

MA3251

STATISTICS AND NUMERICAL METHODS UNIT I

TEST OF HYPOTHESIS

PART-A

1. Write the application of F-test and χ^2 (chi-square test)

F-test

- F-distribution is used to test the equality of the variances.

χ^2 (Chi-square test)

- χ^2 Is used to test the goodness of fit.
- χ^2 Is used to test the independence of attributes.

2. Write two applications of χ^2 test

- χ^2 Is used to test the goodness of fit.
- χ^2 Is used to test the independence of attributes.

3. Write the use of 't' distribution

The t-distribution is used to test the significance of the difference between

- The mean of a small sample and the mean of the population.
- The means of two small samples
- The coefficient of correlation in the small sample and that in the population, assumed zero.

4. Define errors in sampling and critical region.

Type I Error

Reject H_0 when it is true.

Type II Error

Accept H_0 when it is false.

5. Define F- variate.

A random variate F is said to follow Snedecor's F-distribution or simply F-distribution if its probability density function is given by

$$f(F) = \frac{(\gamma_1/\gamma_2)^{\frac{\gamma_1}{2}}}{\beta\left(\frac{\gamma_1}{2}, \frac{\gamma_2}{2}\right)} \cdot \frac{F^{\frac{\gamma_1}{2}-1}}{\left(1 + \frac{\gamma_1 F}{\gamma_2}\right)^{\left(\frac{\gamma_1+\gamma_2}{2}\right)}}, F > 0$$

6. A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance?

Solution:

Given $n_1 = 13$, $n_2 = 15$, $s_1^2 = 3$, $s_2^2 = 2.5$, $\gamma_1 = 12$, $\gamma_2 = 14$

$H_0: S_1^2 = S_2^2$ (i.e. the two samples have been drawn from population with the same Variance)

$H_1: S_1^2 \neq S_2^2$

$$F = \frac{S_1^2}{S_2^2} = \frac{3}{2.5} = 1.2$$

$F_{0.05} (\gamma_1 = 12, \gamma_2 = 14) = 2.53$ and $F < F_{0.05}$

Therefore H_0 is accepted

(i.e) the two samples could have from two normal population with the same variance

7. A machine runs on an average of 125 hours/year. A random sample of 49 machines has an annual average use of 126.9 hours with standard deviation of 8.4 hours. Does this suggest to believe that machines are used on the average more than 125 hours annually at 0.05 level of significance?

Solution:

Given $n = 49$, $\bar{x} = 126.9$, $\mu = 125$, $\sigma = 8.4$

Null Hypothesis: $H_0: \bar{x} = \mu$

Alternative Hypothesis: $H_1: \bar{x} > \mu$ (one-tailed test)

Test statistics is

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{126.9 - 125}{\frac{8.4}{\sqrt{49}}} = 1.583$$

Tabulated value $z_\alpha = 1.645$

$$|z| < z_\alpha$$

Therefore, H_0 is accepted.

(i.e) the difference between \bar{x} and μ is not significant.

8. For 2×2 contingency table, value. (DEC 2012)

a	b
c	d

Write down the corresponding χ^2

Solution:

The value of χ^2 for the 2×2 contingency table is given by

$$\chi^2 = \frac{N(ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

9. Write down t-test significance single mean formula for given sample and population mean \bar{x} and μ respectively.

Solution:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

10. Write down the formula of test statistic t to test the significant difference between the means of large samples.

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{S^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

PART-B

Test of significance for large samples

Z-Test (Sample size is more than or equal to 30)

Working Procedures

1. From the problem context, identify the parameter of interest
2. State the null hypothesis, H_0
3. Specify an appropriate alternative hypothesis, H_1
4. Choose a significant level α
5. Determine an appropriate test statistic
6. State the rejection region for the statistic
7. Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute the value.
8. Conclusion : Decide whether or not, H_0 should be rejected and report that in the problem context

Test for single mean ($n \geq 30$)

1. A sample of 900 members has a mean 3.4 cm and standard deviation 2.61 cm. Is the sample from a large population of mean 3.25 cm? (Test at 5% level of significance. The value of z at 5% level is $|z_\alpha| < 1.96$)

Solution: Given $n = 900$, $\mu = 3.25$, $s = 2.61$, $\bar{x} = 3.4$, $\alpha = 5\%$

1. $H_0 : \mu = 3.25$

2. $H_1 : \mu \neq 3.25$ (Two tail test)
3. $\alpha = 5\%$
4. Critical region :
5. The test statistic

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}} = 1.724$$

6. Conclusion : Cal. Z 1.724 < Tab.Z 1.96
Accept H_0
7. 95% confidence limits are

$$\bar{x} \pm 1.96 \frac{s}{\sqrt{n}} = 3.4 \pm (1.96) \left(\frac{2.61}{\sqrt{900}} \right) = 3.4 \pm 0.1705 = 3.57 \text{ and } 3.2295$$

- 2. The mean breaking strength of the cables supplied by a manufacturer is 1800 with a S.D. of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased In order to test this cl sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we claim at 1% level of significance?**

Solution : Given $n = 50$, $\mu = 1800$, $s = 100$, $\bar{x} = 1850$, $\alpha = 1\%$

1. $H_0 : \mu = 1800$
2. $H_1 : \mu > 1800$ (Use one-tailed test (right))
3. $\alpha = 1\%$
4. Critical region :
5. The test statistic

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1850 - 1800}{\frac{100}{\sqrt{50}}} = 3.54$$

6. Conclusion : Cal. Z 3.54 > Tab.Z 2.33
Reject H_0
We may support the claim of increase in breaking strength

3. An insurance agent has claimed that the average age of policy holders who insure through him is less than the average for all agents which is 30.5 years. A random sample of 100 policy holders who had insured through him reveal that the mean and s.d are 28.8 years and 6.35 years respectively. Test his claim at 5% level of significance.

Solution : Given $n = 100$, $\mu = 30.5$, $s = 6.35$, $\bar{x} = 28.8$, $\alpha = 5\%$

- 1 $H_0 : \mu = 30.5$
- 2 $H_1 : \mu < 30.5$ (Use one-tailed test (left))
- 3 $\alpha = 5\%$
- 4 Critical region :
- 5 The test statistic

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$= \frac{28.8 - 30.5}{\frac{6.35}{\sqrt{100}}}$$

$$= -2.68$$

- 6 Conclusion : Cal. $|Z| = 2.68 > \text{Tab.} Z 1.645$
Reject H_0

Test for Difference of two means ($n \geq 30$)

4. Examine whether the difference in the variability in yields is significant at 5% level of significance for the following:

	Set of 40 plots	Set of 60 plots
Mean yield per plot	1258	1243
S.D per plot	34	28

Solution: Given

$$n_1 = 40, \bar{x}_1 = 1258, s_1 = 34$$

$$n_2 = 60, \bar{x}_2 = 1243, s_2 = 28$$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 \neq \mu_2$
3. $\alpha = 5\%$
4. Critical region:
5. The test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1258 - 1243}{\sqrt{\frac{1156}{40} + \frac{784}{60}}} = 2.32$$

6. Conclusion: Cal. $|Z| = 2.32 > \text{Tab.} Z 1.96$

Reject H_0 .

5. A simple sample of heights of 6400 Englishmen has a mean of 170 cm and a standard deviation of 6.4 cm, while a simple sample of heights of 1600 Australians has a mean of 172 cm and a standard deviation of 6.3 cm. Do the data indicate that Australians are on the average taller than Englishmen?

Solution: Given

$$\begin{aligned} n_1 &= 6400, \bar{x}_1 = 170, s_1 = 6.4 \\ n_2 &= 1600, \bar{x}_2 = 172, s_2 = 6.3 \end{aligned}$$

1. $H_0: \mu_1 = \mu_2$
2. $H_1: \mu_1 < \mu_2$ (One tailed test(Left))
3. $\alpha = 5\%$
4. Critical region:
5. The test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{170 - 172}{\sqrt{\frac{(6.4)^2}{6400} + \frac{(6.3)^2}{1600}}} = -11.32$$

6. Conclusion: Cal. $|Z| = 11.32 > \text{Tab. } Z 1.645$
Reject H_0 .

7. A sample of 100 bulbs of brand A gave a mean lifetime of 1200 hours with a standard deviation of 70 hours, while another sample of 120 bulbs of brand B gave a mean lifetime of 1150 hours with a standard deviation of 85 hours. Can we conclude that brand A bulbs are superior to brand B bulbs?

Solution: Given

$$\begin{aligned} n_1 &= 100, \bar{x}_1 = 1200, s_1 = 70 \\ n_2 &= 120, \bar{x}_2 = 1150, s_2 = 85 \end{aligned}$$

1. $H_0: \mu_1 = \mu_2$
2. $H_1: \mu_1 > \mu_2$ (One tailed test(Right))
3. $\alpha = 5\%$
4. Critical region:
5. The test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1200 - 1150}{\sqrt{\frac{(70)^2}{100} + \frac{(85)^2}{120}}} = 4.79$$

6. Conclusion: Cal. $|Z| = 4.79 > \text{Tab. } Z 1.645$
Reject H_0 .

Test for single mean ($n < 30$)

t – test

7. A sample of ten house owners is drawn and the following values of their incomes are obtained, mean Rs. 6000 and S.D Rs. 650, test the hypothesis that the average income of house owners of the town is Rs. 5500.

Solution:

$$n = 10, \bar{x} = 6000, \mu = 5500$$

1. $H_0: \mu = 5500$
2. $H_1: \mu \neq 5500$ (Two tailed test)
3. $\alpha = 5\%$ with $d.f = n - 1 = 10 - 1 = 9$
4. Critical region:
5. The test statistic

$$t = \frac{x - \mu}{\frac{s}{\sqrt{n - 1}}} = \frac{6000 - 5500}{\frac{650}{\sqrt{10 - 1}}} = 2.31$$

6. Conclusion: Cal. $|t| = 2.31 >$ Tab. $|t| 2.262$
Reject H_0 .

8. A certain injection administered to each of 12 patients resulted in the following increases of blood pressure:

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4

Can it be concluded that the injection will be, in general, accompanied by an increase in BP?

