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Reg. No. :

Question Paper Code : 80213

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Third Semester

Medical Electronics

MA 8352 — LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Biomedical Engineering/Computer and Communication Engineering/Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulation 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Determine whether the vectors $v_1 = (1, 2, 1), v_2 = (2, 1, 0)$ and $v_3 = (1, -1, 2)$ form a linearly independent or linearly dependent in $V_3(R)$.

2. If $V = A \oplus B$, then show that dim V = dim A+ dim B.

3. Define Kernal of T.

4. Obtain the matrix representing the linear transformation $T : V_3(R) \to V_3(R)$ given by $T(a, b, c) = (3a, a-b, 2a+b+c)$ with respect to the standard basis $\{e_1, e_2, e_3\}$.

5. Let V be the vector space of polynomials with linear product given by $<f, g> = \int_0^1 f(t)g(t)\,dt$ where $f(t) = t+2$ and $g(t) = t^2 - 2t - 3$. Find $<f, g>$.

6. Define Adjoint matrix.

7. Find the differential equation of all spheres whose centres lie on the Z- axis.

8. Solve $p^2 + q^2 = x + y$.

9. Write the formula for Half range Fourier sine series.

10. A slightly stretched string of length l has its ends fastened at $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin \frac{3\pi x}{l}$. If it is released from rest from this position, write the boundary conditions.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Let R^+ be the set of all positive real numbers. Define addition and scalar multiplication as follows $u + v = uv$ for all $u, v \in R^+$; $\alpha u = u^\alpha$ for all $u \in R^+$ and $\alpha \in R^+$. Determine whether or not R^+ is a real vector space.

(ii) Prove that $S = \{v_1, v_2, \cdots, v_n\}$ is a linearly dependent set of vectors in V iff there exists a vector $v_k \in S$ such that v_k is a linear combination of the preceding vectors $v_1, v_2, \cdots v_n$. (6 + 10)

Or

(b) (i) If a_1, a_2, a_3 are fixed elements of a field F, then show that the set W of all ordered triads $\{x_1, x_2, x_3\}$ of elements of F, such that $a_1x_1 + a_2x_2 + a_3x_3 = 0$, is a sub space of $V_3(F)$.

(ii) Prove that every linearly independent subset of a finitely generated vector space V (F) is either a basis of V or can be extended to form a basis of V. (8 + 8)

12. (a) (i) Find the linear transformation $T : V_3(R) \to V_3(R)$ determined by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ write the standard basis $\{e_1, e_2, e_3\}$.

(ii) Let V be a vector space over a field F. Let A and B be subspaces of V, then show that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$. (6 + 10)

Or

(b) (i) Let L be a linear transformation from R^3 to R^3 whose matrix representation A with respect to the standard basis is given below. Find the Eigenvalues of L and a basis of Eigenvectors

$$A = \begin{pmatrix} 1 & 3 & -3 \\ 3 & 1 & -3 \\ -3 & -3 & 1 \end{pmatrix}.$$

(ii) If A is an m × n matrix, then prove that $N(A)$ is a sub space of R^n. (8+ 8)

13. (a) (i) Let V be the set of all continuous real valued functions defined on the closed interval [0, 1], then prove that V is a real inner product space with inner product, defined by

$$< f, g >= \int_0^1 f(t)\, g(t)\, dt.$$

(ii) Find the orthogonal basis containing the vector (1, 3, 4) for $V_3(R)$ with the standard inner product. (8 + 8)

Or

(b) State and prove Gram - Schmidt orthogonalisation process. (16)

14. (a) (i) Form the partial differential equation by eliminating the arbitrary functions f and ϕ from $Z = x\, f(y/x) + y\, \phi(x)$,

(ii) Solve $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$. (8 + 8 = 16)

Or

(b) (i) Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$.

(ii) Solve $p^2 + q^2 = x^2 + y^2$. (8 + 8)

15. (a) (i) Express $f(x) = (\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$.

(ii) Show that in $0 \le x \le \pi$, $x(\pi - x) = \frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \cdots\right)$. (8+ 8)

Or

(b) The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time and show that the mid-point of the string always remains at rest. (16)

15. (a) (i) Obtain the Fourier series to represent $f(x)=\frac{1}{4}(\pi-x)^2$ in the interval $0<x<2\pi$.

