Definitions:-

Electrical circuit :- It is a closed path of wires and Electrical components which allows a current through it.

Circuit Diagram :- It is a graphical representation of an Electric circuit, It shows the components and interconnections of the circuit using symbols.

Voltage(v) voltage is the difference in electric potential between two points, that pushes charged electrons (current) through a conducting loop.

$\boxed{\text{Unit is volts (v)}}$

voltage is represented by the letter 'v' or 'E'

Current(I)

Current is the rate of flow of electric charge

$\boxed{\text{Unit is amperes (A)}}$

current is represented by the letter 'I'.

Resistance:-(R)

Resistance is a measure of the opposition to the current flow. in an Electrical circuit

It is represented by the letter 'R'

$\boxed{\text{unit of Resistance} = \Omega \text{ (ohms)}}$

Resistor:-

Components that offer Resistance.

Conductance: (G)

Conductance is the reciprocal of resistance

$$\boxed{G = 1/R}$$

$\boxed{\text{unit of conductance is } \mho \text{ (mho)}}$

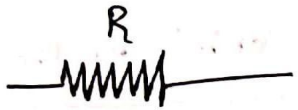
Charge:-

Amount of energy or electrons that pass from one body to another.

Collomb is the unit of Electric charge.

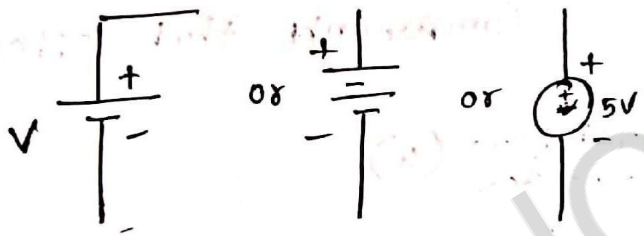
Symbols

Resistance/Resistor:-



unit ohms.

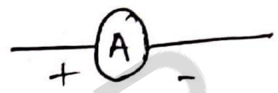
DC voltage Source:-



DC Current Source:-

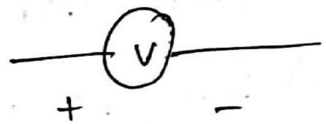
Ammeter:-

* Ammeter is used to measure current
* It should be connected in series



Voltmeter:-

* Voltmeter is used to measure voltage or potential difference between two points in a circuit.



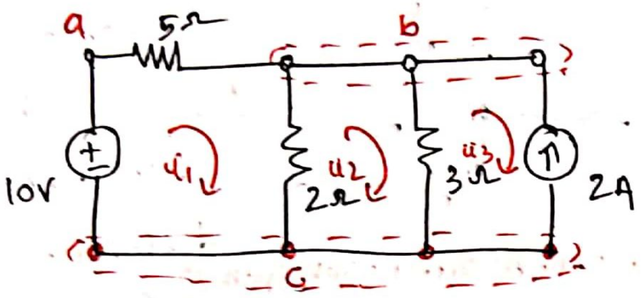
Connection cable:-

Variable Resistor:-

Value of the resistor is variable in such kind of resistors.

Basic Components of Electric Circuits:-

* Branch,
* Nodes.
* Loop's.



Branches:-

A branch represent a single element such as a voltage source or a resistor.

In the above circuit, 10V - voltage source, 2A - current source, 5Ω, 2Ω, & 3Ω Resistors are the branches.

Node:

A Node is a Point of connection between two or more branches.

Point a, b, c are nodes of the circuit.

If a short circuit (a connecting wires) connects two nodes, the two node constitutes a single node. (Even though 3 points are there near point 3 they are considered as single node)

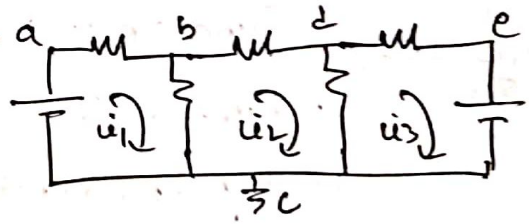
Loop

A loop is a closed path formed by starting at a node, passing through the a set of nodes and returning to the starting node.

abca → forms first loop.

bcb → forms second loop / third loop.

eg.



abca → 1st loop

bdcb → 2nd loop

decd → 3rd loop

$i_1, i_2, \& i_3$ → are called as loop currents.

Series & Parallel Circuits:-

Series

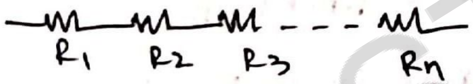
Resistor R_1 & R_2 are said to be in series. because the same current 'I' flows in both the resistors.

$$\boxed{R_{eq} = R_1 + R_2}$$

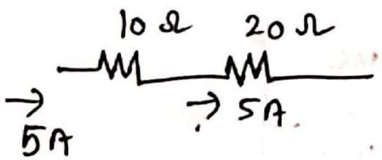
Total Resistance or equivalent Resistance is the sum of the two resistors or 'n' resistors.

$$\boxed{R_{eq} = R_1 + R_2 + \cdots R_n}$$

for 'n' resistors connected in series.



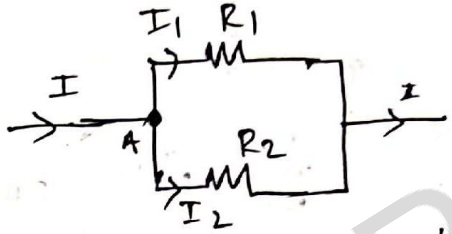
eg.



Same 5A is flowing in both 10 Ω & 20 Ω so they are connected in series.

$R_{eq} = 10 + 20 = 30\,\Omega$

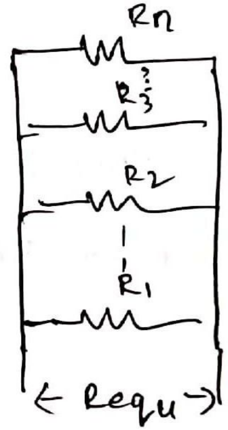
Parallel Circuit / Shunt Circuit:-



Resistors R_1 & R_2 are said to be connected in parallel, because different currents (I_1 in R_1) & (I_2 in R_2) are flowing.

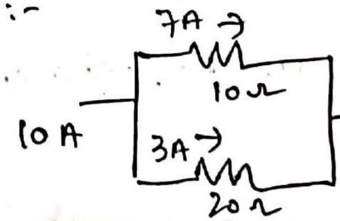
(at point 'A' current gets splitted)

$$\boxed{R_{eq} = \frac{1}{R_1} + \frac{1}{R_2}} \quad \text{or} \quad \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$



$$R_{eq} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$$

eg:-



In this circuit total current enters is 10A, It is ~~splitted~~ splitted & 7A enters 10 Ω & 3A enters 20 Ω, So, 10 Ω & 20 Ω are in parallel.

Ohm's law:-

Ohm's law states that at constant temperature the voltage across a conductor is directly proportional to the current flowing through it.

$$V \propto I$$

$$\boxed{V = IR}$$

V ⇒ voltage $\boxed{V = IR}$ in volts

I ⇒ current $\boxed{I = V/R}$ in amp

R ⇒ Resistance. $\boxed{R = V/I}$ in ohms.

Problem:-

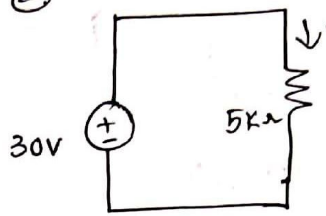
(1) An electric iron draws 2A at 10V, find its resistance

from ohm's law

$$R = \frac{V}{i} = \frac{10}{2} = 5\Omega$$

$$\boxed{R = 5\Omega}$$

(2)



In the circuit shown, calculate the current 'i', conductance 'G' and Power 'p'.

Solution:-

Current:- $i = V/R$

$$= \frac{30}{5\times10^3} \rightarrow (5K = 5\times10^3\,\Omega \text{ or } 5000\,\Omega)$$

$$\boxed{i = 6mA}$$ (6 milli amps)

Conductance G

$$G = 1/R = \frac{1}{5\times10^3} = 0.2\,m\mho \text{ or}$$

0.2 mS

(where S is Siemens)

$$\boxed{G = 0.2\,m\mho}$$

(m → represent milli 10^{-3})

Power (P)

Power $P = V \times I = 30\times(6\times10^{-3})$

$$\boxed{P = 180mW}$$

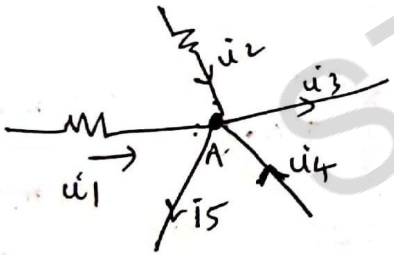
Kirchoff's Law:-

* Kirchoff's Current Law (KCL)
* Kirchoff's Voltage Law (KVL)

Kirchoff's Current Law (KCL)

EC 3251 Circuit Analysis

Kirchoff's current law states that the sum of the currents entering a node is equal to the sum of the current leaving the node. (or) in other words algebraic sum of the currents at node is zero.



In the figure above 'A' is the node,

Currents entering the node are i_1, i_2, i_4 (see the arrow mark for direction)

Currents leaving the node are i_3 & i_5

So according to KCL

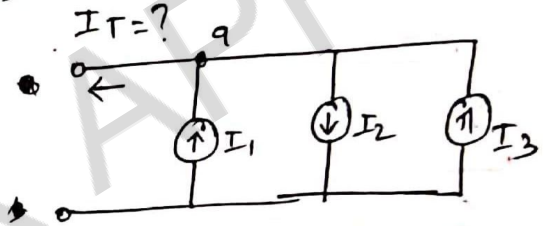
$$\boxed{i_1 + i_2 + i_4 = i_3 + i_5}$$

or

$$\boxed{i_1 + i_2 + i_4 - i_3 - i_5 = 0.}$$

Unit I DC Circuit A

Problem:-



find I_T

Solution

At node 'a' I_1 & I_3 are entering.

I_2 & I_T are leaving

So According to KCL $I_T + I_2 = I_1 + I_3$

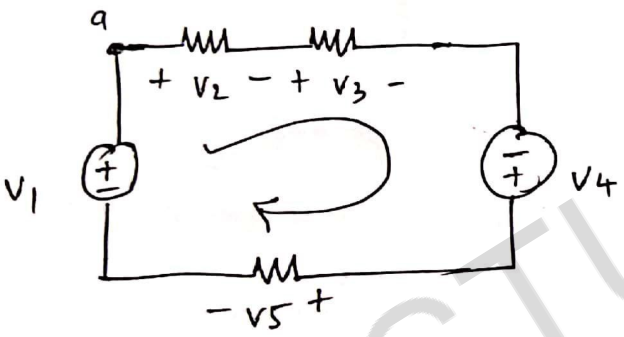
$$\boxed{I_T = I_1 + I_3 - I_2}$$

Kirchoff's voltage Law:- (KVL)

Kirchoff's voltage law states that the algebraic sum of all voltages around a closed path or loop is zero.

or in other words. In a close circuit.

$$\boxed{\text{Sum of the potential raise} = \text{Sum of the potential drops.}}$$



lets starts from point a' & traverse the loop.

$$+V_2 + V_3 - V_4 + V_5 - V_1 = 0$$

$$\boxed{V_1 + V_4 = V_2 + V_3 + V_5}$$

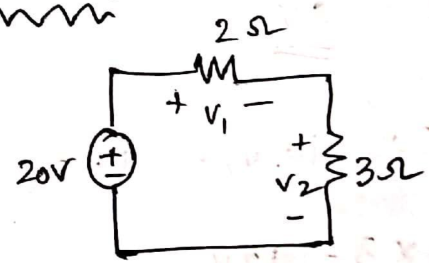
V_1 & V_4 = Potential raise, V_2, V_3 & V_5 - Potential drops.

Concept to remember:-

If current 'I' flows in to a resistor 'R' then a voltage drop V_1 occurs at the resistor R_1 with the polarity shown.

Polarity of the voltage depends on the direction of the current entry.

Problems:



find the voltages V_1 & V_2 for the circuit shown.

Solution:-

To find V_1 & V_2, we need to find the current flowing through the resistors.

* to find the total current we need to find the equivalent resistances of this circuit.

* Resistance R 2Ω & 3Ω are connected in series (since the same current flows)

$$R_{eq} = 5\Omega \quad (2+3)$$

Total current $i = V/R = \frac{20}{5} = \underline{\underline{4A}}$

Voltage drop across 2Ω ie. $V_1 = I \times R_1$

$$4 \times 2 = \underline{\underline{8V}}$$

$$\boxed{V_1 = 8V}$$

Voltage drop across 3Ω ie. $V_2 = I \times R_2$

$$4 \times 3 = \underline{\underline{12V}}$$

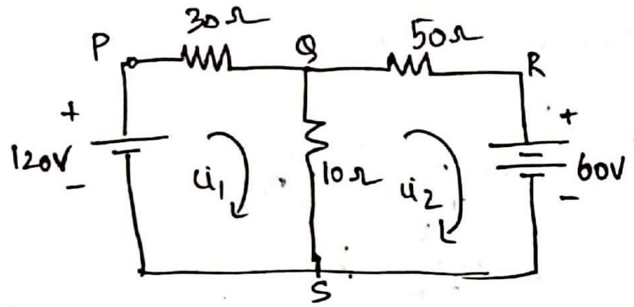
$$\boxed{V_2 = 12V}$$

Verification

According to KVL voltage raise = voltage drops

$$\underbrace{20V}_{\text{Voltage raise}} = \underbrace{8V + 12V}_{\text{voltage drops}}$$

1. Solve the mesh and branch currents for the circuit shown.



Solution:-

Apply KVL at loop1 (PQSP)

$$30i_1 + 10(i_1 - i_2) = 120$$

$$40i_1 - 10i_2 = 120 \quad \text{---} (1)$$

Apply KVL at loop2 (QRSQ)

$$50i_2 + 10(i_2 - i_1) = -60$$

$$-10i_1 + 60i_2 = -60 \quad \text{---} (2)$$

By using Cramer's rule find Δ

$$\Delta = \begin{bmatrix} 40 & -10 \\ -10 & 60 \end{bmatrix} = (60 \times 40) - (-10 \times -10)$$

$$\boxed{\Delta = 2300}$$

$$\Delta_1 = \begin{bmatrix} 120 & -10 \\ -60 & 60 \end{bmatrix} = (120 \times 60) - (-10 \times -60)$$

$$= 7200 - 600$$

$$\boxed{\Delta_1 = 6600}$$

$$\Delta_2 = \begin{bmatrix} 40 & 120 \\ -10 & -60 \end{bmatrix} = (40 \times -60) - (-10 \times 120)$$

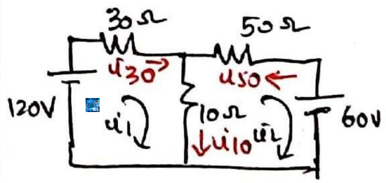
$$= -2400 + 1200$$

$$\boxed{\Delta_2 = -1200}$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{6600}{2300} = 2.86\,A$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{-1200}{2300} = -0.521\,A$$

i_1 & i_2 are loop currents.



i_{30}, i_{50} & i_{10} are called branch currents.

(simply the current at the branches).

$$\boxed{\begin{aligned} i_{30} &= i_1 &= 2.86\,A \\ i_{50} &= 0 - i_2 &= 0.521\,A. \\ i_{10} &= i_{30} + i_{50} &= 3.381\,A \end{aligned}}$$

Note:-

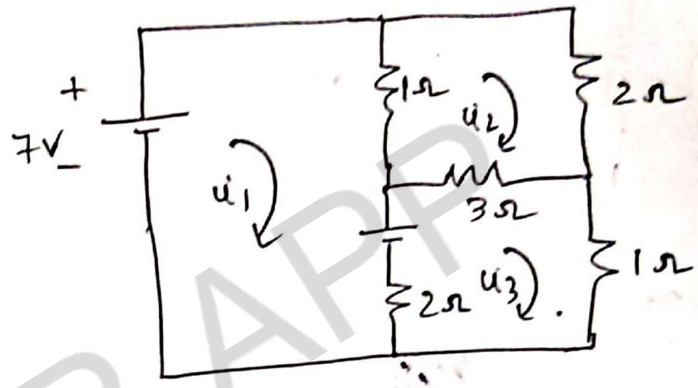
i_{50} is opposite to i_2 = so $i_{50} = -i_2$.

current i_{30} & i_{50} combines & flow in

10Ω resistor so $i_{10} = i_{30} + i_{50}$.

i_1 & i_{30} are same so $i_1 = i_{30}$.
(Refer the circuit).

2) Use the mesh analysis to determine the three mesh currents in the circuit shown.



Three loops are there in this circuit. loop 1, 2, 3.

apply KVL in loop 1:-

$$1(i_1 - i_2) + 2(i_1 - i_3) = 7 - 6$$

$$3i_1 - i_2 - 2i_3 = 1 \quad \text{---} \quad (1)$$

Loop 2

$$1(i_2 - i_1) + 2(i_2) + 3(i_2 - i_3) = 0$$

$$-i_1 + 6i_2 - 3i_3 = 0 \quad \text{---} \quad (2)$$

loop 3

$$3(i_3 - i_2) + 1i_3 + 2(i_3 - i_1) = 6$$

$$-2i_1 - 3i_2 + 6i_3 = 6 \quad \text{--} \quad (3)$$

Matrix representation:-

$$\begin{bmatrix} 3 & -1 & 2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & 2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix}$$

$$= 3(36-9) + 1(-6-6) - 2(3+12)$$

$$= 81 - 12 - 30$$

$$\boxed{\Delta = 39}$$

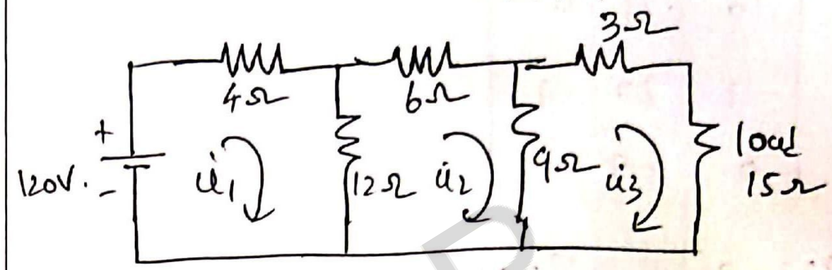
$$\Delta_1 = \begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix} \qquad \Delta_2 = \begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix} \qquad \Delta_3 = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix}$$

$$\boxed{\Delta_1 = 117} \qquad \boxed{\Delta_2 = 78} \qquad \boxed{\Delta_3 = 117}$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{117}{39} = 3A \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{78}{39} = 2A \qquad i_3 = \frac{\Delta_3}{\Delta} = \frac{117}{39} = 3A$$

$$\boxed{i_1 = 3A} \qquad \boxed{i_2 = 2A} \qquad \boxed{i_3 = 3A}$$

3. In the circuit given in figure, obtain the load current and power delivered to the load.



Solution:-

Apply KVL at Loop1

$$4i_1 + 12(i_1 - i_2) = 120$$

$$16i_1 - 12i_2 = 120 \text{ --- } (1)$$

loop2

$$12(i_2 - i_1) + 6i_2 + 9(i_2 - i_3) = 0$$

$$-12i_1 + 27i_2 - 9i_3 = 0 \text{ --- } (2)$$

loop3

$$9(i_3 - i_2) + 3i_3 + 15i_3 = 0$$

$$-9i_2 + 27i_3 = 0 \text{ --- } (3)$$

Matrix representation:-

$$\begin{bmatrix} 16 & -12 & 0 \\ -12 & 27 & -9 \\ 0 & -9 & 27 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 16 & -12 & 0 \\ -12 & 27 & -9 \\ 0 & -9 & 27 \end{vmatrix}$$

$$= 16[729-81] + 12[-324-0] + 0$$

$$\boxed{\Delta = 6480}$$

$$\Delta_3 = \begin{vmatrix} 16 & -12 & 120 \\ -12 & 27 & 0 \\ 0 & -9 & 0 \end{vmatrix}$$

$$\boxed{\Delta_3 = 12960}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{12960}{6480} = 2A$$

$$\boxed{i_3 = 2A}$$

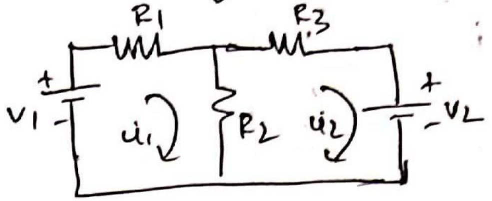
Current through load resistor 15Ω is 2A.

So Power delivered to the load is $P = i^2 R$

$$= 2^2 \times 15$$
$$= 60W$$

$$\boxed{P_{15} = 60W}$$

Mesh Equation by Inspection Method:-



$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

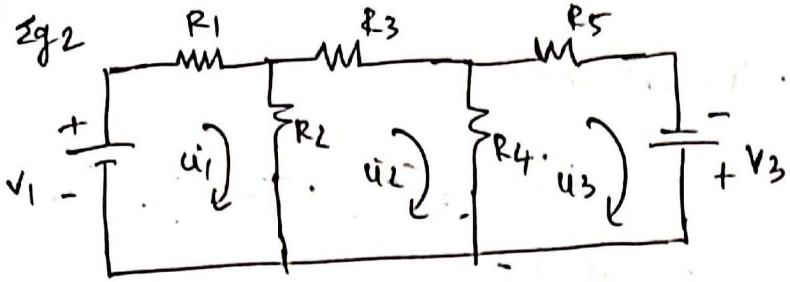
$R_{11} = R_1 + R_2$ (Sum of all resistance in the 1st loop)

$R_{21} = R_{12} = -R_2$ (Total resistance shared by loop one and two, Negative sign indicates the current i_1 & i_2 are in opposite direction)

$R_{22} = R_2 + R_3$ (Sum of all the resistances in the 2nd loop)

$$\begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

Eg 2

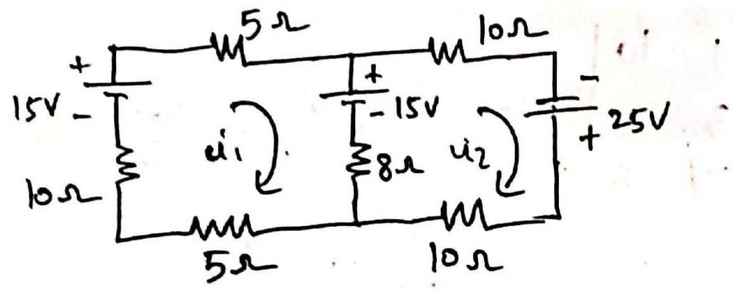


$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} R_1+R_2 & -R_2 & 0 \\ -R_2 & R_2+R_3+R_4 & -R_4 \\ 0 & -R_4 & R_4+R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ V_3 \end{bmatrix}$$

Problem:

1. Find the current through 8Ω resistor



Solution by using mesh Inspection method.

$$\begin{bmatrix} R_{11} & -R_{12} \\ -R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 5+8+5+10 & -8 \\ -8 & 10+10+8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 15-15 \\ 15+25 \end{bmatrix}$$

$$\begin{bmatrix} 28 & -8 \\ -8 & 28 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 40 \end{bmatrix}$$

Apply cramer's rule.

$$\Delta = \begin{vmatrix} 28 & -8 \\ -8 & 28 \end{vmatrix} = 784 - 64 = 720$$

$$\Delta_1 = \begin{vmatrix} 0 & -8 \\ 40 & 28 \end{vmatrix} = 320 \qquad I_1 = \frac{\Delta_1}{\Delta} = \frac{320}{720} = 0.44A$$

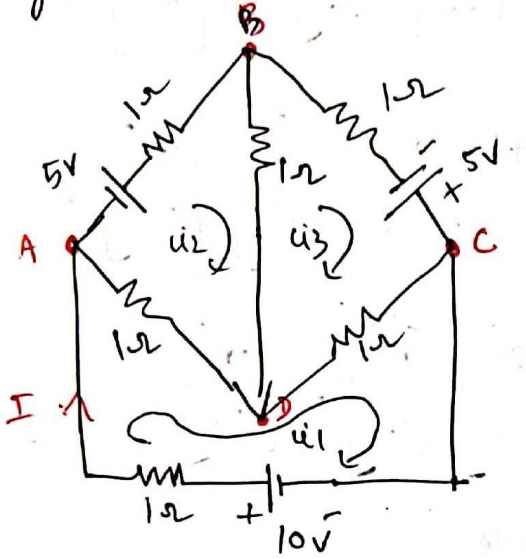
$$\Delta_2 = \begin{vmatrix} 28 & 0 \\ -8 & 40 \end{vmatrix} = 1120 \qquad I_2 = \frac{\Delta_2}{\Delta} = \frac{1120}{720} = 1.55A$$

Current through 8Ω resistor $i_2 - i_1 = 1.55 - 0.44$

$= 1.11A$

$\boxed{I_1 = 0.44A \quad I_2 = 1.55A}$ $\boxed{i_8 = 1.11A}$

2. Determine the currents in bridge circuit by using mesh analysis.



Solution:- By using Inspection method.

$$\begin{bmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} 1+1+1 & -1 & -1 \\ -1 & 1+1+1 & -1 \\ -1 & -1 & 1+1+1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$$

By applying Cramer's rule.

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} = 3[9-1]+1[-3-1]-1[1+3] = 24-4-4$$

$$\boxed{\Delta = 16}$$

$$\Delta_1 = \begin{vmatrix} 10 & -1 & -1 \\ 5 & 3 & -1 \\ 5 & -1 & 3 \end{vmatrix} = 120 \qquad i_1 = \frac{\Delta_1}{\Delta} = \frac{120}{16} = 7.5A$$

$$\boxed{i_1 = 7.5A}$$

$$\Delta_2 = \begin{vmatrix} 3 & 10 & -1 \\ -1 & 5 & -1 \\ -1 & 5 & 3 \end{vmatrix} = 100 = i_2 = \frac{\Delta_2}{\Delta} = \frac{100}{16} = 6.25A$$

$$\boxed{i_2 = 6.25A}$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 10 \\ -1 & 3 & 5 \\ -1 & -1 & 5 \end{vmatrix} = 100 = i_3 = \frac{\Delta i_3}{\Delta} = \frac{100}{16} = 6.25A$$

$$\boxed{i_3 = 6.25A}$$

Current through AD is $i_1 - i_2 = 7.5 - 6.25 = 1.25A$

Current through AB is $i_2 = 6.25A$

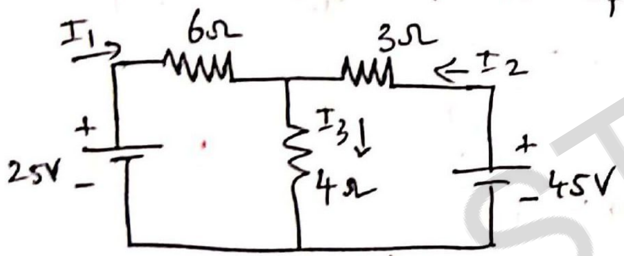
Current through BD is $i_2 - i_3 = 6.25 - 6.25 = 0A$

Current through BC is $i_3 = 6.25A$

Current through CD is $i_1 - i_3 = 7.5 - 6.25 = 1.25A$.

Nodal Method:

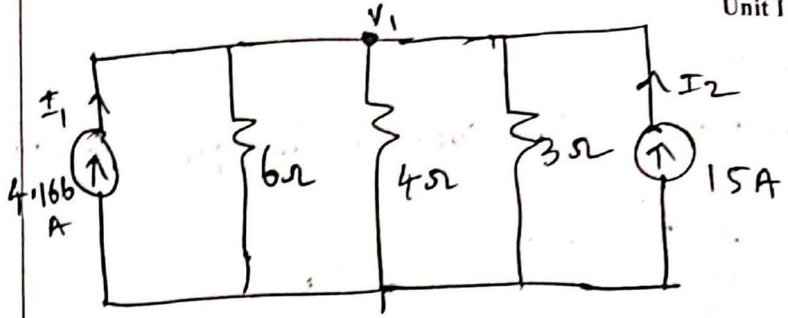
1. Using Nodal Analysis, obtain the currents flowing in all the resistors of the circuit.