Solution: Given

$$n = 12$$

													Total
x	5	2	8	-1	3	0	6	-2	1	5	0	4	31
x^2	25	4	64	1	9	0	36	4	1	25	0	16	185

$$\bar{x} = \frac{\Sigma x}{n} = \frac{31}{12} = 2.58, s^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = \frac{185}{12} - \left(\frac{31}{12}\right)^2 = 8.761 s = 2.96$$

1. $H_0: \mu = 0$
2. $H_1: \mu > 0$ (one tailed test (Right))
3. $\alpha = 5\%$ with $d.f = n - 1 = 12 - 1 = 11$
4. Critical region:
5. The test statistic

$$t = \frac{x - \mu}{\frac{s}{\sqrt{n - 1}}} = \frac{6000 - 5500}{\frac{650}{\sqrt{10 - 1}}} = 2.31$$

6. Conclusion: Cal. $|t| = 2.31 >$ Tab. $|t| 2.262$
Reject H_0 .

9. The mean lifetime of a sample of 25 bulbs is found as 1550 hours with a S.D. of 120 hours. The company which manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% level of significance?

Solution:

$$n = 25, \bar{x} = 1550, s = 120, \mu = 1600$$

1. $H_0: \mu = 1600$
2. $H_1: \mu < 1600$ (one tailed test (left))
3. $\alpha = 5\%$ with $d.f = n - 1 = 25 - 1 = 24$
4. Critical region:
5. The test statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{1550 - 1600}{\frac{120}{\sqrt{25-1}}} = -2.04$$

6. Conclusion: Cal. $|t| = 2.04 > \text{Tab. } |t| 1.711$
Reject H_0 .

Test for Difference of two means ($n < 30$)

t – test

10. Two horses A and B were tested according to the time (in seconds) to run a particular race with the following results :

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	

Test whether the horse A is running faster than B at 5% level.

Solution: Given

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{219}{7} = 31.29, s_1^2 = \frac{\sum x_1^2}{n_1} - \left(\frac{\sum x_1}{n_1}\right)^2 = \frac{6883}{7} - \left(\frac{219}{7}\right)^2 = 4.23$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{169}{6} = 28.17, s_2^2 = \frac{\sum x_2^2}{n_2} - \left(\frac{\sum x_2}{n_2}\right)^2 = \frac{4787}{6} - \left(\frac{169}{6}\right)^2 = 4.28$$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = 5.03$$

1. $H_0: \mu_1 = \mu_2$
2. $H_1: \mu_1 > \mu_2$ (One tailed test (Right))
3. $\alpha = 5\%$ with $d.f = n_1 + n_2 - 2 = 11$
4. Critical region:
5. The test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{31.29 - 28.17}{\sqrt{5.03 \left(\frac{1}{7} + \frac{1}{6}\right)}} = 2.498$$

6. Conclusion: Cal. $|t| = 2.498 > \text{Tab. } t \text{ } 1.796$
 Reject H_0 .

11. The following are the average weekly losses of working hours due to accidents in 10 industrial plants before and after an introduction of a safety program was put into operation

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

Use to 0.05 LOS to test whether the safety program is effective.

Solution: Given

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{538}{10} = 53.8, s_1^2 = \frac{\sum x_1^2}{n_1} - \left(\frac{\sum x_1}{n_1}\right)^2 = \frac{38194}{10} - \left(\frac{538}{10}\right)^2 = 924.96$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{486}{10} = 48.6, s_2^2 = \frac{\sum x_2^2}{n_2} - \left(\frac{\sum x_2}{n_2}\right)^2 = \frac{32286}{10} - \left(\frac{486}{10}\right)^2 = 866.64$$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = 995.33$$

1. $H_0: \mu_1 = \mu_2$
2. $H_1: \mu_1 \neq \mu_2$ (Two tailed test)
3. $\alpha = 5\%$ with d. f = $n_1 + n_2 - 2 = 18$
4. Critical region:
5. The test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{53.8 - 48.6}{\sqrt{995.33 \left(\frac{1}{10} + \frac{1}{10}\right)}} = 0.369$$

6. Conclusion: Cal. $|t| = 0.369 < \text{Tab. } t \text{ } 2.101$
 Accept H_0 .

12. In a certain experiment to compare two types of pig foods A & B, the following results of increase in weights were observed in pigs

Food A	49	53	51	52	47	50	52	53
Food B	52	55	52	53	50	54	54	53

Assuming that the two samples of pigs are independent; can we conclude that food B is better than food A?

Solution: Given

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{407}{8} = 50.875, s_1^2 = \frac{\sum x_1^2}{n_1} - \left(\frac{\sum x_1}{n_1}\right)^2 = \frac{20737}{8} - \left(\frac{407}{8}\right)^2 = 3.86$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{423}{8} = 52.875, s_2^2 = \frac{\Sigma x_2^2}{n_2} - \left(\frac{\Sigma x_2}{n_2}\right)^2 = \frac{22383}{8} - \left(\frac{423}{8}\right)^2 = 2.11$$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = 3.41$$

1. $H_0: \mu_1 = \mu_2$
2. $H_1: \mu_1 < \mu_2$ (One tailed test (Left))
3. $\alpha = 5\%$ with $d. f = n_1 + n_2 - 2 = 14$
4. Critical region:
5. The test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{50.875 - 52.875}{\sqrt{3.41 \left(\frac{1}{8} + \frac{1}{8}\right)}} = -2.17$$

6. Conclusion: Cal. $|t| = 2.17 > \text{Tab. } t \text{ } 1.76$
Reject H_0 .

χ^2 - test

Test for goodness of fit

It enables us to find if the deviation of the experiment from theory is just by chance or is it really due to the inadequacy of the theory to fit the observed data.

It is defined by

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

where O - Observed frequency, E - expected frequency.

Note: In case of fitting,

(i) Binomial distribution $E = N m C_x p^x q^{m-x}$, $d. f = n - 1$

(ii) Poisson distribution $E = N \frac{e^{-\lambda} \lambda^x}{x!}$, $d. f = n - 2$, $\lambda = \frac{\Sigma f_i x_i}{\Sigma f_i}$

13. Five coins are tossed 256 times. The number of heads observed is given below.

Examine if the coins are unbiased, by employing χ^2 goodness of fit.

No. of heads	0	1	2	3	4	5
Frequency	5	35	75	84	45	12

Solution: Given, Number of data $n = 6$.
 Number of coins $m = 5$
 Total number of frequencies $N = 256$

1. H_0 : Binomial is a good fit
2. H_1 : Binomial is not a good fit
3. $\alpha = 5\%$ with $d. f = n - 1 = 5$

4. Table value of $\chi^2 = 11.07$
5. The test statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

x	O	$E = N m C_x p^x q^{m-x} = 256 {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{m-x}$	$\frac{(O - E)^2}{E}$
0	5	8	1.25
1	35	40	0.625
2	75	80	0.3125
3	84	80	0.2
4	45	40	0.625
5	12	8	2
Total	256	256	4.8875

$p = \text{probability of getting head when tossing a coin} = \frac{1}{2}$

$q = \text{probability of not getting head when tossing a coin} = \frac{1}{2}$

$$\therefore \chi^2 = \sum \frac{(O - E)^2}{E} = 4.8875$$

6. Conclusion: Cal. $\chi^2 = 4.8875 < \text{Tab. } \chi^2 11.07$
 Accept H_0
 Therefore binomial distribution is a good fit to the given data.

14. A company keeps records of accidents. During a recent safety review, a random sample of 60 accidents was selected and classified by the day of the week on which they occurred.

Day	Mon	Tue	Wed	Thu	Fri
No. of accidents	8	12	9	14	17

Test whether there is any evidence that accidents are more likely on some days than others.

Solution: Given, Number of data $n = 5$.
 Total number of frequencies $N = 60$

1. H_0 : Given data is a good fit
2. H_1 : Given data is not a good fit
3. $\alpha = 5\%$ with d. f = $n - 1 = 4$
4. Table value of $\chi^2 = 9.488$
5. The test statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

O	$E = \frac{60}{5}$	$\frac{(O - E)^2}{E}$
8	12	1.33

12	12	0	
9	12	0.75	
14	12	0.33	
17	12	2.08	
Total	60	60	4.49

$$\therefore \chi^2 = \sum \frac{(O - E)^2}{E} = 4.49$$

6. Conclusion: $\text{Cal.}\chi^2 = 4.49 < \text{Tab.}\chi^2 9.488$
 Accept H_0
 Therefore given data is a good fit.

15. In 120 throws of a single die, the following distribution of faces was observed.

Face	1	2	3	4	5	6
Frequency	30	25	18	10	22	15

Can you say that the die is biased?

Solution: Given, Number of data $n = 6$.

Total number of frequencies $N = 120$

1. H_0 : Given data is a good fit (Die is unbiased)
2. H_1 : Given data is not a good fit (Die is biased)
3. $\alpha = 5\%$ with $d.f = n - 1 = 5$
4. Table value of $\chi^2 = 11.07$
5. The test statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

O	$E = \frac{120}{6}$	$\frac{(O - E)^2}{E}$	
30	20	5	
25	20	1.25	
18	20	0.2	
10	20	5	
22	20	0.2	
15	20	1.25	
Total	120	120	12.9

$$\therefore \chi^2 = \sum \frac{(O - E)^2}{E} = 12.9$$

6. Conclusion: $\text{Cal.}\chi^2 = 12.9 > \text{Tab.}\chi^2 11.07$
 Reject H_0
 Therefore given data is not a good fit.
 The die is biased.

16. A sample analysis of examination results of 500 students was made. It was found that 220 students have failed, 170 have secured a third class, 90 have secured a second class and the rest, a first class So do these figures support the general belief that the above categories are in the ratio 4 : 3 : 2 : 1 respectively?

Solution: Given, Number of data $n = 4$.
Total number of frequencies $N = 500$

1. H_0 : Given data is a good fit
2. H_1 : Given data is not a good fit
3. $\alpha = 5\%$ with $d.f = n - 1 = 3$
4. Table value of $\chi^2 = 7.815$
5. The test statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

O	E	$\frac{(O - E)^2}{E}$
220	$500 \times \frac{4}{10} = 200$	2
170	$500 \times \frac{3}{10} = 150$	2.667
90	$500 \times \frac{2}{10} = 100$	1
20	$500 \times \frac{1}{10} = 50$	18
Total	500	23.667

$$\therefore \chi^2 = \sum \frac{(O - E)^2}{E} = 23.667$$

6. Conclusion: $\text{Cal.}\chi^2 = 23.667 > \text{Tab.}\chi^2 7.815$
Reject H_0
Therefore given data is not a good fit.

17. The following is the distribution of the hourly number of trucks arriving at a company's warehouse

Trucks arriving per hour	0	1	2	3	4	5	6	7	8
Frequency	52	151	130	102	45	12	5	1	2

Find the mean of the distribution and using its man (Rounded to one decimal) as the parameter λ fit a Poisson distribution. Test for goodness of fit at the LOS $\alpha = 0.05$.

