I 2-MARKS :- UNIT-1 - CLASSIFICATION OF SIGNALS & SYSTEM.

1. State two properties of unit impulse function [N/D-14]

i, Shifting property : $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$.

Here $\delta(t-t_0)$ is time shifted data function.

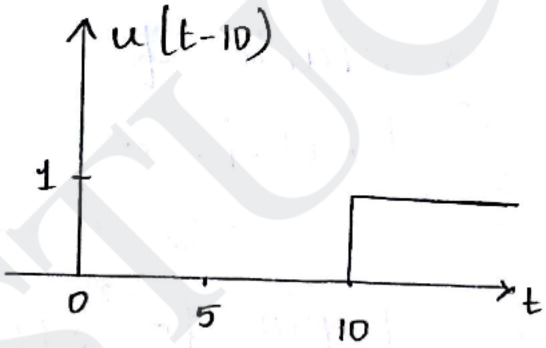
ii, Replication property : $\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(t)$.

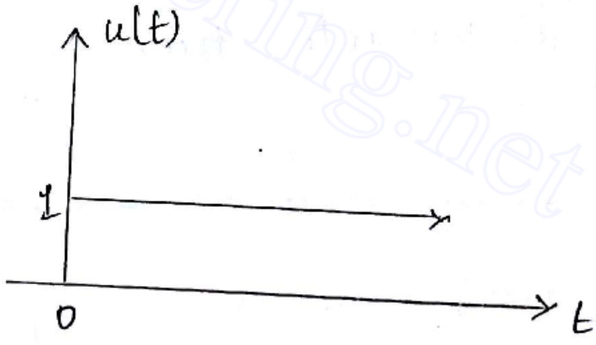
(or) $x(t)\,\delta(t) = x(t)$.

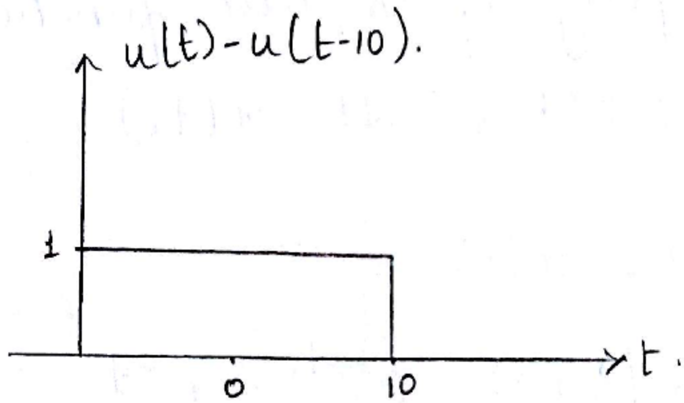
Thus convolution of any function with delta function leaves that function unchanged.

2. Draw the following signals. [N/D-14]

(a) $u(t) - u(t-10)$







3. Define a power signal [A/M -15]

Since wave in infinite length is called power signal. The power signal is not limited in time. If the signal the power signal is not limited in time. If always exists from begining to end and it never ends.

4. How the pulse response of discrete time system is useful in determining its stability and causality? [A/M - 15]

i) Discrete time system is causal if any only if $h(n)=0$, for $n<0$.

ii) Discrete time system is stable if

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty.$$

5. Find the value of the integral $\int_{-\infty}^{\infty} e^{-2t}\delta(t+2)dt$ [N/D - 15]

shifting property of impulse function states that $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$.

Here $t_0 = 2$, $x(t) = e^{-2t}$.

$$\int_{-\infty}^{\infty} e^{-2t}\,\delta(t+2) = e^{-2t_0} = e^{-4}.$$

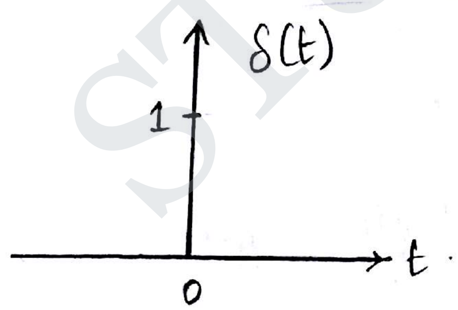
$$\rightarrow \boxed{\int_{-\infty}^{\infty} e^{-2t}\,\delta(t+2)\,dt = e^{4}}$$

6. Give the relation between continuous time unit impulse function $\delta(t)$, step function $u(t)$ and ramp function $r(t)$ [N/D-15]

$$\frac{d}{dt} r(t) = u(t), \quad r(t) = \int u(t)\,dt.$$

$$\frac{d}{dt} u(t) = \delta(t), \quad u(t) = \int \delta(t)\,dt.$$

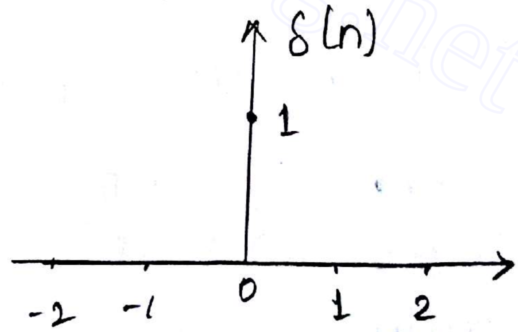
7. Give the Mathematical and graphical representation of a continuous time & discrete time unit impulse function. [N/D-16]



$\delta(t) = 1, \ t = 0.$

$\delta(t) = 0, \ t \neq 0.$

$$\int_{-\infty}^{\infty} \delta(\tau)\,d\tau = 1$$



$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise.} \end{cases}$$

8. State the difference between causal and non causal system [N/D-16]

Causal system depend on past and present inputs for computing present output but non causal system depend only on the past, present and future inputs for computing the present output.

Part-B

II 16 marks:-

1. (a) check whether the following signals are periodic / aperiodic signals. [N/D-14]

(i) $x(t) = \cos 2t + \sin t/5$

(ii) $x(n) = 3 + \cos \pi/2n + \cos 2n$.

(i) Given $x(t) = \cos 2t + \sin t/5$.

here $T_1 = \frac{2\pi}{2} = \pi$, $T_2 = \frac{2\pi}{1/5} = 10\pi$.

ratio of T_1 and T_2 : $\frac{T_1}{T_2} = \frac{\pi}{10\pi} = \frac{1}{10}$

$\therefore$ It is rational. Hence the given signal is periodic.

ii, Given, $x(n) = 3 + \frac{\cos \pi}{2n} + \cos 2n$.

Here, $N_1 = \frac{2\pi m}{\pi/2} = 4m$ if $m=1$.

$\therefore N_1 = 4$

$N_2 = \frac{2\pi n}{2} = \pi m$, if $m=1$.

$\therefore N_2 = \pi$.

The Ratio of N_1 and N_2, $\frac{N_1}{N_2} = \frac{4}{\pi}$

Not a Rational. Hence the given signal is Aperiodic.

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2. Check whether the following is linear, Causal, time invariant and / or stable.

i, $y(n) = x(n) - x(n-1)$ [N/D-14]

Causal / Non Causal:

put $n=1$, $y(1) = x(1) - x(0)$; present & past.

$n=2$, $y(2) = x(2) - x(1)$; presend and past.

As the output depends upon present and past it is causal.

Time Invariant / variant:

Given $y(n) = x(n) - x(n-1)$

output by delayed input.

$y(n-k) = x(n-k) - x(n-1-k)$. → ①

delayed output.

$y(n-k) = x(n-k) - x(n-1-k)$ → ②.

① = ② Time invariant.

Stability:

Given $y(n) = x(n) - x(n-1)$

It has bounded output, hence stable.

Linear / Non-Linear:

Given that, $y(n) = x(n) - x(n-1)$

$a y_1(n) = a x_1(n) - a x_1(n-1)$

$b y_2(n) = b x_2(n) - b x_2(n-1)$

$= a x_1(n) - a x_1(n-1) + b x_2(n) - b x_2(n-1)$

$= x_3(n)$.

Hence the given signal is Linear.

ii, Linear / Non linear.

$$y(t) = \frac{d}{dt} x(t).$$

Given, $y(t) = \frac{d}{dt} x(t)$

$$a y_1(t) = \frac{d}{dt} a x_1(t)$$

$$b y_1(t) = \frac{d}{dt} b x_2(t)$$

$$y_3(t) = a y_1(t) + b y_2(t)$$

$$= \frac{d}{dt} a x_1(t) + b x_2(t).$$

$$= \frac{d}{dt} x_3(t) \quad [\because a x_1(t) + b x_2(t) = x_3(t)].$$

Hence linear.

Causal / Non-causal :-

The given equation is differential so it is causal.

Time variant / Invariant :-

The given system is $y(t) = \frac{d}{dt} x(t)$

The coefficient is constant hence time invariant.

3. Give an account for classification of signal in detail.

signals are basically classified into two types.

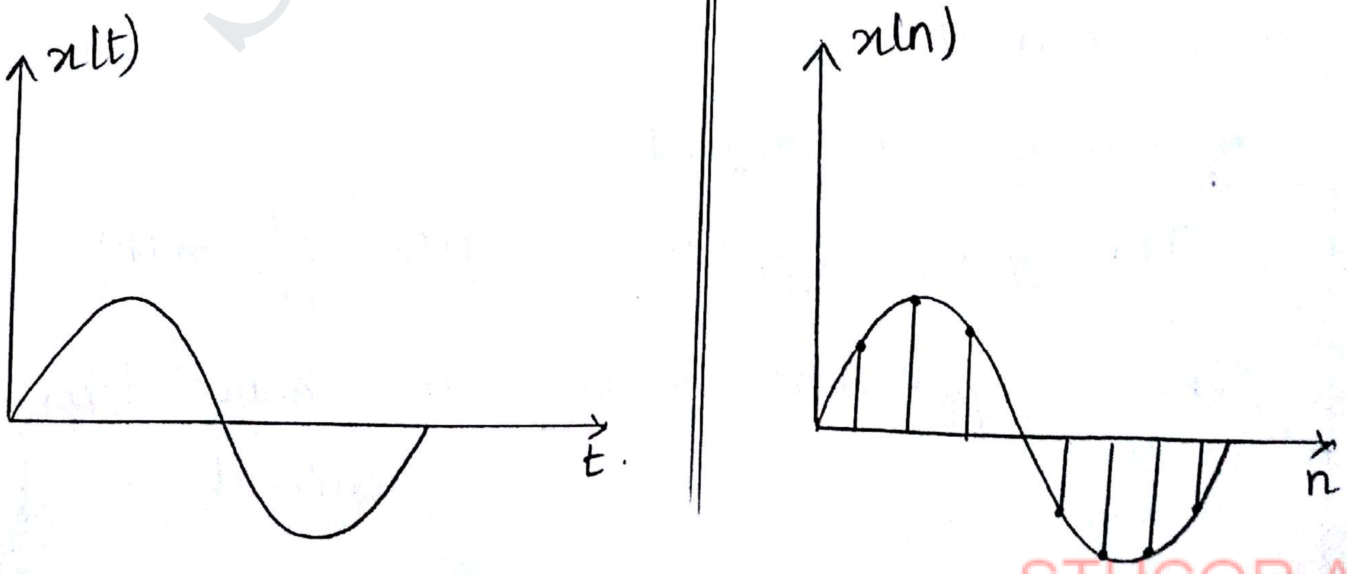
* Continuous time signals (CT signals)
* Discrete time signals (DT signals).

Continuous time signals : (CT signals)

Continuous time signal is defined for all values of time "t" and it is represented by $x(t)$. Here "n" denotes discrete interval of time. "n" ranges from $-\infty$ to $+\infty$.

eg: The below diagram shows an eg. of CT signal & DT signal.

CT signal | DT signal.



Elementary signals :-

Elementary (or) standard signals are used for the analysis of continuous time and discrete time signals.

Elementary signals are listed as follows:-

* Unit step signal
* Unit Impulse signal
* Unit Ramp signal
* Sinusoidal signal
* Exponential signal

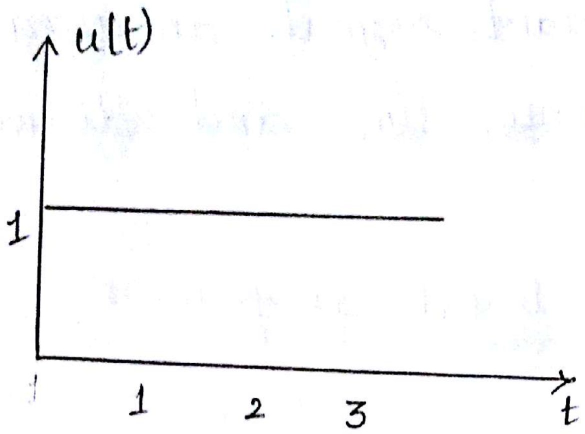
Unit step signal :-

* Unit step signal has amplitude "1" for the positive values of "t" (or) "n"
* Unit step signal has amplitude "0" for the negative values of "t" (or) "n".

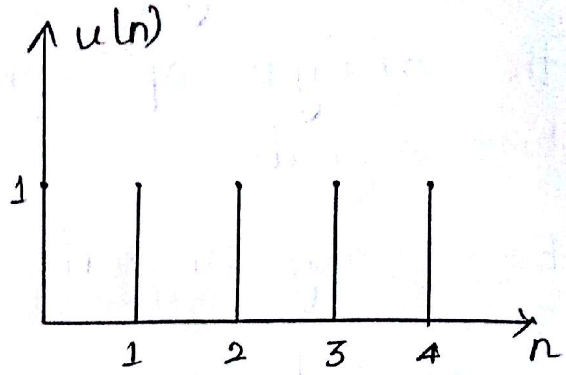
Representation :-

Continuous time (CT) Unit step signal u(t).	Discrete time (DT) Unit step signal u(n).
$u(t) = 1$, when $t \geq 0$ $= 0$, when $t < 0$.	$u(n) = 1$, when $n \geq 0$ $= 0$, when $n < 0$.

CT Unit step signal.



DT Unit step signal



Unit impulse (or) Delta function :-

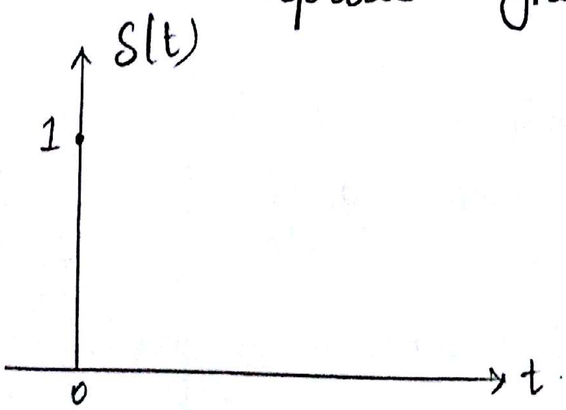
Unit impulse signal $\delta(t)$ (or) unit sample signal $\delta(n)$ has Unit amplitude at $t=0$ (or) at $n=0$.

Representation :-

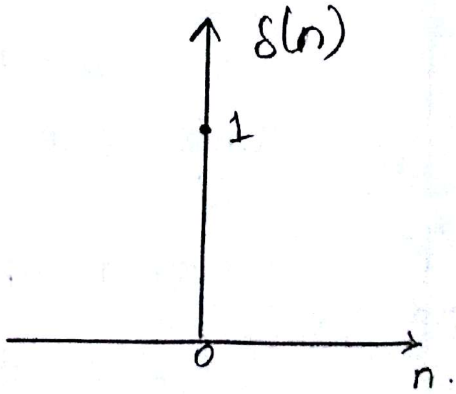
Unit Impulse signal	Unit sample signal.
$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$, when $t=0$.	$\delta(n) = 1$, for $n=0$.
$\delta(t) = 0$, for $t \neq 0$.	$\delta(n) = 0$, for $n \neq 0$.