Solution:- Convert all the voltage sources into equivalent sources.

$$I_1 = \frac{25}{6} = 4.166A$$

$$I_2 = 45/3 = 15A$$



Apply KCL at node V_1

$$4.166 + 15 = \frac{V_1}{6} + \frac{V_1}{4} + \frac{V_1}{3}$$

$$19.166 = V_1\left[\frac{1}{6} + \frac{1}{4} + \frac{1}{3}\right]$$

$$19.166 = V_1[0.75]$$

$$V_1 = \frac{19.166}{0.75} = 25.55V$$

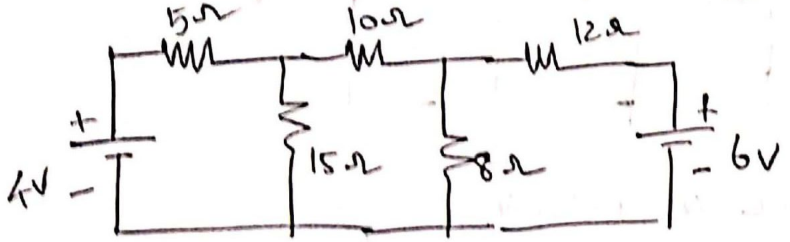
$$\boxed{V_1 = 25.55V}$$

Current through 6Ω resistor $= \frac{V_1}{6} = \frac{25.55}{6} = 4.25A$

Current through 4Ω resistor $V_1/4 = \frac{25.55}{4} = 6.38A$

Current through 3Ω resistor $= \frac{V_1}{3} = \frac{25.55}{3} = 8.51A$

2. Using Nodal Method find Current through 8Ω resistor

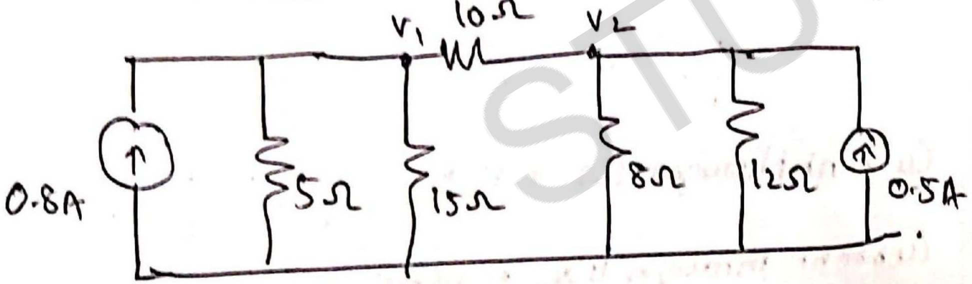


Convert all the voltage sources in to equivalent Current Sources.

$I_1 = 4/5 = 0.8A$

$I_2 = 6/12 = 0.5A$

Circuit becomes



matrix form by inspection method.

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{15}+\frac{1}{10} & -\frac{1}{10} \\ -1/10 & \frac{1}{10}+\frac{1}{8}+\frac{1}{12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.366 & -0.1 \\ -0.1 & 0.308 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.366 & -0.1 \\ -0.1 & 0.308 \end{vmatrix} = 0.112 - 0.01$$

$$\boxed{\Delta = 0.102}$$

$$\Delta_{V_1} = \begin{vmatrix} 0.8 & -0.1 \\ 0.5 & 0.308 \end{vmatrix} = 0.2964$$

$$\Delta V_2 = \begin{vmatrix} 0.366 & 0.8 \\ -0.1 & 0.5 \end{vmatrix} = 0.263$$

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{0.2964}{0.102} = 2.905V$$

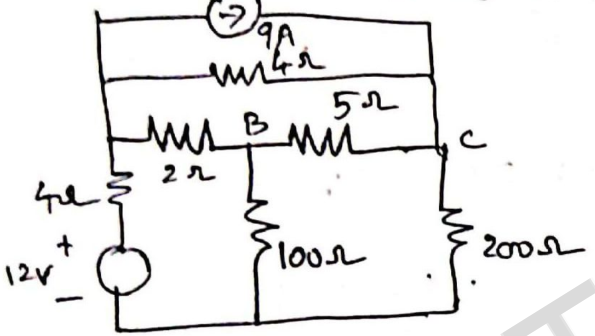
$$\boxed{V_1 = 2.905\,V}$$

$$V_2 = \frac{\Delta V_2}{\Delta} = \frac{0.263}{0.102} = 2.578V$$

$$\boxed{V_L = 2.578V}$$

Current through 8Ω resistor $= \frac{V_2}{8} = \frac{2.578}{8} = 0.322A$

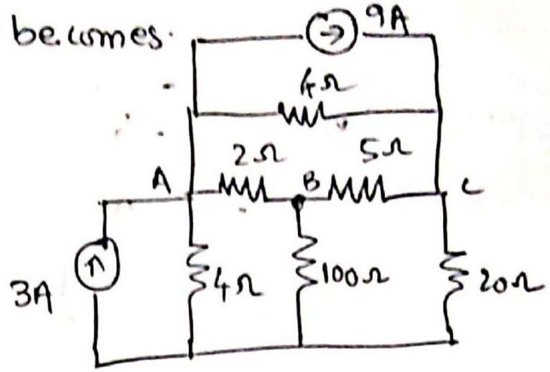
3. Use Nodal Analysis to determine the voltage across 5Ω resistance and the current in the 12V sources.



Solution:- Convert voltage source into current source.

$$I = V/R = 12/4 = 3A$$

Circuit becomes.



At node A, the current is $3-9 = -6A$

At node B, the current is zero.

At node C, the current is 9A

Node Equation in matrix form

$$\begin{bmatrix} \frac{1}{4}+\frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -1/2 & \frac{1}{2}+\frac{1}{100}+\frac{1}{5} & -\frac{1}{5} \\ -1/4 & -1/5 & \frac{1}{5}+\frac{1}{20}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.5 & -0.25 \\ -0.5 & -0.71 & -0.2 \\ -0.25 & -0.2 & 0.5 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

Solve V_B & V_C

$$\Delta = \begin{vmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.2 \\ -0.25 & -0.2 & 0.5 \end{vmatrix}$$

$$= \boxed{\Delta = 0.0956} \qquad 1(0.355-0.04)+0.5(-0.254-0.05) -0.25(0.1+0.1775)$$

$$\Delta V_B = \begin{vmatrix} 1 & -6 & -0.25 \\ -0.5 & 0 & -0.2 \\ -0.25 & 9 & 0.5 \end{vmatrix}$$

$= 1.125$

$\Delta V_B = 1.125$ $\qquad V_B = \frac{\Delta V_B}{\Delta} = \frac{1.125}{0.0956} = 11.76V$

$\boxed{V_B = 11.76V}$

$$\Delta V_C = \begin{vmatrix} 1 & -0.5 & -6 \\ -0.5 & 0.71 & 0 \\ -0.25 & -0.2 & 9 \end{vmatrix}$$

$= 2.475$

$\Delta V_C = 2.475$

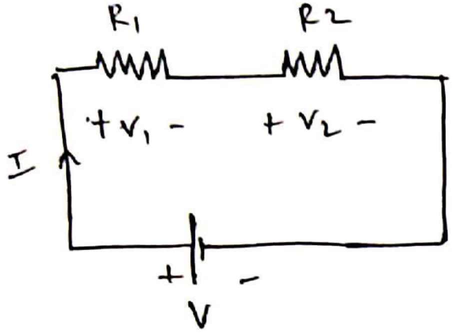
$V_C = \frac{\Delta V_C}{\Delta} = \frac{2.475}{0.0956} = \boxed{V_C = 25.88V}$

Voltage across 5Ω is $V_B - V_C$

$= 11.76 - 25.88$

$\boxed{V_{5\Omega} = -14.12V}$

Voltage Division rule:-



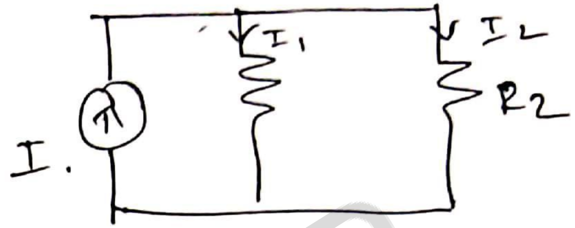
If two resistors are connected in series and if the total voltage applied is V, then the voltage across any one resistance can be found by using the formula

$$= \frac{\text{total voltage} \times \text{Same Resistance in which voltage need to be found}}{\text{Sum of the Resistance.}}$$

$$\boxed{V_1 = \frac{V \times R_1}{R_1 + R_2}}$$

$$\boxed{V_2 = \frac{V \times R_2}{R_1 + R_2}}$$

Current Division Rule:-



If two resistors are connected in parallel and the total current entering the parallel combination is known. Then the current flowing through the resistors can be calculated using

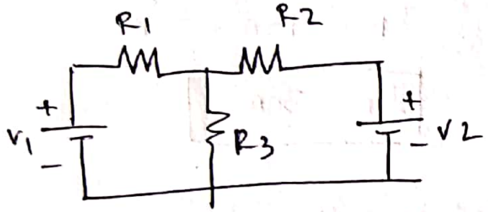
$$\frac{\text{total current} \times \text{Opposite Resistance}}{\text{Sum of the Resistance.}}$$

$$\boxed{I_1 = \frac{I R_2}{R_1 + R_2}}$$

$$\boxed{I_2 = \frac{I R_1}{R_1 + R_2}}$$

Super Position Theorem:-

Super position Theorem states that the response in a circuit with multiple sources is given by the algebraic sum of responses due to individual sources acting alone.

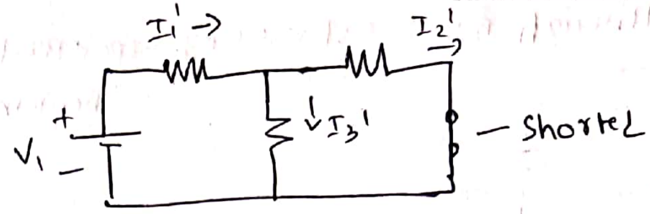


Procedure / steps:

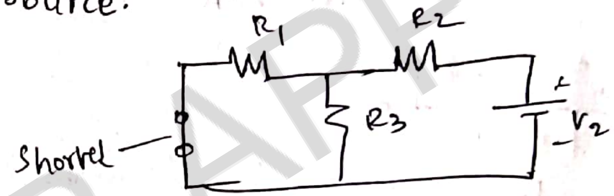
① In a circuit with multiple sources keep only one source active (V_1 - active, V_2 - short)

② Remove all the other sources (voltage sources must be short circuited, current sources must be open circuited)

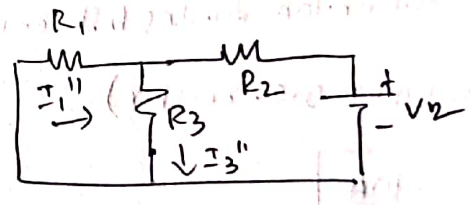
③ Find the response of the circuit for the individual sources and label the term as I_1', I_2' & I_3' etc.



④ Now keep the other source active and remove the first source.



⑤ Now find the response due to the source V_2 and label the current as I_1'', I_2'' & I_3''.



(follow the same procedure if more than two sources present)

⑥ Now find the overall response of the circuit by sum up the individual responses.

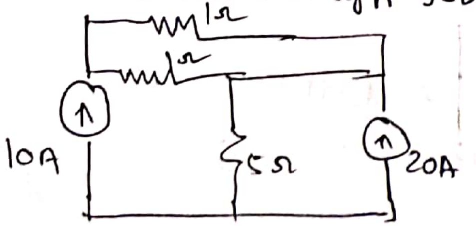
$I_1 = I_1' + I_1''$

$I_2 = I_2' + I_2''$

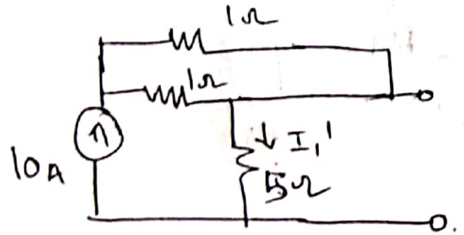
$I_3 = I_3' + I_3''$

[Note: Depending upon the direction assumed sign may vary, once current calculated other parameters like voltage, power etc can be calculated]

① Find the current through 5Ω resistor using Super Position Theorem.



Step1:- Keep 10A Active & Remove 20A



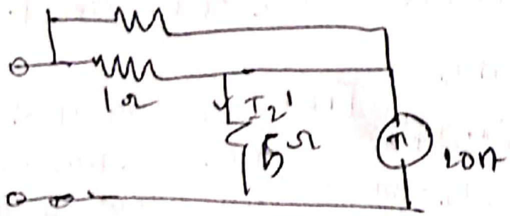
from the fig above current through 5Ω is 10A

(Even though the current 10A divides between two 1Ω it again join & enter 5Ω resistor)

$$\boxed{I_L' = 10A}$$

Step2

Keep 20A Active & Remove 10A



By inspecting the circuit itself we can observe, the current through 5Ω flow is 20A.

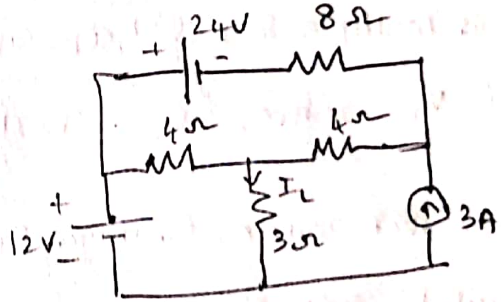
$$\boxed{I_L'' = 20A}$$

So total current through 5Ω resistor $I = I_L' + I_L''$

$$I_L = 10A + 20A$$

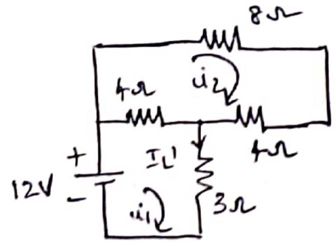
$$\boxed{I_L = 30A}$$

② Using Super position theorem find the current through 3Ω resistor.



Solution:- Step1

Keep 12V source active 24V - Short, 3A - open circuited.



By KVL

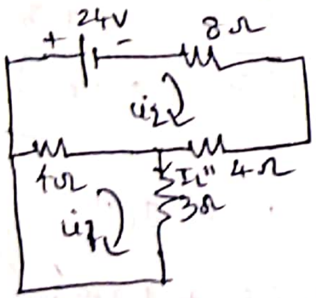
$$-7i_1 + 4i_2 = -12 \quad \text{--- (1)}$$

$$4i_1 - 6i_2 = 0 \quad \text{--- (2)}$$

$\boxed{i_1 = 2A}$ $\boxed{i_2 = 0.5A}$

So $\boxed{I_L' = 2A}$ (since $i_1 = I_L'$ in this circuit)

Step 2: Keep 24V source active & 12V source short circuited and 3A open circuited.



Let I_L'' be the current through 3Ω resistor because of the voltage source 24V

$$16i_1 - 4i_1 = -24 \quad \text{--- (1)}$$

$$7i_1 - 4i_2 = 0 \quad \text{--- (2)}$$

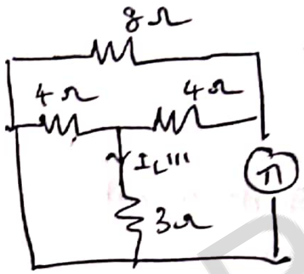
$i_2 = -1.75A$

$i_1 = -1A$

$\boxed{I_L'' = i_1 = -1A}$

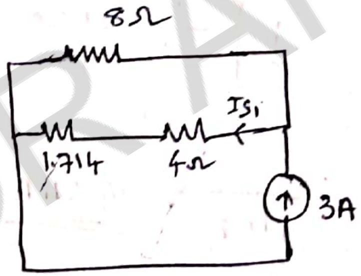
$\boxed{I_L'' = -1A}$

Step 3: Keep 12V & 24V source short circuited and 3A source active.



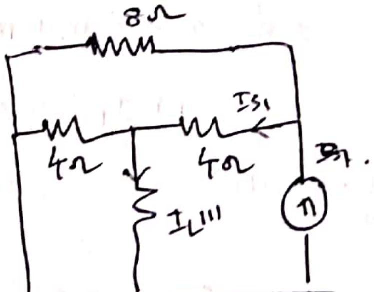
Let $I_L''' = I_{3\Omega}$ due to 3A source.

$$4\Omega \| 3\Omega = \frac{4 \times 3}{4+3} = 1.714\Omega$$



$$I_{S1} = \frac{3 \times 8}{4.714 + 8}$$ (By current division rule)

$\boxed{I_{S1} = 1.887A}$



Current division rule.

$$I_{3\Omega} = I_L''' = \frac{1.887 \times 4}{4+3}$$

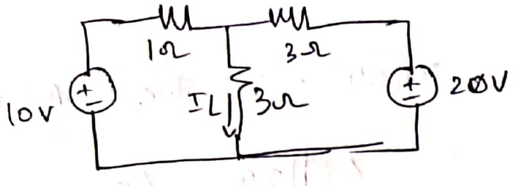
$I_L''' = 1.078A$

So $I_L = I_L' + I_L'' + I_L'''$

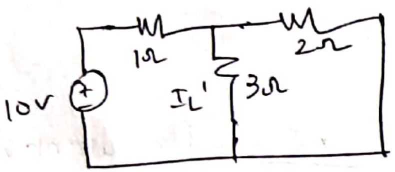
$= 2 - 1 + 1.07 = 3.07A$

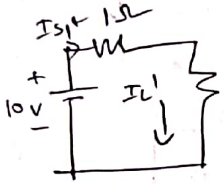
$\boxed{I_L = 3.07A}$

(3) Find the current I_L in the circuit using Super position Theorem



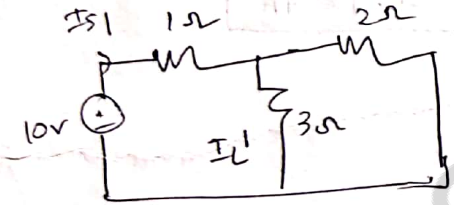
Step1: Keep 10V source Active & 20V short circuit.





$\frac{2\times3}{2+3} = 1.2\Omega$

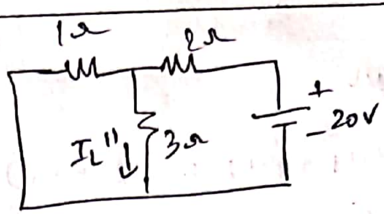
$I_{S1} = \frac{10}{1+1.2} = \underline{\underline{4.54A}}$

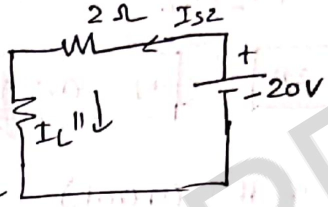


2 & 3 Ω are in parallel so current I_L' can be calculated using current division rule.

$I_L' = \frac{I_{S1} \times 2}{2+3} = \frac{4.54\times2}{2+3} = \underline{\underline{1.8182A}}$

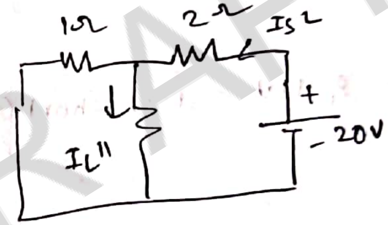
$\boxed{I_L' = 1.8182A}$





$\frac{1\times3}{1+3}$
$= 0.75\Omega$

$I_{S2} = \frac{20}{2+0.75} = 7.27A$



$\cancel{I_{S2} = \frac{20}{2+0.75} = 7.27A}$

$\cancel{I_L'' = I_{S2} \times}$

$I_L'' = I_{S2} \times \frac{1}{1+3}$

$\boxed{I_L'' = 1.818A}$

$I_L = I_L' + I_L''$

$= 1.8182 + 1.8182$

$\boxed{I_L = 3.6364A}$

Thevenin's Theorem:-

Thevenin's Theorem states that a circuit with two terminals can be replaced with an equivalent circuit consisting of a voltage source in series with a resistance or impedance.

Steps:

Steps:- Calculate R_{th} (Thevenin's Equivalent resistance)

Step 1: Remove the load resistor

Step 2: Remove all the voltage sources (short circuit)

Step 3: Remove all the current sources (open circuit)

Step 4: Measure the Resistance across the load terminals, which is the thevenin's resistance. (R_{th})

Steps to calculate V_{th} (Thevenin's Equivalent Voltage) (V_{th})

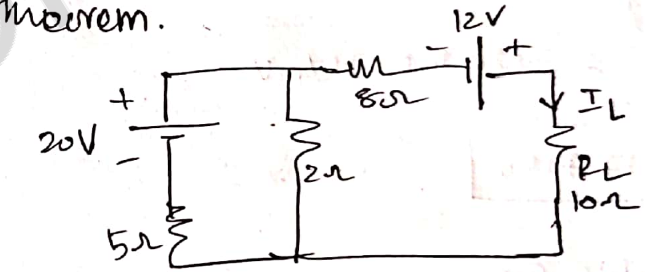
Step 1: Remove the load resistor and then measure the voltage drop across the load terminal, which gives the open circuit voltage also called thevenin's voltage

Step 3: To calculate the current through load resistance use the formula.

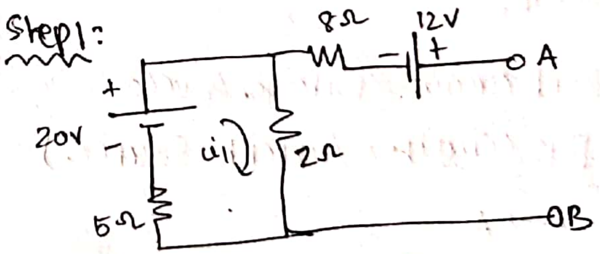
$$\boxed{I_L = V_{th}/R_{th}+R_L}$$

Redraw the circuit as shown above & find I_L.

Problems: Determine the current I_L using Thevenin's theorem.



Step 1:



$2i_1 + 5i_2 = 20$

$i_1 = 20/7 = 2.857A$

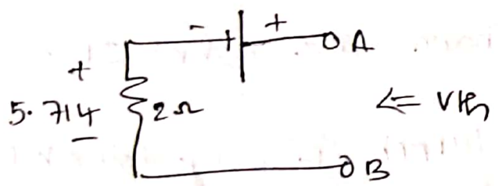
Voltage across 2Ω resistor

$V_{2\Omega} = I_{2\Omega} \times R$

$= 2.857 \times 2$

$\boxed{V_{2\Omega} = 5.714V}$

12 V voltage source is added

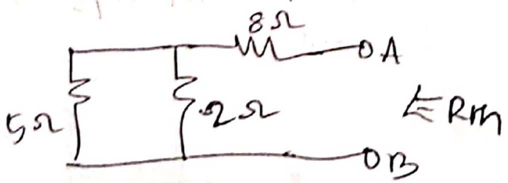


$V_{th} = 12 + V_{2\Omega} = 12V + 5.714 V$

$\boxed{V_{th} = 17.714V}$

Step 2 To calculate R_{th}

Remove load, (Short circuit − voltage source
Open circuit − current source)



$R_{th} = 8 + \frac{5 \times 2}{5+2} = 8 + 1.428 = 9.428\Omega$

$\boxed{R_{th} = 9.428\Omega}$

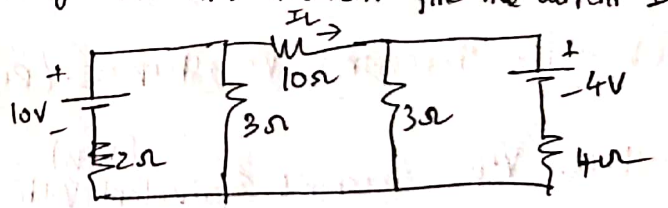
Step 3. To calculate load current I_L

$\boxed{I_L = V_{th} / R_{th} + R_L}$

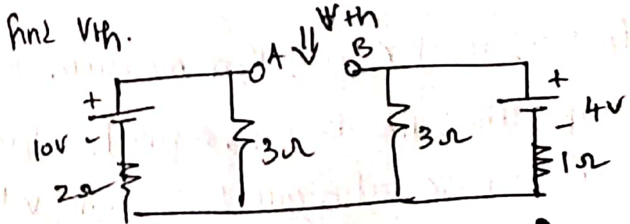
$I_L = \frac{17.714}{9.428 + 10} = 0.9118A$

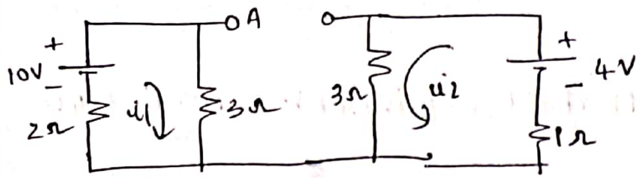
$\boxed{I_L = 0.9118A}$

② Using Thevenin's Theorem find the current I_L in the circuit.



To find V_{th}.





loop1

$3i_1 + 2i_1 = 10$

$5i_1 = 10$

$\boxed{i_1 = 2A}$

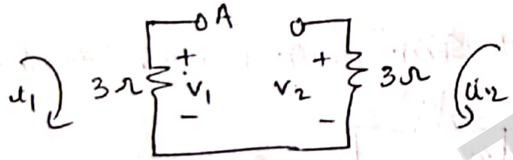
loop2

$3i_2 + 1i_2 = 4$

$4i_2 = 4$

$\boxed{i_2 = 1A}$

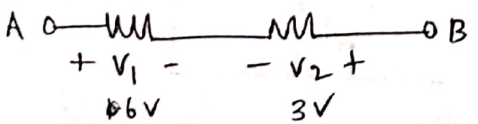
To find voltage drop across A & B, we need to find voltage drop across both the 3Ω resistors.



Voltage $V_1 = i_1 \times 3 = 2 \times 3 = \underline{\underline{6V}}$

$V_2 = i_2 \times 3 = 1 \times 3 = \underline{\underline{3V}}$

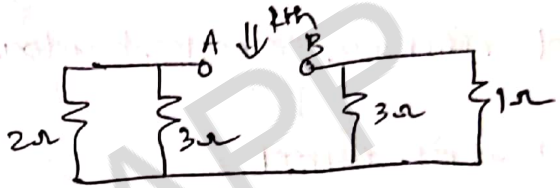
Redraw the circuit for better understanding



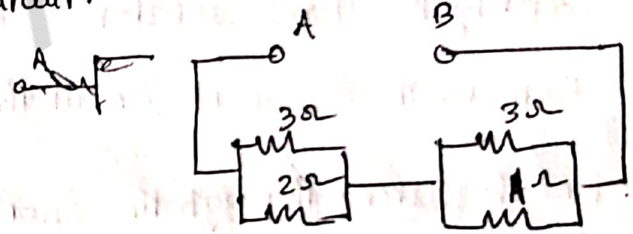
So $V_{AB} = V_{Th} = 6V - 3V = 3V$

$\boxed{V_{Th} = 3V}$

To find Thevenin's Resistance R_{Th}



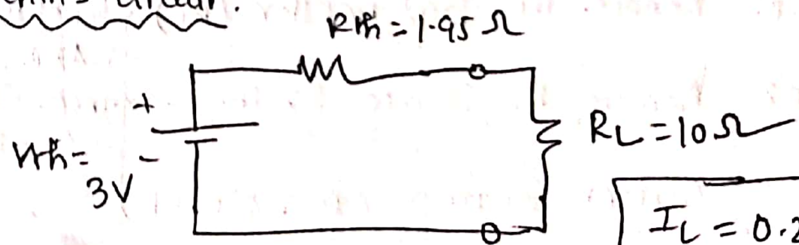
Redraw the circuit.



