xz)\vec{k}$ where S is the open surface of the cube $x = 0, x = 2, y = 0, y = 2, z = 0$ and $z = 2$ above the xy -plane. (16)

13. a) i) Find the analytic function $f(z) = u + iv$ if $u - v = e^x[\cos y - \sin y]$. (8)

ii) Find the bilinear transformation which maps the points $z = -1, 0, 1$ on to the points $w = -1, -i, 1$. Show that under this transformation the upper half of the z-plane maps on to the interior of the unit circle $|w| = 1$. (8)

(OR)

b) i) If $f(z) = u + iv$ is an analytic function then prove that

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(u^p) = p(p-1)(u^{p-2})\,|f'(z)|^2$. (8)

ii) Find the image of the circle $|z - 2i| = 2$ in the complex plane under the transformation $w = \frac{1}{z}$. (8)

14. a) i) Evaluate $\int_C \frac{z^2}{(z^2+1)^2}\,dz$ where C is the circle $|z - i| = 1$ by using Cauchy's integral formula. (8)

ii) Expand $f(z) = \frac{6z+5}{(z+1)z(z-2)}$ in Laurent's series valid for $1 < |z+1| < 3$. (8)

(OR)

b) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\,d\theta$ using contour integration. (16)

15. a) i) Using convolution theorem find the inverse Laplace transform of $\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$. (8)

ii) Find the Laplace transform of $[t \cos t \sinh 2t]$. (8)

(OR)

b) i) Find $L[f(t)]$ if $f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ given $f(t+2) = f(t)$. (8)

ii) Solve $y'' - 3y' + 2y = 1$ given that $y(0) = 0, y'(0) = 1$ by using Laplace transform method. (8)

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Reg. No. :

Question Paper Code : 80209

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Second Semester

Civil Engineering

MA 8251 — ENGINEERING MATHEMATICS – II

(Common to All branches (Except Marine Engineering))

(Regulation 2017)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If λ is the eigenvalue of the matrix A, then prove that λ^2 is the eigenvalue of A^2.

2. If the eigenvalues of the matrix A of order 3 × 3 are 2, 3 and 1, then find the determinant of A.

3. Find the unit normal vector to the surface $x^2 + y^2 = z$ at $(1, -2, 5)$.

4. State Stoke's theorem.

5. Is the function $f(z) = e^z$ analytic.

6. Find the fixed point of the bilinear transformation $w = \frac{1}{z}$.

7. Evaluate $\int_C \sin z \, dz$, where C is the entire complex plane.

8. Define singularity of a function $f(z)$.

9. Find $L[e^{-t} \sin t]$.

10. State sufficient conditions for the existence of Laplace transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem find A^{-1}, if $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $2xy - 2yz + 2xz$ into a canonical form by an orthogonal reduction. (16)

12. (a) (i) Verify Gauss divergence theorem for the vector function $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cuboids bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 1$, and $z = 1$. (10)

(ii) Find the value of n so that the vector $r^n\vec{r}$ is irrotational and solenoidal. (6)

Or

(b) (i) Apply Green's theorem to evaluate $\int_C \left[(2x^2 - y^2)dx + (x^2 + y^2)dy\right]$, where C is the boundary of the area by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$. (8)

(ii) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} + 2xy\vec{j}$, taken around the rectangle bounded by the lines $x = 0, y = 0, x = 1$ and $y = 1$. (8)

13. (a) (i) Determine the analytic function $f(z) = u + iv$, if $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto $w = i, 0, -i$. (8)

Or

(b) (i) Show that the real and imaginary parts of an analytic functions are harmonic. (8)

(ii) Find the image of $|z - 2i| = 2$ under the transformation $w = \dfrac{1}{z}$. (8)

14. (a) (i) If $F(a) = \oint_C \dfrac{(3z^2 + 7z + 1)}{z - a} dz$, where C is $|z| = 2$, then find $F(1 - i)$ and $F'(1 - i)$. (8)

(ii) Using contour integration, evaluate $\int_0^\infty \dfrac{dx}{(x^2 + 1)^2}$. (8)

Or

(b) (i) Obtain the Laurent's series expansion of $f(z) = \dfrac{z^2 - 1}{(z + 2)(z + 3)}$ if $2 < |z| < 3$. (8)

(ii) Evaluate by using contour integration $\int_0^{2\pi} \dfrac{d\theta}{13 + 5\sin\theta}$. (8)

15. (a) (i) Find the Laplace transform of $f(t)$ with period $2a$, where $f(t) = \begin{cases} t, & \text{for } 0 < t < a \\ 2a - t, & \text{for } a < t < 2a \end{cases}$. (8)

(ii) Using convolution theorem, find $L^{-1}\left[\dfrac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right]$. (8)

Or

(b) (i) Find $L\left[\dfrac{\cos 2t - \cos 3t}{t}\right]$. (8)

(ii) Solve $\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} + 2y = e^{3t}$; given that $y(0) = 0, \dfrac{dy}{dt}(0) = 0$. (8)

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ii) Find the image of the circle | z – 2i | = 2 in the complex plane under the transformation $w = \frac{1}{z}$. (8)

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(OR)

b) i) Find L [f(t)] if $f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ given f (t + 2) = f(t). (8)

ii) Solve y'' – 3 y' + 2y = 1 given that y(0) = 0, y'(0) = 1 by using Laplace transform method. (8)