Waveform :-

Unit Impulse signal



Unit sample sgnl.



Shifting property of Unit Impulse signal.

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)$$

Shifting property of Unit Sample signal

$$\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0).$$

Unit Ramp signal:- Unit Ramp signal is linearly growing function for positive values of "t" (or) positive values of "n".

Representation :-

Continuous time unit ramp signal $r(t)$.	Discrete time unit ramp signal $r(n)$.
$r(t) = \begin{cases} t, & \text{for } t \geq 0. \\ 0, & \text{for } t < 0. \end{cases}$ $r(t) = t\,u(t)$	$r(n) = \begin{cases} n, & \text{for } n \geq 0 \\ 0 & \text{for } n < 0. \end{cases}$ $r(n) = n\,u(n).$

Since,

$$u(t) = 1, \text{ for } t \geq 0$$
$$= 0, \text{ for } t < 0.$$

Since,

$$u(n) = 1, \text{ for } n \geq 0$$
$$= 0, \text{ for } n < 0.$$

14.(a) check if $x(t) = 4\cos(3\pi t + \pi/4) + 2\cos 4\pi t$ is periodic. [A/M-15]

$$2\pi f_1 t = 3\pi t.$$

$$f_1 = 3/2 \qquad T_1 = 2/3.$$

$$2\pi f_2 t = 4\pi t.$$

$$f_2 = 2 \qquad T_2 = 1/2.$$

$$\frac{T_1}{T_2} = 4/3$$

since $\frac{T_1}{T_2}$ is rational, the given signal is periodic signal.

(b) For the system $y(n) = \log[x(n)]$, check for linearity, causality, time invariance and stability.

$$y(n) = \log[x(n)].$$

Linearity: since logarthmic function is non-linear the given system is non-linear.

Time variance:- $y(n,k) = \cancel{f}[x(n-k)]$

$$= \log[x(n-k)]$$

Let us delay the output by replacing n by "n-k".

$$y(n,k) = \log[x(n-k)]$$

$y(n,k) = y(n-k)$

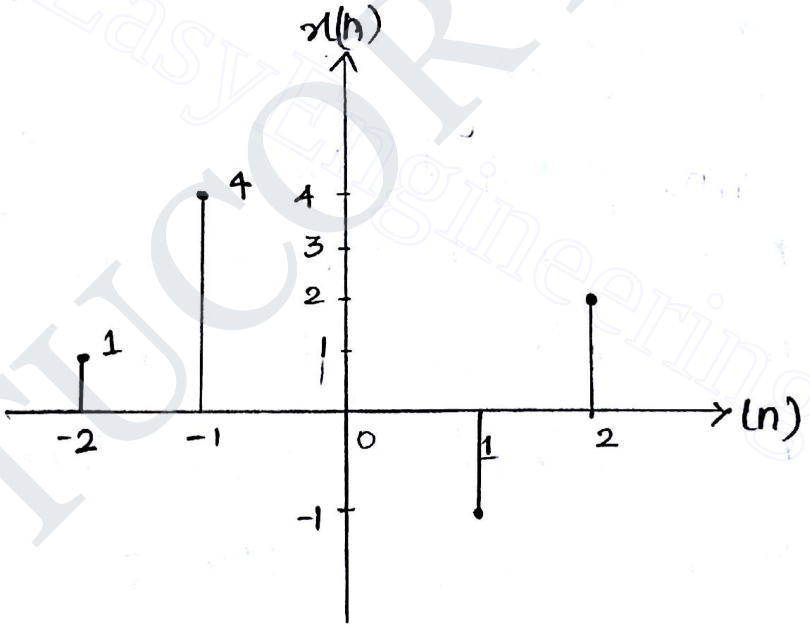
Hence the given system is time invariant system.

Stability: $y_n = \infty$ when $x(n) = 0$.

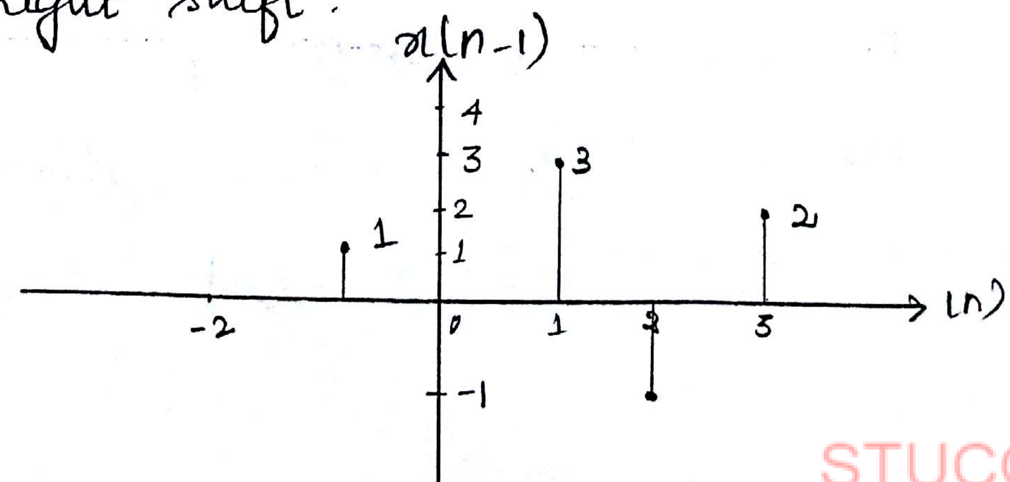
since o/p $y(n)$ is infinity for $x(n) = 0$, the given system is unstable.

15. Give $x(n) = \{1, 4, 3, -1, 2\}$ plot the following signals. [N/D-15]

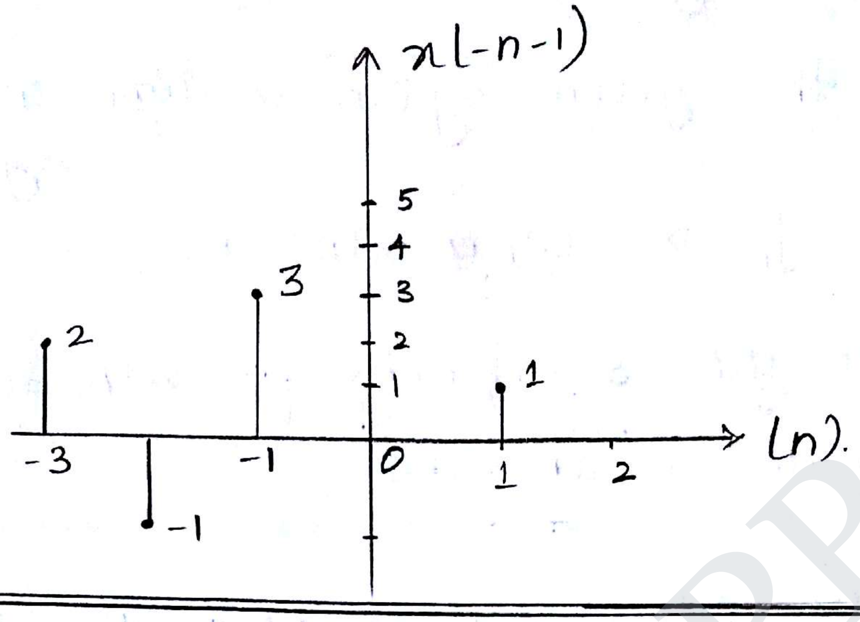
i) $x(-n-1)$



$x(n-1)$ Right shift:



$x(-n-1)$



ii, $x(-n/2)$

Expansion of $x(n)$ by 2.

Folding of $x(n/2)$.

when $n=0$, $x(n/2) = x(0) = 3$.

$n=1$, $x(n/2) = x(1/2) =$ does not exist

$n=2$, $x(n/2) = x(1) = -1$.

$n=3$, $x(n/2) = x(3/2) =$ does not exist

$n=4$, $x(n/2) = x(2) = 2$.

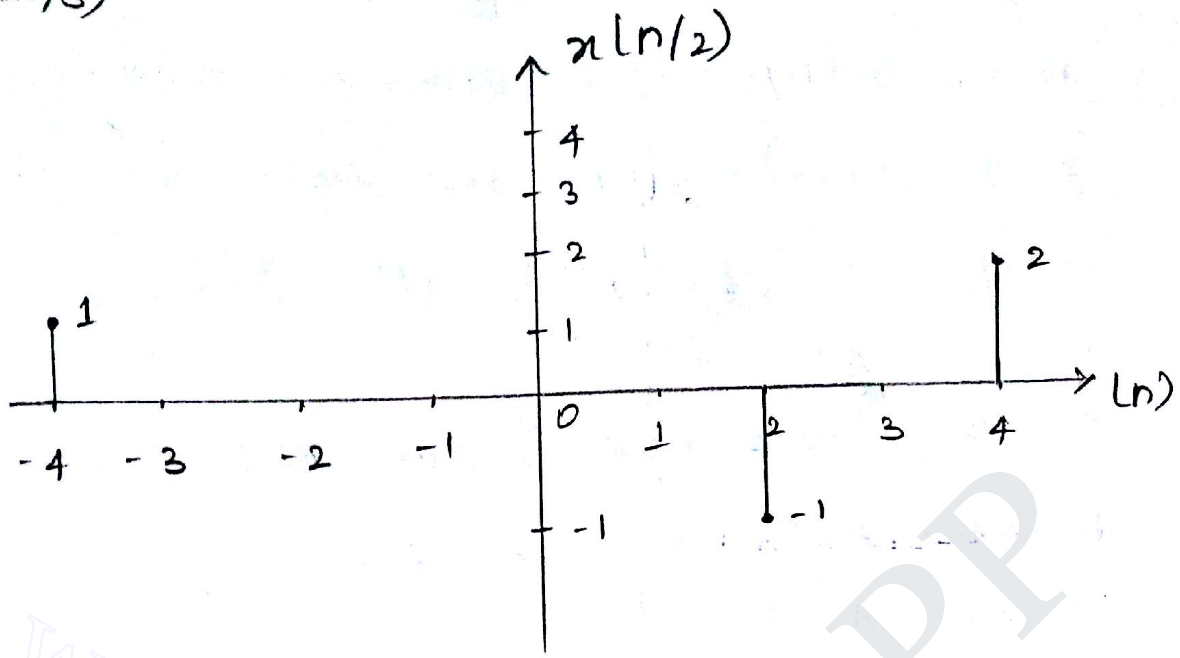
$n=-1$, $x(-1/2) = x(-1) = 4$ does not exist

$n=-2$, $x(n/2) = x(-1) = 4$

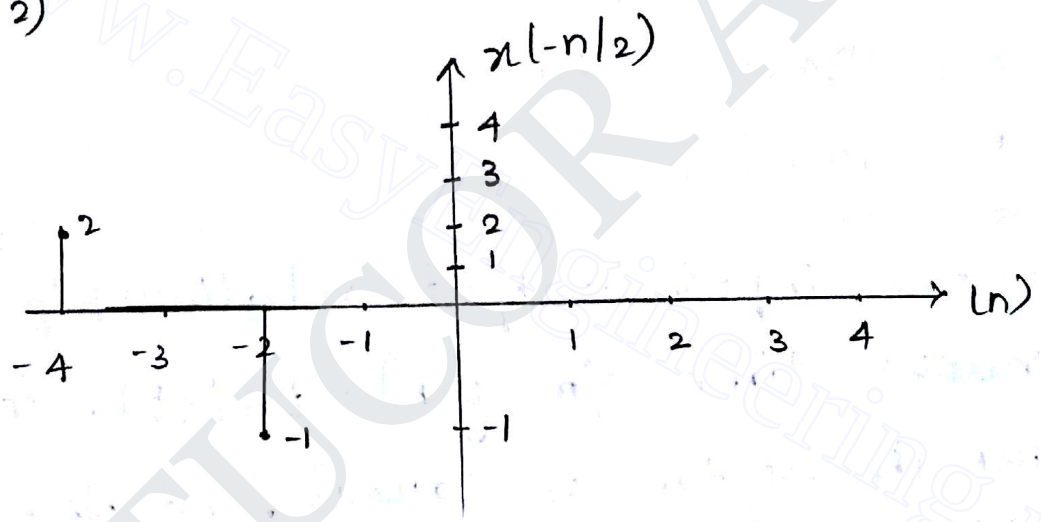
$n=-3$, $x(n/2) = x(-3/2) =$ does not exist

$n=-4$, $x(n/2) = x(-2) = 1$.

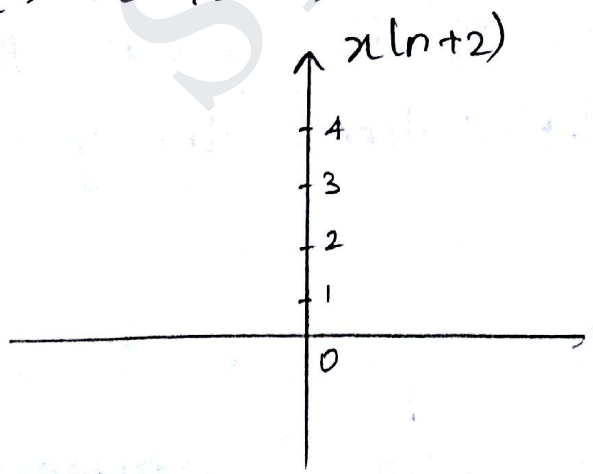
$x(n/2)$

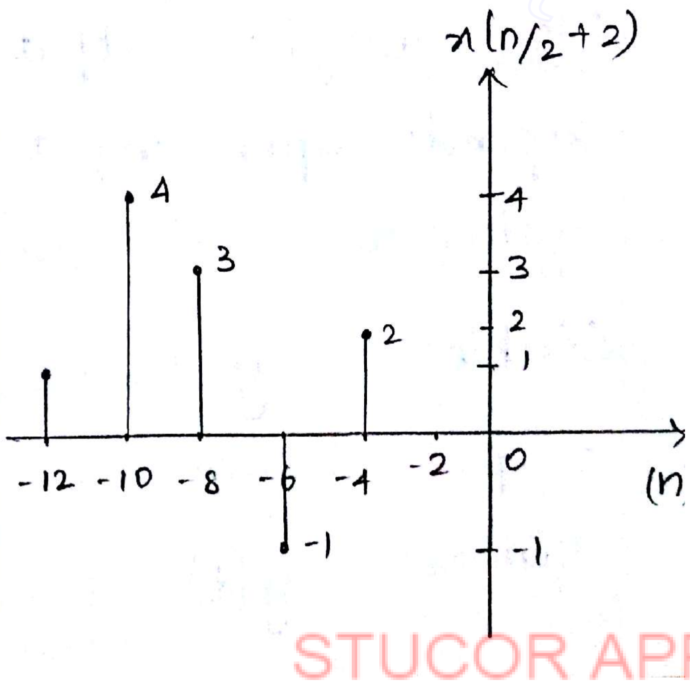


$x(-n/2)$



iii) $x(-n/2+2)$





when $n=0$, $x(n/2+2) = x(2) = 0$.

$n=1$, $x(n/2+2) \Rightarrow$ does not exist

$n=2$, $x(3) =$ does not exist

$n=-1$, $x(2.5) =$ does not exist

$n=-2$, $x(1) = n$.

$n=-3$, $x(n/2+2) \Rightarrow$ does not exist.