$R_{Th} = \left(\frac{3 \times 2}{3+2}\right) + \left(\frac{3 \times 1}{3+1}\right)$

$\boxed{R_{Th} = 1.95\Omega}$

Thevenin's circuit:



$\boxed{I_L = 0.251A}$

$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{3}{1.95 + 10} = 0.251A$

Norton's Theorem:

States that It is possible to simplify any linear circuit to an equivalent circuit with Just a single current source and parallel resistance connected to a load.

Steps: To calculate Norton's current:-

Step1: Identify the load resistor.

Step2: Remove it & short circuit the terminal.

Step3: find the current through the short circuit terminal which is the Norton's current (I_N).

Steps: To calculate R_{th} or R_N (Norton's Resistance)

Step1: Remove the load resistor (R_L) & label the terminal as A&B

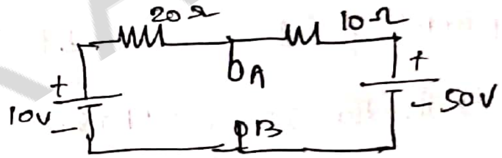
Step 2: Remove the source (voltage source - Short circuit current source - open circuit)

Step3: measure the resistance across the terminals A&B, which gives the Norton's resistance R_N

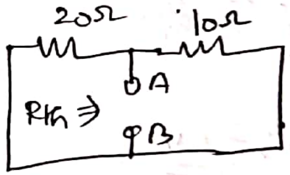
Step3: Draw the Norton's Equivalent circuit & use the formula

$$\boxed{I_L = \frac{I_N \cdot R_N}{R_N + R_L}}$$

Problem:- ① find the thevenin's & Norton's equivalents of the circuit with respect to terminal A & B.

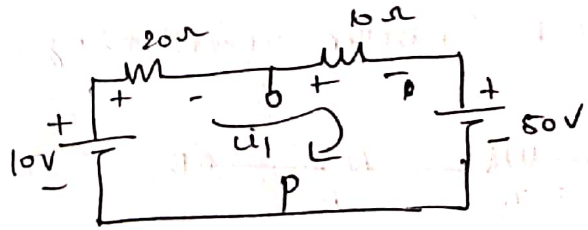


Solution:-
Step1. To find R_{th} Short circuit voltage sources.



$$R_{th} = \frac{20 \times 10}{20+10} = 6.667\Omega$$

Step2 To find V_{th}



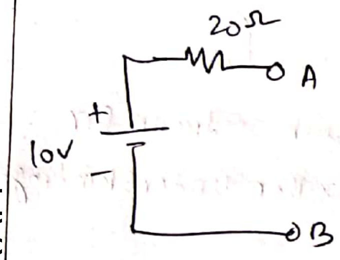
$20i_1 + 10i_1 + 50 - 10 = 0$

$30i_1 = -40$

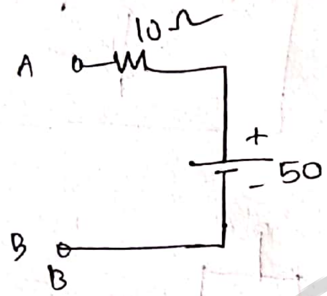
$i_1 = -40/30 = -1.33A$

$\boxed{i_1 = -1.33A}$

Voltage across A & B is either



(or)



We select LHS

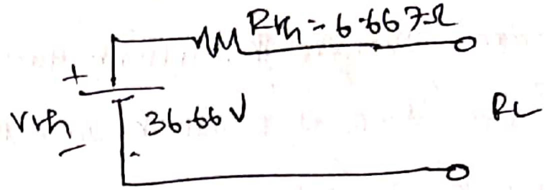
$V_{AB} = 10V -$ voltage across 20Ω

$V_{20\Omega} = i_1 \times 20 = -26.66V$

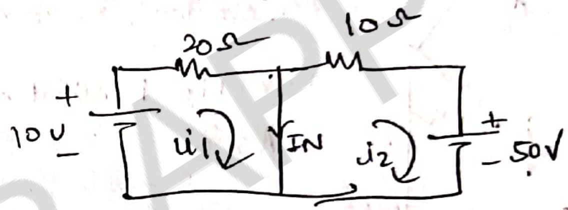
$V_{th} = V_{AB} = 10 - (-26.66V)$

$\boxed{V_{th} = 36.66V}$

Thevenin's Circuit:-



for Norton's Theorem:



loop 1

$20i_1 = 10$

$\boxed{i_1 = 0.5A}$

loop 2

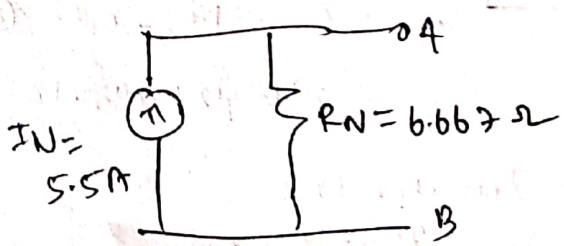
$10i_2 + 50 = 0$

$\boxed{i_2 = -5A}$

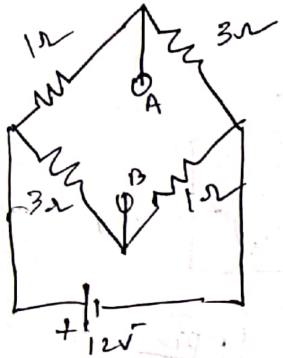
$I_N = i_1 - i_2$

$= 0.5 - (-5) = \boxed{I_N = 5.5A}$

Norton's Equivalent ckt:

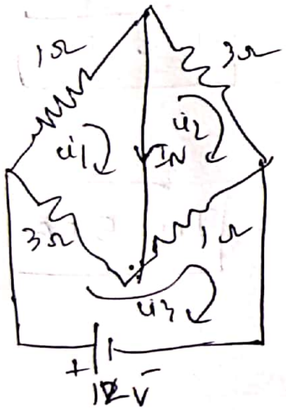


(2) Using Norton's theorem, Determine the current through an ammeter connected across A & B of the circuit, take the resistance of the ammeter as 0.5Ω



1. Remove load resistor & label the terminal as A & B.

2. Short circuit the terminal A & B and find the current through it



$i_1 + 3i_1 - 3i_3 = 0$

$4i_1 - 3i_3 = 0$ — (1)

$4i_2 - 1i_3 = 0$ — (2)

$-3i_1 - i_2 + 4i_3 = 12$ — (3)

$i_1 = 6A$

$i_2 = 2A$

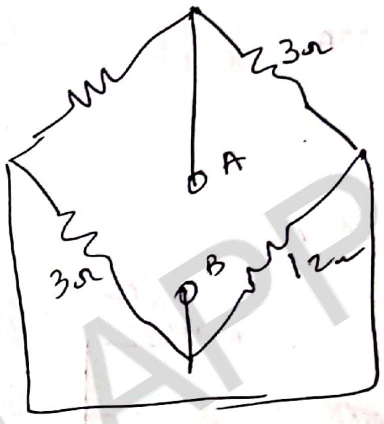
$i_3 = 8A$

$I_N = i_1 - i_2$

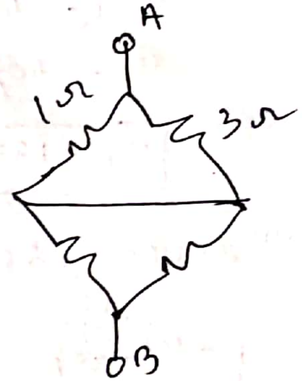
$= 6 - 2 = 4A$

$\boxed{I_N = 4A}$

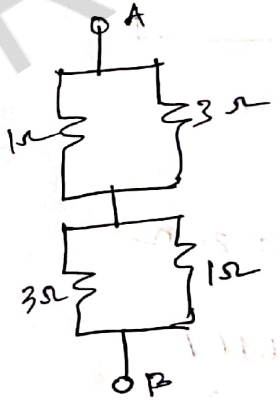
To find Rth or RN



⇒



Circuit redrawn for better understanding



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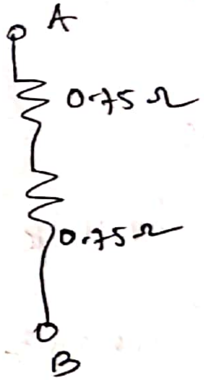
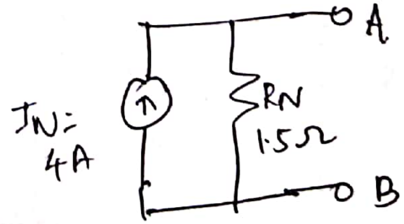


$\boxed{R_N = 1.5\Omega}$

Norton's Equivalent Circuit:



Maximum Power Transfer Theorem (MPTT)

MPTT states that maximum power will be delievered to the load when the load impedance and source impedance are equal.

Steps:

(1.) Find R_{th} (Remove voltage source; current source & load resistance) & label the terminal as A & B)

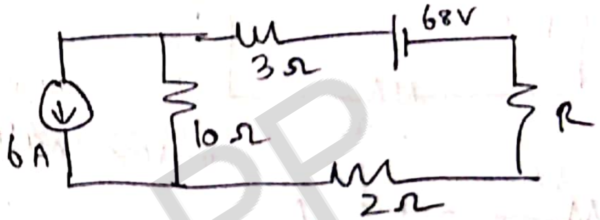
(2.) Find the V_{th} (Thevenin's equivalent voltage) i.e. the voltage across the terminal A & B.

(3) Find maximum power using the formula.

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}}$$

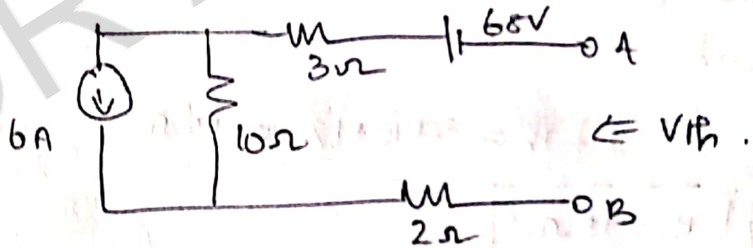
Problems:

(1) Find the value of 'R' for maximum power transfer and calculate the maximum power.

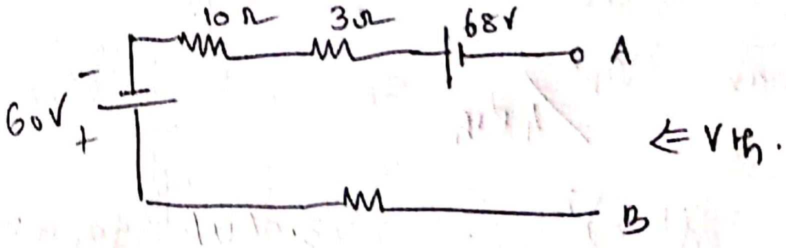


Soln

Step1:- Find R_{th} (Remove R_L)



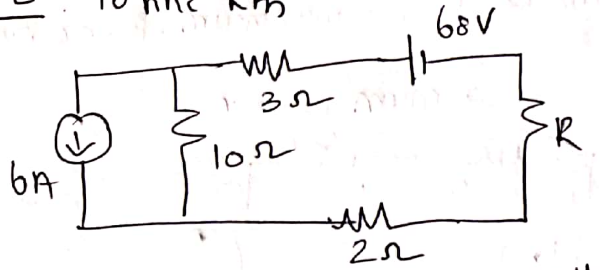
$V = IR$ (By source transformation formula)

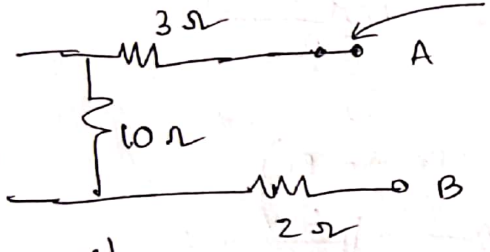


68V & 60V are in series

So $\boxed{V_{th} = 128V}$

Step 2 To find R_{Th}





$R_{Th} = 3+10+12 = 15\Omega$

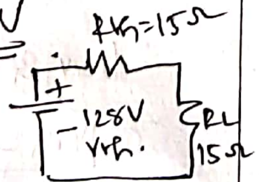
$\boxed{R_{Th} = 15\Omega}$

maximum power P_{max}

$$P_{max} = V_{Th}^2 / 4R_{Th}$$

$$= \frac{(128)^2}{4 \times 15} = 273.06W$$

$\boxed{P_{max} = 273.06W}$ $\boxed{R_{Th} = R_L = 15\Omega}$



Reciprocity Theorem:

The Reciprocity theorem states that in a linear bilateral, single source circuit, the ratio of excitation to response is constant, when the position of the excitation and response are interchanged.

Steps:-

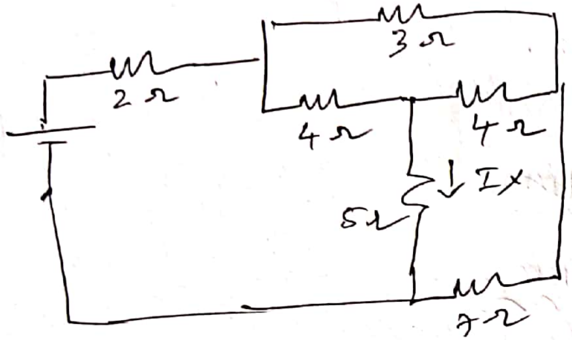
1. Interchange the position of source & response.

   [voltage - Short
   Current - open]

2. Find the Response at the particular Branch.

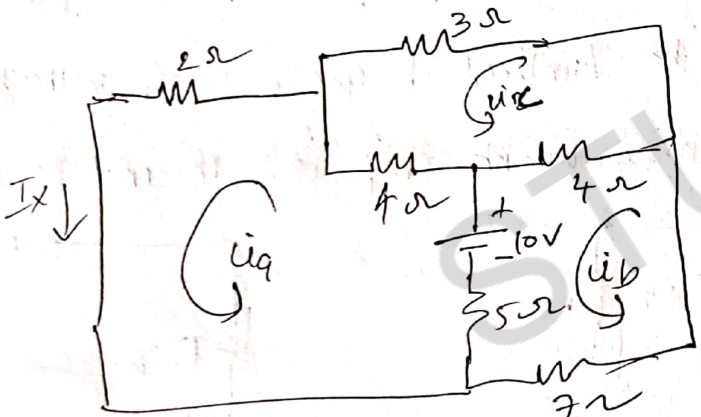
3. Compare it with the response that you got before interchanging.

1. Calculate Ix & prove the reciprocity theorem by Interchanging the position of 10V source & Ix.



Soln:

Interchanging source & response.



$$\begin{bmatrix} 2+4+5 & -5 & -4 \\ -5 & 5+4+7 & -4 \\ -4 & -4 & 3+4+4 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10 \\ -11 \\ 0 \end{bmatrix}$$

$\Delta = 1069$

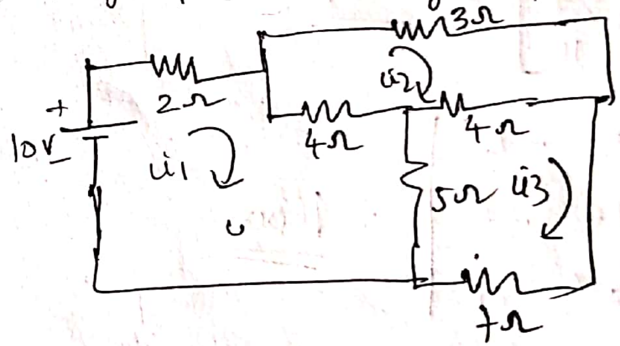
$$\Delta a = \begin{bmatrix} 10 & -5 & -4 \\ -10 & 16 & -4 \\ 0 & -4 & 11 \end{bmatrix} = 890$$

$$I_x = I_a = \frac{\Delta a}{\Delta} = \frac{890}{1069}$$

$$I_x = I_a = \frac{\Delta a}{\Delta} = \frac{890}{1069} = 0.8326A$$

$$\boxed{I_x = I_a = 0.8326A}$$

for original circuit (original position of the voltage source)

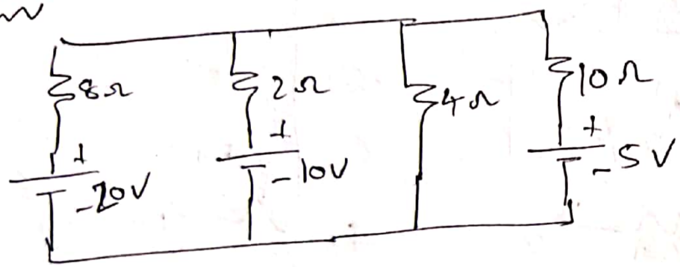


$I_x = I_1 - I_2$

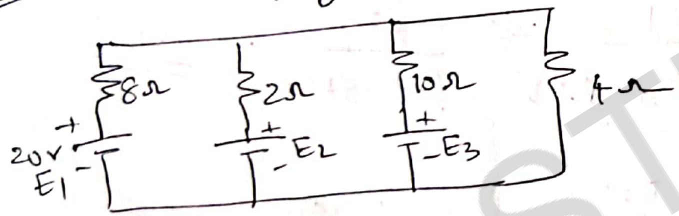
$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots \frac{1}{Z_n}}$$

$$E_{eq} = \left[\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3} \cdots \frac{E_n}{Z_n} \right] Z_{eq}$$

Problem:-



Soln Re-arranging the circuit.



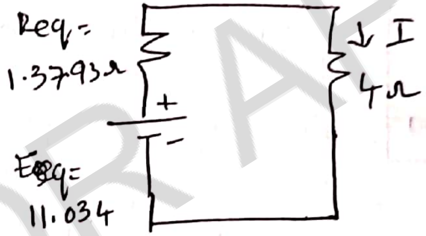
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{8} + \frac{1}{2} + \frac{1}{10}}$$

$$\boxed{R_{eq} = 1.3793\,\Omega}$$

$$E_{eq} = \left[E_1/R_1 + E_2/R_2 + E_3/R_3 \right] \cdot R_{eq}$$

$$= \left[\frac{20}{8} + \frac{10}{2} + \frac{5}{10} \right] \times 1.3793$$

$$\boxed{E_{eq} = 11.0344\ V}$$



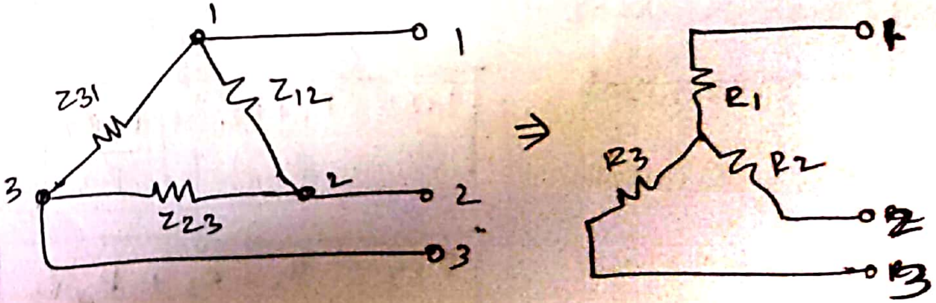
$V = IR$

$I = V/R$

$$= \frac{11.0344}{(1.3793+4)}$$

$$\boxed{I = 2.0513A}$$

Impedance in Star to Delta ($\lambda \to \Delta$) & Delta to Star

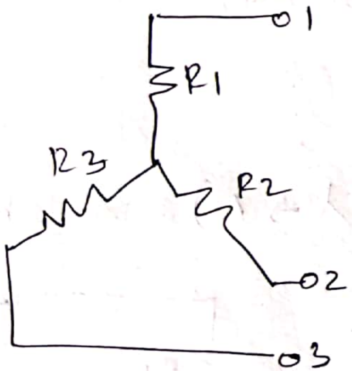


$$\boxed{R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

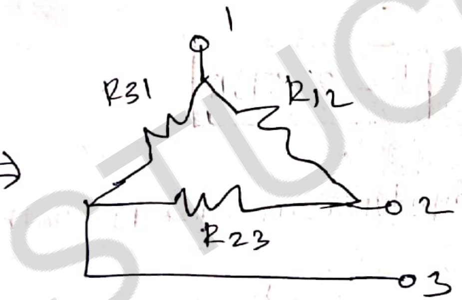
$$\boxed{R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}}$$

$$\boxed{R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

Star to Delta:



$\Rightarrow$

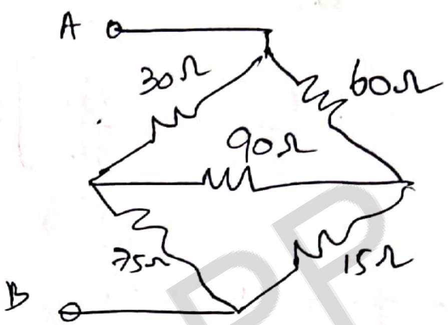


$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}}$$

$$\boxed{R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}}$$

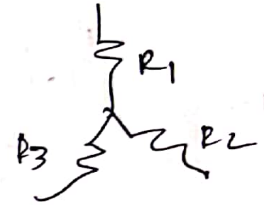
$$\boxed{R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}}$$

Problem: Find the Equivalent resistance of the network.



Soln

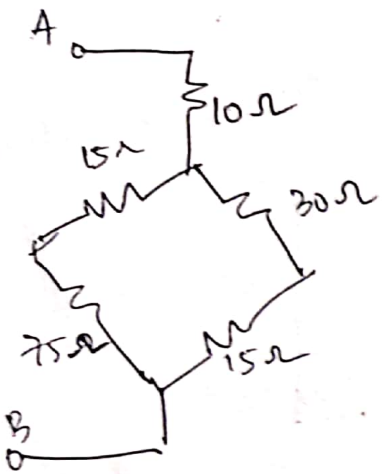
=



$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{31} + R_{23}} = \frac{60 \times 30}{30 + 90 + 60} = \underline{\underline{10\Omega}}$$

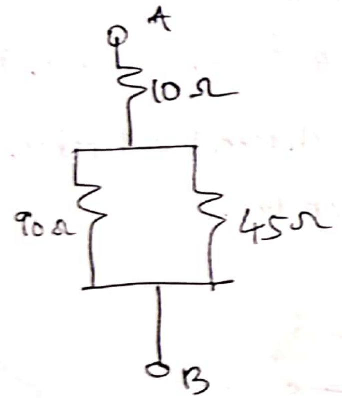
$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{60 \times 90}{30 + 90 + 60} = \underline{\underline{30\Omega}}$$

$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{30 \times 90}{30 + 90 + 60} = \underline{\underline{15\Omega}}$$



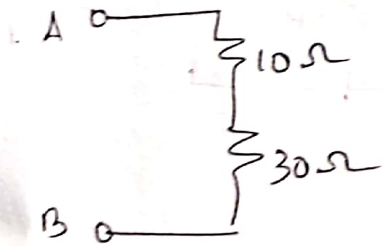
15Ω & 75Ω are in series

30Ω & 15Ω are is series



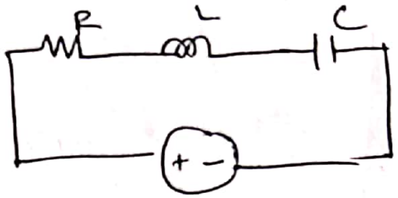
90 & 45 are in Parallel.

$90 \| 45 = 30\Omega$

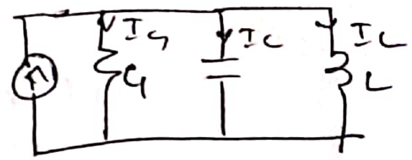


$\boxed{R_{AB} = 40\Omega}$

Duality:



Original network



Dual Network

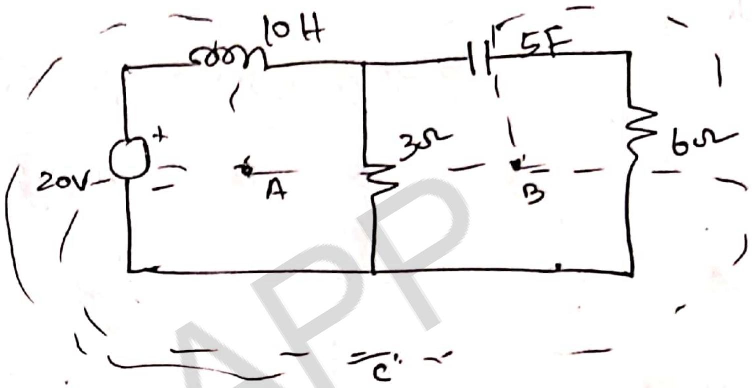
* In an Electrical network, electrical terms are associated in to pairs called as duals.
* A dual of a expression is formed by interchanging voltage & current in the expressions

Dual Elements.

Original	Dual.
Resistance	Conductance
Capacitance	Inductance
Inductance	Capacitance.
Voltage Source	Current Source.

Original	Dual
Current Source	Voltage Source.
Series Branch	Parallel Branch
Mesh equation	Node equation
KCL	KVL
switch closed at t=0	Switch opened at t=0.
Open circuit	Short circuit
Short circuit	Open circuit
Impedance	Admittance.
Reactance	Susceptance.

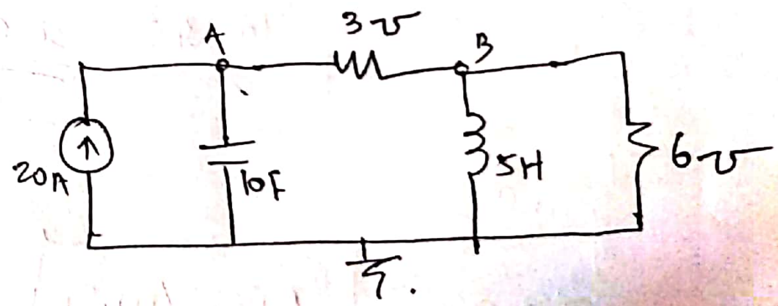
Draw the dual of the network shown.



① Label A & B inside loops & Place reference 'C' node outside

② Draw a dotted line from each node to reference node through all the available branches.

③ Draw its Dual.



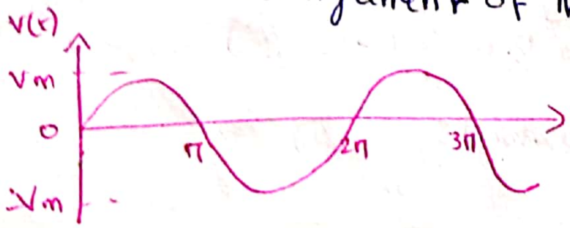
Sinusoid → is a signal that has the form of the cosine or sine function.

$$\boxed{v(t) = V_m \sin\omega t}$$

V_m = amplitude of the sinusoid.

ω = angular frequency in radian/s

ωt = the argument of the sinusoid.



$$\boxed{\omega T = 2\pi}$$

$$\boxed{T = \frac{2\pi}{\omega}}$$ T = Period of sinusoid.

$$\boxed{f = 1/T}$$ $$\boxed{\omega = 2\pi f}$$

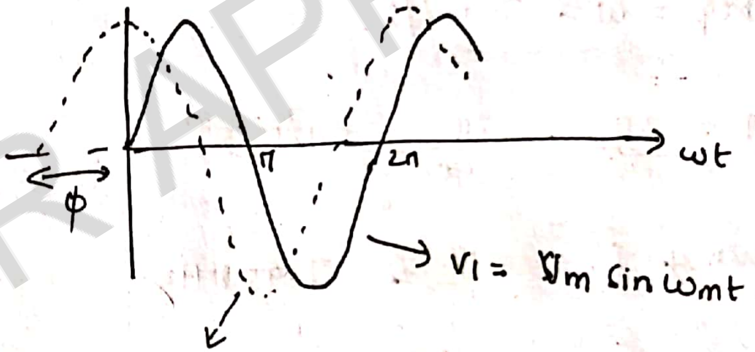
f = frequency in hertz ⇒ or 'f' is also called cycle frequency.

ω = angular frequency.