(ii) Obtain a half range Fourier sine series for $f(x)=\begin{cases}\omega x, & 0\le x\le\frac{1}{2}\\ \omega(l-x), & \frac{1}{2}\le x\le 1\end{cases}$, and find the value of the series $1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\infty$. (8 + 8)

Or

(b) An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. This end is maintened at temperature u_0 at all points and the other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state. (16)

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Question Paper Code : 25140

B.E. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Third Semester

Electronics and Communication Engineering

MA 8352 — LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Electronics and Telecommunication Engineering/ Medical Electronics/Biomedical Engineering/Computer and Communication Engineering)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Determine whether the vectors $v_1=(1,-2,3)$, $v_2=(5,6,-1)$, $v_3=(3,2,1)$ form a linearly dependent or linearly independent set in R^3.

2. What are the possible subspace of R^2?

3. Verify that $T:R^3\to R$, and $T(u)=\|u\|$ is a linear transformation or not.

4. State the dimension theorem for matrices.

5. Let R^2 have the weighted Euclidean inner product defined as $<u,v>=2u_1v_1+3u_2v_2$ and let $u=(1,1), v=(3,2), w=(0,-1)$. Compute the value of $<u+v,3w>$.

6. Let P_2 have the inner product $<p,q>=\int_{-1}^{1}p(x)\,q(x)\,dx$. Find the angle between p and q, where $p=x$ and $q=x^2$ with respect to the inner product on P_2.

7. How the second order partial differential equations are classified?

8. Solve $pq+p+q=0$.

9. Find the value of c, for which $u = e^{-4t}\cos 3x$ is the solution of the equation $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$.

10. State giving reasons whether the function $f(x) = \tan x$ can be expanded in Fourier series in the interval of $(-\pi, \pi)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Let V be the set of all positive real numbers. Define the vector addition and scalar multiplication as follows :

$x + y = xy$ and $kx = x^k$.

Determine whether or not V is a vector space over $\mathbb{R}$ with respect to above operations.

(ii) Determine the basis and dimension of the solution space of the linear homogeneous system $x + y - z = 0; -2x - y + 2z = 0; -x + z = 0$. (8 + 8)

Or

(b) (i) Determine whether the set of all 2 × 2 matrix of the form $\begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix}$, $a, b \in R$, with respect to standard matrix addition and scalar multiplication is a vector space or not? If not, list all the axioms that fail to hold.

(ii) Determine whether the set of vectors $X_1 = (1, 1, 2)$, $X_2 = (1, 0, 1)$ and $X_3 = (2, 1, 3)$ span R^3. (10 + 6)

12. (a) (i) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x, y) = (x + 3y, 0, 2x - 4y)$. Compute the matrix of the transformation with respect to the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$. Find $N(T)$ and $R(T)$. Is T one-to-one? Is T onto. Justify your answer. (8)

(ii) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (2x - y, 3z)$ verify whether T is linear or not. Find $N(T)$ and $R(T)$ and hence verify the dimension theorem. (8)

Or

(b) (i) Let $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ be defined as

$T(f(x)) = f(x) + (x + 1) f'(x)$

Find eigen values and corresponding eigen vectors of T with respect to standard basis of $\mathbb{P}_2(\mathbb{R})$. (8)

(ii) Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3, where $v_1 = (1, 1, 1)$, $v_2 = (1, 1, 0)$ and $v_3 = (1, 0, 0)$. Let $T : R^3 \to R^2$ be the linear transformation such that $T(v_1) = (1, 0)$, $T(v_2) = (2, -1)$ and $T(v_3) = (4, 3)$. Find the formula for $T(x_1, x_2, x_3)$, then use this formula to compute $T(2, -3, 5)$. (8)

13. (a) (i) State the projection theorem.

(ii) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis, where $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$ and $u_3 = (0, 0, 1)$. (4 + 12)

Or

(b) Let the vector space P_2 have the inner product $\langle p, q \rangle = \int_0^1 p(x)\, q(x)\, dx$.

Apply the Gram-Schmidt process to transform the basis $S = \{u_1, u_2, u_3\} = \{1, x, x^2\}$ into an orthonormal basis. (16)

14. (a) (i) Solve : $x(y - z)p + y(z - x)q = z(x - y)$.

(ii) Solve : $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$. (8 + 8)

Or

(b) (i) Form a partial differential equations by eliminating the function f from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

(ii) Solve : $(D^2 + DD' - 6D'^2)z = y\cos x$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. (6 + 10)

Reg. No. :

Question Paper Code : 90337

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Third Semester

Medical Electronics

MA8352 – LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Biomedical Engineering/Computer and Communication Engineering/ Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. If $V = R^3$, then verify whether $W = \{(a_1, a_2, a_3)/2a_1 - 7a_2 + a_3 = 0\}$ is a subspace or not.