$n=-4$, $x(0) = 2$.

$n=-6$, $x(-1) = -1$

$n=-8$ $x(-2) = 3$.

$n=-10$ $x(-3) = 4$

$n=-12$ $x(-4) = 1$.

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6. Given the input output relationship of a continuous time s/m $y(n) = t\,x(-t)$. determine whether the system is causal, stable, linear and time variant. [N/D-15]

Non Causal: output $y(t)$ of the given s/m depends upon input $x(-t)$. Hence the given system is non causual.

Stable: Every bounded input produces bounded output. Hence the given system is stable.

Linear:- $y(t) = t\,x(-t)$.

$y_1(t) = t x_1(-t)$.

$y_2(t) = t x_2(-t)$

condition for linear system.

$$a_1 x_1(t) + a_2 x_2(t) = a_1 y_1(t) + a_2 y_2(t)$$

LHS: $a_1 x_1(t) + a_2 x_2(t) = a_1 t x_1(-t) + a_2 t x_2(-t)$.

RHS: $a_1 y_1(t) + a_2 y_2(t) = a_1 t x_1(-t) + a_2 t x_2(-t)$.

LHS = RHS Hence the given s/m is linear.

Time variant: O/P of the system for the delayed i/p is

$$y(t, t_1) = t x(-t+t_1)$$

O/P is delayed by t_1,

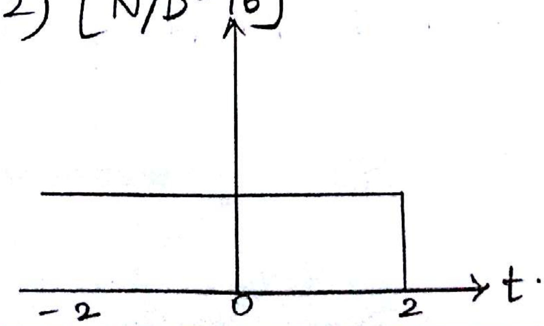
$$y(t-t_1) = (t-t_1) x(-t+t_1)$$

$$y(t, t_1) \neq y(t-t_1)$$

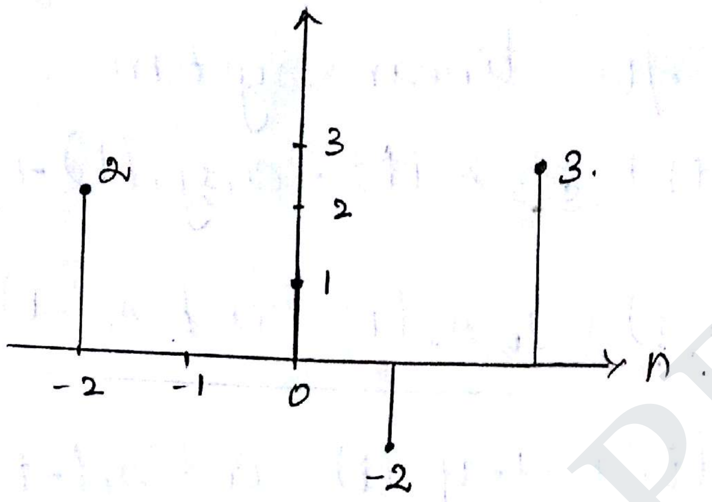
Hence the given system is time variant s/m.

7. Sketch the following signals.

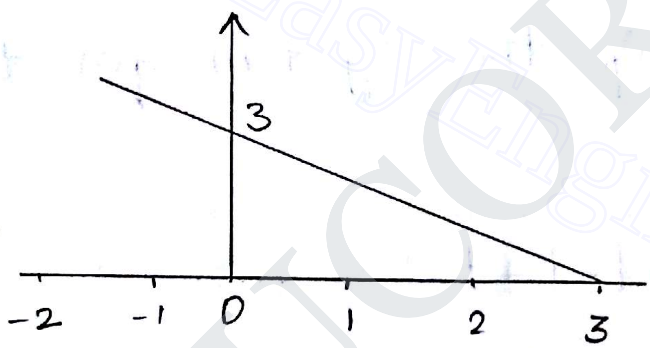
(i) $u(-t+2)$ [N/D-16]



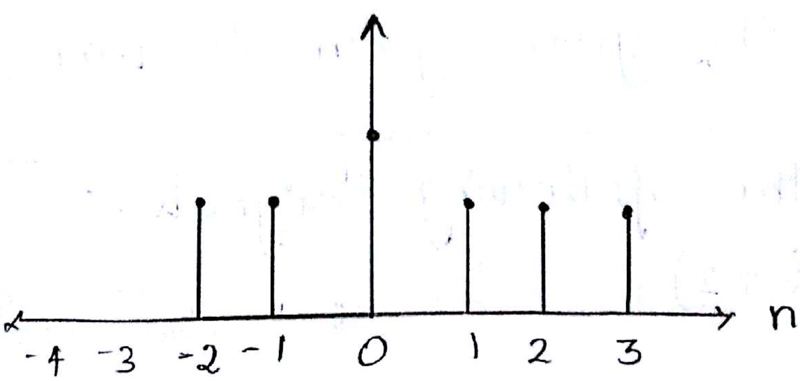
ii) $2\delta[n+2] + \delta[n] - 2\delta[n-1] + 3\delta[n-3]$.



iii) $(-t+3)$



iv) $u[n+2]\, u[-n+3]$.



PART B– Questions (5 X 16 = 80 marks)

UNIT –I

1. Simplifyfor followingBoolean function F, usingQuineMccluskeymethod and verifythe result usingK-map F(A,B,C,D)=∑(0,2,3,5,7,9,11,13,14)(16) [MJ-16]

0000 (0)	0000 (0) ✓	00–0 (0,2) H
0010 (2)	0010 (2) ✓	001– (2,3) G
0011 (3)	0011 (3) ✓	0–11 (3,7) F
0101 (5)	0101 (5) ✓	–011 (3,11) E
0111 (7)	1001 (9)	01–1 (5,7) D
1001 (9)	0111 (7) ✓	–101 (5,13) C
1011 (11)	1011 (11) ✓	1–01 (9,13) B
1101 (13)	1101 (13) ✓	
1100 (14)	1110 (14) A	

The prime implicants are

A 1110

B 1–01

C ~~0~~101

D 01–1

E –011

F 0–11

G 001–

H 00–0

The prime implicants table.

	0	2	3	5	7	9	11	13	14
A									x
B						x		x	
C				x				x	
D				x	x				
E			x				x		
F			x		x				
G		x	x						
H	x	x							

The essential prime implicants A B E H

The minterms covered in A 14

B 9 13

E 3 11

H 0 2

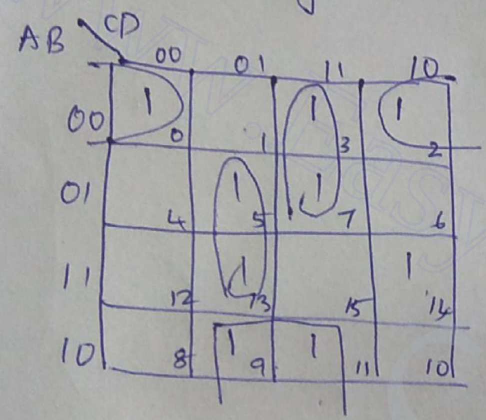
Q11 (contd)

Prime implicant table after removing after removing A B E H

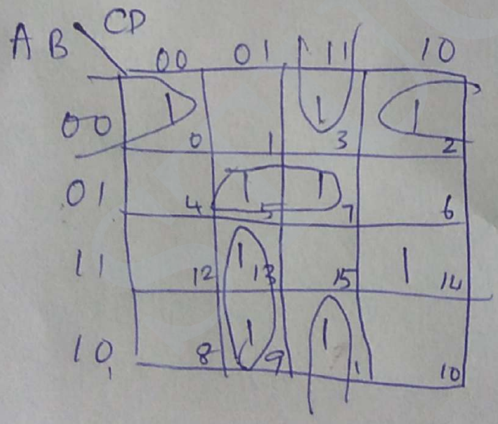
	2	3	5	7	13
C			x		x
D			x	x	
F		x		x	
G	x	x			

$F(A,B,C,D) = ABCD' + AC'D + A'BD + B'CD + A'B'D'$

Karnaugh Map



$F = ABCD' + BC'D + AB'D + A'CD + A'B'D'$



$F = ABCD' + AC'D + A'BD + B'CD + A'B'D'$

2. Minimize the following logic function using K maps and realize using NAND and NOR gates.(10) [N/D – 15]

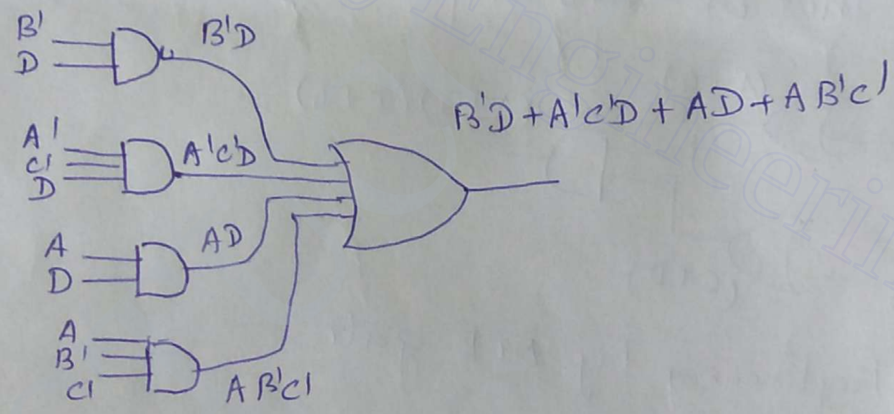
$$F(A,B,C,D)=\sum m(1,3,5,8,9,11,15)+d(2,13).$$

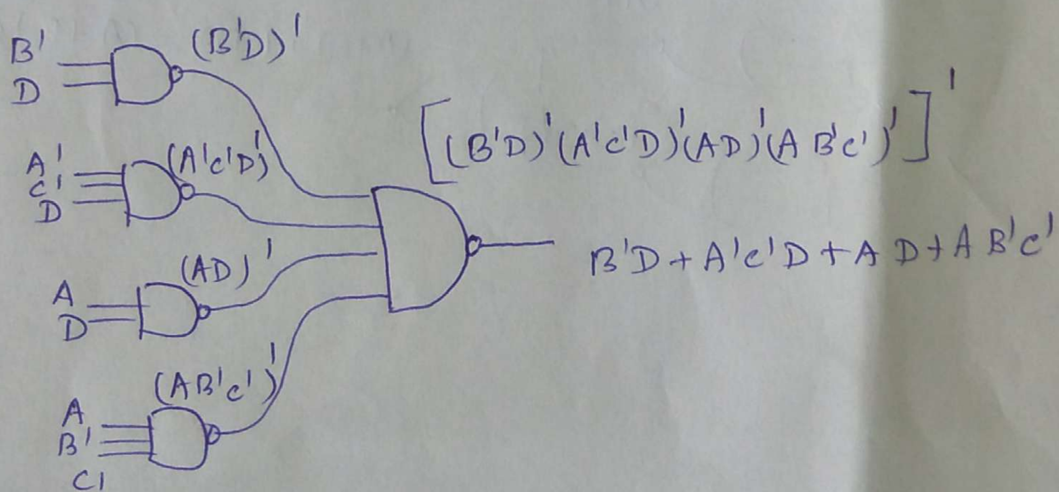
AB \ CD	00	01	11	10
00	(0)	1 (1)	1 (3)	d (2)
01	(4)	1 (5)	(7)	(6)
11	(12)	d (13)	1 (15)	(14)
10	1 (8)	1 (9)	1 (11)	(10)

$F = B'D + A'C'D + A\cancel{C}D$
$+ AB'C'$

Realization using NAND gate.

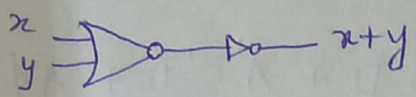
AND - OR logic diagram



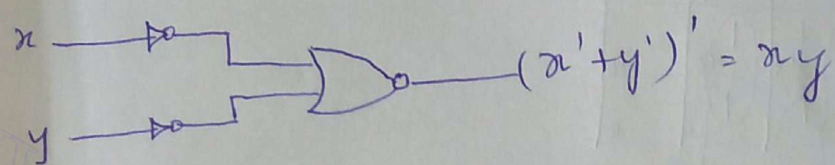


3 Show that if all the gate in a two – level OR-AND gate network are replaced byNORgate, theoutput function does not change. (6) [N/D – 15]

OR function using NOR gate



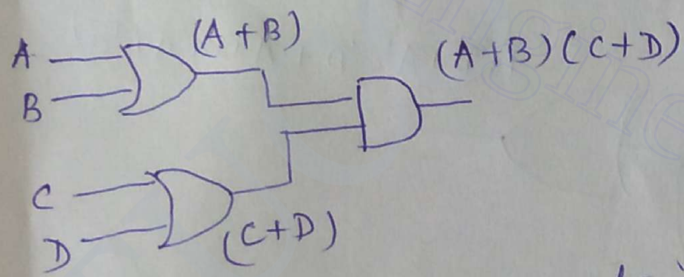
AND function using NOR gate.



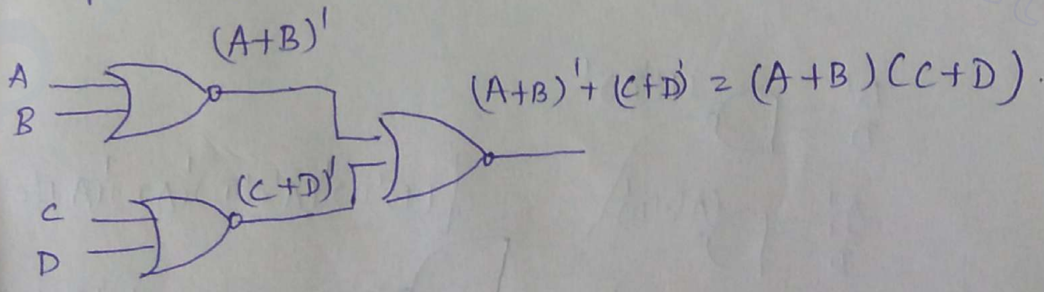
Consider for example the following POS function

$f(A,B,C,D) = (A+B)(C+D)$

The OR-AND implementation is as follows.



Replacing by NOR gate.



04. Simply using Quine Mccluskey method and verify your result using k-map $F=\sum(0,1,2,5,7,8,9,10,13,15)$. (16)[A/M – 15]

The minterms of the function are tabulated

(0) 0000
(1) 0001
(2) 0010
(5) 0101
(7) 0111
(8) 1000
(9) 1001
(10) 1010
(13) 1101
(15) 1111

0000 (0) ✓

0001 (1) ✓
0010 (2) ✓
1000 (8) ✓

0101 (5) ✓
1001 (9) ✓
1010 (10) ✓

0111 (7) ✓
1101 (13) ✓

1111 (15) ✓

000– (0,1) ✓
00–0 (0,2) ✓
–000 (0,8) ✓

0–01 (1,5) ✓
–001 (1,9) ✓
–010 (2,10) ✓
100– (8,9) ✓
10–0 (8,10) ✓

01–1 (5,7) ✓
–101 (5,13) ✓
1–01 (9,13) ✓

–111 (7,15) ✓
11–1 (13,15) ✓

–00– (0,1,8,9)
–0–0 (0,2,8,10)
–00– (0,8,1,9)
–0–0 (0,2,8,10)

––01 (1,5,9,13)
––01 (1,9,5,13)

–1–1 (5,7,13,15)
–1–1 (5,13,7,15)

The prime implicants are

w x y z

A –00– (0,1,8,9)

B –0–0 (0,2,8,10)

C ––01 (1,5,9,13)

D –1–1 (5,7,13,~~7~~,15)

To find the essential prime implicants

	0	1	2	5	7	8	9	10	13	15
A	x	x				x	x			
B	x		x			x		x		
C		x		x			x		x	
D				x	x				x	x

Essential prime implicants are B, D 9 is not included Hence prime implicant Table after rows B & D are removed

05 Express the Boolean functions F = A + BC in a sum of minterms. (10) [A/M – 15]

Consider the term A

(i) $A(B+B') = (AB+AB')(C+C')$

$= ABC + AB'C + AB'C + AB'C'$

(ii) $BC = (A+A')(BC)$

$= ABC + A'BC$.