More general expression for the sinusoid.

$$\boxed{v(t) = V_m \sin(\omega t + \phi)}$$

ϕ = Phase.



The starting point of v_2 occurs first in time, therefore we say that v_2 leads v_1 by ϕ (or) v_1 lags v_2 by ϕ

If $\phi = 0 \Rightarrow v_1 \& v_2$ are in-phase.

$\phi \neq 0 \Rightarrow v_1 \& v_2$ are out-of phase.

1. Find the amplitude, phase & frequency of the sinusoid.

$$v(t) = 12\cos(50t + 10^\circ)$$

Amplitude $= V_m = 12\,V$

Phase $= \phi = 10^\circ$

Angular freq $= \omega = 50\,rad/s$

Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257\,s$

Frequency 'f' $= 1/T = 7.958\,Hz$.

2. Given the sinusoid $5\sin(4\pi t - 60^\circ)$, calculate its amplitude, phase, angular frequency, period and frequency.

Amplitude $= 5V$

Phase $= -60^\circ$

Angular frequency $\omega = 4\pi = 4 \times 3.14 = 12.56\,rad/sec$

Frequency $f = \omega/2\pi = \frac{12.56}{2\times\pi} = \underline{\underline{2\,Hz}}$

1. Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

$v_1 = -10\cos(\omega t + 50^\circ)$

$= 10\cos(\omega t + 50^\circ - 180^\circ)$

$= 10\cos(\omega t - 130^\circ)$ ~~(or) $v_1 = 10\cos(\omega t + 230^\circ)$~~ --- (1)

$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$

$v_2 = 12\cos(\omega t - 100^\circ)$ --- (2). ~~or~~

From (1) & (2) Phase difference between v_1 & v_2 is 30°.

v_2 leads v_1 by 30°.

Phasors:

A Phasor is a complex number that represents the amplitude and phase of a sinusoid.

Rectangular form.

$\boxed{Z = x + jy}$ ⇐ Complex no in rectangular form.

$j = \sqrt{-1}$

$x \Rightarrow$ real part

$y \Rightarrow$ imaginary part

Complex no. can also be written in polar form:-

Polar form:

$$\boxed{Z = r\angle\phi = re^{j\phi}}$$

Complex no. 'Z' can also be written in polar or exponential form.

r = magnitude of 'Z'

ϕ = phase of z

$Z = x + jy \Rightarrow$ Rectangular form

$z = r\angle\phi \Rightarrow$ Polar form

$z = re^{j\phi} \Rightarrow$ Exponential form.

Addition & Subtraction are Better performed in Rectangular form.

Multiplication & Division are better performed in Polar form

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

multiplication:

$$z_1 z_2 = r_1 r_2 \angle\phi_1 + \phi_2$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\phi_1 - \phi_2$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r}\angle{-\phi}$$

Squareroot

$$\sqrt{z} = \sqrt{r}\,\angle{\phi/2}$$

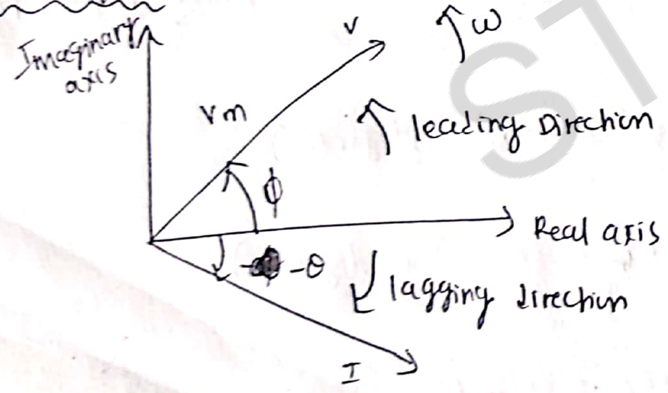
Complex Conjugate:

$$z^* = x - jy = r\angle{-\phi} = re^{-j\phi}$$

$v(t) = V_m \cos(\omega t + \phi)$ = Time domain representation

$V = V_m \angle\phi$ = Phasor domain representation

Phasor-Diagram:



Phasor diagram showing

$$V = V_m \angle\phi \quad \& \quad I = I_m \angle{-\theta}$$

Problems:

Transform the sinusoids to phasors.

① $i = 6\cos(50t - 40°)\,A$

$$\boxed{I = 6\angle{-40°}\,A}$$

② $v = -4\sin(30t + 50°)\,V$

$= 4\cos(30t + 50° + 90°)$ Since. $(-\sin A = \cos(A + 90°))$

$= 4\cos(30t + 140°)\,V$

$$\boxed{V = 4\angle{140°}\,V}$$

③ $v = -7\cos(2t + 40°)\,V$

$= 7\cos(2t + 40 + 180)$

$= 7\cos(2t + 220)\,v$

$$\boxed{V = 7\angle{220°}\,V}$$

(4)

$$i = 4\sin(10t + 10^\circ)\,A$$

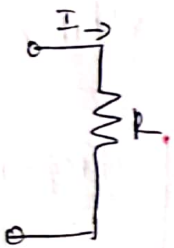
$$= 4\cos(10t + 10^\circ - 90^\circ)\,A$$

$$= 4\cos(10t - 80^\circ)\,A$$

$$\boxed{i = 4\angle -80^\circ\,A}$$

Phasor relationships for circuit elements:-

Resistor



$i = I_m\cos(\omega t+\phi)$ = current

$V = iR$

$V = R\,I_m\cos(\omega t+\phi)$

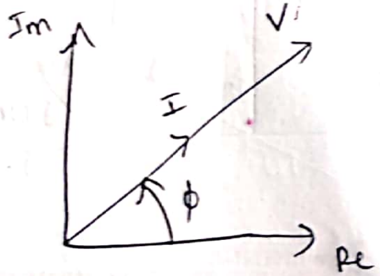
Phasor form:-

$V = R\,I_m\angle\phi$

$I = I_m\angle\phi$

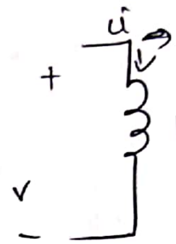
$V = IR.$

Phasor Diagram for Resistor



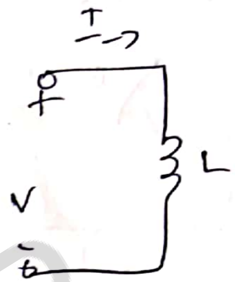
$\boxed{\text{Voltage and current are in-phase.}}$

For Inductor:-



Time Domain

$$\boxed{V = L\,\frac{di}{dt}}$$



freq-domain

$$\boxed{V = j\omega L I}$$

Assume current through Inductor is $i = I_m\cos(\omega t+\phi)$

$$V = L\frac{di}{dt} = -\omega L I_m\sin(\omega t+\phi)$$

$\left[\because -\sin A = \cos(A+90^\circ)\right]$

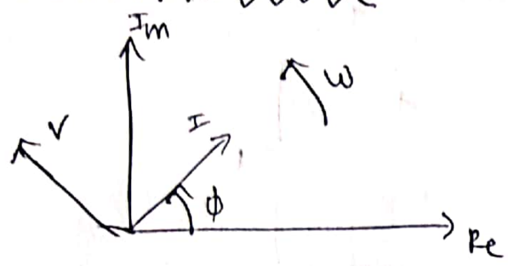
$$V = \omega L I_m\cos(\omega t+\phi+90^\circ)$$

$\left[\because -\sin A = \cos(A+90^\circ)\right]$

$$\boxed{V = \omega L I_m\angle\phi+90^\circ}$$

$$\boxed{I_m\angle\phi = I}$$

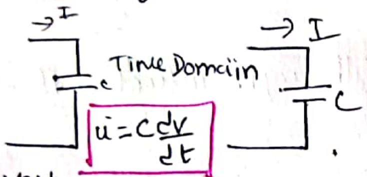
Phasor Diagram for Inductor:-



Voltage & current are 90° out of phase.

Current lags voltage by 90°.

For Capacitor:-



$\boxed{i = C\frac{dv}{dt}}$ (Time Domain)

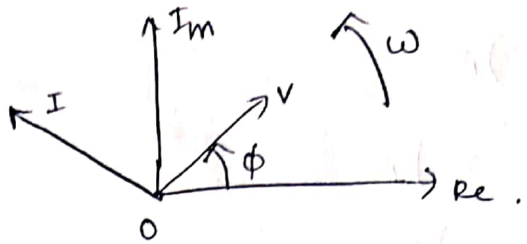
$\boxed{I = j\omega C V}$ (Frequency Domain)

assume the voltage across it is $v = V_m \cos(\omega t + \phi)$

Current through the capacitor is

$$i = C\frac{dv}{dt}$$

$$\boxed{I = j\omega C V \Rightarrow V = \frac{I}{j\omega C}.}$$



Current & voltage are 90° out of phase.

Current leads the voltage by 90°.

Summary of voltage current relationships.

Element	Time Domain	Frequency Domain
R	$v = Ri$	$V = RI$
L	$v = L\,di/dt$	$V = j\omega L I$
C	$i = C\,dv/dt$	$V = I/j\omega C$

Problem:-

The voltage $v = 12\cos(60t + 45^\circ)$ is applied to a 0.1H Inductor, Find the Steady State current through the inductor.

Soln

$V = j\omega L I$ $\quad \omega = 60$ rad/s

$V = 12 \angle 45^\circ$ V

$$I = \frac{V}{j\omega L} = \frac{12 \angle 45^\circ}{j60 \times 0.1} = \frac{12 \angle 45^\circ}{6 \angle 90^\circ}$$

$$\boxed{I = 2 \angle -45^\circ \text{ A}}$$

$$\boxed{I = 2\cos(60t - 45^\circ) \text{A}}$$

(Time Domain)

If voltage $v = 6\cos(100t - 30^\circ)$ is applied to a $50\mu F$ capacitor, calculate the current through it.

$V = I / j\omega C$

$I = V \, j\omega C$

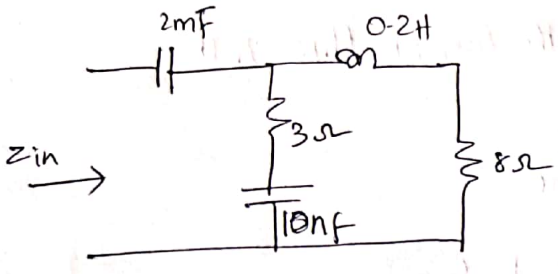
$= 6 \angle -30^\circ \times j\,100 \times 50 \times 10^{-6}$

$$\boxed{I = 30\cos(100t + 60^\circ) \text{mA}}$$

Impedance & Admittance of Passive Elements:-

Element	Impedance	Admittance
R	$Z = R$	$Y = 1/R$
L	$Z = j\omega L$	$Y = 1/j\omega L$
C	$Z = 1/j\omega C$	$Y = j\omega C$.

Find the input impedance of the circuit; operates at $\omega = 50\,rad/s$



$$Z_1 = 1/j\omega C = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\,\Omega$$

$$Z_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\,\Omega$$

$$Z_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\,\Omega$$

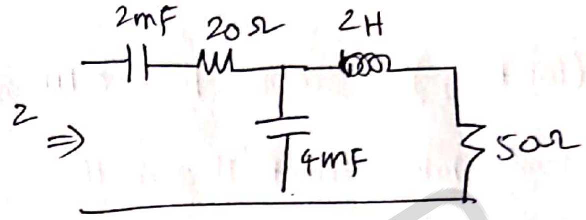
Input Impedance

$$Z = Z_1 + Z_2 \| Z_3$$

$$= -j10 + \frac{(3-j2)(8+j10)}{11+j8}$$

$$\boxed{Z_{in} = 3.22 - j11.07\,\Omega}$$

Determine the input impedance of the circuit at $\omega = 10\,rad/s$



$Z = 32.38 - j73.76\,\Omega$

$$Z_1 = 20 + \frac{1}{j\omega C} = 20 + \frac{1}{j10 \times 2 \times 10^{-3}} = 20 - 50j$$

$$Z_2 = 1/j\omega C = \frac{1}{j10 \times 4 \times 10^{-3}} = -25j$$

$$Z_3 = 50 + j\omega L = 50 + j \times 10 \times 2 = 50 + 20j$$

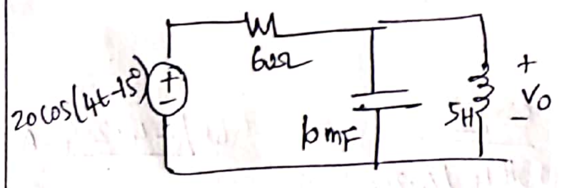
$$Z_1 + (Z_2 \| Z_3)$$

$$(20 - 50j) + \frac{(-25j)(50+20j)}{(-25j + 50 + 20j)} = \frac{1346.29\angle -68.19}{50.24\angle -5.71}$$

$$= 53.85\angle -68.1 + 26.79\angle -63.02$$

$$\boxed{Z = 32.23 - j73.83\,\Omega}$$

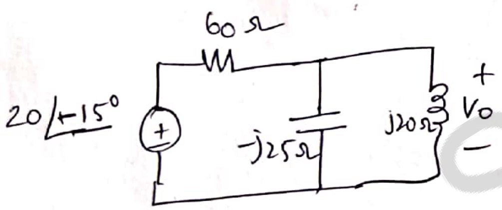
Determine $v_o(t)$ in the circuit.



$V_s = 20\cos(4t - 15^\circ) = 20\angle{-15^\circ}\,V$, $\omega = 4$

$10mF = \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\,\Omega$

$5H = j\omega L = j4 \times 5 = j20\,\Omega$



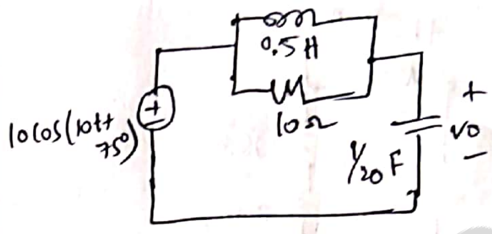
$Z_1 = 60\,\Omega$ $\quad Z_2 = -j25 \| j25 = j100\,\Omega$

$V_o = \frac{Z_2}{Z_1 + Z_2} V_s = \frac{j100}{60 + 100j}(20\angle{-15^\circ})$

$= (0.8575\angle{30.96^\circ})(20\angle{-15^\circ}) = 17.15\angle{15.96^\circ}\,V$

Time Domain $\Rightarrow$ $\boxed{v_o(t) = 17.15\cos(4t + 15.96^\circ)\,V}$

(2) Calculate 'V_o'



$V_s = 10\cos(10t - 15^\circ)\,V$ $\quad (0.5H \| 10\Omega) + (1/20\,F)$

$V_s = 10\angle{-15^\circ}\,V$

$\omega = 10$ rad/sec

~~$Z_1 = 10\Omega$, $Z_2 =$~~

$Z_1 = (0.5H \| 10\Omega)$ $\quad Z_2 = (1/20\,F)$

$0.5H \Rightarrow j\omega L = j10 \times 0.5 =$

$Z_1 = j\omega L = j10 \times 0.5 = 5j$

$Z_2 = 10\,\Omega$

$Z_3 = 1/j\omega C = 1/(j10 \times 1/20) = 1/0.5j = -2j$

$Z = (Z_1 \| Z_2) + Z_3$

$(5j \| 10) + (-2j)$

$= \frac{(10 \times j5)}{(10 + j5)} - 2j$

$= \left(\frac{j50}{10 + j5}\right) - 2j$

$2 + j4 - 2j$

$\boxed{Z_{in} = 2 + j2\,\Omega}$

$I = V/Z_{in} = \frac{10\angle{75^\circ}}{2 + j2} = 3.06 + 1.76j$

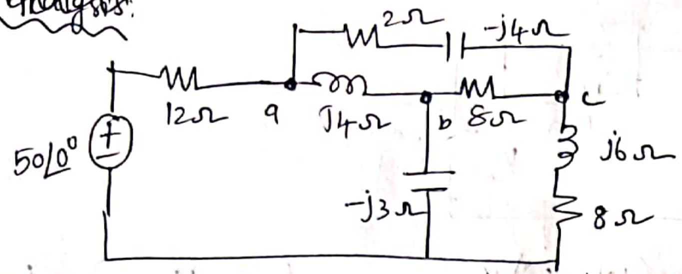
$V_o = I \times Z_3 = (3.06 + 1.76j)(-2j)$

$V_o(t) = 7.06\angle{-60^\circ}$ (or) ~~7.06 cos 10t~~

$V_o(t) = 7.06\cos(10t - 60^\circ)\,V$

Find the current 'I' in the circuit

~~Mesh Analysis~~



Delta network connected to nodes a, b, & c can be converted to the Y network

$$Z_{an} = \frac{j4(2-j4)}{j4+2-j4+8} = \frac{4(4+j2)}{10} = (1.6+j0.8)\,\Omega$$

$$Z_{bn} = \frac{j4(8)}{10} = j3.2\,\Omega,$$

$$Z_{cn} = \frac{8(2-j4)}{10} = (1.6-j3.2)\,\Omega.$$

The total impedance at the source terminals is

$$Z = 12 + Z_{an} + (Z_{bn} - j3) \,||\, (Z_{cn} + j6 + 8)$$

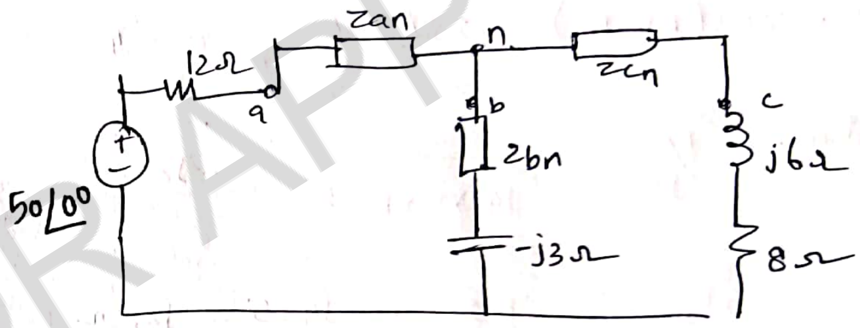
$$= 12 + 1.6 + j0.8 + (j0.2) \,||\, (9.6 + j2.8)$$

$$= 13.6 + j0.8 + \frac{j0.2(9.6+j2.8)}{9.6+j3}$$

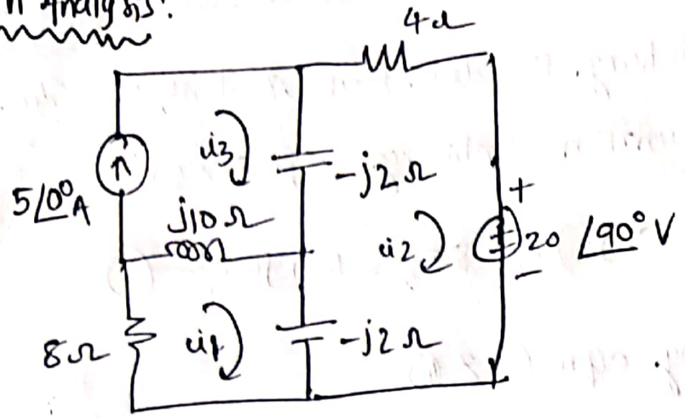
$$= 13.6 + j1 = 13.64 \angle 4.204^\circ \,\Omega$$

The desired current is

$$I = V/Z = \frac{50\angle 0^\circ}{13.64\angle 4.204^\circ} = 3.666 \angle -4.204^\circ \text{ A}$$



Mesh Analysis:



KVL @ loop1

$$(8+j10-j2)I_1-(-j2)I_2-j10I_3=0 \quad \text{--- (1)}$$

KVL @ loop2

$$(4-j2-j2)I_2-(-j2)I_1-(-j2)I_3+20\angle 90^\circ=0 \quad \text{--- (2)}$$

from loop 3

$I_3=5\angle 0^\circ$ — (3) sub this in (1) & (2)

$$(8+j8)I_1+j2I_2=j50 \quad \text{--- (4)}$$

$$j2I_1+(4-j4)I_2=-j20-j10 \quad \text{--- (5)}$$

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta=\begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

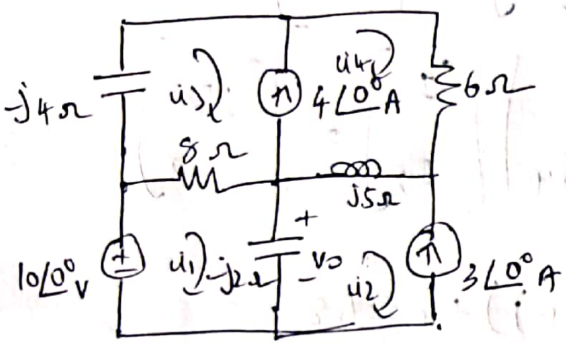
$$=32(1+j)(1-j)+4=68$$

$$\Delta_2=\begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix}=340-j240=416.17\angle -35.22^\circ$$

$$I_2=\Delta_2/\Delta=\frac{416.17\angle -35.22^\circ}{68}=6.12\angle -35.22^\circ \text{A}$$

$$\boxed{I_0=-I_2=6.12\angle 144.78^\circ \text{A}}$$

② Solve for V_0 in the fig.



for loop 1

$$-10+(8-j2)I_1-(-j2)I_2-8I_3=0$$

$$(8-j2)I_1+j2I_2-8I_3=10 \quad \text{--- (1)}$$

for loop 2

$$i_2=-3 \quad \text{--- (2)}$$

for Supermesh

$$(8-j4)I_3-8I_1+(6+j5)I_4-j5I_2=0 \quad \text{--- (3)}$$

From loop 3 & 4

$$I_4=I_3+4 \quad \text{--- (4)}$$

Instead of solving the above four equations, we reduce them to two by elimination combining eqn (1) & (2)

$$(8-j2)I_1-8I_3=10+j6 \quad \text{--- (5)}$$

by combining eqn (3) & (4)

$$-8I_1+(14+j)I_3=-24-j35 \quad \text{--- (6)}$$

$$\begin{bmatrix} 8-j2 & -8 \\ -8 & 14+j \end{bmatrix}\begin{bmatrix} I_1 \\ I_3 \end{bmatrix}=\begin{bmatrix} 10+j6 \\ -24-j35 \end{bmatrix}$$

$$\Delta=\begin{vmatrix} 8-j2 & -8 \\ -8 & 14+j \end{vmatrix}=112+j8-j28+2-64=\underline{50-j20}$$

$$\Delta_1=\begin{vmatrix} 10+j6 & -8 \\ -24-j35 & 14+j \end{vmatrix}=140+j10+j84-6-192-j280$$

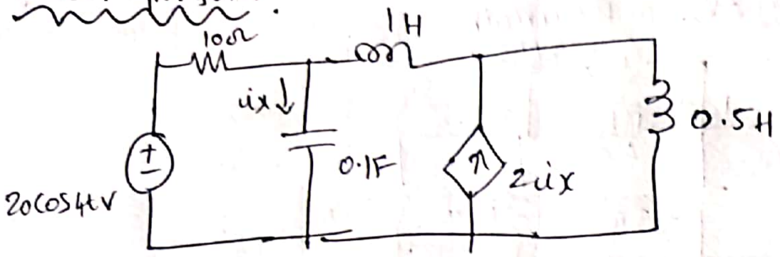
$$=\underline{-58-j186}$$

$$I_1=\frac{\Delta_1}{\Delta}=\frac{-58-j186}{50-j20}=\underline{3.618\angle 274.5^\circ A}$$

$$\text{Voltage } V_0=-j2(I_1-I_2)=-j2(3.618\angle 274.5^\circ+3)$$

$$=-7.2134-j6.568=9.756\angle 222.32^\circ V$$

Nodal Analysis:



First convert to freq. Domain.

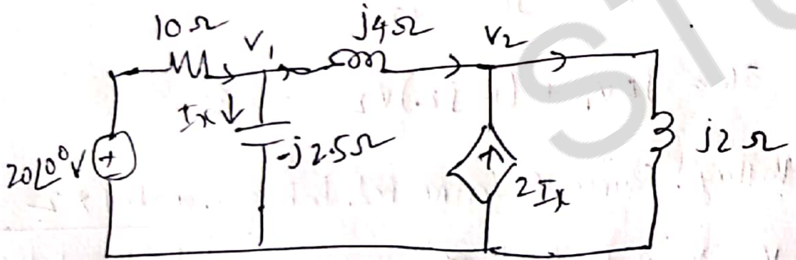
$20\cos 4t = 20\angle 0^\circ$, $\omega = 4$ rad/s.

$1H \Rightarrow j\omega L = j4$

$0.5H \Rightarrow j\omega L = j2$

$0.1F = 1/j\omega C = -j2.5$

freq. Domain Equivalent circuit.



Applying KCL at node 1

$$\frac{20 - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4}$$

$$(1 + j1.5)V_1 + j2.5V_2 = 20 \quad \text{--- (1)}$$

At node2

$$2I_x + \frac{V_1 - V_2}{j4} = V_2/j2$$

But $I_x = V_1/-j2.5$

$$\frac{2V_1}{-j2.5} + \frac{V_1 - V_2}{j4} = V_2/j2$$

Simplifying we get

$$11V_1 + 15V_2 = 0 \quad \text{--- (2)}.$$

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1+j1.52 & 0 \\ 11 & 0 \end{vmatrix} = -220$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15-j5} = 18.97\angle 18.43^\circ\ V$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15-j5} = 13.91\angle 198.3^\circ\ V$$

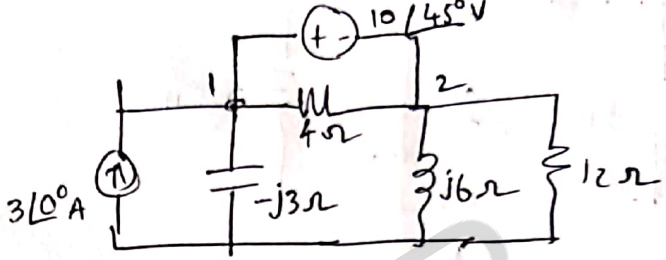
The current I_x is given by

$$I_x = \frac{V_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} =$$

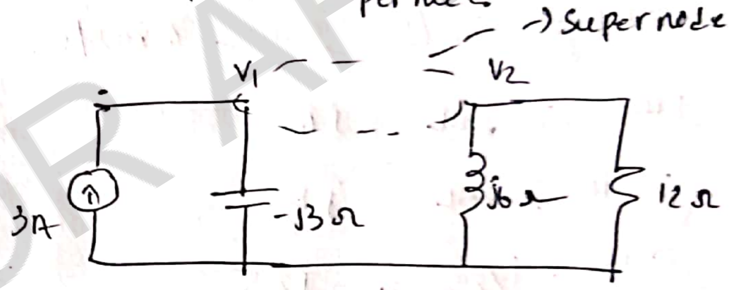
$$= 7.59\angle 108.4^\circ\ A$$

$$\boxed{i_x = 7.59\cos(4t + 108.4^\circ)\ A}$$

② Compute V_1 & V_2 in the circuit.



Node 1 & 2 form a supernode.



Applying KCL

$$3 = \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12}$$

$$36 = j4V_1 + (1-j2)V_2 \quad \text{--- ①}$$

But a voltage source is connected between node 1 & 2

So $V_1 = V_2 + 10\angle 45^\circ$ --- ② $\boxed{V_1 - V_2 = 10\angle 45^\circ}$

Subs ② in ①

$$36 - 40\angle 135^\circ = (1+j2)V_2 \quad \Big| \quad V_2 = 31.41\angle -87.18^\circ\ V$$

From ②.