Solution: Given, Number of data $n = 9$.
Total number of frequencies $N = 500$

1. H_0 : Poisson fit is a good fit
2. H_1 : Poisson fit is not a good fit
3. $\alpha = 5\%$ with $d.f = n - 2 - 2 = 5$

- 4. Table value of $\chi^2 = 11.07$
- 5. The test statistic

$$\lambda = \frac{\sum f_i x_i}{\sum f_i} = \frac{1010}{500} = 2.02 \approx 2\chi^2 = \sum \frac{(O - E)^2}{E}$$

x	0	1	2	3	4	5	6	7	8	Total
O	52	151	130	102	45	12	5	1	2	500
$E = N \frac{e^{-\lambda} \lambda^x}{x!}$	68	135	135	90	45	18	6	2	1	500

In χ^2 test, expected frequency must be greater than or equal to 5.

O	$E = N \frac{e^{-\lambda} \lambda^x}{x!}$	$\frac{(O - E)^2}{E}$
52	68	3.8
151	135	1.9
130	135	0.2
102	90	1.6
45	45	0
12	18	2
8	9	0.1
Total	500	9.6

$$\therefore \chi^2 = \sum \frac{(O - E)^2}{E} = 9.6$$

- 6. Conclusion: Cal. $\chi^2 = 9.6 < \text{Tab.} \chi^2 11.07$
 Accept H_0
 Therefore Poisson fit is a good fit.

χ^2 - test

Test for independence of attributes

Expected frequency in each cell = $\frac{\text{Product of column total and row total}}{\text{whole total}}$,
 $d.f. = (r - 1)(c - 1)$

18. For the 2×2 contingency table

a	b
c	d

the χ^2 - test of independence is $\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}$ where $N = a + b + c + d$.

Proof: Let the two attributes be A and B . Then the 2×2 contingency table is given below :

a	b	$a + b$
c	d	$c + d$

$a + c$	$b + d$	$N = a + b + c + d$
---------	---------	---------------------

O	E	$O - E$
a	$\frac{(a + c)(a + b)}{N}$	$\frac{ad - bc}{N}$
b	$\frac{(b + d)(a + b)}{N}$	$\frac{ad - bc}{N}$
c	$\frac{(a + c)(c + d)}{N}$	$\frac{ad - bc}{N}$
d	$\frac{(b + d)(c + d)}{N}$	$\frac{ad - bc}{N}$
Total	N	N

$$\begin{aligned} \chi^2 &= \sum \frac{(O - E)^2}{E} \\ &= \frac{(ad - bc)^2}{N^2} \left[\frac{N}{(a + c)(a + b)} + \frac{N}{(b + d)(a + b)} + \frac{N}{(a + c)(c + d)} + \frac{N}{(b + d)(c + d)} \right] \\ &= \frac{(ad - bc)^2}{N} \left[\frac{b + d + a + c}{(a + c)(a + b)(b + d)} + \frac{b + d + a + c}{(c + d)(b + d)(a + b)} \right] \\ &= (ad - bc)^2 \left[\frac{b + d + a + c}{(a + c)(a + b)(b + d)(c + d)} \right] \\ &= \frac{N(ad - bc)^2}{(a + c)(b + d)(a + b)(c + d)} \end{aligned}$$

19. Find if there is any association between extravagance in fathers and extravagance in sons from the following data.

	Extravagant fathers	Miserly fathers
Extravagant sons	327	741
Miserly sons	545	234

Determine the coefficient of association also.

Solution: Given, Total number of frequencies $N = 1847$

- H_0 : Data are independent
- H_1 : Data are dependent
- $\alpha = 5\%$ with $d.f = (r - 1)(c - 1) = (2 - 1)(2 - 1) = 1$
- Table value of $\chi^2 = 3.841$
- The test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a + c)(b + d)(a + b)(c + d)}$$

$$= \frac{1847(327 \times 234 - 545 \times 741)^2}{(872)(975)(1068)(779)} = 230.24$$

6. Conclusion: $\text{Cal.}\chi^2 = 230.24 > \text{Tab.}\chi^2 3.841$
 Reject H_0

20. In an investigation into the health and nutrition of two groups of children of different social status, the following results are got

Social status/ Healths	Poor	Rich	Total
Below normal	130	20	150
Normal	102	108	210
Above normal	24	96	120
Total	256	224	480

Discuss the relation between the Health and their social status.

Solution: Given, Total number of frequencies $N = 480$

- H_0 : Data are independent.
- H_1 : Data are dependent
- $\alpha = 5\%$ with $d.f = (r - 1)(c - 1) = (3 - 1)(2 - 1) = 2$
- Table value of $\chi^2 = 5.991$
- The test statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Expected frequency in each cell = $\frac{\text{Product of column total and row total}}{\text{whole total}}$

	O	E	$\frac{(O - E)^2}{E}$
	130	80	31.25
	20	70	35.71
	102	112	0.89
	108	98	1.02
	24	64	25
	96	56	28.57
Total	480	480	122.44

$$\therefore \chi^2 = \sum \frac{(O - E)^2}{E} = 122.44$$

6. Conclusion: $\text{Cal.}\chi^2 = 122.44 > \text{Tab.}\chi^2 5.991$
 Reject H_0

F- Test

Test for difference of variances of two samples.

If s_1^2 and s_2^2 are the variances of two sample of sizes n_1 and n_2 respectively, the estimates of the population variance based on these samples are respectively $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$ and $S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$. The quantities $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ are called the degrees of freedom of these estimates. We want to test if these estimates S_1^2 and S_2^2 are significantly different or if the samples may be regarded as drawn from the same population or from two populations with same variance σ^2 .

F- test statistic

$$F = \frac{S_1^2}{S_2^2} = \frac{\frac{n_1 s_1^2}{n_1 - 1}}{\frac{n_2 s_2^2}{n_2 - 1}}$$

Provided $S_1^2 > S_2^2$

21. A group of 10 rats fed on diet A and another group of 8 rats fed on diet B, recorded the following increase in weight.

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	8		

Find if the variances are significantly different.

Solution: Given

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{64}{10} = 6.4, \quad n_1 = 10, n_2 = 8$$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - \left(\frac{\sum x_1}{n_1}\right)^2 = \frac{512}{10} - \left(\frac{64}{10}\right)^2 = 10.24$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{40}{8} = 5, \quad s_2^2 = \frac{\sum x_2^2}{n_2} - \left(\frac{\sum x_2}{n_2}\right)^2 = \frac{282}{8} - \left(\frac{40}{8}\right)^2 = 10.25$$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(10)(10.24)}{10 - 1} = 11.3777$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{(8)(10.25)}{8 - 1} = 11.7143$$

1. $H_0: \sigma_1^2 = \sigma_2^2$
2. $H_1: \sigma_1^2 \neq \sigma_2^2$
3. $\alpha = 5\%$ with d.f $v_1 = n_2 - 1 = 8 - 1 = 7$ and $v_2 = n_1 - 1 = 10 - 1 = 9$
 $\therefore S_2^2 > S_1^2$

Table value $F = 3.29$

4. The test statistic $F = \frac{S_2^2}{S_1^2} = \frac{11.7143}{11.3777} = 1.02958$

5. Conclusion: Cal. $|F| = 1.022958 < \text{Tab. } F 3.29$
 Accept H_0 .

22. In one sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations

was 102.6. Test whether this difference is significant at 5% level, given that the 5 percent point of F for $v_1 = 7$ and $v_2 = 9$ degrees of freedom is 3.29.

Solution: Given $n_1 = 8, n_2 = 10$

$$\sum(X_1 - \bar{X}_1)^2 = 84.4, \sum(X_2 - \bar{X}_2)^2 = 102.6$$

$$S_1^2 = \frac{\sum(X_1 - \bar{X}_1)^2}{n_1 - 1} = \frac{84.4}{8 - 1} = 12.057$$

$$S_2^2 = \frac{\sum(X_2 - \bar{X}_2)^2}{n_2 - 1} = \frac{102.6}{10 - 1} = 11.42$$

1. $H_0: \sigma_1^2 = \sigma_2^2$
2. $H_1: \sigma_1^2 \neq \sigma_2^2$
3. $\alpha = 5\%$ with d.f $v_1 = n_1 - 1 = 7$ and $v_2 = n_2 - 1 = 9$
 $\therefore S_1^2 > S_2^2$

Table value $F = 3.29$

4. The test statistic $F = \frac{S_2^2}{S_1^2} = \frac{12.057}{11.42} = 1.057$
5. Conclusion: Cal. $|F| = 1.057 < \text{Tab. } F \ 3.29$
 Accept H_0 .

23. Two random samples give the following results

Sample	Size	Sample Mean	Sum of squares of deviations from the mean
I	10	15	90
II	12	14	108

Test whether the samples could have from the same normal population.

Solution: A normal population has two parameters namely the mean μ and the variance σ^2 . If we want to test the samples from the same normal population, we have to test

- (i) Test for difference of variances using F-test
 - (ii) Test for difference of means using t-test
- We have the tests in order

Given $n_1 = 10, n_2 = 12$

$$\sum(X_1 - \bar{X}_1)^2 = 90, \sum(X_2 - \bar{X}_2)^2 = 108$$

$$S_1^2 = \frac{\sum(X_1 - \bar{X}_1)^2}{n_1 - 1} = \frac{90}{10 - 1} = 10$$

$$S_2^2 = \frac{\sum(X_2 - \bar{X}_2)^2}{n_2 - 1} = \frac{108}{12 - 1} = 9.8181$$

(i) **F - TEST**

1. $H_0: \sigma_1^2 = \sigma_2^2$
2. $H_1: \sigma_1^2 \neq \sigma_2^2$
3. $\alpha = 5\%$ with d. f $v_1 = n_1 - 1 = 9$ and $v_2 = n_2 - 1 = 11$
 $\therefore S_1^2 > S_2^2$

Table value $F = 2.9$

4. The test statistic $F = \frac{S_1^2}{S_2^2} = \frac{10}{9.8181} = 1.019$

5. Conclusion: Cal. $|F| = 1.019 < \text{Tab. } F \text{ } 2.9$
 Accept H_0 .

Note : If F – test fails, no need to proceed for t-test

(ii) t-TEST

1. $H_0: \mu_1 = \mu_2$
2. $H_1: \mu_1 \neq \mu_2$ (Two tailed test)
3. $\alpha = 5\%$ with d. f $= n_1 + n_2 - 2 = 20$, Tab. $t = 2.086$
4. Critical region:
5. The test statistic

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{\sum(X_1 - \bar{X}_1)^2 + \sum(X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2} = \frac{90 + 108}{20} = 9.9$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{15 - 14}{\sqrt{9.9 \left(\frac{1}{10} + \frac{1}{12} \right)}} = -1.7336$$

6. Conclusion: Cal. $|t| = 1.7336 > \text{Tab. } t \text{ } 2.08$
 Accept H_0 .

\therefore The samples are taken from same normal population

UNIT II

DESIGN OF EXPERIMENTS

PART A

1. What do you understand by Design of Experiments?

Solution:

By 'experiment' we mean the collection of data (which usually consists of a series of measurement of some feature of an object) for a scientific investigation according to a certain specified sampling procedures.