$F = ABC + AB'C + AB'C + AB'C' + A'BC$.

06. Simplify the following Boolean expression using Boolean algebra.[A/M – 15]

a) $\overline{x}\overline{y}z + \bar{x}yz + x\bar{y}$ (3)

b) $xyz + \bar{x}z + yz$ (3)

a) $x'y'z + x'yz + xy'$

$x'z(y+y') + xy'$

$x'z + xy'$

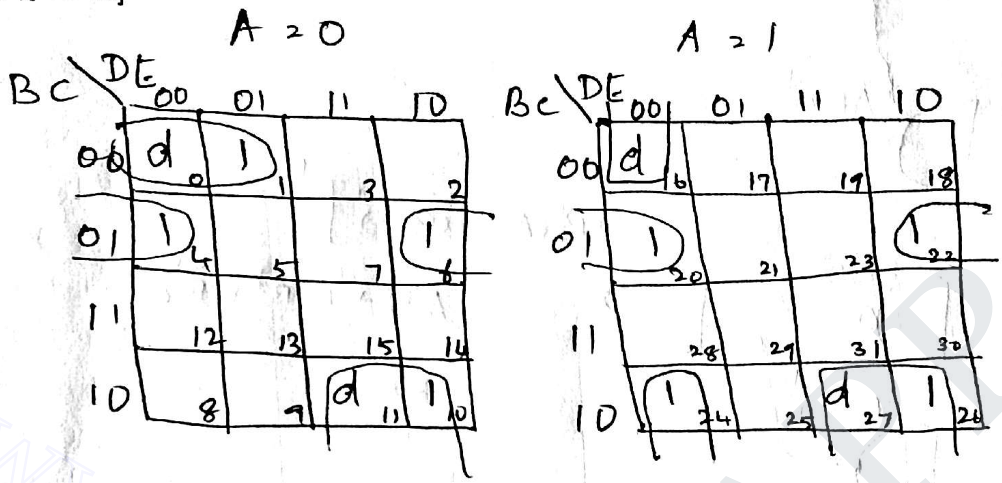
b) $yz(x+1) + \bar{x}z$

$= yz + x'z.$

Contd (6)

	1	2	3	7	8	9	10	11	14	15
A	x					x				
B		x					x			
C				x						x
D		x	x				x	x		
E					x	x	x	x		
F										
G										

Q7. Find the MSOP representation for $F(A,B,C,D,E) = m(1,4,6,10,20,22,24,26) + d(0,11,16,27)$ using K-Map method. Draw the circuit of the minimal expression using only NAND gates. (7)[N/D-16]



(1)

A	B	C	D	E
0	0	1	0	0
0	0	1	1	0
1	0	1	0	0
1	0	1	1	0

$B'CE'$

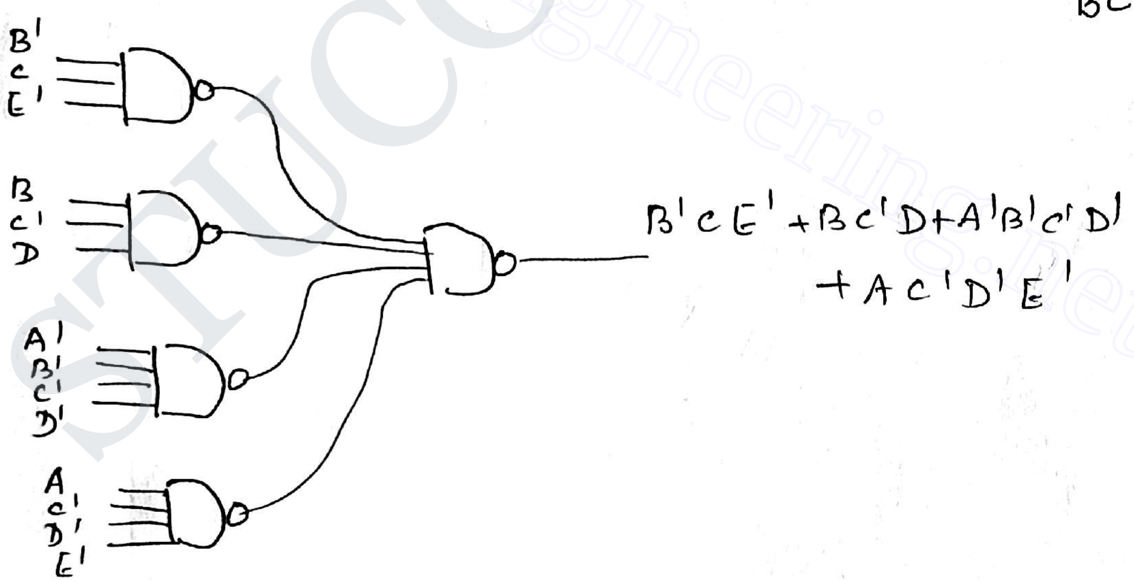
(2)

0	1	0	1	1
0	1	0	1	0
1	1	0	1	1
1	1	0	1	0

$BC'D$

$$F(ABCDE) = B'CE' + BC'D + A'B'C'D' + AC'D'E'$$

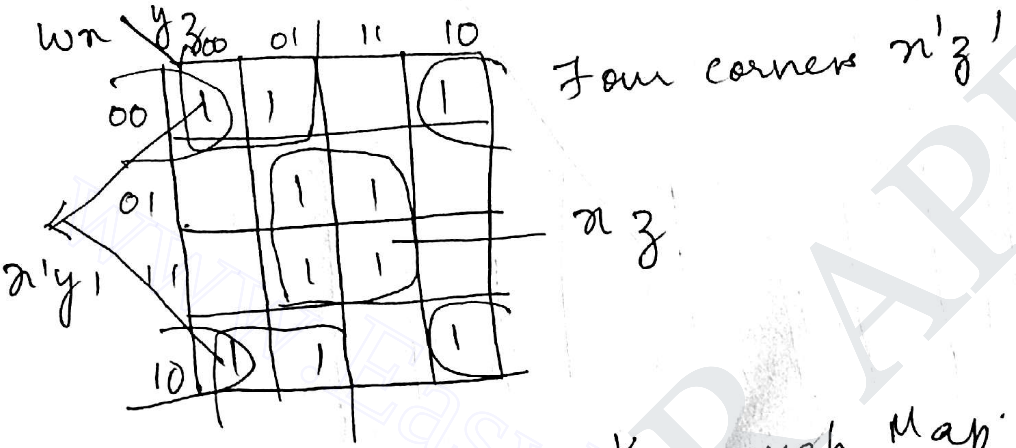
NAND Implementation



Contd. #

	0	1	5	8	9	13
A	×	×		×	×	
C		×	×		×	×

$F = x'z' + x'y' + xz$



Verified using Karnaugh Map.

8. What are the advantages of using tabulation method? Determine the Minimal sum of products for the Boolean expression F=Σ(1,2,3,7,8,9,10,11,14,15) using tabulation method. (13) [N/D-16]

Advantages of using tabulation method.

1) When the number of literals increases beyond 4 the complexity of solution is less than that of Karnaugh Map.

2) There will not be more than one solution for a particular function.

3) Systematic approach to solving problems.

0001	(1)
0010	(2)
0011	(3)
0111	(7)
1000	(8)
1001	(9)
1010	(10)
1011	(11)
1110	(14)
1111	(15)

0001	(1)
0010	(2)
1000	(8)
0011	(3)
1001	(9)
1010	(10)
0111	(7)
1011	(11)
1110	(14)
1111	(15)

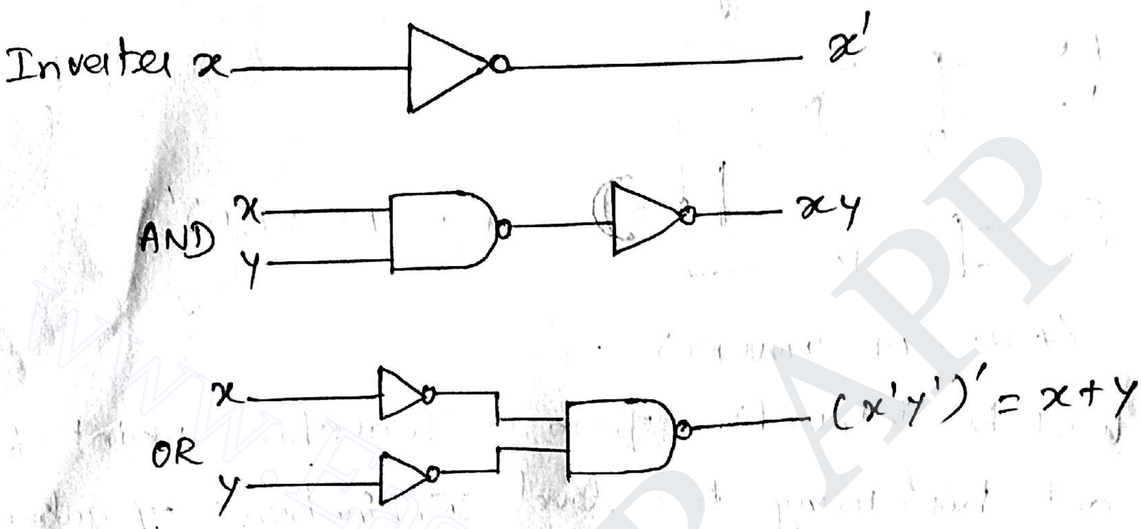
-001	(1,9)	A
001-	(2,3)	✓
-010	(2,10)	B
100-	(8,9)	✓
10-0	(8,10)	✓
0-11	(3,7)	✓
10-1	(9,11)	✓
101-	(10,11)	✓
1-10	(10,14)	✓
-111	(7,15)	C
1-11	(11,15)	✓
111-	(14,15)	✓

-01-	(2,3,10,11)
10--	(8,9,10,11)
10--	(8,10,9,11)
--11	(3,7,11,15)
1-1-	(10,11,14,15)
1-1-	(10,14,11,15)

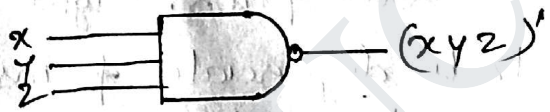
The prime implicants are

A -001
B -010
C -111
D -01-
E 10--
F --11
G 1-1-

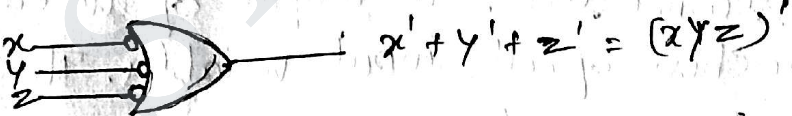
9. Realize NOT, OR, AND gates using universal gates. (8) [N/D – 15]



Logic operations with NAND Gates.

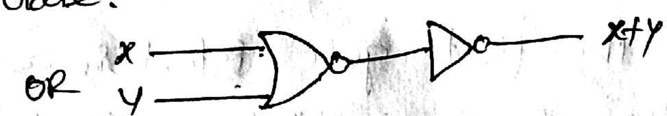


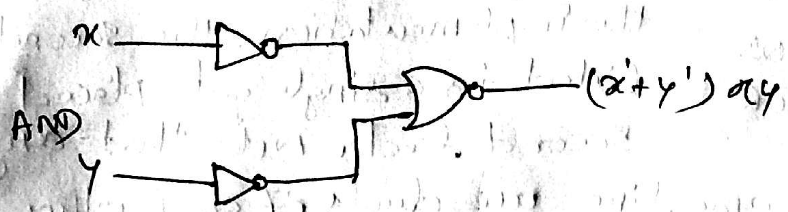
(a) AND – invert



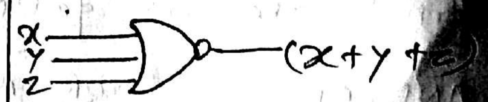
(b) Invert – OR

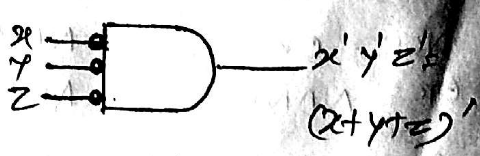
OR Gate:





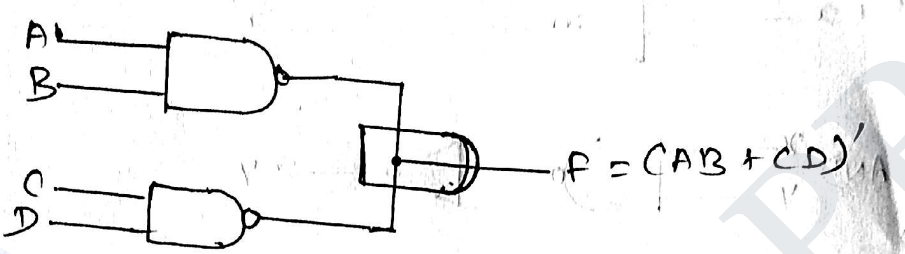
Logic operations with N





10. Discuss about the basic operation of TTL NAND gate. (8) [N/D – 15]

Wired-AND in open-collector TTL NAND gates:



(AND-OR-INVERT)

It will be istructive from a theoretical point of view to find out how many two-level combinations of gates are possible. If we asign one type of gate for the first level and one type for the second level, we find that there are 16 possible combinations of two-level forms. Eight of these combinations are said to be degenerate forms because they degenerate to a single operation. This can be seen from a circuit with AND gates in the first level and an AND gate in the second level. The output of the circuit is merely the AND function of all input variables. The remaining eight nondegenerate forms produce an implementation in sum-of-products form or product-of-sums form. The eight nondegenerate forms are as follows:

AND-OR	OR-AND
NAND-NAND	NOR-NOR
NOR-OR	NAND-AND
OR-NAND	AND-NOR

The first gate listed in each of the forms constitutes a first level in the implementation. The second gate listed is a single gate placed in the second level. Note that any two forms listed on the same line are duals of each other.