$V_1 = V_2 + 10\angle 45^\circ = 25.78\angle -70.48^\circ\ V$ $\boxed{V_1 = 25.78\angle -70.48^\circ\ V}$

Instantaneous power & Average power:

Instantaneous power = instantaneous voltage × instantaneous current

$$\boxed{P(t) = v(t)\,i(t)}$$

The instantaneous power (watts) is the power at any instant of time.

let

$v(t) = V_m \cos(\omega t + \phi_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$

V_m & I_m ⇒ are the amplitudes

ϕ_v & θ_i ⇒ Phase angle of voltage & current respectively.

$$P(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$\cos A \cos B = \tfrac{1}{2}\left\{\cos(A-B) + \cos[A+B]\right\}$$

$$P(t) = \underbrace{\tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant or time dependent}} + \underbrace{\tfrac{1}{2} V_m I_m (2\omega t + \theta_v + \theta_i)}_{\text{Sinusoidal fn}} \quad \text{--- (1)}$$

P(t) can be positive or negative

If $p(t)$ = positive ⇒ Power is absorbed by the circuit

if $p(t)$ = negative ⇒ Power is absorbed by the source.

Average power:

average power in watts, is the average of the instantaneous power over one period.

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt} \quad \text{--- (2)}$$

Subs ② in ①

$$P = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \frac{1}{T}\int_0^T dt + \frac{1}{2} V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

Constant

Sinusoid.

First integral is constant, second is sinusoid, average of a sinusoid over its period is zero.

So Average power is

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v) \Rightarrow$ difference in the phases of the voltage & current.

$P(t)$ = time varying

P = does not depend on time

for instantaneous power we must know $v(t)$ & $i(t)$ (time varying)

But average power can be calculated in when the voltage & current are expressed in ~~freq~~ time domain or freq domain

eg. $V(t) = V_m \angle \theta_v$

$I(t) = I_m \angle \theta_i$

$$P = \frac{1}{2} VI = \frac{1}{2} V_m I_m \angle \theta_v - \theta_i$$

$$= \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

Real part of this equation in average power.

$$P = \frac{1}{2} Re[V I^*]$$

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

When $\theta_v = \theta_i$

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} |I|^2 R$$

(Shows the circuit is purely resistive)

When $\theta_v - \theta_i = 90°$

$$P = \frac{1}{2} V_m I_m \cos 90° = 0$$

Shows that the circuit is purely reactive. Circuit absorbs no average power

A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power

(1) Problem

Given that $v(t) = 120 \cos(377t + 45°)$ V

$i(t) = 10 \cos(377t - 10°)$ A

Find the instantaneous power & average power absorbed by the passive linear ntk.

Instantaneous power

$$P = vi = 1200 \cos(377t + 45°) \cos(377t - 10°)$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$= 600[\cos(754t + 35°) + \cos(55°)]$$

$$\boxed{p(t) = 344.2 + 600 \cos(754t + 35°) W}$$

average power P_{avg}

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120) 10 \cos(45° - (-10°))$$

$$= 600 \cos 55° = \underline{\underline{344.2 W}}$$

② Calculate the instantaneous power & average power absorbed by the passive linear ntk.

$v(t) = 80\cos(10t + 20^\circ) V$

$i(t) = 15\sin(10t + 60^\circ) V$

i(t) can be written as

$i(t) = 15\cos(10t + 60^\circ - 90^\circ) V$

$i(t) = 15\cos(10t - 30^\circ) V$

$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$

$= \frac{1}{2} 80 \times 15 \cos(20 - (-30)) + \frac{1}{2} 80 \times 15 \cos(2 \times 10t + 20 - 30)$

$= \frac{1}{2} 600 \cos(50^\circ) + \frac{1}{2} 600 \cos(20t - 10^\circ)$

$\boxed{p(t) = 385.7 + 600(20t - 10^\circ) W}$

Average Power P_{avg}

$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$= \frac{1}{2} \times 80 \times 15 \cos(20 - (-30))$

$= 600(\cos(50))$

$\boxed{P_{avg} = 385.7 W}$

③ Calculate the average power absorbed by an impedance $Z = 30 - j70\,\Omega$ when a voltage $V = 120\angle 0^\circ$ is applied across it.

$I = V/Z = \frac{120\angle 0^\circ}{30 - j70} = \frac{120\angle 0^\circ}{76.16\angle -66.80^\circ} = 1.576\angle 66.8^\circ A$

$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ)$

$\boxed{P_{avg} = 37.24 W}$

(4) A current $I = 10\angle 30^\circ$ flows through an impedance $Z = 20\angle -22^\circ\,\Omega$, Find the average power delivered to the impedance.

$$V = I \cdot Z = 10\angle 30^\circ \times 20\angle -22^\circ$$

$$V = 200\angle 8^\circ \ V$$

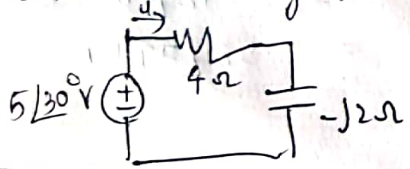
$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \times 200 \times 10 \cos(8 - 30^\circ)$$

$$= 1000 \cos(-22^\circ)$$

$$\boxed{P_{avg} = 927.2\,W}$$

(5) Find the avg. power supplied by the source & the avg power ~~delive~~ absorbed by the resistor



$$I = \frac{5\angle 30^\circ}{4 - j2} = \frac{5\angle 30^\circ}{4.472\angle -26.57} = 1.118\angle 56.57^\circ A$$

$$P_{avg} = \frac{1}{2}(5)(1.118)\cos(30^\circ - 56.57^\circ) = 2.5W$$

Current through resistor is

$$I_R = I = 1.118\angle 56.57^\circ A$$

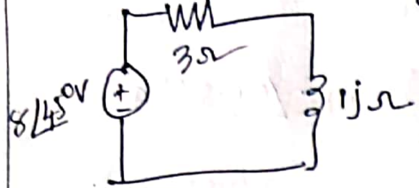
$$V_R = 4 I_R = 4.472\angle 56.57^\circ V$$

Power absorbed by resistor is

$$P = \frac{1}{2}(4.472)(1.118) = 2.5W$$

(Since phase diff bet voltage & current is zero) $(\theta_v - \theta_i = 0)$

which is the same as the average power delivered by the zero average power is absorbed by the capacitor.



Calculate the power (avg) absorbed by the resistor & inductor. Find the avg power supplied by voltage source.

$$I = \frac{8\angle 45^\circ V}{3+j1} = \frac{8\angle 45^\circ}{3.16\angle 18.43^\circ} = 2.53\angle 26.57^\circ \text{ A}$$

Pavg by supply :

$$P_{avg} = \frac{1}{2}\times 8\times 2.53\,\cos(45^\circ - 26.57^\circ)$$

$$\boxed{P_{avg(supply)} = 9.6\text{W}}$$

Current through Resistor

$$\cancel{I = \frac{8\angle 45^\circ}{3} = 2.66\angle 45^\circ \text{ A}}$$

$$\cancel{v = (2.66\angle 45^\circ)\times 3 =}$$

$$\cancel{P_{avg(res)} = \frac{1}{2}\times 8\times 2.66\,\cos(45^\circ - 45^\circ)}$$

$=$

Current through resistor $I = 2.53\angle 26.57^\circ$ A

voltage in resistor $(2.53\angle 26.57^\circ \times 3) =$
$7.59\angle 26.57^\circ$

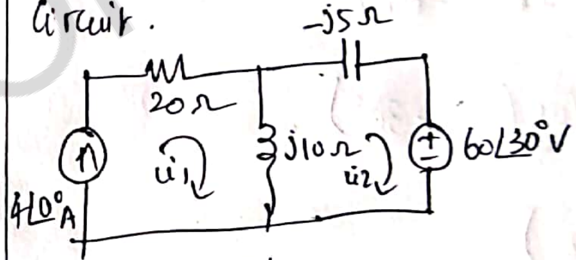
Pow in resistor $= \frac{1}{2}\times \cancel{8\times}\, 7.59\times 2.53\,\cos(26.57 - 26.57)$

$$= \frac{1}{2}\times 7.59\times 2.53$$

$$\boxed{P_{avg\ res.} = 9.6\text{W}}$$

Power absorbed by Inductor is zero.

③ Determine the average power generated by each source and the power absorbed by each passive elements of the circuit.



apply mesh/loop 1

$$i_1 = 4\text{A}$$

From mesh/loop 2

$$(j10 - j5)I_2 - j10I_1 + 60\angle 30^\circ = 0,$$

$$j5I_2 = -60\angle 30^\circ + j40 \Rightarrow I_2 = -12\angle -60^\circ + 8$$

$$= 10.58\angle 79.1^\circ \text{A}$$

For the voltage source, the current flowing from it is I_2

$I_2 = 10.58 \angle 79.1° A$

& voltage is $60\angle 30° V$ so avg power is

$P_5 = \frac{1}{2}(60)(10.58)\cos(30° - 79.1°) =$

$= 207.8W$

This avg power is absorbed by the source, inview of the direction of I_2 & polarity of the voltage source the ckt is delivering average power to the voltage source.

for the current source: $I_1 = 4\angle 0°$.

$V_1 = 20I_1 + j10(I_1 - I_2) = 80 + j10(4-2-j10.39)$

$= 183.9 + j20$.

$= 184.984 \angle 6.21° V$

The avg power supplied by the current source is

$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21° - 0) = -367.8W$

negative sign indicates the current source is supplying power to the circuit.

For the resistor the current through it is $I_1 = 4\angle 0°$

Voltage across it is $20 I_1 = 80\angle 0°$;

$P_2 = \frac{1}{2}(80)(4) = 160W$

For the capacitor, the current through it is $I_2 = 10.58\angle 79.1°$

& the voltage across it is $-j5I_2 = (5\angle -90°)(10.58\angle 79.1°)$

$= 52.9 \angle 79.1° - 90°$.

avg power absorbed by capacitor is

$P_4 = \frac{1}{2}(52.9)(10.58)\cos(-90°) = 0$

Inductor, current is $(I_1 - I_2) = 2 - j10.39 = 10.58 \angle -79.10$

Voltage across it is $j10(I_1 - I_2) = 10.58 \angle -79.1 + 90^\circ$

Hence the average power absorbed by the inductor is

$$P_3 = \frac{1}{2}(105.8)(10.58)\cos 90^\circ = 0$$

Notice that the inductor & the capacitor absorb zero average power and that the tot. power supplied by the current source equals the power absorbed by the resistor & the voltage source or

$$P_1 + P_2 + P_3 + P_4 + P_5 = -367.8 + 160 + 0 + 0 + 207.8 = 0$$

indicating that power is conserved.

Effective or Rms value:

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

$\boxed{I_{eff} = I_{rms}}$ $\boxed{V_{eff} = V_{rms}}$

$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$ $\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$

average power can be written in terms of rms values

$$\boxed{P = I_{rms}^2 R = \frac{V_{rms}^2}{R}}$$

Apparent Power & Power Factor:-

Apparent power (in VA) is the product of the rms values of voltage and current. $\boxed{\text{units (Volt-Amp)}}$

Power factor:- Ratio of ~~apparent~~ average power to apparent power

$$\boxed{P_f = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

$\theta_v - \theta_i =$ Power factor angle.

Power factor is the cosine of the phase difference between voltage & current, It is also the cosine of the angle of load

(1) A Series-connected load draws a current $i(t) = 4\cos(100\pi t + 10°)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20°)$ V, find the apparent power & the power factor of the load, Determine the element values that form the Series-connected load.

Apparent power $S = V_{rms} \cdot I_{rms} \; \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = \underline{240\ VA}$

Power factor $Pf = \cos(\theta_v - \theta_i) =$

$= \cos(-20-10) = 0.866$ (leading)

(Because current leads voltage)

$Z = V/I = \frac{120\angle -20°}{4\angle 10°} = 25.98 - j15\,\Omega$

$Pf = \cos(-30°) = 0.866$ (leading).

load impedance 'Z' can be modeled by a $25.98\,\Omega$ resistor in Series with a Capacitor

$X_C = -15 = -1/\omega C$

$C = 1/15\omega = \frac{1}{15 \times 100\pi} = 212.2\mu F$

(2) Obtain Power factor & the apparent power of a load whose impedance is $Z = 60 + j40\,\Omega$ $v(t) = 150\cos(377t + 10°)$ V

~~Apparent power S = Vrms Irms~~ :

$I(t) = \frac{V(t)}{Z} = \frac{150\angle 10°}{60 + j40} = \frac{150\angle 10°}{72.11\angle 33.69}$

$I(t) = \cancel{2.08\angle 23.835} \quad 2.08\angle -23.69$

$S = V_{rms} I_{rms} = \frac{150}{\sqrt{2}} \times \frac{2.08}{\sqrt{2}} = \underline{156\,VA}$

$Pf = \cos(\theta_v - \theta_i) =$

$= \cos(10 + 23.69) = 0.832$ (lagging)

(Because current lags voltage)

Complex power: in (VA)

is the Product of the rms voltage phasor and the complex conjugate of the rms current phasor, As a complex quantity its real part is real power 'p' & its imaginary part is reactive power Q.

$$\boxed{\text{Complex power} = S = P + jQ = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i}$$

$$\boxed{\text{Apparent power } S = |S| = V_{rms} I_{rms} = \sqrt{P^2 + Q^2}}$$

$$\boxed{\text{Real power} = P = Re(S) = S\cos(\theta_v - \theta_i)}$$

$$\boxed{\text{Reactive Power} = Q = Im(S) = S\sin(\theta_v - \theta_i)}$$

$$\boxed{\text{Power factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

Problem:

1. The voltage across a load is $v(t) = 60\cos(\omega t - 10°)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50°)$ A

Find (a) complex & apparent powers

(b) real & reactive power

(c) power factor & the load impedance.

(a)

$V_{rms} = \frac{60}{\sqrt{2}} \angle -10°$ $\quad I_{rms} = \frac{1.5}{\sqrt{2}} \angle 50°$

Complex power $= V_{rms} I_{rms}^* \left(\frac{60}{\sqrt{2}} \angle -10°\right)\left(\frac{1.5}{\sqrt{2}} \angle -50°\right)$

$$\boxed{\text{Complex Power} = 45 \angle -60° \text{ VA}}$$

$$\boxed{\text{Apparent power } S = |S| = 45 \text{ VA}}$$

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(b) we can express the complex power in rectangular form

$$S = 45\angle -60^\circ = 45[\cos(-60) + j\sin(-60^\circ)]$$

$$= 22.5 - j38.97$$

Since $S = P + jQ$, the real power is

$$\boxed{\text{Real power } P = 22.5\,W}$$

while reactive power is

$$\boxed{Q = -38.97\,VAR}$$

(c) The power factor

$$pf = \cos(-60^\circ) = 0.5 \text{ (leading)}$$

$$Z = \frac{V}{I} = \frac{60\angle -10^\circ}{1.5\angle +50^\circ} = 40\angle -60^\circ\ \Omega$$

(2) For a load $V_{rms} = 110\angle 85^\circ V$, $I_{rms} = 0.4\angle 15^\circ A$

Determine complex power, apparent power, real power, reactive power, power factor & load impedance.

Complex power

$$S = V_{rms} I_{rms}^*$$

$$S = 110\angle 85^\circ \times 0.4\angle -15^\circ$$

$$\boxed{S = 44\angle 70^\circ\ VA}$$

Apparent power:

$$\boxed{|S| = 44\ VA}$$

real power

Complex power in rectangular form

$$15.04 + j41.3\ W$$

$$S = P + jQ$$

Real power $P = 15.04W$

Reactive power $Q = 41.3\ VAR$

Power factor

$$Pf = \cos(\theta_v - \theta_i) \quad \text{or} \quad Pf = \cos(\theta)$$

from complex power

$$= \cos(85 - 15)$$

$$= \cos(70^\circ)$$

$$\boxed{Pf = 0.342 \text{ lagging}}$$

Impedance

$$Z = \frac{V}{I} = \frac{V_{rms} \times \sqrt{2}}{I_{rms} \times \sqrt{2}} = \frac{110\angle 85}{0.4\angle 15^\circ}$$

$$= 275\angle 70^\circ\ \Omega$$

$$\boxed{Z = 94.05 + j258.4\ \Omega}$$

~~(3) for a load, Vrms = 110∠85° V, Irms = 0.4~~

(3) A load Z draws 12 KVA at a p.f of 0.856 lagging from a 120V rms sinusoidal source. Calculate the average & reactive power's delivered to the load, (b) Peak current and the (c) load Impedance.

Given that $Pf = \cos\theta = 0.856$, we obtain the power angle as $\theta = \cos^{-1} 0.856 = 31.13^\circ$

If apparent power is $S = 12{,}000\ VA$ the average or real power is

$$\boxed{P = S\cos\theta = 12000 \times 0.856 = 10.272\ KW}$$

while the Reactive power is

$$\boxed{Q = S\sin\theta = 12000 \times 0.517 = 6.204\ KVA}$$

(b) Since the PF is lagging, the complex power is

$$\boxed{S = P + jQ = 10.272 + j6.204\ KVA}$$

From $S = V_{rms} I_{rms}^*$

$$I_{rms}^* = \frac{S}{V_{rms}} = \frac{10,272 + j6204}{120\angle 0^\circ}$$

$$= 85.6 + j51.7\,A$$

$$= 100\angle 31.13^\circ\,A$$

Thus $I_{rms} = 100\angle -31.13^\circ$ & the peak current is

$$I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 100 = 141.4\,A$$

The load impedance

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{120\angle 0^\circ}{100\angle -31.13^\circ} = 1.2\angle 31.13^\circ\,\Omega$$

which is an inductive impedance.

UNIT-4

TRANSIENTS AND RESONANCE IN RLC CIRCUIT

Here We will examine two types of Simple circuits:

→ A Circuit Comprising a resistor and Capacitor

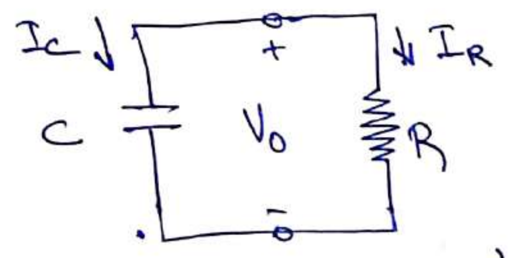
→ and a Circuit Comprising a resistor and an Inductor.

These are called RC and RL.

The Source-free RC Circuit:

* A Source-free RC Circuit Occurs when its DC source is suddenly dis-connected. The energy already stored in the capacitor is released to the resistors.

→ Consider a Series Combination of a resistor and an Initially Charged Capacitor.



→ Objective is to determine the Circuit response, assume to be the Voltage $v(t)$ across the capacitor.

Since capacitor initially charged, we can assume that at time $t=0$, the initial Voltage is,

$v(0) = V_0$.

When the Corresponding Value of energy stored

$$w(0) = \frac{1}{2} C V_0^2$$

Applying KCL at the node of the Circuit,

$$i_C + i_R = 0$$

We know that $i_C = C\frac{dv}{dt}$ and $i_R = \frac{v}{R}$

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

by rearranging

$$\frac{dv}{v} = -\frac{1}{RC} dt$$

Integrating both sides,

$$\boxed{\frac{dx}{x} = \ln x}$$

$$\ln v = -\frac{t}{RC} + \ln A$$

$A \rightarrow$ integration constant, thus,

$$\frac{\ln v}{\ln A} = -\frac{t}{RC}$$

Taking Powers of e produce

$$\boxed{e^{\ln x} = x}$$

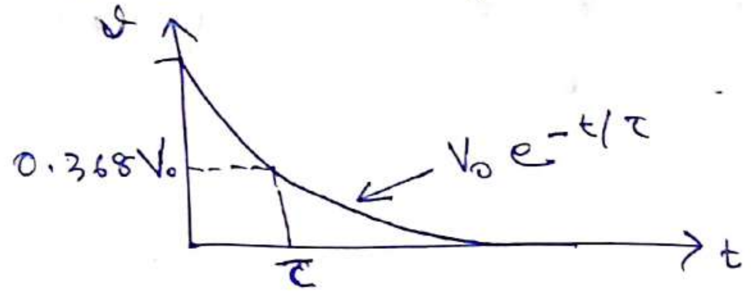
$$\frac{v}{A} = e^{-t/RC}$$

$$\therefore v(t) = A e^{-t/RC}$$

at initial Condition $v(0) = A = V_0$

$$\boxed{v(t) = V_0 e^{-t/RC}}$$

This shows the voltage Response of the RC Circuit is an exponential decay of the initial voltage.



The Response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source, it is called the natural response of the circuit.

From the graph, As t - increases the voltage decreases toward zero.

The rapidity with which the voltage decreases is expressed in terms of the time constant (τ).

that, $t = \tau$

then $\boxed{V(t) = V_0 e^{-t/RC}}$

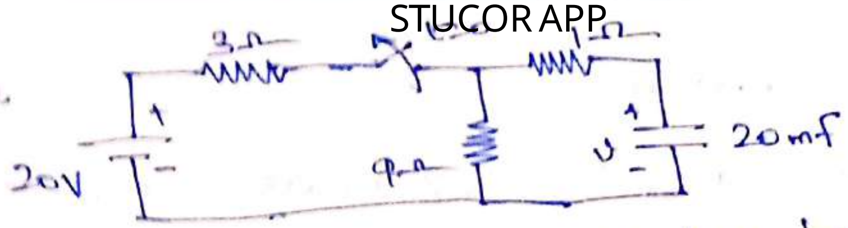
where $\tau = RC$

$$\boxed{V(t) = V_0 e^{-t/\tau}} = V_0 e^{-1}$$

$$= 0.368 V_0$$

In terms of time constant

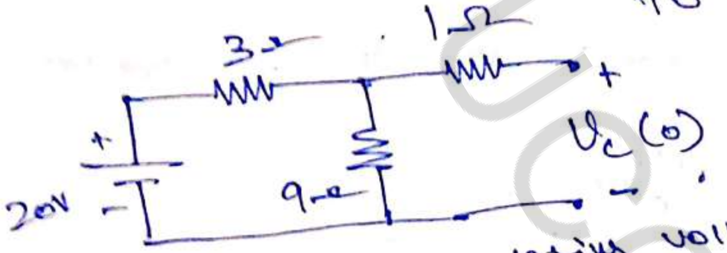
$$\boxed{v(t) = V_0 e^{-t/\tau}}$$



The Switch in the Circuit has been closed for a long time, and its opened at $t=0$. find $v(t)$ for $t \geq 0$. Calculate the intial energy stored in the Capacitor.

Solution:

for $t<0 \rightarrow$ Switch closed, the Capacitor is an Open Circuit, to DC.



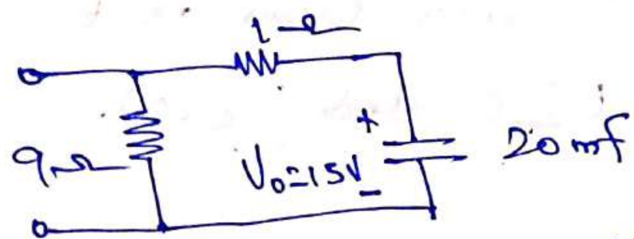
using voltage division Rule

$$V_c(t) = 20 \times \frac{9}{9+3} = 15\text{ V} \quad t<0$$

Voltage across a Capacitor cannot change instantaneously, the voltage across the Capacitor at $t=0^-$ is Same at $t \geq 0$

$$V_c(0) = V_0 = 15\text{ V}.$$

for $t>0$, the switch is opened,



Source free RC circuit.

$R_{eq} = 1+9 = 10\,\Omega$

time Constant,

$$\tau = R_{eq} C = 10 \times 20 \times 10^{-3} = 0.2\ s$$

the voltage across the capacitor for $t \geq 0$ is

$$v(t) = V_c(0)\, e^{-t/\tau}$$

$$= 15\, e^{-t/0.2}\ V$$

$$\boxed{v(t) = 15\, e^{-5t}\ V}$$

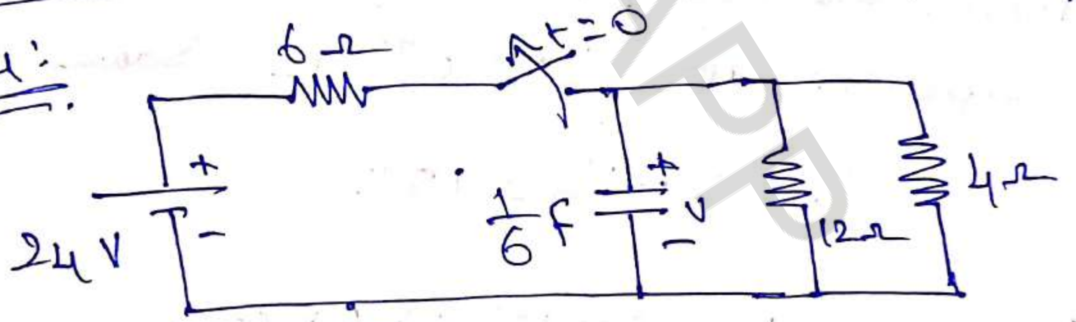
Initial energy stored in the capacitor is

$$W_c(0) = \frac{1}{2} C\, v_c^2(0)$$

$$= \frac{1}{2} \times 20 \times 10^{-3} \times 15^2$$

$$\boxed{W_c(0) = 2.25\ J}$$

Ex:4:

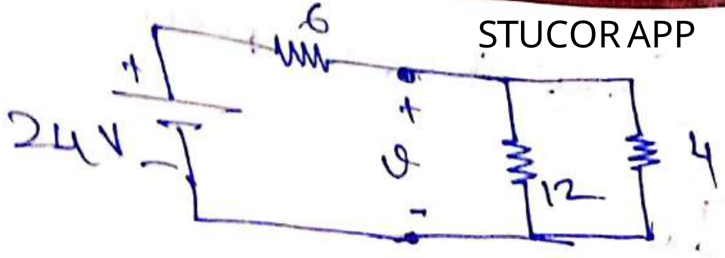


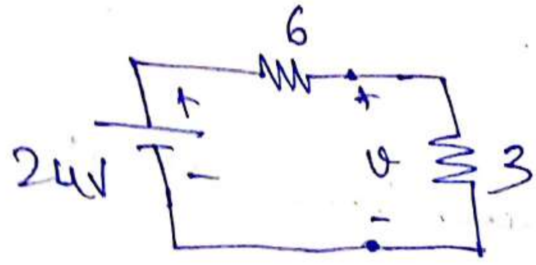
If the switch opens at $t = 0$, find $v(t)$ for $t \geq 0$ and $W_c(0)$.

Solution:

for $t < 0 \rightarrow$ Switch Closed

Capacitor at Open Circuited.





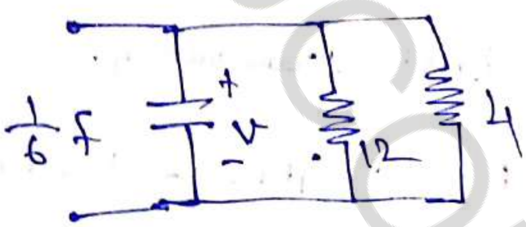
$\Rightarrow 24 \times \frac{3}{6+3}$

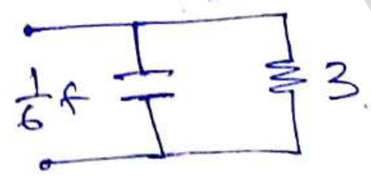
$= 8\ V$

$v_c(t) = 8\ V \quad t < 0$

for $t \geq 0$ Switch Opened

$v_c(0) = V_0 = 8\ V$





$R_{eq} = 3$

$\tau = RC = 3 \times \frac{1}{6} = 0.5\ s$

Voltage across capacitor $t \geq 0$

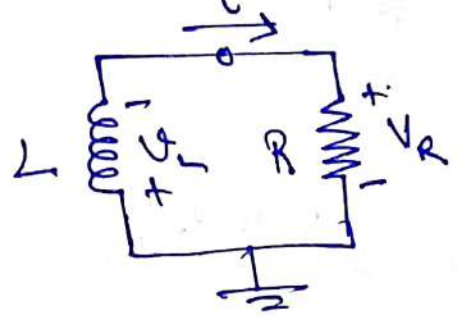
$v(t) = v_c(0)\, e^{-t/0.5}$

$v(t) = 8\, e^{-2t}\ V$

$W_c(0) = \frac{1}{2} C V_c^2(0) = \frac{1}{2} \times \frac{1}{6} \times 8^2$

$= 5.33\ J$

THE SOURCE FREE RL CIRCUIT



Our goal to determine the circuit Response.