2. Distinguish between experimental variables and extraneous variables.

Solution:

Consider the example of agricultural experiment which may be performed to verify the claim that particular manure has got the effect of increasing the yield of paddy. Here the quantity of manure used and amount of yield of paddy are known as experimental variables. And factors such as rainfall, quality of soil and quality of seeds (will also affect the yield of paddy, which are not under study) are called extraneous variables.

3. What is the aim of design of experiments?

Solution:

In any statistical experiment we will have both experimental variables and extraneous variables. The aim of the design of experiments is to control extraneous variables and hence to minimize the error so that the results of experiments could be attributed only to the experimental variables.

4. State the basic principles of design of experiments.

(Or)

What are the basic principles of design of experiments?

Solution:

1. Randomization 2. Replication 3. Local control.

5. What do you mean by experimental group and control group?

Solution:

Consider an agricultural experiment. To analyze the effect of a manure in the yield of paddy, we use the manure in some plots of same size (group of experimental units) is called experimental group and the some other group of plots in which the manure is used and which will provide a basis for comparison is called the control group.

6. Explain the techniques used in the local control of extraneous variables.

Solution:

(i). By **grouping**, we mean combining sets of homogeneous plots into groups, so that different manures may be used in different groups.

(ii). By **blocking**, we mean assigning the same number of plots in different blocks

(iii).By **balancing**, we mean adjusting the procedures of grouping, blocking and assigning the manures in such a manner that a balanced configuration is obtained.

7. Explain replication.

Solution:

In order to study the effects of different manures on the yield are studied, each manure is used in more than one plot. In other words, we resort to replication which means repetition.

8. Name the basic designs of experiments.

Solution:

1. Completely Randomized Design (ANOVA one way classification)
2. Randomized Block Design (ANOVA two way classification)
3. Latin Square Design (ANOVA three way classification)

9. Explain ANOVA.

Solution:

ANOVA enables us to divide the total variation (represented by variance) in a group into parts which are accounted to different factors and a residual random variation which could be accounted for by any of these factors. The variation due to any specific factor is compared with the residual variation for significance, and hence the effects of the factors are concluded.

10. Explain CRD.

Solution:

Consider an experiment of agriculture in which “h” treatments (manures) and “n” plots are available. To control the extraneous variables treatment “1” should be replicated on “n₁” plots, treatment 2 should be replicated on “n₂” plots and so on. To reduce the error we have to randomize this process that is which n₁ plots be used treatment 1 and so on. For this we number the plots (from 1 to n) and write the numbers on cards and shuffle well. Now, we select n₁ cards (as cards are selected at random the numbers will not be in order) on which treatments 1 will be used and so on. This process and design is called completely randomized design.

11. Explain RBD.

Solution:

Consider an experiment of agriculture in which effects of “k” treatments on the yield of paddy used. For this we select “n” plots. If the quality of soil of these “n” plots is known, then these plots are divided into “h” blocks (each with one quality). Each of these “h” blocks are divided into “k” times (n = hk) and in each one of this “k” plots are applied the “k” treatments in a perfectly randomized manner such that each treatment occurs only once in any of the block. This design is called randomized block design.

12. Explain LSD.

Solution:

Consider an agricultural experiment, in which n^2 plots are taken and arranged in the form of an $n \times n$ square, such that plots in each row will be homogeneous as far as possible with respect to one factor of classification, say, quality of soil and plots in each column will be homogeneous with respect to another factor of classification, say, seed quality. Then “ n ” treatments are given to these plots such that each treatment occurs only once in each row and only once in each column. The various possible arrangements obtained in this manner are known as Latin squares of order “ n ” and the design is called Latin Square Design.

13. Is a 2 X 2 Latin Square Design possible? Why?

Solution:

In a 2 X 2 LSD, the degree of freedom for the residual variation is $(n - 1)(n - 2) = 0$ which is not possible. Therefore 2 X 2 LSD is not possible.

14. Compare LSD and RBD.

Solution:

RBD	LSD
1. The number of replications of each 2. Can be performed in square field only 3. Number of treatments must be 4. It controls two extraneous variables	1. There is no such restriction treatment is equal to the number of treatments. 2. Can be performed in square or rectangular field. 3. No such restriction 4. Controls only one extraneous variable.

15. Write down the ANOVA table for one way classification. (MAY 2012)

Solution:

Analysis of variance table for One Way Classification

Source of variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	Variance Ratio	Table value of F	
					5%	1%
Columns	SSC	C-1	$MSC = \frac{S}{C-1}$	$F_C = \frac{M}{M}$		
Error	SSE	N-C	$MSE = \frac{S}{N-C}$			
Total	TSS					

PART B

1. The following are the numbers of mistakes made in 5 successive days of 4 technicians working for a photographic laboratory:

Technician I (X ₁)	Technician II (X ₂)	Technician III (X ₃)	Technician IV (X ₄)
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

Test at the level of significance = 0.01, whether the differences among the 4 sample means, can be attributed to chance.

Solution: H_0 = There is no significant difference between the technicians (columns)
 H_1 = There is significant difference between the technicians (columns)

We shift the origin to 10

Technicians									
I X ₁	II X ₂	III X ₃	IV X ₄	Total	X ₁ ²	X ₂ ²	X ₃ ²	X ₄ ²	
-4	4	0	-1	-1	16	16	0	1	
4	-1	2	2	7	16	1	4	4	
0	2	-3	-2	-3	0	4	9	4	
-2	0	5	0	3	4	0	25	0	
1	4	1	1	7	1	16	1	1	
Total = -1	9	5	0	13	37	37	39	10	

Step 1: $N = 20, T = 13, C.F = \frac{T^2}{N} = \frac{1}{2} = 8.45$

Step 2: $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$
 $= 37+37+39+10-8.45$
 $= 114.55$

Step 3 : $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

N_1 = Number of elements in each column

$$= \frac{(-1)^2}{5} + \frac{(9)^2}{5} + \frac{(5)^2}{5} + \frac{(0)^2}{5} - 8.45$$

$$= 12.95$$

Step 4 : SSE = TSS – SSC

$$= 114.55 - 12.95$$

$$= 101.6$$

Analysis of variance table for One Way Classification

Source of variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	Variance Ratio	Table value of F	
					5%	1%
Columns	SSC=12.95	C-1 =4-1 =3	MSC = $\frac{S}{C-1} = \frac{1.9}{3}$ = 4.317	F _C = $\frac{M}{M}$ = $\frac{6.35}{4.317}$ = 1.471	F _C (16,3) = 8.68	26.89
Error	SSE=101.6	N-C = 20-4 =16	MSE = $\frac{S}{N-C} = \frac{1.6}{1}$ = 6.35			
Total	TSS=114.55					

Conclusion

Cal F_c = 1.471 < Tab. F_c (16, 3) = 8.68 [There is a no significant difference]

2. An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following “cleanliness” readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines:

	Engine 1	Engine 2	Engine 3	Total
Detergent A	45	43	51	139
Detergent B	47	46	52	145
Detergent C	48	50	55	153
Detergent D	42	37	49	128
Total	182	176	207	565

Perform the ANOVA and test at 0.01 level of significance, whether there are differences in the detergents or in the engines.

Solution: Changing the origin by 50

Detergents	Engines						
	I X ₁	II X ₂	III X ₃	Total	X ₁ ²	X ₂ ²	X ₃ ²
Y ₁	-5	-7	1	-11	25	49	1
Y ₂	-3	-4	2	-5	9	16	4
Y ₃	-2	0	5	3	4	0	25
Y ₄	-8	-13	-1	-22	64	169	1
Total	-18	-24	7	-35	102	234	31

Step 1 : $N = 12$, $T = -35$, $C.F = \frac{T^2}{N} = \frac{(-35)^2}{12} = 102.08$

Step 2 : $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$
 $= 102+234+31-102.08$
 $= 264.92$

Step 3 : $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} - \frac{T^2}{N}$
 $N_1 =$ Number of elements in each column
 $= \frac{(-1)^2}{4} + \frac{(-2)^2}{4} + \frac{(7)^2}{4} - 102.08$
 $= 135.17$

Step 4 : $SSR = \frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \frac{(\sum Y_4)^2}{N_4} - \frac{T^2}{N}$
 $N_1 =$ Number of elements in each row
 $= \frac{(-1)^2}{4} + \frac{(-5)^2}{4} + \frac{(3)^2}{4} + \frac{(-2)^2}{4} - 102.08$
 $= 110.91$

Step 5 : $SSE = TSS - SSC - SSR$
 $= 264.92-135.17-110.91$
 $= 18.84$

Analysis of variance table for Randomized Block Design

Source of variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	Variance Ratio	Table value of F
					1%
Blocks	SSC=135.17	c-1 =3-1 =2	MSC= $\frac{S}{c-1}$ $=\frac{135.17}{2}$ =67.585	F _C = $\frac{M}{M}$ $=\frac{67.585}{3.14}$ =21.52	F _C (2,6) =10.92
Treatments	SSR=110.91	r-1 =4-1 =3	MSR= $\frac{S}{r-1}$ $=\frac{110.91}{3}$ =36.97	F _R = $\frac{M}{M}$ $=\frac{36.97}{3.14}$ =11.77	F _R (3,6) =9.78
Error	SSE=18.84	(c-1)(r-1) =6	MSE= $\frac{S}{(c-1)(r-1)}$ $=\frac{18.84}{6}$ =3.14		
Total	TSS=264.92	11			

Conclusion

- (i) Cal F_C = 21.52 > Tab. F_C(2,6) = 10.92 [There is a significant difference]
- (ii) Cal F_R = 11.77 > Tab. F_R(3,9) = 9.78 [There is a significant difference]

3. Four varieties A, B, C, D of a fertilizer are tested in randomized block design with 4 replication. The plot yields in pounds are as follows:

	1	2	3	4
1	A 12	D 20	C 16	B 10
2	D 18	A 14	B 11	C 14
3	B 12	C 15	D 19	A 13
4	C 16	B 11	A 15	D 20

Analyze the experimental yield

Solution: Arrange A, B, C, D row – wise

Detergents	Engines								
	Enne				Total	X ₁ ²	X ₂ ²	X ₃ ²	X ₄ ²
	I X ₁	II X ₂	III X ₃	IV X ₄					
A Y ₁	12	14	15	13	54	144	196	225	169
B Y ₂	12	11	11	10	44	144	121	121	100
C Y ₃	16	15	16	14	61	256	225	256	196
D Y ₄	18	20	19	20	77	324	400	361	400
Total	58	60	61	57	236	868	942	963	865

Step 1 : $N = 16, T = 236, C.F = \frac{T^2}{N} = \frac{(236)^2}{16} = 3481$

Step 2 : $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$
 $= 868 + 942 + 963 + 865 - 3481$
 $= 157$

Step 3 : $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \frac{(\sum X_4)^2}{N_4} - \frac{T^2}{N}$
 $N_1 = \text{Number of elements in each column}$
 $= \frac{(58)^2}{4} + \frac{(60)^2}{4} + \frac{(61)^2}{4} + \frac{(57)^2}{4} - 3481$
 $= 2$

Step 4 : $SSR = \frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \frac{(\sum Y_4)^2}{N_4} - \frac{T^2}{N}$
 $N_1 = \text{Number of elements in each row}$
 $= \frac{(54)^2}{4} + \frac{(44)^2}{4} + \frac{(61)^2}{4} + \frac{(77)^2}{4} - 3481$
 $= 144.5$

Step 5 : $SSE = TSS - SSC - SSR$
 $= 157 - 2 - 144.5$
 $= 10.5$

Analysis of variance table for Randomized Block Design

Source of variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	Variance Ratio	Table value of F
					5%
Blocks	SSC=2	c-1 =4-1 =3	MSC= $\frac{S}{c-1}$ $=\frac{2}{3}$ =0.67	F _C = $\frac{M}{M}$ $=\frac{1.17}{0.67}$ = 1.75	F _C (9,3) =8.81
Treatments	SSR=144.5	r-1 =4-1 =3	MSR= $\frac{S}{r-1}$ $=\frac{144.5}{3}$ =48.17	F _R = $\frac{M}{M}$ $=\frac{48.17}{1.17}$ = 41.17	F _R (3,9) =3.86
Error	SSE=10.5	(c-1)(r-1) =9	MSE= $\frac{S}{(c-1)(r-1)}$ $=\frac{10.5}{9}$ =1.17		
Total	TSS=157	15			

Conclusion

- (i) Cal F_C = 1.75 < Tab. F_C(9,3) = 8.81 [There is a no significant difference]
- (ii) Cal F_R = 11.77 > Tab. F_R(1,9) = 9.78 [There is a significant difference]

4. The following is a Latin square of a design, when 4 varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You may carry out suitable change of origin and scale.

A 105	B 95	C 125	D 115
C 115	D 125	A 105	B 105
D 115	C 95	B 105	A 115
B 95	A 135	D 95	C 115

Solution: Subtract 100 and then divided by 5, we get

A 1	B -1	C 5	D 3
C 3	D 5	A 1	B 1
D 3	C -1	B 1	A 3
B -1	A 7	D -1	C 3

	I X ₁	II X ₂	III X ₃	IV X ₄	Total	X ₁ ²	X ₂ ²	X ₃ ²	X ₄ ²
Y ₁	1	-1	5	3	8	1	1	25	9
Y ₂	3	5	1	1	10	9	25	1	1
Y ₃	3	-1	1	3	6	9	1	1	9
Y ₄	-1	7	-1	3	8	1	49	1	9
Total	6	10	6	10	32	20	76	28	28

Step 1 : $N = 16, T = 32, C.F = \frac{T^2}{N} = \frac{3^2}{1} = 64$

Step 2 : $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$
 $= 20+76+28+28-64$
 $= 88$

Step 3 : $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

$N_1 =$ Number of elements in each column

$= \frac{(6)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(10)^2}{4} - 64$
 $= 4$

Step 4: $SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

$N_2 =$ Number of elements in each row

$= \frac{(8)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(8)^2}{4} - 64$
 $= 2$

To find SSK:

Arrange the elements in the order of treatment

A	1	1	3	7	12
B	-1	1	1	-1	0
C	5	3	-1	3	10
D	3	5	3	-1	10

Step 5: $SSK = \frac{(\sum A)^2}{N_2} + \frac{(\sum B)^2}{N_2} + \frac{(\sum C)^2}{N_2} + \frac{(\sum D)^2}{N_2} - \frac{T^2}{N}$

$N_2 =$ Number of elements in each row

$$= \frac{(1)^2}{4} + \frac{(0)^2}{4} + \frac{(1)^2}{4} + \frac{(1)^2}{4} - 64$$

$$= 22$$

Step 6 : SSE = TSS – SSC – SSR - SSK

$$= 88 - 4 - 2 - 22$$

$$= 60$$

Analysis of variance table for Latin Square Design

Source of variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	Variance Ratio	Table value of F 5%
Columns	SSC=4	c-1 =4-1 =3	MSC= $\frac{S}{c-1}$ $=\frac{4}{3}$ =1.33	F _C = $\frac{M}{M}$ $=\frac{10}{1.33}$ = 7.52	F _C (6,3) = 8.94
Rows	SSR=2	c-1 =4-1 =3	MSR= $\frac{S}{c-1}$ $=\frac{2}{3}$ =0.67	F _R = $\frac{M}{M}$ $=\frac{10}{0.67}$ = 14.9	F _R (6,3) = 8.94
Treatments	SSK=22	c-1 =4-1 =3	MSK= $\frac{S}{c-1}$ $=\frac{22}{3}$ =7.33	F _K = $\frac{M}{M}$ $=\frac{10}{7.33}$ = 1.36	F _K (6,3) = 8.94
Error	SSE=60	(c-1)(r-2) =6	MSE= $\frac{S}{(c-1)(r-2)}$ $=\frac{6}{6}$ =10		

Conclusion

- (i) Cal F_C = 7.52 < Tab. F_C (6,3) = 8.94 [There is a no significant difference]
- (ii) Cal F_R = 14.9 > Tab. F_R (6,3) = 8.94 [There is a significant difference]
- (iii) Cal F_K = 1.36 < Tab. F_K (6,3) = 8.94 [There is a no significant difference]

5. In a Latin square experiment given below are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers *A, B, C, D, E*. Analyze the data for variations

B 25	A 18	E 27	D 30	C 27
A 19	D 31	C 29	E 26	B 23
C 28	B 22	D 33	A 18	E 27
E 28	C 26	A 20	B 25	D 33
D 32	E 25	B 23	C 28	A 20

Solution: Subtract 100 and then divided by 5, we get

B 0	A 18	E 27	D 30	C 27
A -6	D 31	C 29	E 26	B 23
C 3	B 22	D 33	A 18	E 27
E 3	C 26	A 20	B 25	D 33
D 7	E 25	B 23	C 28	A 20

	I X ₁	II X ₂	III X ₃	IV X ₄	V X ₅	Total	X ₁ ²	X ₂ ²	X ₃ ²	X ₄ ²	X ₅ ²
Y ₁	0	-7	2	5	2	2	0	49	4	25	4
Y ₂	-6	6	4	1	-2	3	36	36	16	1	4
Y ₃	3	-3	8	-7	2	3	9	9	64	49	4
Y ₄	3	1	-5	0	8	7	9	1	25	0	64
Y ₅	7	0	-2	3	-5	3	49	0	4	9	25
Total	7	-3	7	2	5	18	103	95	113	84	101

Step 1 : $N = 25, T = 18, C.F = \frac{T^2}{N} = \frac{1^2}{2} = 12.96$

Step 2 : $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 + \sum X_5^2 - \frac{T^2}{N}$
 $= 103 + 95 + 113 + 84 + 101 - 12.96$
 $= 483.04$

Step 3 : $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} + \frac{(\sum X_5)^2}{N_1} - \frac{T^2}{N}$

$N_1 =$ Number of elements in each column

$= \frac{(7)^2}{5} + \frac{(-3)^2}{5} + \frac{(7)^2}{5} + \frac{(2)^2}{5} + \frac{(5)^2}{5} - 12.96$

$$= 14.24$$

$$\text{Step 4: SSR} = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} + \frac{(\sum Y_5)^2}{N_2} - \frac{T^2}{N}$$

N_2 = Number of elements in each row

$$= \frac{(2)^2}{5} + \frac{(3)^2}{5} + \frac{(3)^2}{5} + \frac{(7)^2}{5} + \frac{(3)^2}{5} - 12.96$$

$$= 3.04$$

To find SSK:

Arrange the elements in the order of treatment

						TOTAL
A	-6	-7	-5	-7	-5	-30
B	0	-3	-2	0	-2	-7
C	3	1	4	3	2	13
D	7	6	8	5	8	34
E	3	0	2	1	2	8

$$\text{Step 5: SSK} = \frac{(\sum A)^2}{N_2} + \frac{(\sum B)^2}{N_2} + \frac{(\sum C)^2}{N_2} + \frac{(\sum D)^2}{N_2} + \frac{(\sum E)^2}{N_2} - \frac{T^2}{N}$$

N_2 = Number of elements in each row

$$= \frac{(-30)^2}{5} + \frac{(-7)^2}{5} + \frac{(13)^2}{5} + \frac{(34)^2}{5} + \frac{(8)^2}{5} - 12.96$$

$$= 454.64$$

$$\text{Step 6 : SSE} = \text{TSS} - \text{SSC} - \text{SSR} - \text{SSK}$$

$$= 483.04 - 14.24 - 3.04 - 454.64$$

$$= 11.12$$

Analysis of variance table for Latin Square Design

Source of variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	Variance Ratio	Table value of F 5%
Columns	SSC=14.24	c-1 =5-1 =4	$MSC = \frac{S}{c-1} = \frac{14.24}{4} = 3.56$	$F_C = \frac{M}{M} = \frac{3.56}{0.927} = 3.84$	$F_C(4,12) = 3.26$
Rows	SSR=3.04	c-1 =5-1 =4	$MSR = \frac{S}{c-1} = \frac{3.04}{4} = 0.76$	$F_R = \frac{M}{M} = \frac{0.927}{0.76} = 1.22$	$F_R(12,4) = 5.91$
Treatments	SSK=454.64	c-1 =5-1 =4	$MSK = \frac{S}{c-1} = \frac{454.64}{4} = 113.66$	$F_K = \frac{M}{M} = \frac{113.66}{0.927} = 122.6$	$F_K(4,12) = 3.26$
Error	SSE=11.12	(c-1)(r-2) =12	$MSE = \frac{S}{(c-1)(r-2)} = \frac{11.12}{12} = 0.927$		