UNIT - 3 LINEAR TIME INVARIANT CONTINUOUS TIME SYSTEMS

2 - MARKS

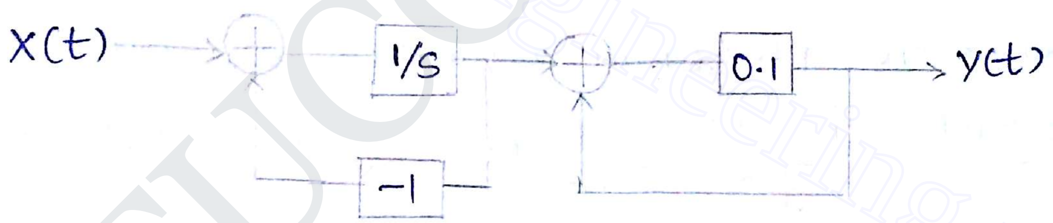
1. Draw the block diagram of the LTI system described by $\frac{dy(t)}{dt} + y(t) = 0.1\,x(t)$? [N-13]

$$\frac{dy(t)}{dt} + y(t) = 0.1\,x(t)$$

$$sY(s) + Y(s) = 0.1\,X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{0.1}{s+1}$$

Cascade form:



2. Find $y(n) = x(n-1) * \delta(n+2)$ [N-13]

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)$$

$$y(n) = x(n-1) * \delta(n+2)$$

$$y(n) = \sum_{K=-\infty}^{\infty} x(K-1) * \delta(n+2-K) \Big|_{K=n+2}$$

$$= x(n+2-1)$$

$$= x(n+1)$$

3. Given $x(t) = \delta(t)$ Find $X(S)$ and $X(\omega)$ [A-15]

$\delta(t) \Rightarrow$ unit Impulse signal

$X(S) \Rightarrow$ Laplace transform of $x(t)$

$X(\omega) \Rightarrow$ Fourier Transform of $x(t)$

$X(S) \rightarrow L\{\delta(t)\} = 1$

$X(\omega) = F\{\delta(t)\} = 1$

4. State the convolution Integral ? [A-15]

Convolution Integral relates Input, output and unit impulse response of the continous time system. Convolution can be used to resolve inter connected systems using convolution properties.

5. Given the differential equation representation of a System $\frac{d^2y(t)}{dt^2} + 2\frac{d}{dt}y(t) - 3y(t) = 2x(t)$. Find the frequency response $H(S)$ [N-15]

Taking Fourier Transform of the differential Equation.

$$(j\omega)^2 Y(\omega) + 3j\omega(\omega)Y + 2Y(\omega) = X(\omega)$$

$$Y(\omega)\left[(j\omega)^2 + 3j\omega + 2\right] = X(\omega)$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{1}{(j\omega)^2 + 3j\omega + 2}$$

$$H(\omega) = \frac{1}{(j\omega+1)(j\omega+2)} = \frac{A}{j\omega+1} + \frac{B}{j\omega+2}$$

$$H(\omega) = \frac{1}{j\omega+1} + \frac{1}{j\omega+2}$$

6. Convolve the following signals $u(t-1)$ and $\delta(t-1)$

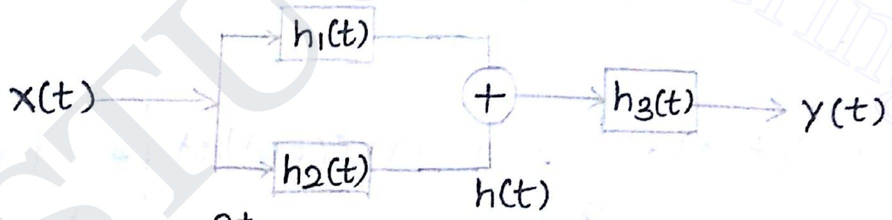
$$u(t-1) * \delta(t-1) = u(t-2)$$ [N-16]

7. Given $H(S) = \frac{S}{S^2+2S+1}$ Find the differential equation Representation of the system. [N-16]

$$x(t) = \frac{d^2y(t)}{dt^2} + 2\frac{d}{dt}y(t) + y(t).$$

16 MARKS :

1. Find the overall impulse of the following system [N-13]



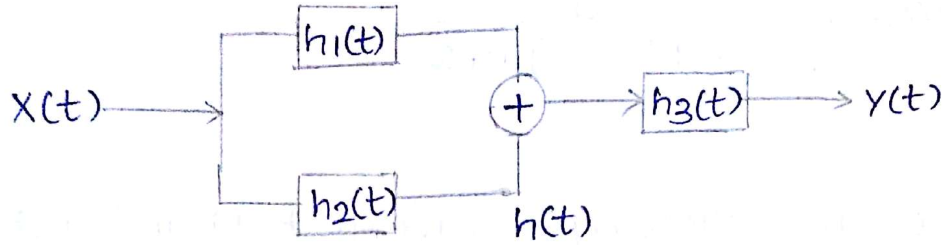
Here $h_1(t) = e^{-2t}u(t)$

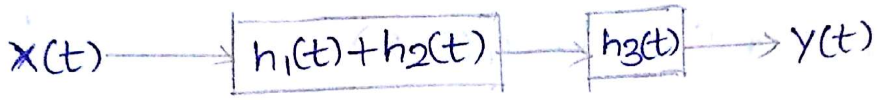
$h_2(t) = \delta(t) - \delta(t-1)$

$h_3(t) = \delta(t)$

Also find output of the system for the input $x(t) = u(t)$ using convolution Integral.

<u>Sol</u>





$$h_1(t) = e^{-2t}$$

$$h_2(t) = \delta(t) - \delta(t-1)$$

$$h_3(t) = \delta(t)$$

$$h_0(t) = h_1(t) \times h_3(t) + h_2(t) \times h_3(t)$$

$$= \int_{\lambda=0}^{t} h_1(\lambda) h_3(t-\lambda)\, d\lambda + \int_{\lambda=0}^{t} h_2(\lambda)\, h_3(t-\lambda)\, d\lambda$$

$$= \int_{\lambda=0}^{t} e^{-2\lambda} \delta(t-\lambda)\, d\lambda + \int_{\lambda=0}^{t} [\delta(\lambda) - \delta(\lambda-1)]\, \delta(t-\lambda)\, d\lambda$$

$$= \int_{\lambda=0}^{t} e^{-2\lambda} \delta(t-\lambda)\, d\lambda \Big|_{\lambda=t} + \int_{\lambda=0}^{t} \delta(\lambda)\, \delta(t-\lambda) d\lambda \Big| + - \int_{\lambda=0}^{t} \delta(\lambda-1)\delta(t-$$

$$h_0(t) = e^{-2t} + \delta(t) + \delta(t-1)$$

overall response

$$h_0(t) = e^{-2t} + \delta(t) + \delta(t-1)$$

$$y(t) = x(t) \times h_0(t)$$

$$x(t) = u(t)$$

$$y(t) = \int_0^t e^{-2(t-\lambda)} d\lambda + \int_0^t \delta(t-\lambda)\, d\lambda \Big|_{\lambda=t} + \int_0^t \delta(t-\lambda-1)\, d\lambda \Big|_{\lambda=t}$$

$$= e^{-2t}\left[\frac{e^{2\lambda}}{2}\right]_0^t + u(t) + u(t-1)$$

$$= e^{-2t}\left[e^{2\lambda}-1\right] + u(t) + u(t-1)$$

$$y(t) = \frac{1}{2}\left[1-e^{-2t}\right] + u(t) + u(t-1)$$

$$y(t) = \frac{1}{2}\left[1-e^{-2t}\right] + 1 + 1$$

$$y(t) = \frac{5-e^{-2t}}{2}.$$

2. An LTI system is represented by $\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 4y(t) = x(t)$ with initial conditions $y(0^-) = 0$; $y'(0^-) = 1$ Find the output of the system, when the input is $x(t) = e^{-t}u(t)$. [N-13]

Sol. $\frac{d^2y(t)}{dt^2} \xleftrightarrow{LT} s^2Y(s)$, $\frac{dy(0^+)}{dt} - sY(0^+)$

$\frac{d^2y(t)}{dt^2} \xleftrightarrow{LT} sY(s) - (y)(0^+)$, $Y(s) = \frac{1}{s+1}$

$$= s^2Y(s) - sy(0^+) - y'(0^+) + 4\left[sY(s) + y(0^+)\right]$$

$y'(0^+) = 0$; $y'(0^+) = 1$

$4Y(s) = X(s)$

$s^2Y(s) - 1 + 4sY(s) + 4Y(s) = X(s)$

$$Y(s) = \frac{1}{s^2+4s+3} \quad \frac{1}{s+1}$$

$$Y(S) = \frac{1}{(S+3)(S+1)^2}$$

$$\frac{1}{(S+3)(S+1)^2} = \frac{A}{S+3} + \frac{B}{S+1} + \frac{C}{(S+1)^2}$$

By using partial fractional expansion

$$1 = A(S+1)^2 + B(S+3)(S+1) + C(S+3)$$

$$1 = A(-2)^2$$

$$A = \frac{1}{4}$$

$$S=-1, \quad 1 = C(2)$$

$$C = \frac{1}{2}$$

$S=0$

$$1 = A(1)^2 + B(3) + C(3)$$

$$1 = \frac{1}{4} + 3B + \frac{1}{2} \times 3$$

$$1 = \frac{7}{4} + 3B$$

$$B = -\frac{1}{7}$$

$$Y(S) = \frac{1}{4(S+3)} - \frac{1}{7(S+1)} + \frac{1}{2(S+1)^2}$$

$$y(t) = \frac{1}{4}e^{-3t} - \frac{1}{7}e^{-t} + \frac{1}{2}te^{-t}\,u(t)$$

3 (i) Solve the differential equation $(D^2+3D+2)\,y(t) = D\,x(t)$ using the input $x(t) = 10e^{-3t}$ and with initial condition $y(0^+) = 2$ and $y(0^+) = 3$. [A-15]

Q1.

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx}{dt}$$

$$\left[s^2Y(s) - sY(0^-) - \frac{dy(t)}{dt}\right] + 3\left[sY(s) - Y(0^-)\right] + 2Y(t) = sX(s)$$

Applying Initial conditions:

$$\left[s^2Y(s) - 2s - 3\right] + 3\left[sY(s) - 2\right] + 2Y(s) = sX(s)$$

$$s^2Y(s) - 2s - 3 + 3sY(s) - 6 + 2Y(s) = sX(s)$$

$$s^2Y(s) + 3sY(s) + 2Y(s) - 2s - 9 = sX(s)$$

$$Y(s)\left[s^2+3s+2\right] - 2s - 9 = \frac{10s}{s+3}$$

$$Y(s) = \frac{10s}{(s^2+3s+2)(s+3)} + 2s + 9$$

$$Y(s) = \frac{12s+9}{(s+3)(s^2+3s+2)}$$

$$Y(s) = \frac{12s+9}{(s+3)(s+1)(s+2)}$$

Applying partial Fraction.

$$Y(s) = \frac{12s+9}{(s+3)(s+1)(s+2)}$$

$$= \frac{A}{S+1} + \frac{B}{S+2} + \frac{C}{S+3}$$

$$12S+9 = A(S+2)(S+3) = B(S+1)(S+3) + C(S+1)(S+2$$

when $S=-1$

$$A = -3/2 \quad B = 15 \quad C = \frac{-27}{2}$$

$$Y(S) = \frac{-3/2}{S+1} + \frac{15}{S+2} - \frac{27/2}{S+3}$$

Taking Inverse Laplace Transform.

$$y(t) = -3/2\, e^{-t} u(t) + 15\, e^{-2t} u(t) - \frac{27}{2} e^{-3t} u($$

(ii) Draw the block diagram representation for

$$H(S) = \frac{4S+28}{S^2+6S+5}$$

[A-15]

Sol. Transfer Function $H(S) = \dfrac{4S+28}{S^2+6S+5}$

$$H(S) = \frac{Y(S)}{X(S)} = \frac{4S+28}{S^2+6S+5}$$

$$\frac{Y(S)}{X(S)} = \frac{\frac{4S}{S^2} + \frac{28}{S^2}}{\frac{S^2}{S^2} + \frac{6S}{S^2} + \frac{5}{S^2}}$$

$$\frac{Y(S)}{X(S)} = \frac{\frac{4}{S} + \frac{28}{S^2}}{1 + \frac{6}{S} + \frac{5}{S^2}}$$

$$Y(s)\left[1+6/s+5/s^2\right] = X(s)\left[\frac{4}{s}+\frac{28}{s^2}\right]$$

$$Y(s)+\frac{6Y(s)}{s}+\frac{5Y(s)}{s} = \frac{4X(s)}{\cancel{s}}+\frac{28X(s)}{s^2}$$

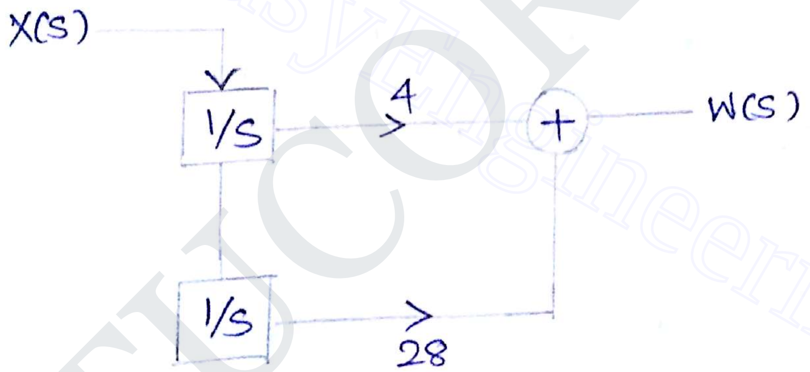
$$\frac{4X(s)}{s}+\frac{28X(s)}{s^2} = W(s)$$

Then

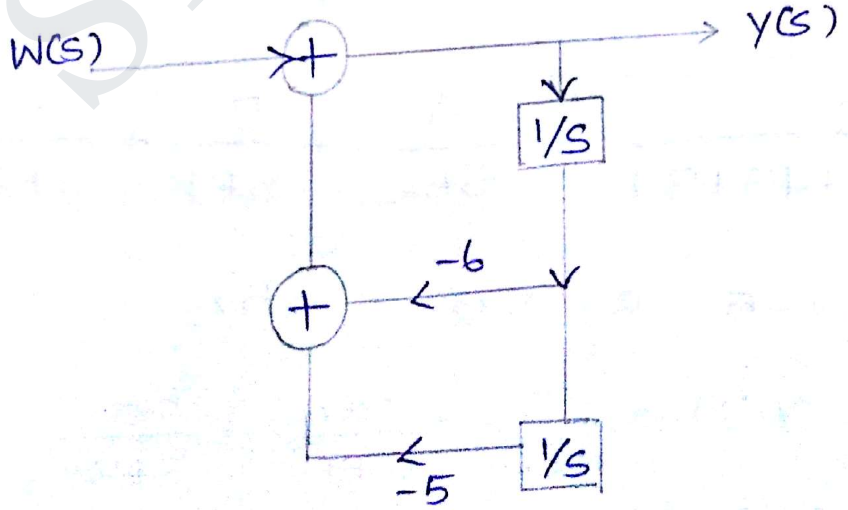
Let $Y(s)+\frac{6Y(s)}{s}+\frac{5Y(s)}{s^2} = W(s)$ .... (1)

$$Y(s) = W(s)-\frac{6Y(s)}{s}+\frac{5Y(s)}{s^2} \quad \text{.....(2)}$$

Block diagram representation of (1)



Block diagram representation of (2)



4. For LTI system with $H(s) = \frac{s+5}{s^2+4s+3}$ Find the differential equation. Find the system output $y(t)$ to the input $x(t) = e^{-2t}u(t)$. [A-15] 5

Sol. $H(s) = \frac{Y(s)}{X(s)} = \frac{s+5}{s^2+4s+3}$

$$Y(s)(s^2+4s+3) = X(s)(s+5)$$

$$s^2Y(s) + 4sY(s) + 3Y(s) = sX(s) + 5X(s)$$ 01

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 5x(t)$$

$$s^2Y(s) + 4sY(s) + 3Y(s) = sX(s) + 5X(s)$$

$$s^2Y(s) + 4sY(s) + 3Y(s) = \frac{s}{s+2} + \frac{5}{s+2}$$

$$Y(s)[s^2+4s+3] = \frac{3}{s+2} + \frac{5}{s+2}$$

$$Y(s) = \frac{s}{s+2} + \frac{5}{s+2} \cdot \frac{1}{s^2+4s+3}$$

$$Y(s) = \frac{s}{s+2} + \frac{5}{(s+2)(s^2+4s+3)}$$

$$\frac{5}{s+2(s^2+4s+3)} = \frac{A}{s+2} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$A = -5 \quad B = 5/2 \quad C = 5/2$$

$$Y(s) = \frac{-5}{s+2} + \frac{5/2}{s+1} + \frac{5/2}{s+3}$$

Taking Inverse Laplace Transform

$$y(t) = -5e^{-2t}u(t) + 5/2e^{-t}u(t) + 5/2e^{-3t}u(t) + te^{-2t}u($$

5 The input and output of a causal LTI System are related, by the differential equation $\frac{d^2y(t)}{dt^2} + 6\frac{d}{dt}y(t) + 8y(t) = 2x(t)$

i) Find the impulse response h(t)

ii) Find the response y(t) of the system if $x(t) = u(t)$.