We select the inductor current as the response in order to take advantage of the idea that the inductor current cannot change instantaneously.

Assume at $t=0$ the inductor has initial current I_0,

$$i(0) = I_0.$$

The corresponding energy stored in the inductor,

$$w(0) = \frac{1}{2} L I_0^2.$$

Apply KVL around the loop,

$$V_L + V_R = 0$$

but, $V_L = L\frac{di}{dt}$ & $V_R = iR$, thus

$$L\frac{di}{dt} + Ri = 0$$

for simplification, divide by 'L'

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

Rearranging terms and integrating gives

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L} dt$$

$$\ln i \Big]_{I_0}^{i(t)} = -\frac{Rt}{L}\Big]_0^t$$

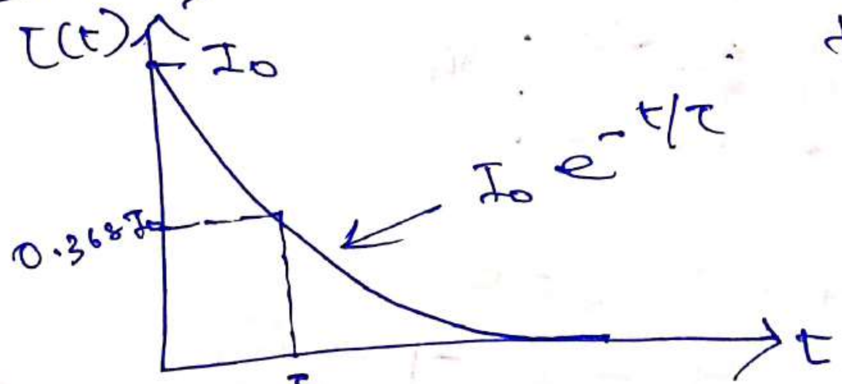
$$\ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L}$$

Taking e power

$$\therefore i(t) = I_0 e^{-Rt/L}$$

This shows that the natural response of the RL circuit is an exponential decay of the initial current.



time constant for RL

$$\tau = \frac{L}{R}$$

$$\boxed{i(t) = I_0 e^{-t/\tau}}$$

then we can find voltage across the resistor as

$$\boxed{V_R(t) = iR = I_0 R e^{-t/\tau}}$$

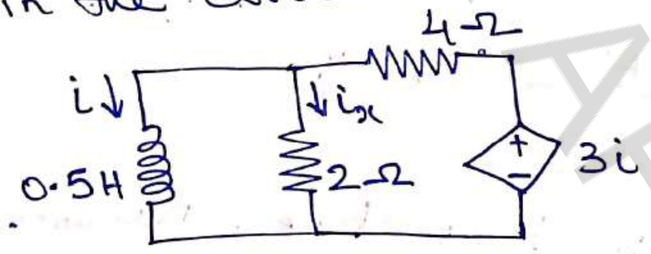
Power dissipate in the Resistor

$$\boxed{P = V_R i = I_0^2 R e^{-2t/\tau}}$$

$$\left[= I_0 R e^{-t/\tau} \times I_0 e^{-t/\tau} = I_0^2 R e^{-2t/\tau} \right]$$

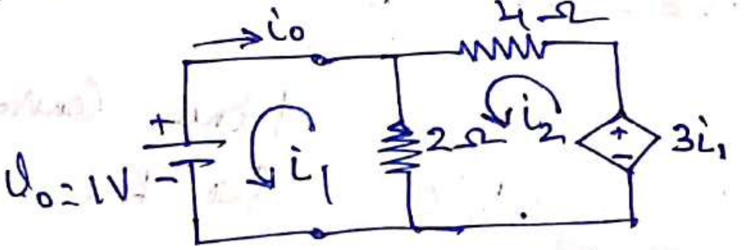
Problem:

Assuming that $i(0) = 10A$, calculate $i(t)$ and $i_x(t)$ in the circuit.



Solution:

Because of the dependent source, we insert a voltage source with $V_0 = 1V$,



Apply KVL:

$$1 + 2(i_1 - i_2) = 0 \quad (1)$$

$4i_2 + 2(i_2 - i_1)$

$4i_2 + 2i_2 - 2i_1 - 3i_1 = 0$

$6i_2 - 5i_1 = 0 \quad (2)$

$i_2 = \frac{5}{6} i_1$

Sub i_2 value in (1)

$2i_1 - \cancel{2}\left(\frac{5}{6}\right)i_1 = -1$

$i_1\left[\cancel{2 - \frac{10}{6}}\right] + 1 = 0$

$i_1\left[\frac{2}{6}\right] + 1 = 0$

$2i_1 - 2i_2 = -1$

$-5i_1 + 6i_2 = 0$

$\Delta = \begin{vmatrix} 2 & -2 \\ -5 & 6 \end{vmatrix} = 12 - 10 = 2$

$\Delta_1 = \begin{vmatrix} -1 & -2 \\ 0 & 6 \end{vmatrix} = -6$

$i_1 = \frac{\Delta_1}{\Delta} = \frac{-6}{2} = -3\,A$

$\therefore i_0 = -i_1 = -(-3) = \underline{3\,A}$

$\therefore R_{eq} = R_{th} = \frac{V_0}{I_0} = \frac{1}{3}\,\Omega$

Then the time constant is

$$\tau = \frac{L}{R_{eq}} = \frac{0.5}{1/3} = 1.5\,s$$

the current through the inductor is,

$$i(t) = i(0)e^{-t/\tau} = \underline{\underline{10e^{-t/1.5}}}\,A \quad t > 0$$

Voltage across the inductor is

$$v = L\frac{di}{dt} = 0.5\frac{d\,10e^{-t/1.5}}{dt}$$

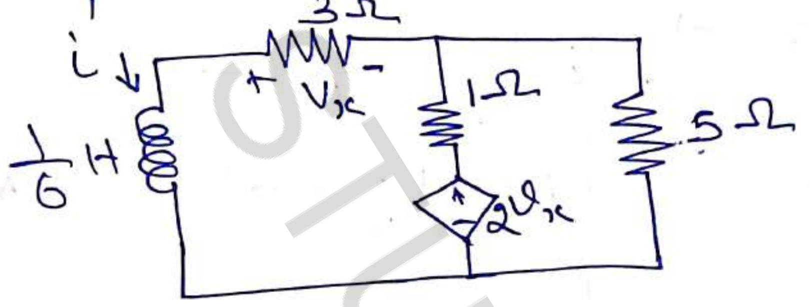
$$= (0.5)(10)\left(-\frac{1}{1.5}\right)e^{-t/1.5}$$

$$v = -3.33e^{-t/1.5}\,V$$

Since inductor and 2 Ω resistor are in Parallel,

$$i_x(t) = \frac{v}{2} = \frac{-3.33\,e^{-t/1.5}}{2} = -1.666\,e^{-t/1.5}\ \text{A} \quad t>0.$$

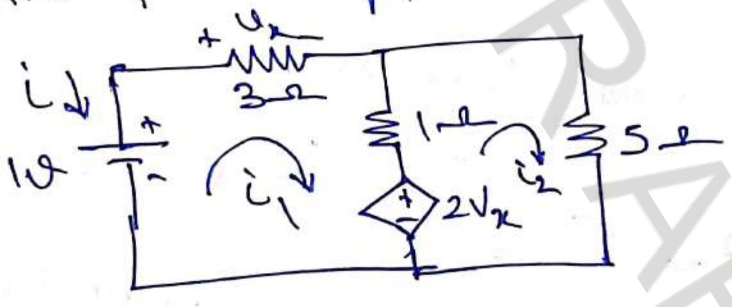
Ex 2: find i and V_x in the circuit in figure, Let $i(0) = 5$ A



Solution: Given: $i(0) = 5$

find: i and V_x

first find R_{eq},



Apply KVL

$-1 + 3i_1 + 1(i_1 - i_2) + 2V_x = 0$

$4i_1 - i_2 + 2(3i_1) = -1$

$10i_1 - i_2 = +1$ —— (1)

Loop 2

$1[i_2 - i_1] + 5i_2 - 2V_x = 0$

$i_2 - i_1 + 5i_2 - 2(3i_1) = 0$

$6i_2 - 7i_1 = 0$

$-7i_1 + 6i_2 = 0$ (2)

$$10i_1 - i_2 = +1$$
$$-7i_1 + 6i_2 = 0$$

y

$$\Delta = \begin{vmatrix} 10 & -1 \\ -7 & 6 \end{vmatrix} = 60 - 7 = 53$$

$$\Delta_1 = \begin{vmatrix} +1 & -1 \\ 0 & 6 \end{vmatrix} = +6$$

$$i_1 = \frac{+6}{53} = +0.113$$

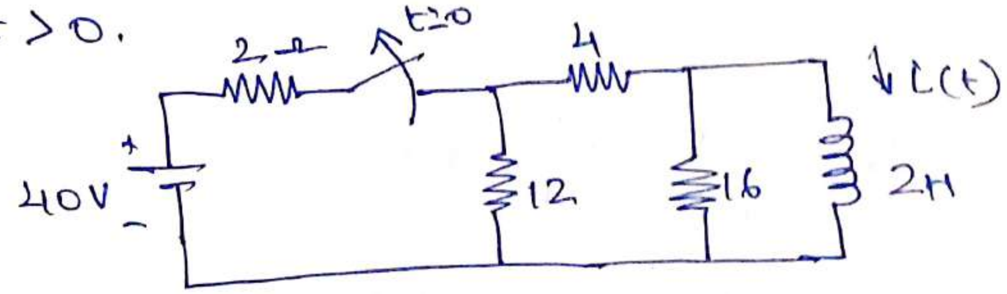
$$R_{eq} = R_{th} = \frac{V_0}{I_0} = \frac{1}{+0.113} = 8.8\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1/6}{8.8} = 0.0189\cancel{8}$$

$$i(t) = i(0)\, e^{-t/\tau}$$
$$i(t) = 5\, e^{-t/0.0189} = 5\, e^{-53t} \text{ A}$$

$$V_x = 3 \times i(t)$$
$$= 3 \times \left[-5 e^{-53t}\right]$$
$$V_x = -15 e^{-53t} \text{ V}$$

Ex 3: The switch in the circuit has been closed for a long time. At $t=0$ the switch is opened. Calculate $i(t)$ for $t>0$.

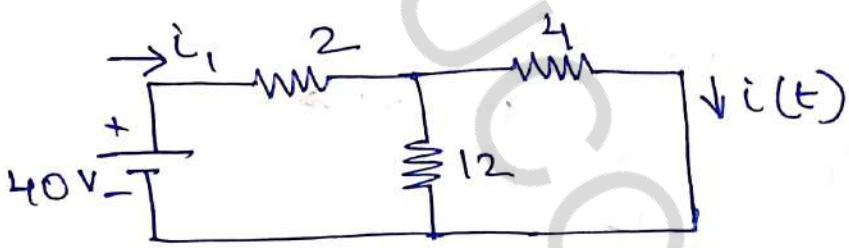


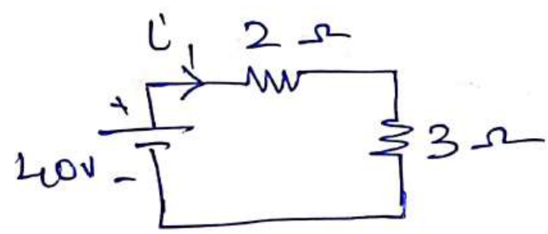
Solution:

at $t<0 \rightarrow$ Switch closed,

Inductor act as short circuit.

$\Rightarrow$ 16 Ω Resistor is short circuited.





$$i_1 = \frac{40}{5} = 8\text{ A}$$

and $i(t)$ obtain by current division rule

$$i(t) = 8 \times \frac{12}{12+4} = 6\text{A} \quad t<0$$

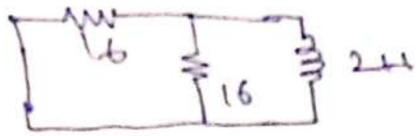
Since current through an inductor cannot change instantaneously.

$$i(0) = i(0^-) = 6\text{ A} \quad \text{at } t=0$$

When $t>0 \rightarrow$ Switch opened, voltage source dis connected.

Source free RL circuit

Req =>



$= \frac{16 \times 16}{16+16} = 8\,\Omega$

∴ time constant

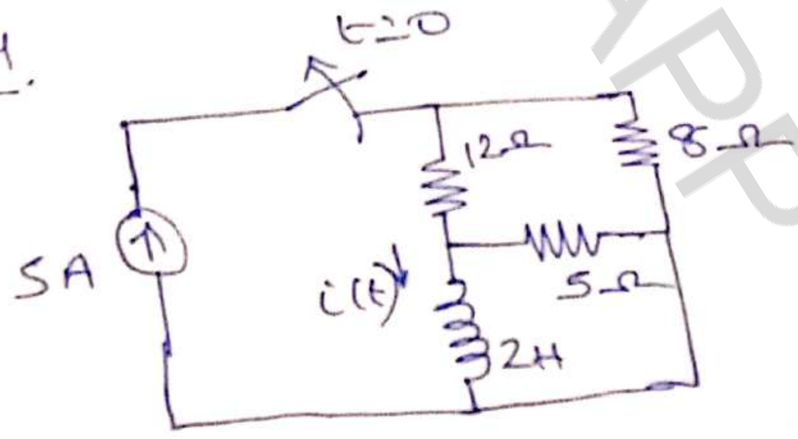
$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\ s$

Thus

$i(t) = i(0)\, e^{-t/\tau}$

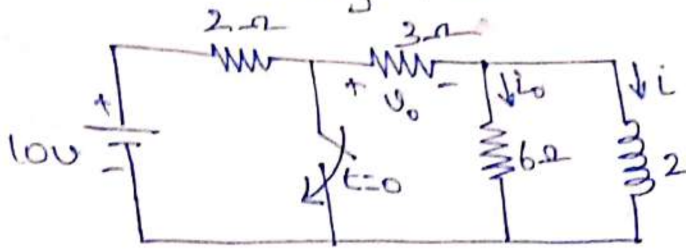
$i(t) = 6\, e^{-4t}\ A \qquad t > 0$

Ex: 4



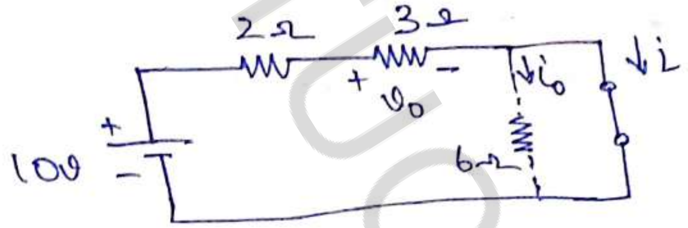
Ans: $i(t) = 2e^{-2t}$ A

Ex: 7.5 In the circuit shown, find i_o, v_o & i for all time, assuming that the switch was open for a long time.



Solution:

⇒ for $t<0$ → Switch is open, then DC source connected to Inductor, so it behaves like short circuit.



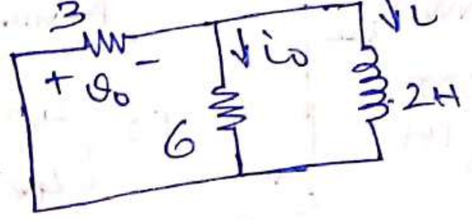
$$i_o = 0 \; ; \; t<0$$

$$i = \frac{10}{3+2} = 2A \; ; \; t<0$$

$$v_o = i \times 3 = 2 \times 3 = 6V \; ; \; t<0$$

then $\underline{\underline{i(0) = 2A}}$.

⇒ for $t>0$ → Switch is closed, so that voltage source is short circuited.



$$i(t) = i(0)\, e^{-t/\tau}$$

$$R_{eq} = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{2} = 1 \text{ Sec}$$

$i(t) = 2e^{-t/1}\ A$

$\therefore i(t) = 2e^{-t}\ A$ at $t \geq 0$

Need to find i_o i.e. Current through $6\,\Omega$ Resistor

by Current division Rule,

i_o or $i_6 = (-2e^{-t}) \times \frac{3}{9}$

[Note: $i(t)$ is reverse direction when current through in $6\,\Omega$]

$i_o(t) = -\frac{2}{3} e^{-t}\ A$

at $t > 0$

$v_o(t) = v_L = L\frac{di}{dt}$

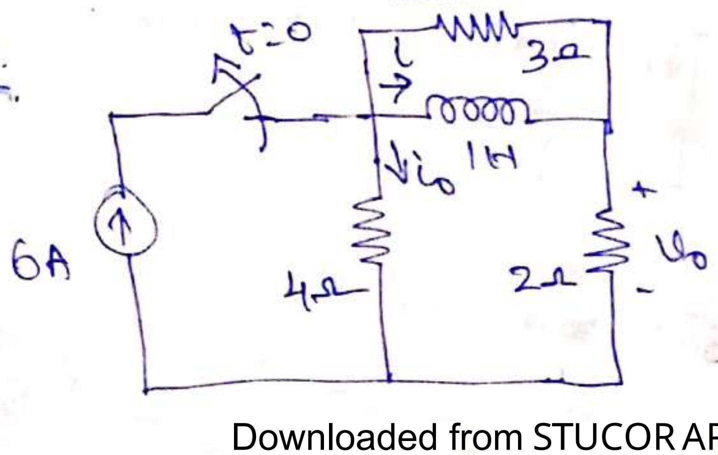
$= 2 \times \frac{d(-2e^{-t})}{dt}$

$= 2 \times [-2(-1)e^{-t}]$

$= 2 \times 2e^{-t}$

$v_o(t) = 4e^{-t}\ V$

Ex:



Answers

$i: \begin{cases} 4A & t < 0 \\ 4e^{-2t}\ A & t \geq 0 \end{cases}$

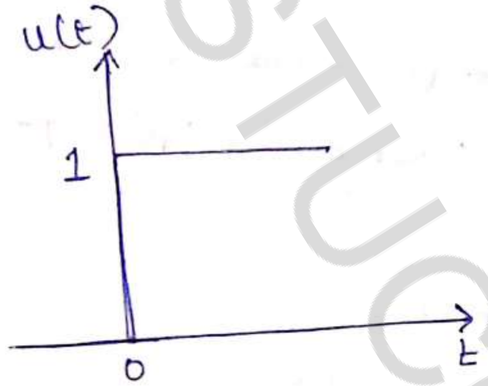
$v_o = \begin{cases} 4V & t < 0 \\ -\frac{8}{3}e^{-2t}\ V & t > 0 \end{cases}$

$i_o = \begin{cases} 2A & t < 0 \\ -\frac{4}{3}e^{-2t}\ A & t > 0 \end{cases}$

Unit Step function:

The Unit Step function $u(t)$ is '0' for Negative values of 't' and '1' for Positive Value of 't'.

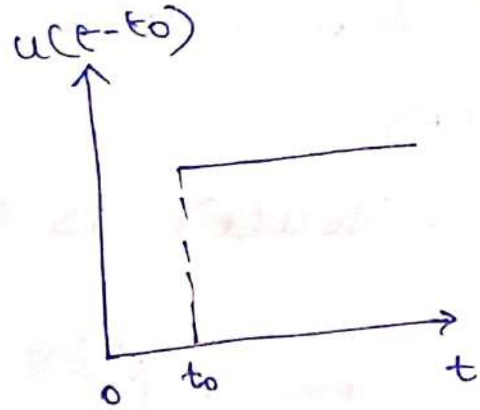
i.e. $$u(t) = \begin{cases} 0 & , t < 0 \\ 1 & , t > 0 \end{cases}$$



The Unit Step function is Undefined at $t=0$, Where it changes abruptly from 0 to 1.

→ If the abrupt changes occurs at $t=t_0$, then the Unit Step function becomes,

$$u(t-t_0) = \begin{cases} 0 & , t < t_0 \\ 1 & , t > t_0 \end{cases}$$

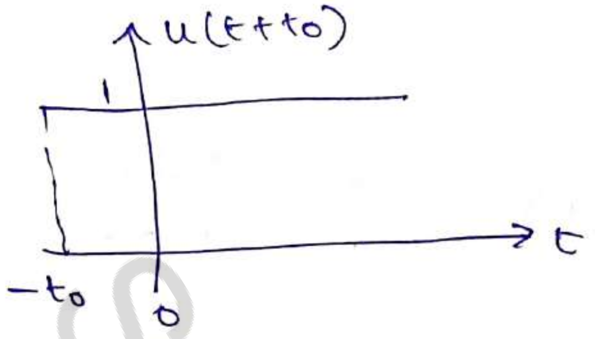


that $u(t)$ delayed by t_0 Seconds.

→ If the change is at $t=-t_0$, the Unit Step function becomes,

$$u(t+t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$

i.e $u(t)$ is advanced by t_0 seconds.



We use the Step function to represent an abrupt change in Voltage or Current.

examp, the voltage,

$$v(t) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

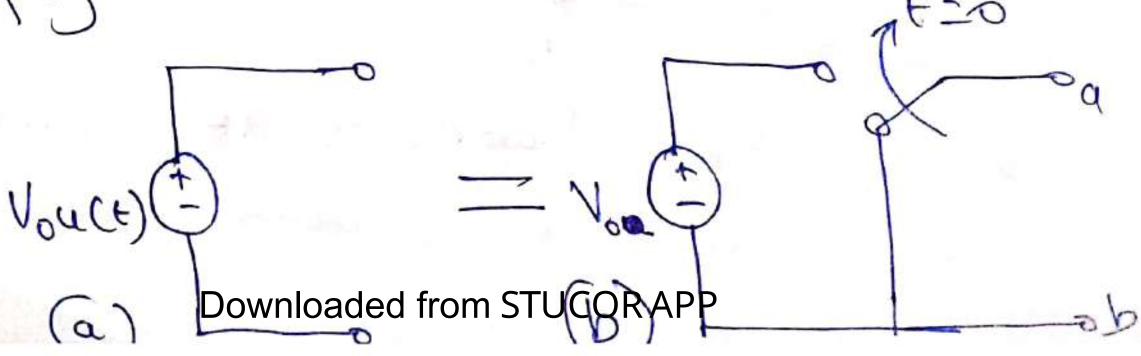
it expressed in terms of Unit Step function as,

$$v(t) = V_0 u(t-t_0)$$

If $t_0 = 0$, then, Step voltage,

$$v(t) = V_0 u(t).$$

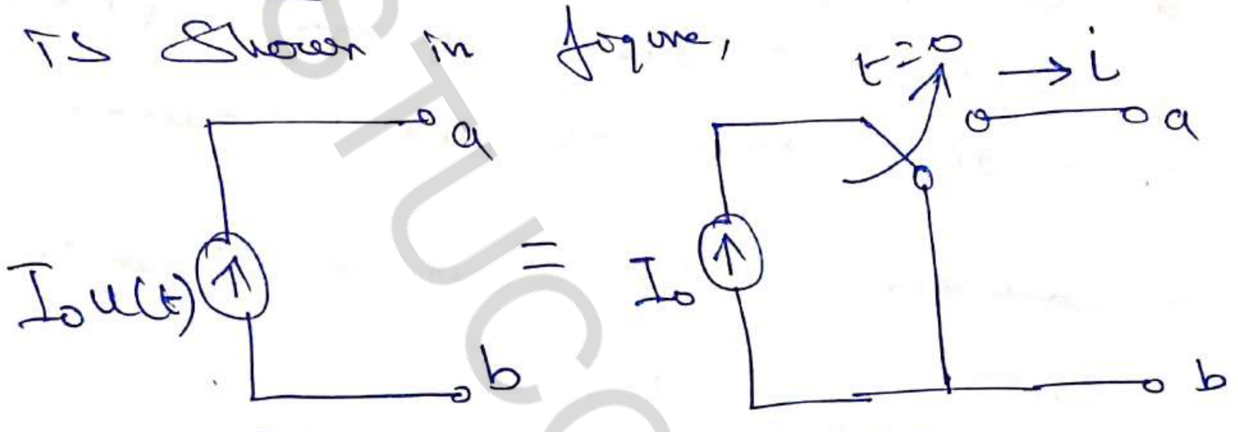
A voltage source $V_0 u(t)$ is shown in figure.



(a) (b)

from the figure (b) it is evident that terminals a-b are short circuited $v = 0$ for $t < 0$, and $v = V_0$ at terminals for $t > 0$.

Similarly, a current source of $I_0 u(t)$ is shown in figure,



(a) (b)

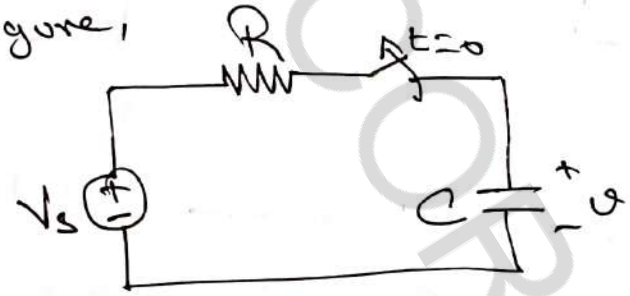
from figure (b), Noticed that terminal a & b open circuited $i = 0$ for $t < 0$, and $i = I_0$ flow for $t > 0$.

STEP RESPONSE OF AN RC CIRCUIT

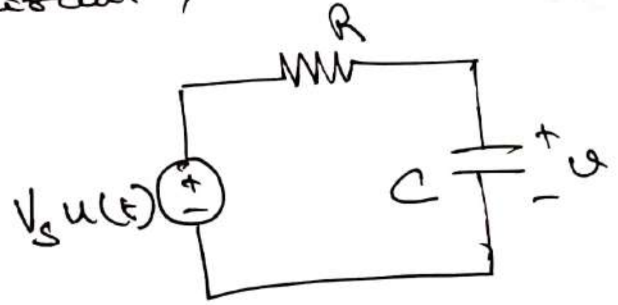
When the dc source of an RC circuit is suddenly applied, the voltage or current source can be modeled as a step function and the response is known as a step response.

→ The step response is the response of the circuit due to sudden application of a dc voltage or current source.

→ Consider the RC circuit shown in figure,



Which can be replaced by the circuit,



$V_s \rightarrow$ is a constant dc voltage source

Assume an initial voltage V_0 on the capacitor, the voltage on the capacitor cannot change instantaneously,

$$V(0^-) = V(0^+) = V_0$$

Apply KCL,

$$i_R + i_C = i_0$$

$$\frac{v}{R} + C\frac{dv}{dt} = \frac{V_s u(t)}{R}$$

$$\frac{v}{R} - \frac{V_s u(t)}{R} + C\frac{dv}{dt} = 0$$

$$\frac{v - V_s u(t)}{R} + C\frac{dv}{dt} = 0$$

divide by 'C'

$$\frac{v - V_s u(t)}{RC} + \frac{dv}{dt} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s u(t)}{RC}$$

where v is voltage across capacitor at $t > 0$. So $u(t) = 1$ at $t > 0$.

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}$$

Rearranging terms,

$$\frac{dv}{dt} = -\frac{(v - V_s)}{RC}$$

$$\frac{dv}{v - V_s} = -\frac{dt}{RC}$$

Integrating both sides

$$\int_{V_0}^{v(t)} \frac{dv}{v - V_s} = \int^{t} -\frac{dt}{RC}$$

$$\ln(v - V_s)\Big]_{V_0}^{v(t)} = -\frac{t}{RC}\Big]_0^t$$

$$\ln[v(t) - V_s] - \ln[V_0 - V_s] = -\frac{t}{RC} + 0$$

$$\ln\frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

Taking exponential of both sides,

$$\frac{v - V_s}{V_0 - V_s} = e^{-t/RC}$$

where $\tau = RC$

$$\frac{v - V_s}{V_0 - V_s} = e^{-t/\tau}$$

$$v - V_s = (V_0 - V_s)e^{-t/\tau}$$

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \quad t > 0.$$

Thus,

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}$$

This is known as Complete Response of the RC circuit to a sudden application of DC voltage source. Capacitor is initially charged.