Conclusion

- (i) Cal $F_C = 3.84 > \text{Tab. } F_C(4,12) = 3.26$ [There is a significant difference]
- (ii) Cal $F_R = 1.22 < \text{Tab. } F_R(12,4) = 5.91$ [There is a no significant difference]
- (iii) Cal $F_K = 122.6 > \text{Tab. } F_K(4,12) = 3.26$ [There is a significant difference]

6. Find out the main effects and interactions in the following 2² – factorial experiment and write down the analysis of variance table:

	(1) 00	a 10	b 01	ab 11
Block I	64	25	30	6
II	75	14	50	33
III	76	12	41	17
IV	75	33	25	10

Solution : Taking deviation from $y = 37$,

Treatment combination	Blocks				Total	X_1^2	X_2^2	X_3^2	X_4^2
	I X_1	II X_2	III X_3	IV X_4					
(Y ₁) (1)	27	38	39	38	142	729	1444	1521	1444
(Y ₂) a	-12	-23	-25	-4	-64	144	529	625	16
(Y ₃) b	-7	13	4	-12	-2	49	169	16	144
(Y ₄) ab	-31	-4	-20	-27	-82	961	16	400	729
Total	-23	24	-2	-5	-6	1883	2158	2562	2333

Step 1: $N = 16, T = -6, C.F = \frac{T^2}{N} = \frac{36}{16} = 2.25$

Step 2: $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$
 $= 1883 + 2158 + 2562 + 2333 - 2.25$
 $= 8933.75$

Step 3: $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$
 $N_1 = \text{Number of elements in each column}$
 $= \frac{(-23)^2}{4} + \frac{(24)^2}{4} + \frac{(-2)^2}{4} + \frac{(-5)^2}{4} - 2.25$
 $= 281.25$

Step 4: $SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$
 $N_2 = \text{Number of elements in each row}$
 $= \frac{(142)^2}{4} + \frac{(-64)^2}{4} + \frac{(-2)^2}{4} + \frac{(-82)^2}{4} - 2.25$
 $= 7744.75$

Step 5: $SSE = TSS - SSC - SSR$

$$= 8933K - 281 - 774475$$

$$= 907.75$$

For 2^2 experiment

$$SS_A = \frac{[a+a -b-(1)]^2}{1}$$

$$= \frac{[-6 -8 +2-1]^2}{1}$$

$$= 5112.25$$

$$SS_B = \frac{[b+a -a-(1)]^2}{1}$$

$$= \frac{[-2-8 +6 -1]^2}{1}$$

$$= 1640.25$$

$$SS_{AB} = \frac{[a +(1)-a-b]^2}{1}$$

$$= \frac{[-8 +1 +6 +2]^2}{1}$$

$$= 992.25$$

Analysis of variance table for the 2² experiment

Source of variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	Variance Ratio	Table value of F	
					5%	1%
Blocks	SSC=281.5	3	MSC=93.83	$F_C = \frac{M}{M} = \frac{100.86}{93.83} = 1.075$	$F_{C(9,3)} = 8.81$	27.35
Treatments	SSR=7744.75	3	MSR=2581.58	$F_R = \frac{M}{M} = \frac{2581.88}{100.86} = 25.6$	$F_{R(3,9)} = 3.86$	6.99
A	SSA=5112.25	1	MSA=5112.25	$F_A = \frac{M}{M} = \frac{5112.25}{100.86} = 50.69$	$F_{A(1,9)} = 5.12$	6.99
B	SSB=1640.25	1	MSB=1640.25	$F_B = \frac{M}{M} = \frac{1640.25}{100.86} = 16.26$	$F_{B(1,9)} = 5.12$	6.99
AB	SSAB=992.25	1	MSAB=992.25	$F_{AB} = \frac{M}{M} = \frac{992.25}{100.86} = 9.84$	$F_{AB(1,9)} = 5.12$	6.99
Error	SSE =907.75	9	MSE=100.86			

Conclusion

(iv) Cal $F_A = 50.69 > \text{Tab. } F_{A(1,9)} = 5.12$

(v) Cal $F_B = 16.26 > \text{Tab. } F_{B(1,9)} = 5.12$

(vi) Cal $F_{AB} = 9.84 > \text{Tab. } F_{AB}(1,9) = 5.12$

PITTSBURGH MATHEMATICS DEPARTMENT

UNIT – III

SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

PART - A

1. Give Newton Raphson iterative formula.

Solution:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 0,1,2,\dots$$

2. How will you find a negative root of a polynomial equation by Iteration method?

Solution:

Let $f(x) = 0$ be the polynomial. The negative root of this polynomial can be obtained by finding the positive root of $f(-x) = 0$.

3. State the order (rate) of convergence and convergence condition for Newton Raphson method.

Solution:

The order of convergence of Newton Raphson method is 2

(quadratic) and convergence condition is $|f(x)f''(x)| < [f'(x)]^2$.

4. Show that Newton Raphson formula to find \sqrt{a} can be expressed

in the form
$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right)$$

Solution:

Let $x = \sqrt{a} \Rightarrow x^2 = a$

(i.e.) $f(x) = x^2 - a \Rightarrow f(x_n) = x_n^2 - a, f'(x_n) = 2x_n$

By Newton Raphson method,

$$\begin{aligned}
 x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{(x_n^2 - a)}{2x_n} \\
 &= \frac{x_n^2 + a}{2x_n} \\
 &= \frac{1}{2} \left(x_n + \frac{a}{x_n} \right).
 \end{aligned}$$

5. Derive Newton's algorithm for finding the p^{th} root of a number N .

Solution:

Let $x = N^{1/p}$
 $\Rightarrow x^p = N \Rightarrow x^p - N = 0$

i.e. $f(x) = x^p - N \Rightarrow f(x_n) = x_n^p - N, f'(x_n) = px_n^{p-1}$

By Newton Raphson method,

$$\begin{aligned}
 x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^p - N}{px_n^{p-1}} \\
 &= \frac{(p-1)x_n^p + N}{px_n^{p-1}}.
 \end{aligned}$$

6. Establish an iteration formula to find the reciprocal of a positive number N by Newton Raphson method.

Solution:

Let $x = 1/N$

$$\Rightarrow N = \frac{1}{x} \Rightarrow \frac{1}{x} - N = 0$$

(i.e.) $f(x) = \frac{1}{x} - N \Rightarrow f(x_n) = \frac{1}{x_n} - N, f'(x_n) = -\frac{1}{x_n^2}$

By Newton Raphson method,

$$\begin{aligned}
 x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\frac{1}{x_n} - N}{-\frac{1}{x_n^2}} = x_n + x_n^2 \left(\frac{1}{x_n} - N \right) \\
 &= x_n (2 - Nx_n).
 \end{aligned}$$

7. Derive Newton's formula to find the cube root of a positive number k.

Solution:

$$\text{Let } x = \sqrt[3]{k}$$

$$f(x) = x^3 - k \Rightarrow f(x_n) = x_n^3 - k, f'(x_n) = 3x_n^2$$

By Newton Raphson method,

$$\begin{aligned} x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - k}{3x_n^2} \\ &= \frac{2x_n^3 + k}{3x_n^2} \\ &= \frac{1}{3} \left(2x_n + \frac{k}{x_n^2} \right) \end{aligned}$$

8. State the principle used in Gauss-Jordan method.

Solution:

In the equation $AX = B$, the matrix A is transformed into an identity matrix.

9. Give the sufficient condition of convergence of Gauss Seidel method (or) Gauss Jacobi method.

Solution:

The absolute value of the leading diagonal element is greater than the sum of the absolute values of the other elements in that row, which is called diagonally dominant.

10. Write the conditions for convergence in Gauss Seidel iterative technique. (or) When the method of iteration will be useful?

Solution:

The coefficient matrix should be diagonally dominant.

11. State Gauss Seidel method.

Solution:

As soon as a new value for a variable is found by iteration it is used immediately in the following equations. This method is called Gauss Seidel method.

12. Why Gauss Seidel method is a better method than Jacobi's iterative method.

Solution:

Since the immediately calculated values in Gauss Seidel of the unknowns are used, the convergence in Gauss Seidel method will be more rapid than in Gauss Jacobi method. The rate of convergence of Gauss Seidel method is roughly twice that of Gauss Jacobi method.

13. Write the first iteration values of x,y,z when the equations $7x+6y-z = 85$, $6x+15y+2z = 72$, $x+y+5z = 110$ are solved by Gauss Seidel method.

Solution:

Here the coefficient matrix is diagonally dominant. Then

$$x = \frac{1}{27}(85 - 6y + z) \dots\dots(1)$$

$$y = \frac{1}{15}(72 - 6x - 2z) \dots\dots(2)$$

$$z = \frac{1}{5}(110 - x - y) \dots\dots(3)$$

First Iteration

Put $y = 0, z = 0$ in (1), we get $x = 3.148$

Put $x = 3.148, z = 0$ in (2), we get $y = 3.451$

Put $x = 3.148, y = 3.451$ in (3), we get $z = 20.662$

14. Distinguish between direct and iteration method of solving simultaneous equations.

Solution:

Direct method

Iterative method

i) We get exact solution

i) Approximate solution

ii) Simple, take less time

ii) Time consuming laborious.

15. Compare Gauss Elimination and Gauss Jordan methods for solving linear systems of the form $AX=B$.

Solution:

In Gauss Elimination method, the coefficient matrix reduced to upper triangular matrix and we get the solution by back substitution whereas in Gauss Jordan method, the coefficient matrix reduces to an unit or identity matrix and we get the solution without using back substitution.

16. Solve $3x+y = 2, x+3y = -2$ by Gauss Seidel iteration method.

Solution:

In sixth iteration, we get $x = 1, y = -1$.

17. Find the dominant eigen value of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method.

Solution:

Dominant eigen value = 5.3722

18. On what type of equations Newton's method can be applicable?

Solution:

Newton's method can be applicable to the solution of both algebraic and transcendental equation and can be also used when the roots are complex.

19. By Gauss elimination method solve $x + y = 2$ and $2x + 3y = 5$.

Solution:

The augmented matrix is

$$[A,B] = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} R_2 = R_2 - 2R_1$$

By back substitution, $x + y = 2$ ----(1)

$$y = 1$$

(1) becomes, $x + 1 = 2$

$$x = 1$$

Hence $x = 1, y = 1$.

20. Why Gauss Seidel iteration is a method of successive corrections?

Solution:

Because we replace approximations by corresponding new ones as soon the latter have been computed.