[N-15]

Sol. Taking Fourier Transform on given differential equation.

$$(j\omega)^2 Y(\omega) + 6j\omega Y(\omega) + 8Y(\omega) = 2X(\omega)$$

$$Y(\omega)\left[(j\omega)^2 + 6j\omega + 8\right] = 2X(\omega)$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{2}{(j\omega)^2 + 6j\omega + 8}$$

$$= \frac{2}{(j\omega+4)(j\omega+2)}$$

$$H(\omega) = \frac{A}{j\omega+4} + \frac{B}{j\omega+2}$$

$$H(\omega) = \frac{-1}{j\omega+4} + \frac{1}{j\omega+2}$$

Taking Inverse Fourier Transform on both sides of above equation.

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

$$h(t) = \left[e^{-2t} - e^{-4t}\right]u(t).$$

6. Convolve the following signals. [N-16]

(i) $x(t) = e^{-3t} u(t)$

$h(t) = u(t+3)$

Sol. $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

$$= \int_{-\infty}^{\infty} e^{-3\tau} u(\tau)\, u(t+3-\tau) d\tau$$

In this case the product $u(\tau)\, u(t+3-\tau)$ exits for $0 < \tau < t+3$, so the limits of integration changes to

$$y(t) = \int_{0}^{t+3} e^{-3\tau} d\tau$$

$$y(t) = \begin{cases} \dfrac{1-e^{-3(t+3)}}{3}, & t > -3 \\ 0, & t < -3 \end{cases}$$

(ii) A system is described by differential equation $\frac{d^2}{dt^2} y(t) + 6 \frac{d}{dt} y(t) + 8y(t) = \frac{d}{dt} x(t) + x(t)$. Find the Transfer function and the output signal $y(t)$ for $x(t) = \delta(t)$

Sol. Taking Laplace Transform on both sides

$$s^2 Y(s) + 6s\, Y(s) + 8 Y(s) = s X(s) + X(s)$$

$$[s^2+6s+8]\, Y(s) = [1+s]\, X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+1}{s^2+6s+8}$$

$$H(S) = \frac{S+1}{(S+4)(S+2)} = \frac{A}{S+4} + \frac{B}{S+2}$$

Solving for A & B.

$$A = 3/2 \quad B = -1/2$$

$$H(S) = \frac{3/2}{S+4} + \frac{-1/2}{S+2}$$

$$Y(S) = H(S)\,X(S)$$

Laplace Transform of $\delta(t)$ is 1.

$$\delta(t) \xleftrightarrow{Lt} 1$$

$$Y(S) = H(S) \; 1$$

$$Y(S) = 3/2 \frac{1}{S+4} - 1/2 \frac{1}{S+2}$$

$$y(t) = 3/2\, e^{4t} u(t) - 1/2\, e^{-2t} u(t).$$

7. Using graphical Method convolve

$x(t) = e^{-2t} u(t)$ with $h(t) = u(t+2)$ [N-16]

Sol

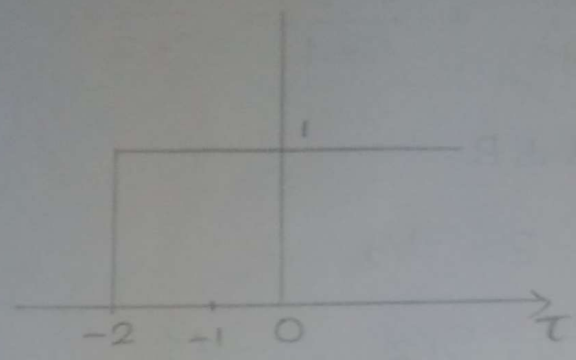
$$x(t) = e^{-2t} u(t)$$

$$h(t) = u(t+2)$$

$$y(t) = x(t) \times h(t)$$

Case-I : $t=0$

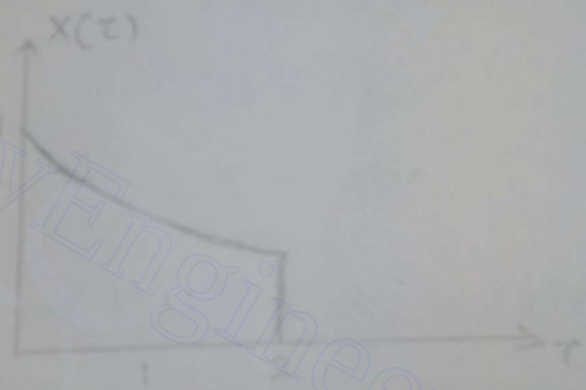
$$y(0) = \int_{-\infty}^{\infty} x(z)\, h(-z)\, dz$$



Case II, t>0

$$y(t) = \int_{0}^{t-1} e^{-2t}\,dt$$

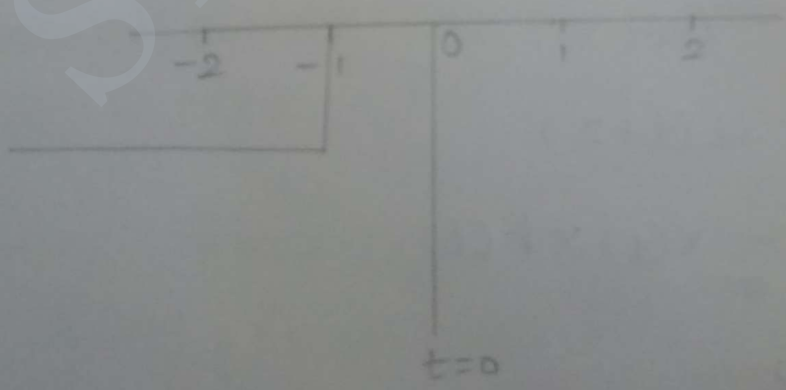
$$= \frac{1}{2}\left[1-e^{-2(t-1)}\right] \text{ for } t>0$$



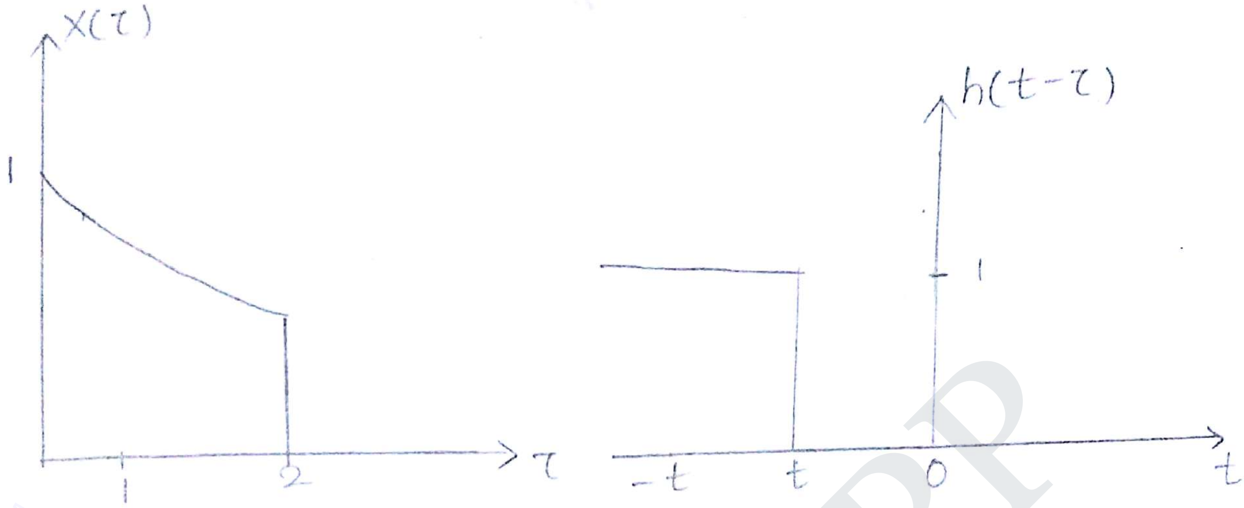
Case III: t<0

$$y(t) = \int_{-\infty}^{\infty} x(z)h(t-z)\,dz$$

$$x(z) = e^{-2z}$$



RESULT :



$$y(t) = \begin{cases} \frac{1}{2}(1 - e^{-2t}) & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Unit - IV

1. $x(n) = \delta(n) + \delta(n-1)$. Find the DTFT (N/D - 14)

$$x(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n} + \sum_{n=-\infty}^{\infty} \delta(n-1) e^{-j\omega n}$$

$$= 1 + \sum_{n=-\infty}^{\infty} \delta(n-1) e^{-j\omega n}$$

$$= 1 + e^{-j\omega n}$$

2. Time folding Property of z - Transform. (N/D - 14)

if $x(z) = z[x(n)]$ then

$$z\{x(-n)\} = X(z^{-1})$$

Proof: $z\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$.

let $l = -n$ Then

$$= z\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(l) (z^{-1})^{-l}$$

$$= X(z^{-1}).$$

3. Determine the Nyquist sampling rate for $x(t)$ $= \sin(200\pi t) + 3\sin^2(120\pi t)$ (A/M - 15)

Let $x(t) = \sin(2\pi f_1 t) + 3\sin^2(2\pi f_2 t)$

Here $f_1 = 100$, $f_2 = 60$

Nyquist sampling rate $= 240$ Hz.

4. List the methods used for finding inverse z transform. (A/M-15)

* Long division
* Partial Fraction expansion.
* Residue theorem.

5. State the need for sampling. (N/D-15)

In signal processing, sampling is the reduction of a continuous signal to a discrete signal.

6. Distinguish between Recursive and Non Recursive system. (N/D-15)

Recursive	Non Recursive
When the output $y(n)$ of the depends upon present and past input as well as past output, then it is called recursive system.	When the output $y(n)$ of the system depends upon present and past input, the it is called non-recursive system.

7. Find the Nyquist rate of the signal $x(t) = \sin 200\pi t - \cos 100\pi t$. (N/D-16)

Nyquist rate $\Omega_s = 2\Omega_B$

$$\Omega_s = 2 \times 200\pi = 400\pi.$$

8. Find the z-transform of the signal and its associated ROC $x(n) = \{2, -1, \underset{\uparrow}{3}, 0, 2\}$. (N/D-16)

$$X(z) = 2z^2 - z + 3 + 2z^{-2}$$

ROC : Entire z plane except $z=0$ and $z=\infty$.

16 marks :

Part - B :

1. State and Prove Samping theorem for a band limited signal. (N/D-14)

Solution :

A bandlimited signal of finite energy, which has no frequency components higher than W hertz, is completely described by specifying the values of the signal at instants of time seperated by $\frac{1}{2W}$ seconds.

A bandlimited signal of finite energy, which has no frequency components higher than W hertz, may be completely recovered from the knowledge of its samples taken at the rate of 2W samples per second.

The first part of above statement tells about sampling of the signal and second part tells above reconstuction of the signal.

$f_s \geq 2W$

f_s = Sampling frequency

W = Higher frequency content.

Proof of sampling theorem :

Representation of $x(t)$ interms of its samples.

Reconstruction of $x(t)$ from its samples.

Part I : Representations of $x(t)$ interms of its samples. $x(nT_s)$.

Step 1 : Define $x_\delta(t)$

Step 2 : Fourier transform of $x_\delta(t)$ i.e, $X_\delta(f)$

Step 3 : Relation between $X(f)$ and $X_\delta(f)$

Step 4 : Relation between $x(t)$ and $x(nT_s)$

Step 5 : Define $x_\delta(t)$.

The sampled signal $x_\delta(t)$ is given as

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\, \delta(t - nT_s)$$

Step 2 : FT of $x_\delta(t)$ ie. $X_\delta(f)$

Taking FT of equation

$$X_\delta(f) = FT\left[\sum_{n=-\infty}^{\infty} x(t) \cdot \delta(t - nT_s)\right].$$

$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s).$$

Since convolution is linear

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f-nf_s)$$

$$= f_s \sum_{n=-\infty}^{\infty} X(f-nf_s) \quad \text{By shifting Property}$$

$$= f_s X(f-2f_s) + f_s X(f-f_s) + f_s X(f) + f_s X(f-f_s) + \ldots$$

$X(z) = \dfrac{z^{-1}}{1-0.25z^{-1}-0.375z^{-2}}$ find universe z-transform for (i) ROC $|z| > 0.75$. (ii) ROC $|z| < 0.5$. (N/D-14)

Given $X(z) = \dfrac{z^{-1}}{1-0.25z^{-1}-0.375z^{-2}}$

$$= \frac{z^{-1}}{z^{-2}(z^2-0.25z-0.375)}$$

$$= \frac{z}{(z-0.75)(z+0.5)}.$$

Apply Partial Fraction,

$$\frac{X(z)}{z} = \frac{A}{z-0.75} + \frac{B}{z+0.5}.$$

$$A = (z-0.75)\left.\frac{1}{(z-0.75)(z+0.5)}\right|_{z=-0.5}.$$

$A = 0.8$.

$$B = (z-0.5) \left. \frac{1}{(z-0.75)(z+0.5)} \right|_{z=-0.5}.$$

$B(-0.8)$

$$\frac{X(z)}{z} = \frac{0.8}{z-0.75} - \frac{0.8}{z+0.5}.$$

$$X(z) = 0.8 \frac{z}{z-0.75} - 0.8 \frac{z}{z+0.5}.$$

(i) ROC is $|z| > 0.75$.

It is causal esquence

$\therefore x(n) = 0.8(0.75)^n u(n) - 0.8(0.5)^n u(n)$.

ii) ROC $|z| < 0.5$.

Now causel sequence.

$\therefore x(n) = 0.8(0.75)^n u(n) - 0.8(0.5)^n u(n-1)$.

3. Prove the following DTFT Properties.

i) $nx(n) \leftrightarrow \frac{jd\,X(\Omega)}{d\Omega}$ (A/M-15)

$$F\{x(n)\} = X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}.$$

Differentiating both sides with respect to 'w' we get

$$\frac{d}{dw}[X(w)] = \frac{d}{dw}\left\{\sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}\right\}.$$

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{d}{dw} e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} x(n) (-jn) e^{-jwn}$$

$$= -j \left\{ \sum_{n=-\infty}^{\infty} n \left[x(n) e^{-jwn} \right] \right\}$$

$$\therefore \sum_{n=-\infty}^{\infty} n\, x(n) e^{-jwn} = F[n x(n)]$$

$$= j \frac{d}{dw} [X(w)].$$

ii) $x(n) e^{j\Omega_c n} \longleftrightarrow X(\Omega - \Omega_c)$. (A/M-15)

$$F\{x(n) \cdot e^{jw_0 n}\} = \sum_{n=-\infty}^{\infty} \{x(n) \cdot e^{jw_0 n}\} e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j(w-w_0)n}$$

$$= X(w - w_0)$$

This property is the dual of the time shifting property.