If we assume capacitor is uncharged initially $V_0 = 0$, so the equation becomes,

$$v(t) = \begin{cases} 0 , & t < 0 \\ V_s (1 - e^{-t/\tau}) , & t > 0 \end{cases}$$

Can be written alternatively as,

$$\boxed{v(t) = V_s (1 - e^{-t/\tau}) u(t)}$$

This is the complete step response of the RC circuit when the capacitor is initially uncharged.

→ The current through the capacitor is obtained from above equation, by using

$i(t) = C \, dv/dt$

$$i(t) = C \frac{dv}{dt} = C \frac{d V_s (1 - e^{-t/\tau}) u(t)}{dt}$$

$$\tau = RC \quad t > 0, \quad u(t) = 1$$

$$i(t) = (C)(V_s)\left[-\left(-\frac{1}{\tau}\right) e^{-t/\tau}\right]$$

$$i(t) = \frac{C}{\tau} V_s e^{-t/\tau}$$

$$\boxed{i(t) = \frac{V_s}{R} e^{-t/\tau} u(t)}$$

$\rightarrow v(t)$

Complete Response = Natural response + Forced Response
(Stored energy) (independent Source)

Complete Response = Transient Response + Steady State Response
(temporary part) (Permanent part)

Natural Response = Transient Response

Forced Response = Steady State Response

A Complete Response may be written as,

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}}$$

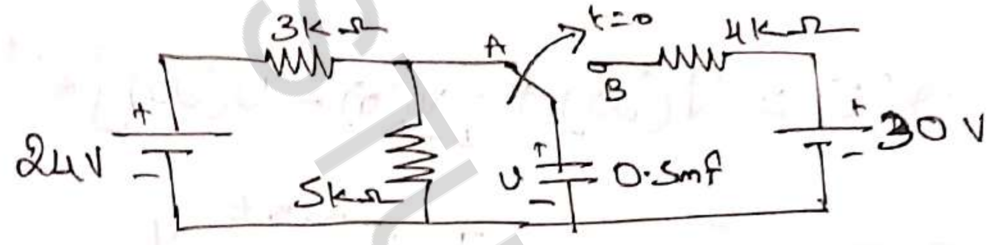
$v(0) \rightarrow$ initial voltage

$v(\infty) \rightarrow$ final or Steady State value.

To find Step Response of an RC Circuit Requires three things,

1. The initial Capacitor Voltage $v(0)$.. $t<0$
2. The final Capacitor Voltage $v(\infty)$ } $t>0$
3. The time constant τ.

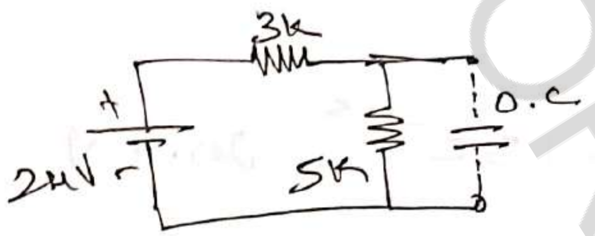
Ex:1. The Switch in figure has been in position A for a long time. At $t=0$, the Switch moves to B. Determine $v(t)$ for $t>0$ and Calculate its value at $t = 1s$ and $4s$.



Solution:

* for $t<0 \rightarrow$ Switch is at position 'A'.

Capacitor acts like Open circuit.



Voltage across 5k is Voltage across capacitor.

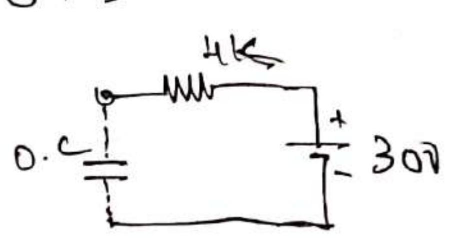
Using voltage division Rule

$$v(0^-) = 24 \times \frac{5}{3+5} = 15\,V$$

* the capacitor voltage cannot change instantaneously, so,

$$v(0) = v(0^-) = v(0^+) = 15\,V$$

* for $t>0 \rightarrow$ Switch is in position 'B'.



$R_{th} = 4k\,\Omega$

$\wedge \qquad \neg v(t)$

time Constant,

$$\tau = R_{th} C = 4\times10^3 \times 0.5\times10^{-3} = 2\,s$$

→ the capacitor acts Like Open Circuit, to dc at Steady State,

$$V(\infty) = 30V. \qquad t > 0$$

Thus,

$$v(t) = V(\infty) + [V(0) - v(\infty)]\, e^{-t/\tau}$$

$$= 30 + [15 - 30]\, e^{-0.5t} \; V$$

$$v(t) = 30 - 15e^{-0.5t} \; V$$

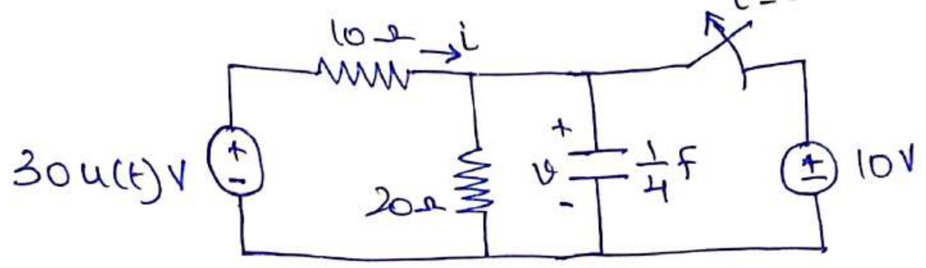
At $t = 1$

$$v(1) = 30 - 15e^{-0.5} = 20.9\,V$$

At $t = 4$,

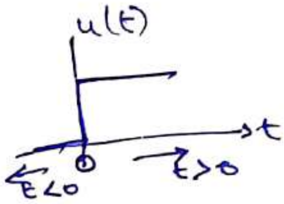
$$v(4) = 30 - 15e^{-2} = 27.97\,V$$

Ex: 2: In figure the Switch has been closed for a long time and is Opened at $t=0$. find i and v for all time.



Solution:

for $t<0 \rightarrow$ Switch closed

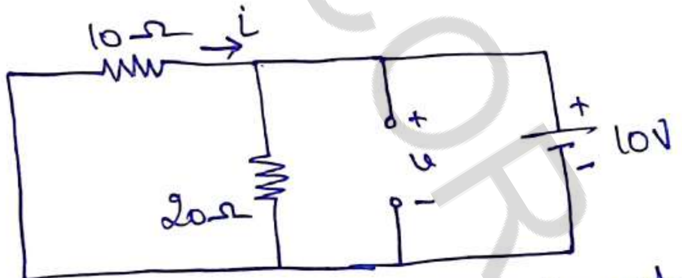


at $t<0$ the $30u(t) = 0$

So 10 V source connected with capacitor

It behaves Open Circuited.

$$30u(t) = \begin{cases} 0 & t<0 \\ 30 & t>0 \end{cases}$$



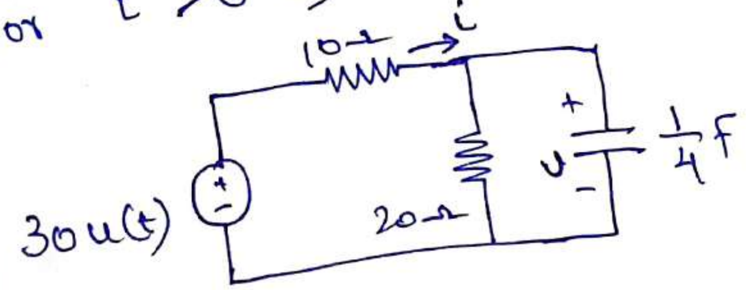
No current flow through 20Ω Resistor,

$$i = -\frac{v}{10} = -\frac{10}{10} = -1\,A \quad ; \quad v = 10\,V$$

Capacitor voltage cannot change instantaneously

So, $\quad v(0) = v(0^-) = 10V$

for $t>0 \rightarrow$ Switch Opened.



After a long time, the circuit reaches Steady State & Capacitor acts like an Open Circuit again.

Steady State voltage, $v(\infty)$ obtain using Voltage division,

$$v(\infty) = 30 \times \frac{20}{20+10} = 20\,V$$

find R_{th} by,

$$R_{th} = \frac{10 \times 20}{10+20} = \frac{200}{30} = \frac{20}{3}\,\Omega$$

time Constant,

$$\tau = R_{th} C = \frac{20}{3} \times \frac{1}{4} = \frac{20}{12} = \frac{5}{3}\,s$$

Thus,

$$v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$$

$$= 20 + [10 - 20]\, e^{-(3/5)t}$$

$$v(t) = (20 - 10\, e^{-0.6t})\,V$$

To Obtain i,

$i \rightarrow$ Sum of Current through $20\,\Omega$ and Capacitor.

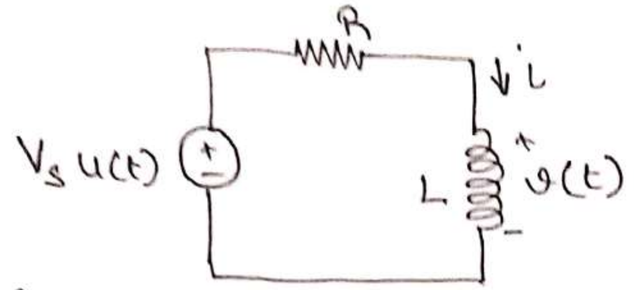
$$i = \frac{v}{20} + C\frac{dv}{dt}$$

$$= \frac{20 - 10\, e^{-0.6t}}{20} + \frac{1}{4} \times \frac{d\,[20 - 10\, e^{-0.6t}]}{dt}$$

$$= \qquad\qquad = (1 + e^{-0.6t})\,A$$

STEP RESPONSE OF AN RL CIRCUIT

Consider the RL circuit,



Our goal is to find the inductor current i as the circuit response.

Let the response be the sum of the Natural current and the forced current.

$$i = i_n + i_f \quad \text{—} \ (1)$$

i_n → Natural response

i_f → forced response

We know that Natural response is the decaying exponential.

$$i_n = A e^{-t/\tau} \quad \text{—} \ (2)$$

$A \rightarrow$ is a constant

forced response is the value of the current long time after switch is closed. At the time the inductor becomes short circuit and the voltage across it is zero. The entire source voltage V_s appears across R, thus,

$$i_f = \frac{V_s}{R} \quad \text{—} \ (3)$$

Sub equation (2) & (3) in (1)

$$i = A e^{-t/\tau} + \frac{V_s}{R} \quad \text{—} \ (4)$$

We now determine the constant A from the value of 'i'. Let I_0 be initial current, The current through the inductor cannot change instantaneously,

$$i(0^+) = i(0^-) = I_0 \quad (5)$$

Thus at $t=0$, then equation (4) becomes,

$$I_0 = A + \frac{V_s}{R}$$

then, $A = I_0 - \frac{V_s}{R}$

Substituting 'A' in equation (4)

$$i(t) = \frac{V_s}{R} + \left[I_0 - \frac{V_s}{R}\right] e^{-t/\tau} \quad (5)$$

This is the complete response of the RL circuit.

The response in equation (5) may be written as,

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau} \quad (6)$$

$i(0) \rightarrow$ initial value

$i(\infty) \rightarrow$ final value

To find the step response of an RL circuit requires three things,

1. The initial inductor current $i(0)$ at $t=0^+$
2. final inductor current $i(\infty)$.
3. The time constant τ.

$\Big\}\ t>0$

If switching takes place at time $t=t_0$ instead of $t=0$, equation becomes,

$$i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-(t-t_0)/\tau}$$

If $I_0 = 0$, then

$$i(t) = \begin{cases} 0, & t<0 \\ \frac{V_s}{R}(1-e^{-t/\tau}), & t>0 \end{cases}$$

$$\therefore\ i(t) = \frac{V_s}{R}(1-e^{-t/\tau})u(t)$$

This is the step response of the RL circuit with no initial inductor current.

The voltage across the inductor is obtained by,

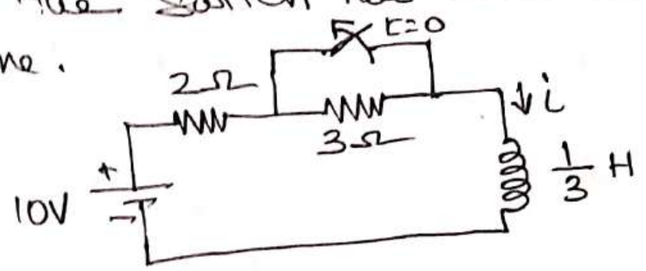
$$v(t) = L\frac{di}{dt} = V_s \frac{L}{\tau R} e^{-t/\tau}$$

where $\tau = \frac{L}{R}$

$$\therefore\ v(t) = V_s e^{-t/\tau} u(t)$$

Ex 1:

Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time.



Solution:

→ For $t < 0$, → the switch is closed position, So, the 3Ω resistor is short-circuited, & inductor acts like a short circuit.

The current through the inductor at $t = 0^-$

$$i(0^-) = \frac{10}{2} = 5\,A$$

Since the inductor current cannot change instantaneously,

$$i(0) = i(0^-) = i(0^+) = 5\,A.$$

→ for $t > 0$, the switch is open, 2Ω & 3Ω are in series,

$$i(\infty) = \frac{10}{3+2} = 2A \text{ at } t > 0.$$

Thevenin Resistance across inductor,

$$R_{th} = 2 + 3 = 5\,\Omega.$$

for time constant,

$$\tau = \frac{L}{R_{th}} = \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}\,s.$$

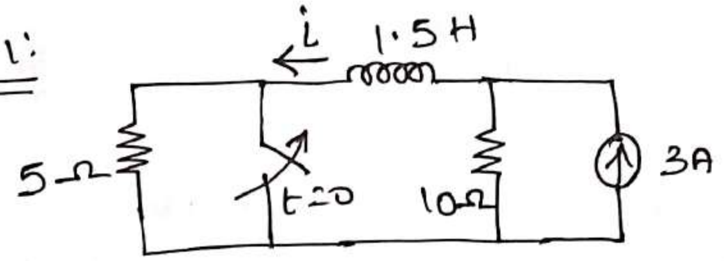
Thus,

$$i(t) = i(\infty) + [i(0) - i(\infty)]\,e^{-t/\tau}$$

$$= 2 + (5-2)e^{-15t}$$

$$i(t) = 2 + 3e^{-15t} \text{ A} \quad t > 0.$$

<u>PRC Prob 1:</u>



The Switch in fig has been closed for a long time. It opens at t=0 find $i(t)$ for $t>0$.

<u>Solution:</u> for $t<0$ - Switch closed, so inductor short circuited and 5Ω & 10Ω are short circuited (no current flow through 5Ω & 10Ω).

$$i = 3A \quad t<0.$$

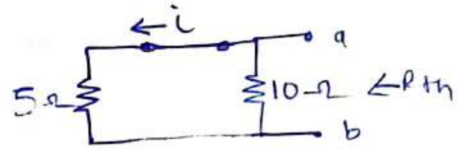
So initial current through inductor cannot change instantaneously,

$$i(0) = i(0^-) = 3A \quad t=0$$

for $t>0$ Switch opened, inductor like short circuit, the current i_1 by CDR

$$i(\infty) = 3 \times \frac{10}{10+5} = 2A \quad t>0$$

Thevenin Resistance, R_{th},



$$R_{th} = \cancel{\frac{5 \times 10}{15}} = \cancel{\frac{50}{15}} = \cancel{3.33\Omega}$$

$$= 5 + 10 = 15\Omega$$

time constant,

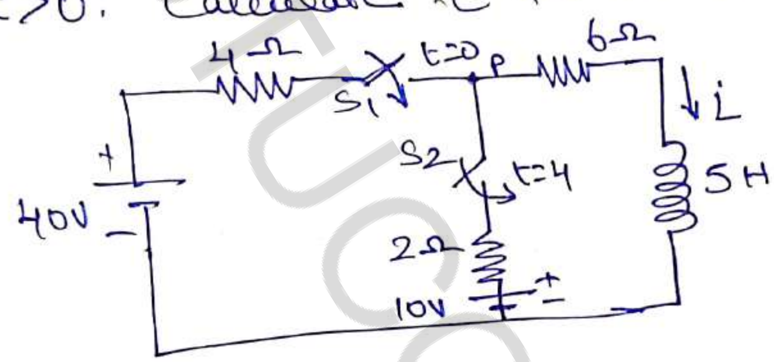
$$\tau = \frac{L}{R_{th}} = \frac{1.5}{\cancel{3.33}\ 15} = \cancel{0.45}\ 0.1 \text{ s}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau}$$

$$= 2 + [3 - 2]\, e^{-t/0.1}$$

$$i(t) = (2 + e^{-10t})\,A, \quad t > 0.$$

Ex2: At $t=0$, Switch 1 in fig is closed, and Switch 2 is closed 4 s later, find $i(t)$ for $t>0$. Calculate i for $t = 2s$ and $t = 5s$



Solution: Need to consider three time intervals, $t<0$, $0<t<4$, $t>4$ Separately.

→ for $t<0$ → Switches S_1 & S_2 are open, so $i = 0$.

$$i(0^-) = i(0) = 0$$

→ for $0 < t < 4 \Rightarrow S_1$ is closed so 4Ω & 6Ω are in Series.

$$i(\infty) = \frac{40}{4+6} = \underline{\underline{4\,A}}.$$

$$R_{th} = 4+6 = 10\,\Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{5}{10} = \frac{1}{2} = 0.5\,s$$

Then,

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$i(t) = 4 + (0-4)e^{-2t} = 4(1-e^{-2t})\,A$$

$$0 < t < 4$$

→ for $t > 4$, S_2 is closed, 10 Volt source is connected. This sudden change does not affect the inductor current because the current cannot change abruptly. The initial current is,

$$i(4) = i(4^-) = 4(1-e^{-8}) \simeq \underline{\underline{4\,A}}$$

To find $i(\infty)$, let v be the voltage at node P in fig, using KCL,

$$\frac{40-V}{4} + \frac{10-V}{2} = \frac{V}{6}$$

$$\frac{40}{4} - \frac{V}{4} + \frac{10}{2} - \frac{V}{2} = \frac{V}{6}$$

$$10 + 5 - \frac{V}{4} - \frac{V}{2} - \frac{V}{6} = 0$$

$$-V\left[\frac{1}{4}+\frac{1}{2}+\frac{1}{6}\right] = -15 \Rightarrow v = 16.3636$$

$$i = \frac{v}{6} = \frac{16.3636}{6} = 2.727\,A$$

$$\therefore \quad \underline{\underline{i(\infty) = 2.727\,A}}$$

Thevenin resistance at inductor terminal,

$$R_{th} = [4 \| 2] + 6 = \frac{4 \times 2}{6} + 6 = \frac{22}{3}\,\Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{5}{22/3} = \frac{15}{22} \text{ s}$$

$$\therefore \; i(t) = i(\infty) + [i(4) - i(\infty)] e^{-(t-4)/\tau} \qquad t \geq 4$$

[Note: $(t-4)$ is time delay]

$$i(t) = 2.727 + (4 - 2.727) e^{-(t-4)/\tau}$$

$$= 2.727 + 1.273 e^{-(t-4)/0.6818}$$

$$i(t) = 2.727 + 1.273 e^{-1.4667(t-4)}, \quad t \geq 4$$

$$\therefore \; i(t) = \begin{cases} 0, & t < 0 \\ 4(1 - e^{-2t}) & 0 < t < 4 \\ 2.727 + 1.273 e^{-1.466(t-4)}, & t > 4 \end{cases}$$

At: $t = 2$

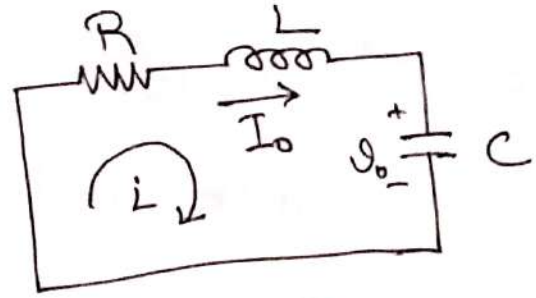
$$i(2) = 4(1 - e^{-4}) = 3.93 \text{ A}$$

$t = 4$

$$i(5) = 2.727 + 1.273 e^{-1.4667}$$

$$= \underline{\underline{3.02 \text{ A}}}$$

THE SOURCE-FREE SERIES RLC CIRCUIT



Consider Series RLC Circuit,

initial Capacitor voltage $\rightarrow V_0$
initial Inductor Current $\rightarrow I_o$ } at $t=0$

→ the initial value of the derivative of i,

$$\frac{di(0)}{dt} = -\frac{1}{L}[RI_o + V_0]$$

[This equation Used to find A_2 Constant].

→ Characteristic equation of i,

[roots of equation]

$$S_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$S_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

where $\alpha = \frac{R}{2L}$ & $\omega_0 = \frac{1}{\sqrt{LC}}$

$\therefore \boxed{S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}}$ & $\boxed{S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}}$

Thus, Natural response of the Series RLC circuit is,

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

A_1 find by initial value $i(0)$

A_2 find by $\frac{d\,i(0)}{dt}$

Based of S_1 & S_2 there are three types of solutions

(1.) If $\alpha > \omega_0 \rightarrow$ Overdamped Case

Response is,

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

(2) Critically Damped Response

$\alpha = \omega_0$

$$S_1 = S_2 = -\alpha = -\frac{R}{2L}$$

Response is,

$$i(t) = (A_2 + A_1 t) e^{-\alpha t}$$

(3) $\alpha < \omega_0$ Under damped Case

$$S_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\omega_d$$

$$S_2 = -\alpha - \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha - j\omega_d$$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \rightarrow$ damping frequency.

$\omega_0 \rightarrow$ Undamped frequency

Response is,

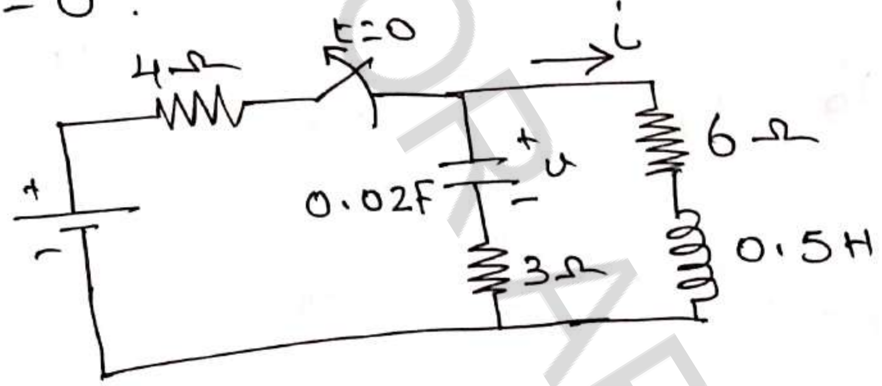
$$\boxed{i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)}$$

time Constant $\rightarrow 1/\alpha$

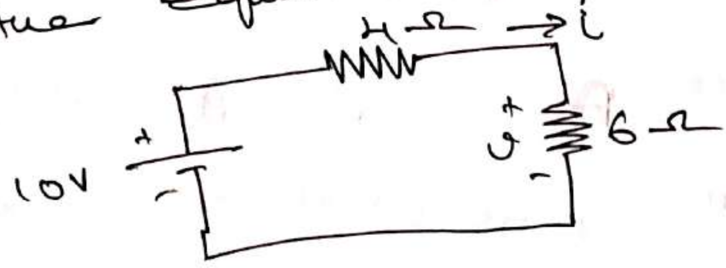
Period $\rightarrow T = 2\pi/\omega_d$

Example Problem 1:

find $i(t)$ in the Circuit. Assume that the circuit has reached Steady State at $t=0^-$.



Solution: for $t<0 \rightarrow$ Switch is closed. The Capacitor behaves like open circuit, & Inductor acts like Short Circuit, then the equivalent circuit is,



initial Current through Inductor $i(0) = \frac{10}{4+6} = 1A$ and

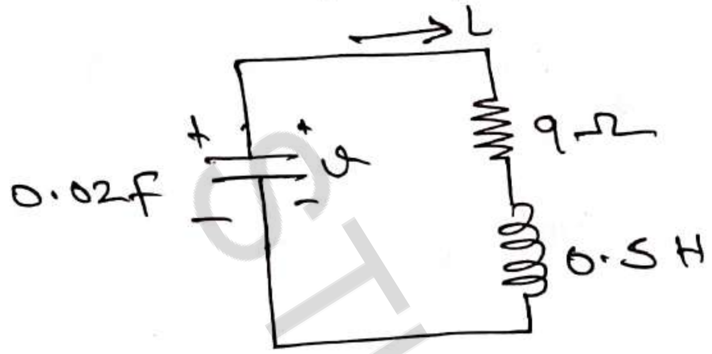
Voltage across ~~Capacitor~~ $v(0) = 6 \times 1 = 6V$ [Voltage across 6Ω]

$i(0) \rightarrow$ Current through inductor

$v(0) \rightarrow$ Voltage across Capacitor.

* for $t > 0$, $\rightarrow$ Switch Opened, equivalent circuit is,

6Ω & 3Ω are in Series.



Source free RLC Circuit

Roots are calculated as,

$$\alpha = \frac{R}{2L} = \frac{9}{2 \times 0.5} = 9$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 0.02}} = 10$$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -9 \pm \sqrt{9^2 - 10^2} = -9 \pm \sqrt{81 - 100}$$

$$S_{1,2} = -9 \pm j4.359 \quad \boxed{S_{1,2} = -\alpha \pm j\omega_d}$$

$\alpha < \omega_0 \rightarrow$ Underdamped

$$i(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

$$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)$$

A_1 & A_2 find by initial conditions.

At $t=0$,

$$i(0) = 1 = A_1$$

then A_2 find by,

$$① \rightarrow \left.\frac{di}{dt}\right|_{t=0} = -\frac{1}{L}\left[R\,i(0) + v(0)\right]$$

$$= -2\left[9-6\right] = -6\ A/s$$

$v(0) = V_0 = -6 \rightarrow$ because of
[opposite direction of capacitor voltage & 6 Ω resistor voltage.]

Now, derivate $i(t)$

$$② \rightarrow \frac{di}{dt} = -9e^{-9t}\left(A_1 \cos 4.359t + A_2 \sin 4.359t\right)$$

$$+ e^{-9t}(4.359)\left[-A_1 \sin 4.359t + A_2 \cos 4.359t\right]$$

Imposing the two equations, ① & ② at $t=0$

$$-6 = -9\left[A_1 + 0\right] + 4.359\left[0 + A_2\right]$$

$$\boxed{A_1 = 1}$$

$$-6 = -9\left[1\right] + 4.359\left[A_2\right]$$

$$A_2 = \frac{-6+9}{4.359} = \frac{3}{4.359} = 0.6882$$

Sub values A_1 & A_2 in $i(t)$

Complete response,

$$i(t) = e^{-9t}\left(\cos 4.359t + 0.6882 \sin 4.359t\right) \text{ A}$$

Source free Parallel RLC ckt.