21. What are the merits of Newton's method of iteration?

Solution:

Newton's method is successfully used to improve the result obtained by other methods. It is applicable to the solution of equations involving algebraical functions as well as transcendental functions.

22. Give two direct methods to solve a system of linear equations.

Solution:

Gauss Elimination method and Gauss Jordan method.

23. Compare Gauss Elimination with Gauss Seidel method. (DEC 2012)

Solution:

Gauss Elimination

i. Direct method

ii. Used to find inverse of equations only

iii. Diagonally dominant condition is insisted.

Gauss Seidel

i. Indirect method

ii. Used to solve system of the matrix also.

iii. Diagonally dominant condition is not insisted.

24. Find inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by Gauss Jordan method.

Solution:

$$\begin{aligned} AI &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} \end{aligned}$$

$$\text{Hence } A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

25. Mention the order and condition for the convergence of Newton-Raphson method. (DEC 2012).

26. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 6 decimal places. (MAY 2012).

27. Solve the equations $A+B+C = 6$, $3A+3B+4C = 20$, $2A+B+3C = 13$ using Gauss elimination method. (MAY 2012).

28. Perform four iterations of Newton – Raphson method to find the smallest positive root of the equation of $f(x) = x^3 - 5x + 1 = 0$

29. Solve the equations $10x - y + 2z = 2$; $x + 10y - z = 3$; $2x + 3y + 10z = 7$ using Gauss elimination method

PART – B

1. Solve $x \log_{10} x = 12.34$ with $x_0 = 10$ using Newton's method. (MAY 2011)

2. Find the negative root of the equation $\sin x = 1 + x^3$ by using Newton Raphson method.

3. Solve the following equation by Gauss Elimination method

$$10x - 2y + 3z = 23$$

$$2x + 10y - 5z = -33$$

$$3x - 4y + 10z = 41$$

4. Solve the equation by Gauss Jordan method :

$$2x_1 + x_2 + 4x_3 = 4$$

$$x_1 - 3x_2 - x_3 = -5$$

$$3x_1 - 2x_2 + 2x_3 = -1$$

5. Find the inverse of $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ using Gauss Jordan method. (MAY 2011)

6. Find the inverse of $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{bmatrix}$ using Gauss Jordan method.

7. Solve by Gauss Jacobi method :

$$x + y + 54z = 110$$

$$27x + 6y - z = 85 \quad (\text{MAY 2011})$$

$$6x + 15y + 2z = 72$$

8. Solve by Gauss Siedel method

$$x + y + 54z = 110$$

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72 \quad (\text{DEC 2012})$$

9. Solve by Gauss Siedel method

$$10x + 2y + z = 9$$

$$x + 10y - z = -22 \quad (\text{MAY 2009})$$

$$-2x + 3y + 10z = 22$$

10. Find the dominant (largest) eigen value and the corresponding eigen

vector of $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by power method. (DEC 2010)

11. Find the dominant eigen value and the corresponding eigen vector of

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ by power method. Find also the least latent root (smallest eigen value) and hence the third eigen value.}$$

12. Find the numerically largest eigen value of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by power method and the corresponding eigen vector. (MAY 2009)

13. Find the largest eigen value and eigen vector of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ by power method.

14. Solve by Gauss Jacobi iteration method correct to 2 decimal places.

$$10x + y - z = 11.19$$

$$x + 10y + z = 28.08$$

$$-x + y + 10z = 35.61$$

(MAY 2009)

15. Solve the following system using Gauss elimination method.

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 10$$

$$7x_1 + 10x_2 + 5x_3 + 2x_4 = 40$$

$$13x_1 + 6x_2 + 2x_3 - 3x_4 = 34 \text{ (DEC 2010)}$$

$$11x_1 + 14x_2 + 8x_3 - x_4 = 64$$

16. Solve the following system using Gauss Jordan method.

$$2x_1 + 2x_2 - x_3 + x_4 = 4, \quad 4x_1 + 3x_2 - x_3 + 2x_4 = 6, \quad 8x_1 + 5x_2 - 3x_3 + 4x_4 = 12$$

$$3x_1 + 3x_2 - 2x_3 + 2x_4 = 6$$

17. Find the negative root of the equation $x^2 + 4 \sin x = 0$ by Newton's method.

18. Solve by Gauss Seidel iteration the given system of equations starting with (0,0,0,0) as solution. Do 5 iterations only

$$4x_1 - x_2 - x_3 = 2, \quad -x_1 + 4x_2 - x_4 = 2, \quad -x_1 + 4x_3 - x_4 = 1, \quad -x_2 - x_3 + 4x_4 = 1$$

(DEC 2010)

19. Find the dominant eigen value of

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \text{ by power method. (DEC 2012)}$$

20. If $A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$, find A^{-1} by Gauss-Jordan method. (DEC 2012)

21. Solve the following system using Gauss elimination method.

$$x+y+z=9, \quad 2x-3y+4z=13, \quad 3x+4y+5z=40 \quad (\text{DEC 2012})$$

22. Solve by Gauss elimination method.

$$3x+4y+5z=18, \quad 2x-y+8z=13, \quad 5x-2y+7z=20 \quad (\text{MAY 2013})$$

23. Solve by Gauss-Seidal method the following system

$$28x+4y-z=32, \quad x+3y+10z=24, \quad 2x+17y+4z=35 \quad (\text{MAY 2013})$$

24. Solve the system of equations by Gauss-Jordan method

$$x+y+z+w=1, \quad 2x-3y+2z-w=-5, \quad 3x+2y+3z+4w=7, \quad x-2y-3z+2w=5 \quad (\text{MAY 2013})$$

25. Using power method, find all the eigen values of $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ (MAY 2013)

26. Given $f(2)=5, f(2.2)=5.5$ find the linear interpolating polynomial using lagrange interpolation.

UNIT – IV

INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

PART - A

1. The n^{th} order difference of a polynomial of n^{th} degree is _____ and $(n+1)^{\text{th}}$ order of the same degree is _____

Solution:

Constant and zero.

2. Define the operators E and Δ and show that $E = 1 + \Delta$.

Solution:

$$Ef(x) = f(x+h) \text{ and } \Delta f(x) = f(x+h) - f(x)$$

$$\text{We have } \Delta f(x) = f(x+h) - f(x)$$

$$= Ef(x) - f(x)$$

$$= (E - 1)f(x)$$

$$\Delta = E - 1 \text{ (or) } E = 1 + \Delta.$$

3. What is the relation between the shift operator E and the backward difference operator ∇ ?

Solution:

$$\nabla = 1 - E^{-1}$$

4. What will be the first difference of a polynomial of degree 4?

Solution:

We get a polynomial of degree 3.

5. $\nabla + E^{-1} =$ _____

Solution:

$$\nabla + E^{-1} = 1 - E^{-1} + E^{-1} = 1.$$

6. Given $u_0 = 1, u_1 = 11, u_2 = 21, u_3 = 28, u_4 = 29$. Find $\Delta^4 u_0$.

Solution:

x	y = u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$	$\Delta^4 u_x$
0	1				
1	11	10	0		
2	21	10	-3	-3	
3	28	7	-6	-3	0
4	29	1			

Hence $\Delta^4 u_0 = 0$.

7. If $u_1 = 1, u_3 = 17, u_4 = 43, u_5 = 89$. Find the value of u_2 .

Solution:

Let the missing term be y_1 .

x	$y = u_x$	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$	$\Delta^4 u_x$
1	1				
2	y_1	$y_1 - 1$	$-2y_1 + 18$		
3	17	$17 - y_1$	$y_1 - 43$	$3y_1 - 61$	$-4y_1 + 124$
4	43	26	20	$-y_1 + 63$	
5	89	46			

By assumption, we have $-4y_1 + 124 = 0$, $y_1 = 31$.

8. Define interpolation and extrapolation?

Solution:

The process of computing the value of a function inside the given range is called interpolation. The process of computing the value of a function outside the given range is called extrapolation.

9. State Newton's formula on interpolation. When it is used?

Solution:

Newton's forward interpolation formula is

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \dots$$

where $u = \frac{x - x_0}{h}$

This formula is used mainly for interpolating the values of y near the beginning of a set of tabular values. Newton's backward interpolation formula is

$$y = y_n + u\nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n + \dots$$

where $u = \frac{x - x_n}{h}$

This formula is used mainly for interpolating the values of y near the end of a set of tabular values.

10. State Newton's divided difference formula.

Solution:

$$y = y_0 + (x - x_0)\Delta y_0 + (x - x_0)(x - x_1)\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)\Delta^3 y_0 + \dots$$

11. State Lagrange’s interpolation formula

Solution:

$$\begin{aligned}
 y = f(x) = & \frac{(x - x_1)(x - x_2)(x - x_3) \dots (x - x_n)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3) \dots (x_0 - x_n)} y_0 \\
 & + \frac{(x - x_0)(x - x_2)(x - x_3) \dots (x - x_n)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3) \dots (x_1 - x_n)} y_1 \\
 & + \frac{(x - x_0)(x - x_1)(x - x_3) \dots (x - x_n)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3) \dots (x_2 - x_n)} y_2 \\
 & + \dots + \\
 & + \frac{(x - x_0)(x - x_1)(x - x_2)(x - x_3) \dots (x - x_{n-1})}{(x_n - x_0)(x_n - x_1)(x_n - x_2)(x_n - x_3) \dots (x_n - x_{n-1})} y_n
 \end{aligned}$$

12. What advantage has Lagrange’s formula over Newton?

Solution:

Newton’s formula can be used only when the values of the Independent variable x are equally spaced. But Lagrange’s interpolation formula can be used whether the values of the independent variable x are equally spaced or not. Lagrange’s formula can be used for inverse interpolation also, while Newton’s formula cannot be used.

13. What is the disadvantage in practice in applying Lagrange’s interpolation formula?

Solution:

It requires close attention to sign and there is always a chance of committing some error due to a number of positive and negative signs in the numerator and denominator.

14 What is the difference between Newton divided difference Formula and Lagrange’s formula.

Solution:

Lagrange’s formula is merely a variant of Newton divided difference formula.

15. What is inverse interpolation?

Solution:

It is the process of finding the values of x corresponding to a value of y, not present in the table.