4. A continuous time sinusoid $\cos(2\pi f t + \theta)$ is sampled at a rate $f_s = 1000$ Hz. Determine the resulting signal samples if the input signal frequency f is 400 Hz, 600 Hz and 100 Hz respectively. (A/M-15)

Given :

$x(t) = \cos(2\pi f t + \theta)$

$f_s = 1000$ Hz.

I/P signal frequency ranges are 400 Hz, 600Hz and 1000 Hz respectively.

Solution :

I case :

$f_m = 400$ Hz

$f_s = 2f_m$

$= 2 \times 400$

$f_s = 800$ samples/sec.

II case :

$f_m = 600$ Hz.

$f_s = 2f_m$

$= 2 \times 600$

$f_s = 1200$ samples/sec.

III case :

$f_s = 2f_m$

$= 2 \times 1000$

$f_s = 2000$ samples/sec.

5. Find the DTFT of $x(n) = \left(\frac{1}{2}\right)^{n-2} u(n-2)$ (A/M-15)

$$X(\omega) = F\{x(n)\} = \sum_{n=-\infty}^{\infty} \left[\left(\frac{1}{2}\right)^{n-2} u(n-2)\right] e^{-j\omega n}$$

$$= \sum_{n=2}^{\infty} \left(\frac{1}{2}\right)^{n-2} e^{-j\omega n}.$$

$$= e^{-j2\omega} + \frac{1}{2} e^{-j3\omega} + \left(\frac{1}{2}\right)^2 e^{-j4\omega} + \cdots$$

$$= e^{-j2\omega} \left[1 + \frac{1}{2} e^{-j\omega} + \left(\frac{1}{2}\right)^2 e^{-j2\omega} + \cdots\right]$$

$$= \frac{e^{-j2\omega}}{1 - \left(\frac{1}{2}\right) e^{-j\omega}}$$

6. Using suitable z transform properties find $X(z)$ if $x(n) = (n-2)\left(\frac{1}{3}\right)^{n-2} u(n-2)$ (A/M-15)

Solution:

$$x(n) = (n-2)\left(\frac{1}{3}\right)^{n-2} u(n-2)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (n-2)\left(\frac{1}{3}\right)^{n-2} u(n-2) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (n-2)\left(\frac{1}{3}\right)^{n}\left(\frac{1}{3}\right)^{-2} u(n-2) z^{-n}$$

$$= \left(\frac{1}{3}\right)^{-2} \sum_{n=-\infty}^{\infty} (n-2) \left(\frac{1}{3}\right)^{n} u(n-2) z^{-n}.$$

$$\text{Let } (x, z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{n} z^{-n}$$

$$= \sum_{n=+2}^{\infty} \left(\frac{1}{3} z^{-1}\right)^{n}$$

$$= \frac{\left(\frac{1}{3} z^{-1}\right)^{2} - \left(\frac{1}{3} z^{-1}\right)^{\infty+1}}{1 - \frac{1}{3} z^{-1}}$$

$$= \frac{\left(\frac{1}{3} z\right)^{2} - \left(\frac{1}{3} z\right)^{1}}{\frac{3z^{-1}}{3z}} = \frac{(3z)^{2} - 1}{3z - 1}$$

$$= z \left\{ \sum_{n=2}^{\infty} n \left(\frac{1}{3}\right)^{n} z^{-n} \right\} = -z \frac{d}{dz} \left\{ \frac{(3z)^{2} - 1}{3z - 1} \right\}$$

$$= \left(\frac{1}{3}\right)^{-2} (-z) \frac{d}{dz} \left\{ \frac{(3z)^{2} - 1}{3z - 1} \right\}$$

$$= \left(\frac{1}{3}\right)^{-2} (-z) \left\{ \frac{(3z-1)(3^{2} \cdot 2z) - (3z)^{2} \cdot 3}{(3z-1)^{2}} \right\}$$

$$= \left(\frac{1}{3}\right)^{-2} \left\{ \frac{54z^{3} - 18z^{2} - 27z^{2} - 3z}{3z - 1} \right\}.$$

$$X(z) = -\left(\frac{1}{3}\right)^{-2} \left\{ \frac{27z^{3} - 18z^{2} - 3z^{2}}{3z - 1} \right\}.$$

State and Prove any two Properties of DTFT and any two properties of z-Transform. (N/D - 15)

Properties of DTFT:

Linearity Property: $F[ax_1(n) + bx_2(n)] = aX_1(\omega) + bX_2(\omega)$

Proof: $F\{ax_1(n) + bx_2(n)\} = \sum_{n=-\infty}^{\infty} [ax_1(n) + bx_2(n)]e^{-j\omega n}$

$$= \sum_{n=-\infty}^{\infty} ax_1(n) \cdot e^{-j\omega n} + \sum_{n=-\infty}^{\infty} bx_2(n) \cdot e^{-j\omega n}$$

$$= aX_1(\omega) + bX_2(\omega).$$

Time Reversal Property:

$$F\{x(n)\} = X(\omega)$$

$$F\{x(-n)\} = X(-\omega)$$

Proof: $F\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n) \cdot e^{-j\omega n}$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(-\omega)n}$$

$$= X(-\omega).$$

Properties of z-Transform :

Linearity :

$$x_1(n) \xleftrightarrow{z} X_1(z)$$

$$x_2(n) \xleftrightarrow{z} X_2(n) .$$

Proof :

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [a_1 x_1(n) + a_2 x_2(n)] z^{-n} .$$

$$= \sum_{n=-\infty}^{\infty} a_1 x_1(n) z^{-n} + \sum_{n=-\infty}^{\infty} a_2 x_2(n) z^{-n} .$$

$$= a_1 \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} + a_2 \sum_{n=-\infty}^{\infty} x_2(n) z^{-n} .$$

$$X(z) = a_1 X_1(z) + a_2 X_2(z) .$$

Time Reversal :

$$x(n) \xleftrightarrow{z} X(z)$$

$$x(-n) \xleftrightarrow{z} X(z^{-1})$$

Proof

$$z\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(m) z^{m}$$

$$= \sum_{n=-\infty}^{\infty} x(m) z^{-1})^{m} .$$

$$= X(z^{-1}) \quad ROC; \; r_1 < |z^{-1}| < r_2 .$$

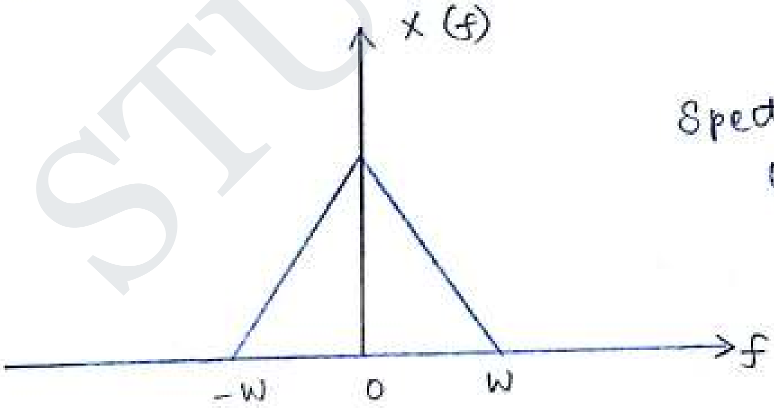
8. Discuss the effects of undersampling a signal using necessary diagrams. (N/D-16)

While proving sampling theorem we considered that $f_s = 2W$ consider the case of $f_s < 2W$, then spectrum of $X_\delta(f)$ will be modified as follows:

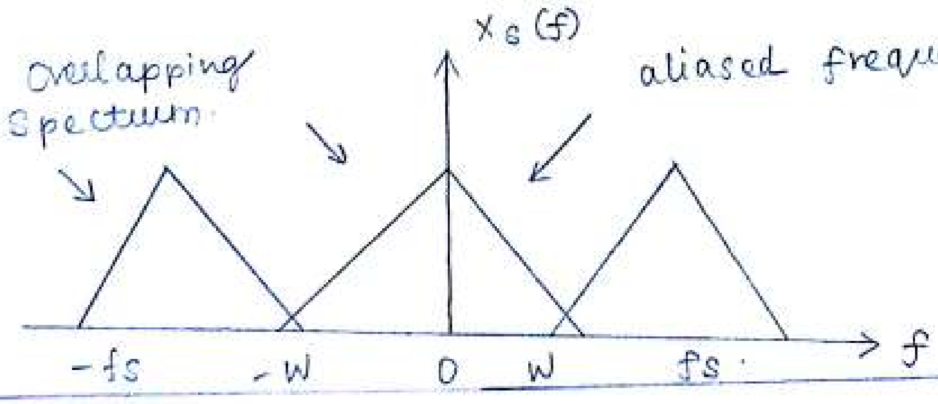
i) The spectrum located at $X(f), X(f-f_s), X(f-2f_s)$ ... overlap on each other.

ii) consider the spectrums of $X(f)$ and $X(f-f_s)$ is magnified.

iii) The high frequency near 'w' in $X(f-f_s)$ overlap with low frequencies $(f_s - W)$ in $X(f)$.



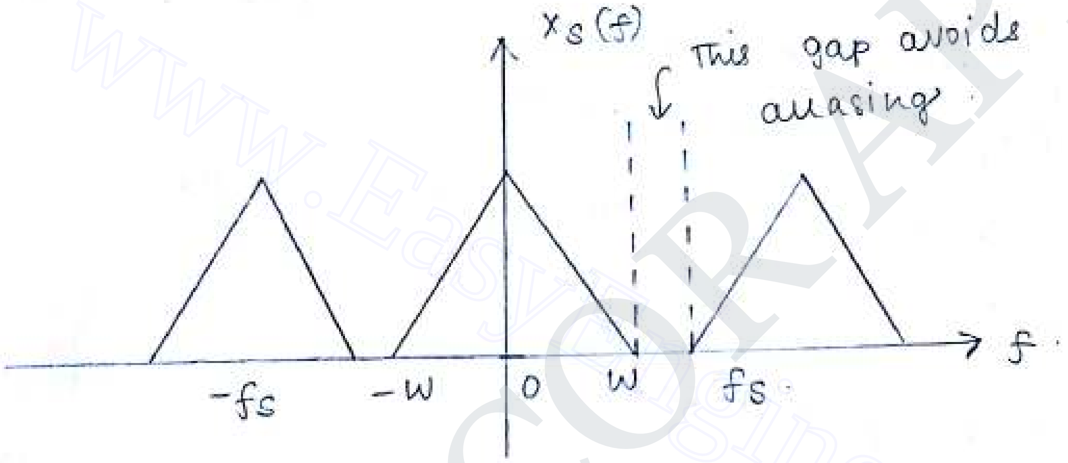
Spectrum of original signal.



Different ways to avoid aliasing:

i) Sampling rate $f_s \geq 2W$:

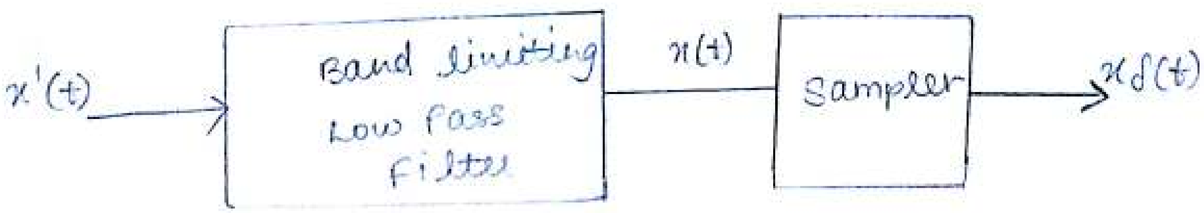
When the sampling rate is made higher than 2W, then the spectrums will not overlap and there will be sufficient gap between the individual spectrum.



$f_s \geq 2W$ Avoids aliasing by creating a band gap.

ii) Bandlimiting the signal:

The sampling rate is $f_s = 2W$. There can be for components higher than 2W. These components creating aliasing.



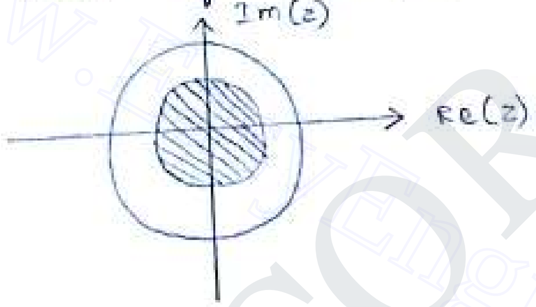
9. Find the z-transform of $x[n] = a^n u(n) - b^n u(-n-1)$ and specify its ROC. (N/D-16)

$$a^n u(n) \xleftrightarrow{ZT} \frac{z}{z-a}$$

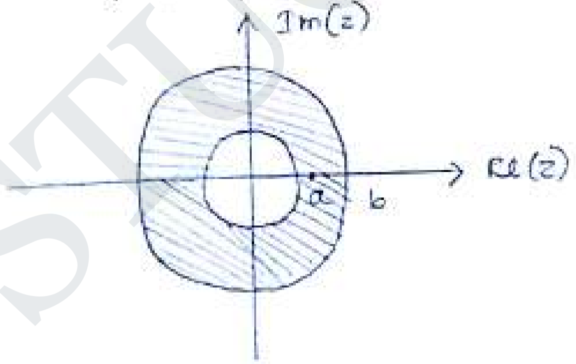
$$-b^n u(-n-1) \xleftrightarrow{ZT} \frac{z}{z-b}$$

case 1 : $|b| < |a|$.

The ROC do not overlap so z transform do not exit if $|b| < |a|$



case 2 : $|b| > |a|$



ROC : $|a| < |z| < |b|$

$$x(z) = \frac{z}{z-a} + \frac{z}{z-b}, \quad ROC : |a| < |z| < |b|.$$

UNIT-5

LINEAR TIME INVARIANT - DISCRETE TIME SYSTEMS

① Give the impulse response of a linear time invariant time as $h(n) = \sin \pi n$

The value of sine lies between 0 to 1 for any value of N hence bounded input produces bounded output. Thus the system is stable.

② In terms of ROC, state the condition for an LTI discrete time system to be causal and stable.

If $x(n)$ is causal sequence then the ROC is entire z plane except at $z=0$. The ROC of an LTI stable system contain the unit circle.

③ Name the basic building blocks used in LTIDT system block diagram.