2. Find the dimension of W, where $W = \{(x_1, x_2, x_3)/x_1 + x_2 + x_3 = 0\}$.

3. Let $T : P_3(R) \to P_2(R)$ be a linear transformation defined by $T(f(x)) = f'(x)$. Let B_1 and B_2 be the standard bases for $P_3(R)$ and $P_2(R)$ respectively. Then find [T].

4. Test the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \in M_{2\times 2}(R)$ for diagonalizable.

5. Let $V = R^2$ and $S = \{(1,0), (0, 1)\}$. Check whether S is orthonormal basis or not.

6. Find the conjugate transpose of $A = \begin{pmatrix} i & 1+2i \\ 2 & 3+4i \end{pmatrix}$.

7. Form the partial differential equation by eliminating the arbitrary function from $z = e^{x-y} \cdot f(x + y)$.

8. Find the complete integral of the partial differential equation $z = px + qy + p^2 - q^2$.

9. State Dirichlet's conditions for Fourier series of f(x) defined in the interval $c \le x \le c + 2l$.

10. Write all three possible solutions of one dimensional heat equation.

PART – B (5×16=80 Marks)

11. a) i) Determine the given set in $P_4(R)$ is linearly dependent or linearly independent for $x^4 - x^3 + 5x^2 - 8x + 6$, $-x^4 + x^3 - 5x^2 + 5x - 3$, $x^4 + 3x^2 - 3x + 5$ and $2x^4 + x^3 + 4x^2 + 8x$. (8)

ii) Let $S = \{v_1, v_2, v_3\}$ where $v_1 = (1, -3, -2)$, $v_2 = (-3, 1, 3)$, $v_3 = (-2, -10, -2)$. Verify whether S forms a basis or not. (8)

(OR)

b) i) Verify whether the first polynomial can be expressed as a linear combination of the other two in P_3 (R) for the given $x^3 - 8x^2 + 4x$, $x^3 - 2x^2 + 3x - 1$ and $x^3 - 2x + 3$. (8)

ii) Let W_1 and W_2 be subspaces of V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ (or) $W_2 \subseteq W_1$. (8)

12. a) i) Let $T : R^3 \to R^2$ be defined by $T(x, y, z) = (2x, -y, 3z)$. Verify whether T is linear or not. Find N(T) and R(T) and hence verify the dimension theorem. (8)

ii) Let $T : P_2(R) \to P_2(R)$ be defined as $T[f(x)] = f(x) + (x + 1) f'(x)$. Find eigenvalues and corresponding eigenvectors of T with respect to standard basis of $P_2(R)$. (8)

(OR)

b) i) Test for diagonalizability of the matrix $A = \begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ 6 & -6 & 0 \end{bmatrix}$ and if A is diagonalizable, find the invertible matrix Q such that $Q^{-1}AQ = D$. (8)

ii) Let T be the linear operator on R^3 defined by $T\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{bmatrix} 4a_1 + a_3 \\ 2a_1 + 3a_2 + 2a_3 \\ a_1 + 4a_3 \end{bmatrix}$. Determine the eigenspace of T corresponding to each eigenvalue. Let B be the standard ordered basis for R^3. (8)

13. a) i) Let R^3 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis, where $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$ and $u_3 = (0, 0, 1)$. (10)

ii) Let $S = \{(1, 1, 0), (1, -1, 1), (-1, 1, 2)\}$ be an orthogonal set then orthonormal set is $\left\{\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{3}}(1,-1,1), \frac{1}{\sqrt{6}}(-1,1,2)\right\}$ both are basis of R^3. Let $x = (2, 1, 3) \in R^3$. Express x as a linear combination of orthogonal set S and orthonormal set. (6)

(OR)

b) i) Use the least square approximation to find the best fit with a linear function and hence compute the error for the following data (–3, 9), (–2, 6), (0, 2) and (1, 1). (10)

ii) Compute the orthogonal complement of $S = \{(1, 0, i), (1, 2, 1)\}$ in C^3. (6)

14. a) i) Solve $z = p^2 + q^2$. (8)

ii) Find the complete integral of $p^2y(1 + x^2) = qx^2$. (8)

(OR)

b) i) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$. (8)

ii) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = x + y$. (8)

15. a) i) Find the cosine series for $f(x) = x - x^2$ in the interval $0 < x < 1$. (8)

ii) Obtain the sine series for $f(x) = x$ in $0 < x < \pi$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. (8)

(OR)

b) i) An finitely long uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. This end is maintained at a temperature u_0 at all points and other edges are kept at zero temperature. Determine the temperature at any point of the plate in the steady state. (8)

ii) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any time and at any distance from the end $x = 0$. (8)

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