$(\alpha > \omega_0)$ Over damped

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$(\alpha = \omega_0)$. Critically Damped case

$$v(t) = (A_1 + A_2 t) e^{-\alpha t}$$

$(\alpha < \omega_0)$ under damped case.

$$s_{1,2} = -\alpha \pm j\omega_d$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$v(t) = e^{-\alpha t} (A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

$$\frac{V_0}{R} + I_0 + C \frac{dv(0)}{dt} = 0$$

$$\frac{dv(0)}{dt} = \frac{-(V_0 + R I_0)}{RC}$$

In the parallel ckt find $v(t)$ for $t>0$, assuming $v(0) = 5V$, $i(0) = 0$, $L = 1H$ & $C = 10mF$

Consider these cases: $R = 1.923\Omega$, $R = 5\Omega$, $R = 6.25\Omega$

Case 1

if $R = 1.923\Omega$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 1.923 \times 10 \times 10^{-3}} = \underline{\underline{26}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10 \times 10^{-3}}} = \underline{\underline{10}}$$

Since $\alpha > \omega_0$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2, -50$$

& the corresponding response is

$$v(t) = A_1 e^{-2t} + A_2 e^{-50t}$$

Initial conditions to get A_1 & A_2

$$v(0) = 5 = A_1 + A_2$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = \frac{-5+0}{1.923 \times 10 \times 10^{-3}} = \underline{\underline{-260}}$$

By differentiating

$$\frac{dv}{dt} = -2A_1 e^{-2t} - 50A_2 e^{-50t}$$

At $t=0$

$$-260 = -2A_1 - 50A_2$$

$A_1 = -0.2083$

$A_2 = 5.208$

Subs A_1 & A_2.

$$\boxed{v(t) = -0.2083 e^{-2t} + 5.208 e^{-50t}}$$

Case 2 $R = 5\Omega$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 5 \times 10 \times 10^{-3}} = 10$$

While $\omega_0 = 10$ remains the same.

$\alpha = \omega_0 = 10$, the response is critically damped,

$S_1 = S_2 = -10.$

$$v(t) = (A_1 + A_2 t) e^{-10t}$$

To get A_1 & A_2

$$v(0) = 5 = A_1$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + R\,i(0)}{RC} = -\frac{5+0}{5 \times 10 \times 10^{-3}} = -100$$

By diff.

$$\frac{dv}{dt} = (-10A_1 - 10A_2 t + A_2) e^{-10t}$$

At $t=0$

$$-100 = -10A_1 + A_2$$

$A_1 = 5$

$A_2 = -50$

$$v(t) = (5 - 50t) e^{-10t} \, V$$

Case 3

When $R = 6.25\,\Omega$

$$\alpha = 1/2RC = \frac{1}{2 \times 6.25 \times 10 \times 10^{-3}} = \underline{\underline{8}}$$

While $\omega_0 = 10$ remains the same, As $\alpha < \omega_0$ in this case the response is underdamped. The roots of the characteristic eqn are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -8 \pm j6$$

hence $v(t) = (A_1 \cos 6t + A_2 \sin 6t)\, e^{-8t}$

we obtain A_1 & A_2

$$V(0) = 5 = A_1$$

$$\frac{dV(0)}{dt} = \frac{-V(0) + R\,i(0)}{RC} = \frac{-5+0}{6.25 \times 10 \times 10^{-3}} = \underline{\underline{-80}}$$

But differentiation

$$\frac{dv}{dt} = (-8A_1 \cos 6t - 8A_2 \sin 6t - 6A_1 \sin 6t + 6A_2 \cos 6t)e^{-8t}$$

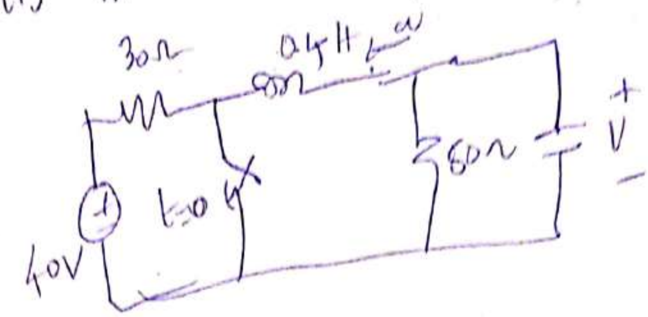
At $t = 0$,

$$-80 = -8A_1 + 6A_2$$

$A_1 = 5$, $A_2 = -6.667$ Thus

$$v(t) = (5 \cos 6t - 6.667 \sin 6t)\, e^{-8t}$$

Find V(t), for t > 0 in the RLC ckt.



Sln t < 0

$$V(0) = \frac{50 \times 40}{50+30} = \underline{\underline{25V}}$$

initial current through the inductor

$$i(0) = -\frac{40}{30+50} = \underline{\underline{-0.5A}}$$

$$\frac{dV(0)}{dt} = -\frac{V(0) + R\,i(0)}{RC} = \frac{25 - 50 \times 0.5}{50 \times 20 \times 10^{-6}} = \underline{\underline{0}}$$

t > 0 Switch is closed, 30 Ω & V/ge. ckt seperm.

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 50 \times 20 \times 10^{-6}} = \underline{\underline{500}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4 \times 20 \times 10^{-6}}} = \underline{\underline{354}}$$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -500 \pm \sqrt{250000 - 124{,}997.6} = -500 \pm 354$$

$$S_1 = \underline{\underline{-854}} \qquad S_2 = \underline{\underline{-146}}$$

Since $\alpha > \omega_0$. we have overdamped response

$$V(t) = A_1 e^{-854t} + A_2 e^{-146t}$$

At $t=0$

$$V(0) = 25 = A_1 + A_2$$

$$A_2 = 25 - A_1$$

Taking the derivative of $V(t)$

$$\frac{dV}{dt} = -854 A_1 e^{-854t} - 146 A_2 e^{-146t}$$

$$\frac{dV(0)}{dt} = 0 . \quad -854 A_1 - 146 A_2$$

$$0 = 854 A_1 + 146 A_2$$

$$A_1 = \underline{\underline{-5.156}}$$

$$A_2 = \underline{\underline{30.16}}$$

$$V(t) = -5.156 e^{-854t} + 30.16 e^{-146t} \text{ V}$$

Example 13.3

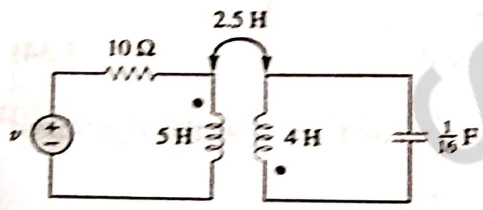


Figure 13.16
For Example 13.3.

Consider the circuit in Fig. 13.16. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time $t = 1$ s if $v = 60\cos(4t + 30^\circ)$ V.

Solution:

The coupling coefficient is

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

indicating that the inductors are tightly coupled. To find the energy stored, we need to calculate the current. To find the current, we need to obtain the frequency-domain equivalent of the circuit.

$$60\cos(4t + 30^\circ) \quad \Rightarrow \quad 60\underline{/30^\circ}, \quad \omega = 4 \text{ rad/s}$$

$$5 \text{ H} \quad \Rightarrow \quad j\omega L_1 = j20\ \Omega$$

$$2.5 \text{ H} \quad \Rightarrow \quad j\omega M = j10\ \Omega$$

$$4 \text{ H} \quad \Rightarrow \quad j\omega L_2 = j16\ \Omega$$

$$\frac{1}{16} \text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j4\ \Omega$$

The frequency-domain equivalent is shown in Fig. 13.17. We now apply mesh analysis. For mesh 1,

$$(10 + j20)\mathbf{I}_1 + j10\mathbf{I}_2 = 60\underline{/30^\circ} \tag{13.3.1}$$

For mesh 2,

$$j10\mathbf{I}_1 + (j16 - j4)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 \tag{13.3.2}$$

Substituting this into Eq. (13.3.1) yields

$$\mathbf{I}_2(-12 - j14) = 60\underline{/30^\circ} \quad \Rightarrow \quad \mathbf{I}_2 = 3.254\underline{/160.6^\circ} \text{ A}$$

and

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 = 3.905\underline{/-19.4^\circ} \text{ A}$$

In the time-domain,

$$i_1 = 3.905 \cos(4t - 19.4^\circ), \qquad i_2 = 3.254 \cos(4t + 160.6^\circ)$$

At time $t = 1$ s, $4t = 4$ rad $= 229.2^\circ$, and

$$i_1 = 3.905 \cos(229.2^\circ - 19.4^\circ) = -3.389 \text{ A}$$
$$i_2 = 3.254 \cos(229.2^\circ + 160.6^\circ) = 2.824 \text{ A}$$

The total energy stored in the coupled inductors is

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$
$$= \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$

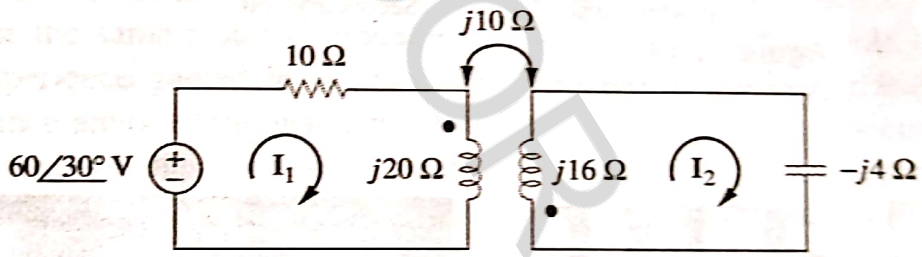


Figure 13.17
Frequency-domain equivalent of the circuit in Fig. 13.16.

Practice Problem 13.3

For the circuit in Fig. 13.18, determine the coupling coefficient and the energy stored in the coupled inductors at $t = 1.5$ s.

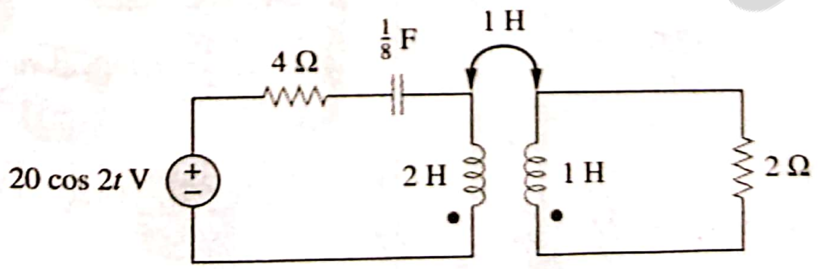


Figure 13.18
For Practice Prob. 13.3.

Answer: 0.7071, 9.85 J.

Example 13.7

An ideal transformer is rated at 2400/120 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Solution:

(a) This is a step-down transformer, since $V_1 = 2{,}400\text{ V} > V_2 = 120\text{ V}$.

$$n = \frac{V_2}{V_1} = \frac{120}{2{,}400} = 0.05$$

(b)

$$n = \frac{N_2}{N_1} \quad \Rightarrow \quad 0.05 = \frac{50}{N_1}$$

or

$$N_1 = \frac{50}{0.05} = 1{,}000 \text{ turns}$$

(c) $S = V_1 I_1 = V_2 I_2 = 9.6$ kVA. Hence,

$$I_1 = \frac{9{,}600}{V_1} = \frac{9{,}600}{2{,}400} = 4\text{ A}$$

$$I_2 = \frac{9{,}600}{V_2} = \frac{9{,}600}{120} = 80\text{ A} \quad \text{or} \quad I_2 = \frac{I_1}{n} = \frac{4}{0.05} = 80\text{ A}$$

Practice Problem 13.7

The primary current to an ideal transformer rated at 3300/110 V is 3 A. Calculate: (a) the turns ratio, (b) the kVA rating, (c) the secondary current.

Answer: (a) 1/30, (b) 9.9 kVA, (c) 90 A.

Example 13.8

For the ideal transformer circuit of Fig. 13.37, find: (a) the source current $\mathbf{I}_1$, (b) the output voltage $\mathbf{V}_o$, and (c) the complex power supplied by the source.

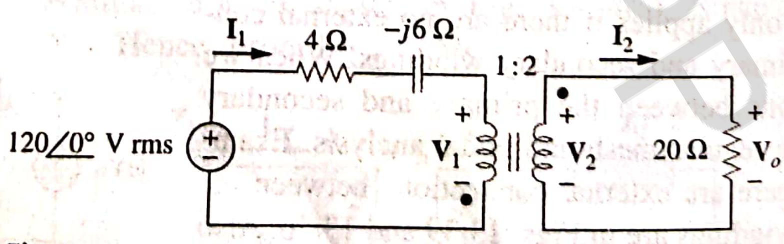


Figure 13.37
For Example 13.8.

Solution:

(a) The 20-Ω impedance can be reflected to the primary side and we get

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\ \Omega$$

Thus,

$$\mathbf{Z}_{in} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\underline{/-33.69^\circ}\ \Omega$$

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{\mathbf{Z}_{in}} = \frac{120\underline{/0^\circ}}{10.82\underline{/-33.69^\circ}} = 11.09\underline{/33.69^\circ}\ \text{A}$$

(b) Since both $\mathbf{I}_1$ and $\mathbf{I}_2$ leave the dotted terminals,

$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\underline{/33.69^\circ}\ \text{A}$$

$$\mathbf{V}_o = 20\mathbf{I}_2 = 110.9\underline{/213.69^\circ}\ \text{V}$$

(c) The complex power supplied is

$$\mathbf{S} = \mathbf{V}_s\mathbf{I}_1^* = (120\underline{/0^\circ})(11.09\underline{/-33.69^\circ}) = 1{,}330.8\underline{/-33.69^\circ}\ \text{VA}$$

Practice Problem 13.8

In the ideal transformer circuit of Fig. 13.38, find $\mathbf{V}_o$ and the complex power supplied by the source.

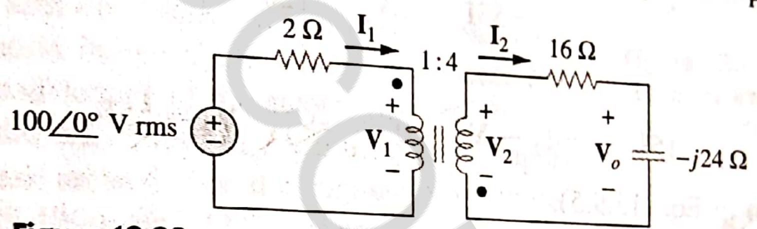


Figure 13.38
For Practice Prob. 13.8.

Answer: $178.9\underline{/116.56^\circ}$ V, $2{,}981.5\underline{/-26.56^\circ}$ VA.

Example 13.9

Calculate the power supplied to the 10-Ω resistor in the ideal transformer circuit of Fig. 13.39.

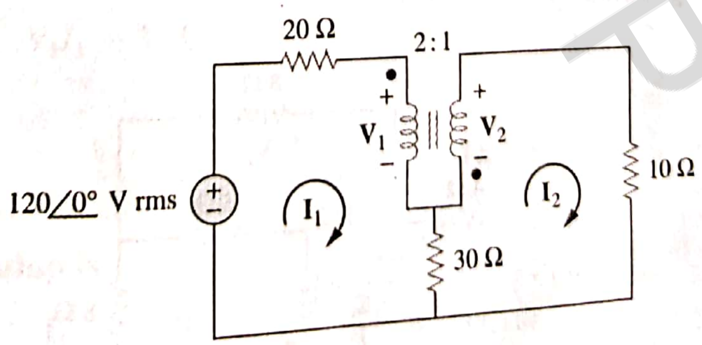


Figure 13.39
For Example 13.9.

Solution:
Reflection to the secondary or primary side cannot be done with this circuit: there is direct connection between the primary and secondary sides due to the 30-Ω resistor. We apply mesh analysis. For mesh 1,

$$-120 + (20 + 30)\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 0$$

or

$$50\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 120 \quad (13.9.1)$$

For mesh 2,

$$-\mathbf{V}_2 + (10 + 30)\mathbf{I}_2 - 30\mathbf{I}_1 = 0$$

or

$$-30\mathbf{I}_1 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \quad (13.9.2)$$

At the transformer terminals,

$$\mathbf{V}_2 = -\frac{1}{2}\mathbf{V}_1 \quad (13.9.3)$$

$$\mathbf{I}_2 = -2\mathbf{I}_1 \quad (13.9.4)$$

(Note that $n = 1/2$.) We now have four equations and four unknowns, but our goal is to get $\mathbf{I}_2$. So we substitute for $\mathbf{V}_1$ and $\mathbf{I}_1$ in terms of $\mathbf{V}_2$ and $\mathbf{I}_2$ in Eqs. (13.9.1) and (13.9.2). Equation (13.9.1) becomes

$$-55\mathbf{I}_2 - 2\mathbf{V}_2 = 120 \quad (13.9.5)$$

and Eq. (13.9.2) becomes

$$15\mathbf{I}_2 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \quad \Rightarrow \quad \mathbf{V}_2 = 55\mathbf{I}_2 \quad (13.9.6)$$

Substituting Eq. (13.9.6) in Eq. (13.9.5),

$$-165\mathbf{I}_2 = 120 \quad \Rightarrow \quad \mathbf{I}_2 = -\frac{120}{165} = -0.7272 \text{ A}$$

The power absorbed by the 10-Ω resistor is

$$P = (-0.7272)^2(10) = 5.3 \text{ W}$$

Practice Problem 13.9

Find $\mathbf{V}_o$ in the circuit of Fig. 13.40.

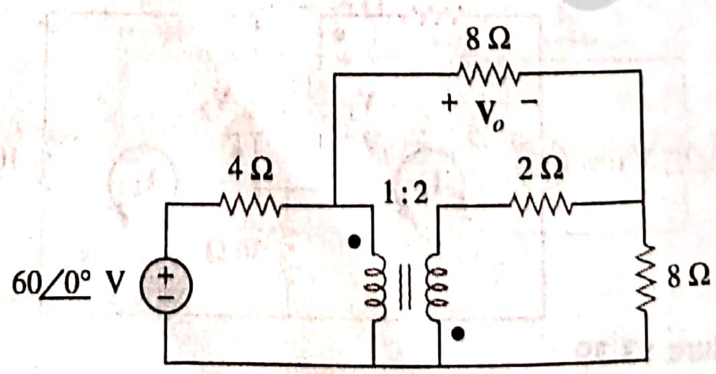


Figure 13.40
For Practice Prob. 13.9.

Answer: 24 V.

Example 13.4

In the circuit of Fig. 13.24, calculate the input impedance and current $\mathbf{I}_1$. Take $\mathbf{Z}_1 = 60 - j100\ \Omega$, $\mathbf{Z}_2 = 30 + j40\ \Omega$, and $\mathbf{Z}_L = 80 + j60\ \Omega$.

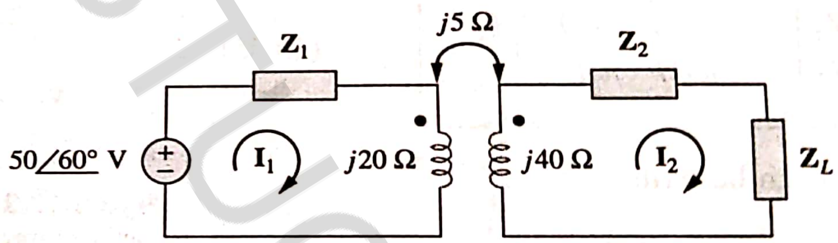


Figure 13.24
For Example 13.4.

Solution:
From Eq. (13.41),

$$\begin{aligned}\mathbf{Z}_{\text{in}} &= \mathbf{Z}_1 + j20 + \frac{(5)^2}{j40 + \mathbf{Z}_2 + \mathbf{Z}_L}\\ &= 60 - j100 + j20 + \frac{25}{110 + j140}\\ &= 60 - j80 + 0.14\underline{/-51.84^\circ}\\ &= 60.09 - j80.11 = 100.14\underline{/-53.1^\circ}\ \Omega\end{aligned}$$

Thus,

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_{in}} = \frac{50\underline{/60^\circ}}{100.14\underline{/-53.1^\circ}} = 0.5\underline{/113.1^\circ}\text{ A}$$

Practice Problem 13.4

Find the input impedance of the circuit in Fig. 13.25 and the current from the voltage source.

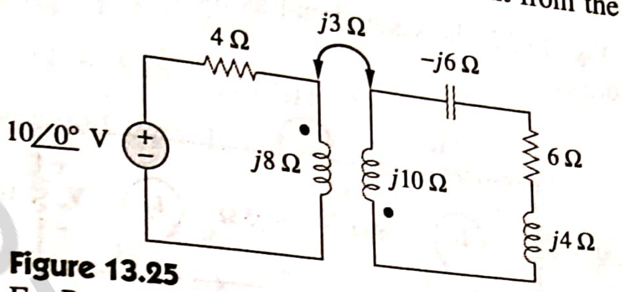


Figure 13.25
For Practice Prob. 13.4.

Answer: $8.58\underline{/58.05^\circ}\ \Omega$, $1.165\underline{/-58.05^\circ}$ A.

Example 13.5

Determine the T-equivalent circuit of the linear transformer in Fig. 13.26(a).

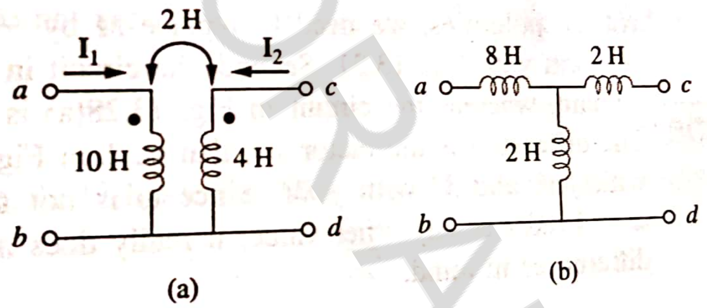


Figure 13.26
For Example 13.5: (a) a linear transformer, (b) its T-equivalent circuit.

Solution:
Given that $L_1 = 10$, $L_2 = 4$, and $M = 2$, the T-equivalent network has the following parameters:

$$L_a = L_1 - M = 10 - 2 = 8\text{ H}$$

$$L_b = L_2 - M = 4 - 2 = 2\text{ H}, \qquad L_c = M = 2\text{ H}$$

The T-equivalent circuit is shown in Fig. 13.26(b). We have assumed that reference directions for currents and voltage polarities in the primary and secondary windings conform to those in Fig. 13.21. Otherwise, we may need to replace M with $-M$, as Example 13.6 illustrates.

Practice Problem 13.5

For the linear transformer in Fig. 13.26(a), find the Π equivalent network.

Answer: $L_A = 18$ H, $L_B = 4.5$ H, $L_C = 18$ H.

Example 13.6

Solve for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_o$ in Fig. 13.27 (the same circuit as for Practice Prob. 13.1) using the T-equivalent circuit for the linear transformer.

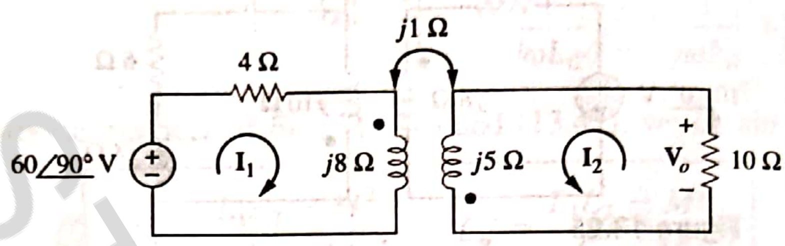


Figure 13.27
For Example 13.6.

Solution:
Notice that the circuit in Fig. 13.27 is the same as that in Fig. 13.10 except that the reference direction for current $\mathbf{I}_2$ has been reversed, just to make the reference directions for the currents for the magnetically coupled coils conform with those in Fig. 13.21.

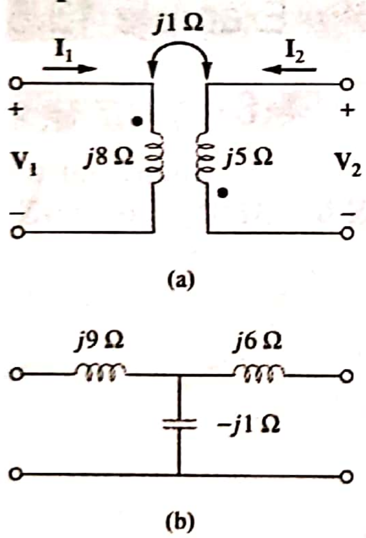


Figure 13.28
For Example 13.6: (a) circuit for coupled coils of Fig. 13.27, (b) T-equivalent circuit.

We need to replace the magnetically coupled coils with the T-equivalent circuit. The relevant portion of the circuit in Fig. 13.27 is shown in Fig. 13.28(a). Comparing Fig. 13.28(a) with Fig. 13.21 shows that there are two differences. First, due to the current reference directions and voltage polarities, we need to replace M by $-M$ to make Fig. 13.28(a) conform with Fig. 13.21. Second, the circuit in Fig. 13.21 is in the time-domain, whereas the circuit in Fig. 13.28(a) is in the frequency-domain. The difference is the factor $j\omega$; that is, L in Fig. 13.21 has been replaced with $j\omega L$ and M with $j\omega M$. Since ω is not specified, we can assume $\omega = 1$ rad/s or any other value; it really does not matter. With these two differences in mind,

$$L_a = L_1 - (-M) = 8 + 1 = 9 \text{ H}$$

$$L_b = L_2 - (-M) = 5 + 1 = 6 \text{ H}, \qquad L_c = -M = -1 \text{ H}$$

Thus, the T-equivalent circuit for the coupled coils is as shown in Fig. 13.28(b).

Inserting the T-equivalent circuit in Fig. 13.28(b) to replace the two coils in Fig. 13.27 gives the equivalent circuit in Fig. 13.29, which can be solved using nodal or mesh analysis. Applying mesh analysis, we obtain

$$j6 = \mathbf{I}_1(4 + j9 - j1) + \mathbf{I}_2(-j1) \tag{13.6.1}$$

and

$$0 = \mathbf{I}_1(-j1) + \mathbf{I}_2(10 + j6 - j1) \tag{13.6.2}$$

From Eq. (13.6.2),

$$\mathbf{I}_1 = \frac{(10 + j5)}{j}\mathbf{I}_2 = (5 - j10)\mathbf{I}_2 \tag{13.6.3}$$

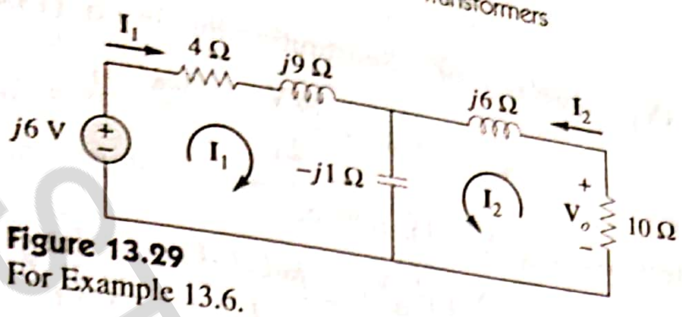


Figure 13.29
For Example 13.6.

Substituting Eq. (13.6.3) into Eq. (13.6.1) gives

$$j6 = (4 + j8)(5 - j10)\mathbf{I}_2 - j\mathbf{I}_2 = (100 - j)\mathbf{I}_2 \simeq 100\mathbf{I}_2$$

Since 100 is very large compared with 1, the imaginary part of $(100 - j)$ can be ignored so that $100 - j \simeq 100$. Hence,

$$\mathbf{I}_2 = \frac{j6}{100} = j0.06 = 0.06\underline{/90^\circ} \text{ A}$$

From Eq. (13.6.3),

$$\mathbf{I}_1 = (5 - j10)j0.06 = 0.6 + j0.3 \text{ A}$$

and

$$\mathbf{V}_o = -10\mathbf{I}_2 = -j0.6 = 0.6\underline{/-90^\circ} \text{ V}$$

This agrees with the answer to Practice Prob. 13.1. Of course, the direction of $\mathbf{I}_2$ in Fig. 13.10 is opposite to that in Fig. 13.27. This will not affect $\mathbf{V}_o$, but the value of $\mathbf{I}_2$ in this example is the negative of that of $\mathbf{I}_2$ in Practice Prob. 13.1. The advantage of using the T-equivalent model for the magnetically coupled coils is that in Fig. 13.29 we do not need to bother with the dot on the coupled coils.

Practice Problem 13.6

Solve the problem in Example 13.1 (see Fig. 13.9) using the T-equivalent model for the magnetically coupled coils.

Answer: $13\underline{/-49.4^\circ}$ A, $2.91\underline{/14.04^\circ}$ A.