* Adder
* Constant multiplier
* Signal multiplier
* Delay elements

④ Write the nth order difference equation.

$$P_n(t)\, y^{(n)} + P_{n-1}(t)\, y^{(n-1)} + \dots + P_1(t)\, y^{(t)} + P_0^{(t)} y = G(t)$$

⑤ Find the x transform and its associated ROC for $x(n) = \{1, -1, 2, 3, 4\}$

$x(0) = 1,\ x(1) = -1,\ x(2) = 2,\ x(3) = 3,\ x(4) = 4.$

By definition, $X(z) = \sum_{n=4}^{\infty} x(n)\, z^{-n}$

$$= \sum_{n=4}^{4} x(n)\, z^{-n}$$

$= x(0)z^{0} + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4}$

$X(z) = 1 - z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4}$

$$X(z) = 1 - \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \frac{4}{z^4}$$

⑥ Convolve the following signals, $x(n) = \{1,1,3\}$ and $h(n) = \{1,4,-1\}$

$$y(n) = x(n) * h(n)$$

$$\begin{array}{rccccc}
x(n) = & & & 1 & 1 & 3 \\
h(n) = & & & 1 & 4 & -1 \\
\hline
 & & & -1 & -1 & -3 \\
 & & 4 & 4 & 12 & \times \\
 & 1 & 1 & 3 & \times & \times \\
\hline
y(n) & 1 & 5 & 6 & 11 & -3
\end{array}$$

$$y(n) = \{1, 5, 6, 11, -3\}$$

⑦ Convolve the following sequences $x[n] = \{1,2,3\}$, $h[n] = \{1,1,2\}$.

$$\begin{array}{rccccc}
x(n) * h(n) = & & & 1 & 2 & 3 \\
 & & & 1 & 1 & 2 \\
\hline
 & & & 2 & 4 & 6 \\
 & & 1 & 2 & 3 & \\
 & 1 & 2 & 3 & & \\
\hline
 & 1 & 3 & 7 & 7 & 6 \\
\hline
\end{array}$$

$$y(n) = \{1, 3, 7, 7, 6\}$$

⑧ Given the system function $H(z) = 2 + 3z^{-1} + 4z^{-3} - 5z^{-4}$. Determine the impulse response $h[n]$.

Impulse response $h(n) = \{\underset{\uparrow}{2}, 3, 0, 4, -5\}$

PART-B

1) Compute $y(n) = x(n) + h(n)$ where

$x(n) = \left(\frac{1}{2}\right)^{-n}, u(n-2) h(n) = u(n-2)$

Solution:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{2} \left(\frac{1}{2}\right)^{-n} z^{-n}$$

$$= \sum_{n=0}^{2} \left(\frac{z}{2}\right)^{-n}$$

$$X(z) = 1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}$$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{n=0}^{2} z^{-n}$$

$$Y(z) = X(z) H(z)$$

$$= (1 + 2z^{-1} + 4z^{-2})(1 + z^{-1} + z^{-2})$$

$$= 1 + z^{-1} + z^{-2} + 2z^{-1} + 2z^{-2} + 2z^{-3} + 4z^{-2} + 4z^{-3} + 4z^{-4}$$

$$= 1 + 3z^{-1} + 7z^{-2} + 6z^{-3} + 4z^{-4}$$

$$Y(z) = 1 + 3z^{-1} + 7z^{-2} + 6z^{-3} + 4z^{-4}$$

2) LTI discrete time system $y(n) = \frac{3}{2} y(n-1) - \frac{1}{2} y(n-2) + x(n) + x(n-1)$ given an input $x(n) = \mu(n)$

(i) Find the transfer function of the system.

(ii) Find the impulse response of the system.

SOLUTION:

$$\frac{3}{2} y(n-1) - \frac{1}{2} y(n-2) + x(n) + x(n-1)$$

Taking z-transform on both sides,

$$Y(z) = \frac{3}{2} z^{-1} Y(z) - \frac{1}{2} z^{-2} Y(z) + z^{-1} X(z) + X(z)$$

$$Y(z) - \frac{3}{2} z^{-1} Y(z) - \frac{1}{2} z^{-1} Y(z) = X(z) + z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}\right] = X(z)\left[1 + z^{-1}\right]$$

Transfer function: $\dfrac{Y(z)}{X(z)} = \dfrac{1+z^{-1}}{1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}}$

(ii) Impulse Response:

$$x(n) = \delta(n)$$

$$\therefore x(z) = 1$$

$$y(n) - \frac{3}{2} y(n-1) + \frac{1}{2} y(n-2) = x(n) + x(n-1)$$

Apply z-transform,

$$Y(z) - \frac{3}{2} z^{-1} Y(z) + \frac{1}{2} z^{-2} Y(z) = X(z) + z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}\right] = X(z)\left[1 + z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}}$$

$$Y(z) = \frac{1+z^{-1}}{1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}}$$

$$Y(z) = \frac{(1+z^{-1})}{z^{-2}\left(\frac{1}{z^{-2}} - \frac{3}{2} \frac{z^{-1}}{z^{2}} + \frac{1}{2}\right)}$$

$$= \frac{z^{2}(1+z^{-1})}{z^{2} - \frac{3}{2} z + \frac{1}{2}} = \frac{z^{2}+2}{z^{2} - \frac{3}{2} z + \frac{1}{2}}$$

$$Y(z) = \frac{z(1+z)}{(z-1)(z-1/2)}$$

Apply partial fraction,

$$\frac{z(1+z)}{(z-1)(z-1/2)} = \frac{A}{z-1} + \frac{B}{z-1/2}$$

$$\frac{Y(z)}{z} = \frac{1+z}{(z-1)(z-1/2)}$$

$$(1+z) = A\left(z-\frac{1}{2}\right) + B(z-1)$$

put $z = \frac{1}{2}$

$$1+\frac{1}{2} = B\left(-\frac{1}{2}\right)$$

$$B = -3$$

put $z = 1$

$$2 = A\left(\frac{1}{2}\right); A = 4$$

$$\frac{Y(z)}{z} = \frac{4}{z-1} - \frac{3}{z-\frac{1}{2}}$$

$$Y(z) = 4\frac{z}{z-1} - 3\frac{z}{z-\frac{1}{2}}$$

Apply inverse Z-transform,

$$y(n) = 4u(n) - 3\left(\frac{1}{2}\right)^n u(n)$$

3 (i) Determine the impulse response and step response if $y(n)+y(n-1)-2y(n-2) = x(n-1)+2x(n-2)$ for unit input signal.

SOLUTION:

(i) System function H(z)

Given difference equation is

$$y(n)+y(n-1)-2y(n-2) = x(n-1)+2x(n-2)$$

Taking z transform of above equation

$$Y(z)+z^{-1}Y(z)-2z^{-2}Y(z) = z^{-1}X(z)+2z^{-2}X(z)$$

$$Y(z)\left[1+z^{-1}-2z^{-2}\right] = X(z)\left[z^{-1}+2z^{-2}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{z^{-1}+2z^{-2}}{1+z^{-1}-2z^{-2}}$$

$$H(z) = \frac{z^{-1}+2z^{-2}}{1+z^{-1}-2z^{-2}}$$

(ii) Impulse response h(n)

By taking inverse z transform of H(z), we can find h(n).

$$H(z) = \frac{(z^{-1}+2z^{-2})z^2}{z^2+z-2}$$

$$= \frac{(z^{-1}+2z^{-2})z^2}{z^2+z-2}$$

Divide by z,

$$\frac{H(z)}{z} = \frac{(z^{-1}+2z^{-2})z}{z^2+z-2}$$

$z^2+2z-z-2$

$z(z+2)\ -1(z+2)$

$$\frac{H(z)}{z} = \frac{A}{(z+2)} + \frac{B}{(z-1)}$$

$$A = (z+2)\left.\frac{H(z)}{z}\right|_{z=-2}$$

$$A = \cancel{(z+2)}\left.\frac{(z^{-1}+2z^{-2})z}{\cancel{(z+2)}(z-1)}\right|_{z=-2}$$

$$= \frac{\left(-\frac{1}{2}+\frac{2}{4}\right)(-2)}{(-2-1)}$$

$$= \frac{0}{-3} = 0$$

$$B = (z-1)\left.\frac{H(z)}{z}\right|_{z=1}$$

$$B = \cancel{(z-1)} \left. \frac{(z^{-1}+2z^{-2})z^2}{(z+2)\cancel{(z-1)}} \right|_{z=1}$$

$$= \frac{(1+2/1)z^2}{(1+2)} = \frac{3}{3} = 1$$

$$\therefore B = 1$$

After substituting the values of A and B,

$$\frac{H(z)}{z} = \frac{0}{(z+2)} + \frac{1}{(z-1)}$$

$$\therefore H(z) = \frac{z}{z-1} = \frac{z\,1}{(1-z^{-1})z}$$

Taking inverse z transforms

$$h(n) = u(n)$$

(iii) Step Response:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(z^{-1}+2z^{-2})z^2}{z^2+z-2}$$

$$Y(z) = \frac{(z^{-1}+2z^{-2})z^2}{z^2+z-2} X(z)$$

For step input $x(n) = u(n)$

Taking z transform of above equation

$$X(z) = \frac{1}{1-z^{-1}}$$

$$X(z) = \frac{z}{z-1}$$

$$Y(z) = \frac{(z^{-1}+2z^{-2})z^2}{z^2+z-2} \cdot \frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{(z^{-1}+2z^{-2})z^{2}}{z^{2}+z-2} \cdot \frac{1}{z-1}$$

$$\frac{Y(z)}{z} = \frac{A}{(z+2)} + \frac{B}{(z-1)} + \frac{C}{(z-1)}$$

$$A = 0 \quad B = 3 \quad C = 3$$

$$\frac{Y(z)}{z} = \frac{0}{(z+2)} + \frac{3}{(z-1)} + \frac{3}{(z-1)}$$

$$Y(z) = \frac{3z}{(z-1)} + \frac{3z}{(z-1)}$$

$$Y(z) = \frac{6z}{(z-1)} = \frac{6z^{2}}{1-z^{-1}}$$

Taking inverse z transform,

(ii) Find the convolution sum between $x(n)=\{1,4,3,2\}$ and $h(n)=\{1,3,2,1\}$

$x(0)=1$ $\quad$ $h(0)=1$

$x(1)=4$ $\quad$ $h(1)=3$

$x(2)=3$ $\quad$ $h(2)=2$

$x(3)=2$ $\quad$ $h(3)=1$

	$h(0)$	$h(1)$	$h(2)$	$h(3)$
$x(0)$	1	3	2	1
$x(1)$	4	12	8	4
$x(2)$	3	9	6	3
$x(3)$	2	6	4	2

$y(n) = \{1, 7, 17, 20, 16, 7, 2\}$

4 (i) Causal system has $x(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2)$ and $y(n) = \delta(n) - \frac{3}{4}\delta(n-1)$. Find the impulse response and output if $x(n) = \left(\frac{1}{2}\right)^n u(n)$.

Solution:

$$H(z) = \frac{Y(z)}{X(z)}$$

$$X(z) = 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}$$

$$Y(z) = 1 - \frac{3}{4}z^{-1}$$

$$\frac{Y(z)}{X(z)} = \frac{1 - 3/4\, z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$H(z) = \frac{4z - 3}{8z^2 + 2z - 1}$$

$h(n)$ is calculated by taking inverse z transform of above expression.

(ii) Compare recursive and non recursive systems.

Non-recursive system:

When the output $y(n)$ of the system depends upon present and past inputs, then it is called non recursive system.

Recursive system:

When the output $y(n)$ of the system depends upon present and past input as well as past output then it is called recursive system.

(5) Consider an LTI system with impulse response $h(n) = \alpha^n u(n)$ and the input to this system is $x(n) = \beta^n u(n)$ with $|\alpha|$ and $|\beta| < 1$. Determine the response $y(n)$.

(i) when $\alpha = \beta$ (ii) when $\alpha \neq \beta$ using DTFT.

Solution:

(i) $H(\omega) = F\{\alpha^n u(n)\}$

$$= \sum_{n=-\infty}^{\infty} \alpha^n u(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^n = \frac{1}{1-\alpha e^{-j\omega}}$$

(ii) Response $y(t)$ of the system.

$x(t) = u(t)$

$X(\omega) = \frac{1}{j\omega}$

$$Y(\omega) = \frac{2}{(j\omega+4)(j\omega+2)} \times (\omega)$$

$$Y(\omega) = \frac{2}{(j\omega)(j\omega+4)(j\omega+2)}$$

$$Y(\omega) = \frac{A}{j\omega} + \frac{B}{j\omega+4} + \frac{C}{j\omega+2}$$

$$A = j\omega Y(\omega)\Big|_{j\omega=0}$$

$$= j\omega \cdot \frac{2}{j\omega(j\omega+4)(j\omega+2)}\Big|_{j\omega=0}$$

$A = \frac{1}{4}$ $B = -1$ $C = 1$

$$Y(\omega) = \frac{1/4}{j\omega} - \frac{-1}{j\omega+4} + \frac{1}{j\omega+2}$$

Taking Inverse Fourier transform

$$y(t) = \frac{1}{4}u(t) - e^{-4t}u(t) + e^{-2t}u(t)$$

$$y(t) = \left[\frac{1}{4} - e^{-4t} + e^{-2t}\right]u(t)$$

$$H(\omega) = \frac{1}{1-\alpha e^{-j\omega}}$$

$$X(\omega) = F\{\beta^n u(n)\}$$

$$= \sum_{n=-\infty}^{\infty} \beta^n u(n) . e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (\beta e^{-j\omega})^n$$

$$X(\omega) = \frac{1}{1-\beta e^{-j\omega}}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$$Y(\omega) = X(\omega)H(\omega)$$

$$Y(\omega) = \frac{1}{1-\alpha e^{-j\omega}} \cdot \frac{1}{1-\beta e^{-j\omega}}$$

6. Convolute the following signals

$$x[n] = u[n] - u[n-3]$$

$$h[n] = (0.5)^n u[n]$$

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \{u(k) - u(k-3)\}\{(0.5)^{n-k} u(n-k)\}$$

$$= (0.5)^n \sum_{K=-\infty}^{\infty} (0.5)^{-K} u[K] u[n-K] - (0.5)^{-K} u(K-3) u(n-K)]$$

The product $u(K)\,u(n-K)$ exists for $0 < K < n$ and the product $u(K-3)\,u(n-K)$ exists for $3 < K < n$ so we change the limits to

$$y(n) = (0.5)^n \left[\sum_{K=0}^{n} (0.5)^{-K} - \sum_{K=3}^{n} (0.5)^{-K} \right] u(n)$$

$$y(n) = \begin{cases} 0 & n < 0 \\ (0.5)^n \sum_{K=0}^{n} (0.5)^{-K} & n = 0,1,2 \\ (0.5)^n \sum_{K=0}^{2} (0.5)^{-K} & n \geq 3 \end{cases}$$

$$y(n) = \{1, 3/2, 7/4, 7/8, \ldots\}$$

(8) Determine whether the given system is stable by finding H(z) and plotting the pole-zero diagram.

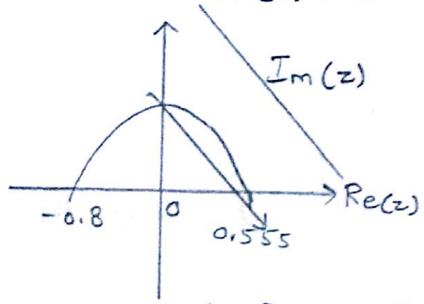
$$y[n] = 2y[n-1] - 0.8y[n-2] + x[n] + 0.8x[n-1]$$

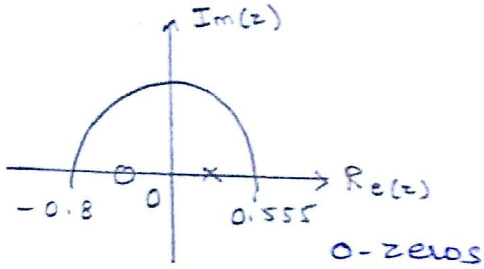
Taking z transform on both the sides

$$Y[z] = 2z^{-1}Y(z) - 0.8z^{-2}Y(z) + X(z) + 0.8z^{-1}x(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 0.8z^{-1}}{-0.8z^{-2} + 2z^{-1} + 1}$$

Zeros of H(z) are $z = 6$ and $z = -0.8$ poles of H(z): $z = 1.445$ & $z = 0.555$





Since one pole is outside the unit circle, the system is unstable