16. What is the Lagrange’s formula to find y if three sets of values (x₀,y₀), (x₁,y₁), (x₂,y₂) are given?

Solution:

$$y = f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

17. Construct a linear interpolating polynomial given the point $(x_0, y_0), (x_1, y_1)$.

Solution:

$$y = f(x) = \frac{(x - x_1)}{(x_0 - x_1)} y_0 + \frac{(x - x_0)}{(x_1 - x_0)} y_1 = \frac{(y_0 - y_1)x + (x_0 y_1 - x_1 y_0)}{x_0 - x_1}$$

18. Write the Lagrange's formula for inverse interpolation.

Solution:

$$x = f(y) = \frac{(y - y_1)(y - y_2)(y - y_3) \dots (y - y_n)}{(y_0 - y_1)(y_0 - y_2)(y_0 - y_3) \dots (y_0 - y_n)} x_0 + \frac{(y - y_0)(y - y_2)(y - y_3) \dots (y - y_n)}{(y_1 - y_0)(y_1 - y_2)(y_1 - y_3) \dots (y_1 - y_n)} x_1 + \frac{(y - y_0)(y - y_1)(y - y_3) \dots (y - y_n)}{(y_2 - y_0)(y_2 - y_1)(y_2 - y_3) \dots (y_2 - y_n)} x_2 + \dots + \frac{(y - y_0)(y - y_1)(y - y_2)(y - y_3) \dots (y - y_{n-1})}{(y_n - y_0)(y_n - y_1)(y_n - y_2)(y_n - y_3) \dots (y_n - y_{n-1})} x_n$$

19. Find the quadratic polynomial that fits $y(x) = x^4$ at $x = 0, 1, 2$.

Solution:

The following data is

$$x : 0 \quad 1 \quad 2$$

$$y=x^4 : 0 \quad 1 \quad 16$$

By Lagrange's formula

$$y = f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

$$y = f(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)} \cdot 0 + \frac{(x-0)(x-2)}{(1-0)(1-2)} \cdot 1 + \frac{(x-0)(x-1)}{(2-0)(2-1)} \cdot 16$$

$$y = -x(x-2) + 8x(x-1)$$

$$y(x) = 7x^2 - 6x.$$

20. Use Lagrange's formula to find the quadratic polynomial that takes these values

$$\begin{matrix} x : & 0 & 1 & 3 \\ y : & 0 & 1 & 0 \end{matrix}$$

Then find y(2).

Solution:

By Lagrange's formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$y = f(x) = \frac{(x-1)(x-3)}{(0-1)(0-3)} \cdot 0 + \frac{(x-0)(x-3)}{(1-0)(1-3)} \cdot 1 + \frac{(x-0)(x-1)}{(3-0)(3-1)} \cdot 0$$

$$y(x) = \frac{x^2 - 3x}{-2}$$

Hence y(2) = 1.

21. Obtain the interpolation quadratic polynomial for the given data by using Newton forward difference formula

$$\begin{matrix} X : & 0 & 2 & 4 & 6 \\ Y : & -3 & 5 & 21 & 45 \end{matrix}$$

Solution:

The difference table is

x	y = f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-3	8	8	
2	5	16	8	0
4	21	24		
6	45			

Newton Forward Interpolation formula is

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \dots$$

where $u = \frac{x - x_0}{h} = \frac{x - 0}{2} = \frac{x}{2}$

$$y = -3 + (x/2)(8) + \frac{(x/2)(x/2-1)}{2!} (8) + 0$$

$$y = -3 + 4x + x(x-2)$$

$$y = x^2 + 2x - 3.$$

22. Show that the divided differences are symmetrical in their arguments.

Solution:

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = f(x_1, x_0).$$

23. What is the nature of nth divided differences of a polynomial of nth degree?

Solution:

The nth divided differences of a polynomial of nth degree is constant.

24. Find the second divided differences with arguments a,b,c if f(x) = 1/x.

Solution:

The divided difference table is

x	y = 1/x	Δy	$\Delta^2 y$
a	1/a	-1/ab	1/abc
b	1/b	-1/bc	
c	1/c		

25. If f(x) = 1/x², find f(a,b) and f(a,b,c) by using divided differences.

Solution:

The divided difference table is

x	y = 1/x ²	Δy	$\Delta^2 y$
a	1/a ²		$(ab + bc + ca) / a^2b^2c^2$
b	1/b ²	$-(a+b)/a^2b^2$	
c	1/c ²	$-(b+c)/b^2c^2$	

26. Using Lagrange's formula, find the polynomial through (0,0),(1,1),(2,2).

Solution:

Polynomial is $-x^2 + 2x + x^2 - x = x$ (Solve it)

27. By differentiating Newton forward and backward difference formula, find the first derivative of the function f(x).

Solution:

Newton forward interpolation formula is

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots$$

$$\text{where } u = \frac{x-x_0}{h}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{h} \left[\Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0 + \frac{2u^3-9u^2+11u-3}{12}\Delta^4 y_0 + \dots \right]$$

Newton backward interpolation formula is

$$y = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \dots$$

$$\text{where } u = \frac{x-x_n}{h}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{h} \left[\nabla y_n + \frac{2u+1}{2}\nabla^2 y_n + \frac{3u^2+6u+2}{6}\nabla^3 y_n + \frac{2u^3+9u^2+11u+3}{12}\nabla^4 y_n + \dots \right]$$

28. Using Newton's forward and backward difference formula, write the formula for the first and second derivative at the starting value $x = x_0$ and at the end value $x = x_n$ upto the fourth order difference term?

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2} \left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 - \dots \right]$$

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h} \left[\nabla y_n + \frac{1}{2} \nabla^2 y_n + \frac{1}{3} \nabla^3 y_n + \frac{1}{4} \nabla^4 y_n + \dots \right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2} \left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \dots \right]$$

29. Find the error in the derivative of $f(x) = \cos x$ by computing directly and using the approximation $f'(x) = \frac{f(x+h) - f(x-h)}{2h}$ at $x = 0.8$ choosing $h = 0.1$

Solution:

$$f(x) = \cos x \Rightarrow f'(x) = -\sin x \Rightarrow f'(0.8) = -0.717$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$f'(0.8) = \frac{f(0.8+0.1) - f(0.8-0.1)}{2(0.1)}$$

$$= \frac{f(0.9) - f(0.7)}{0.2}$$

$$= \frac{\cos(0.9) - \cos(0.7)}{0.2}$$

$$= -0.716$$

$$\text{Error} = -0.001$$

30. When can numerical differentiation be used?

Solution:

When the function is given in the form of table of values instead of giving analytical expression we use numerical differentiation.

31. If $f(x) = a^x$ ($a \neq 0$) is given for $x = 0, 0.5, 1$. Show by numerical differentiation that $f'(0) = 4\sqrt{a} - a - 3$.

Solution:

For $x = 0, 0.5, 1$, the values of $y = f(x) = a^x$ are $a^0, a^{0.5}, a^1$
 (i.e.) $1, \sqrt{a}, a$

x	$y = a^x$	Δy	$\Delta^2 y$
0	1		
0.5	\sqrt{a}	$\sqrt{a} - 1$	
1	a	$a - \sqrt{a}$	$a - 2\sqrt{a} + 1$

$$\begin{aligned} \left(\frac{dy}{dx}\right)_{x=0} &= f'(0) = \frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \dots \right] \\ &= \frac{1}{0.5} \left[\sqrt{a} - 1 - \frac{1}{2} (a - 2\sqrt{a} + 1) \right] \\ &= 2 \left[2\sqrt{a} - \frac{a}{2} - \frac{3}{2} \right] \\ &= 4\sqrt{a} - a - 3. \end{aligned}$$

32. Write down the Newton – cotes quadrature formula.

Solution:

$$\int_{x_0}^{x_n} f(x) dx = h \left\{ n y_0 + \frac{n^2}{2} \Delta y_0 + \frac{1}{2} \left(\frac{n^3}{3} - \frac{n^2}{2} \right) \Delta^2 y_0 + \frac{1}{6} \left(\frac{n^4}{4} - n^3 + n^2 \right) \Delta^3 y_0 + \dots \right\}$$

33. What is the geometrical interpretation of Trapezoidal rule?

Solution:

We are finding the area of the curve enclosed by $y = f(x)$, the X-axis, the ordinates $x = a$ and $x = b$ by using the area of trapezium.

34. Using Trapezoidal rule evaluate $\int_0^\pi \sin x dx$ by dividing the range into 6 equal

parts.

Solution:

$$h = \frac{\pi - 0}{6} = \frac{\pi}{6}$$

When $h = \frac{\pi}{6}$, the values of $y = \sin x$ are

x :	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π
y=sinx:	0	0.5	.8660	1	.8660	0.5	0

Trapezoidal rule is

$$\int_0^{\pi} \sin x dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})]$$

$$= \frac{\pi}{6(2)} [(0 + 0) + 2(0.5 + 0.8660 + 1 + 0.8660 + 0.5)]$$

$$= 0.9770$$

35. Write down the Trapezoidal rule to evaluate $\int_1^6 f(x) dx$ with $h = 0.5$

Solution:

Trapezoidal rule is

$$\int_1^6 f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})]$$

$$= \frac{0.5}{2} [(y_0 + y_{10}) + 2(y_1 + y_2 + y_3 + \dots + y_9)]$$

36. Why is Trapezoidal rule so called?

Solution:

The Trapezoidal rule is so called, because it approximates the integral by the sum of n trapezoids.

38. How the accuracy can be increased in Trapezoidal rule of evaluating a given definite integral?

Solution:

If the number of points of the base segment $b-a$, (the range of integration) is increased, a better approximation to the area given by the definite integral will be obtained.

39. When does Simpson's rule or Simpson's one-third rule give exact result?

Solution:

Simpson's rule will give exact result, if the entire curve $y = f(x)$ is itself a parabola.

40. What are the truncation errors in Trapezoidal and Simpson's rules of numerical integration?

Solution:

Error in the Trapezoidal rule is $-\frac{h^3}{12} f''(\theta)$. Error in the Trapezoidal rule is of

the order h^2 . Error in the Simpson's one-third rule is $-\frac{h^5}{90} f^{IV}(\theta)$. Error in the Simpson's one-third rule is of the order h^4 .

41. Can you use Simpson's rule for the following data:

x :	7.47	7.48	7.49	7.50	7.51	7.52	
f(x) :	1.93	1.95	1.98	2.01	2.03	2.06	Why?

Solution:

We cannot use Simpson's rule, since the number of ordinates is 6 (even).

42. Why Simpson's one-third rule is called a closed formula.

Solution:

Since the end point ordinates y_0 and y_n are included in the Simpson's one-third rule, it is called closed formula.

43. Using Simpson's rule find $\int_0^4 e^x dx$ given $e^0 = 1, e^1 = 2.72, e^2 = 7.39,$

$e^3 = 20.09, e^4 = 54.6$

Solution:

The following data is

x :	0	1	2	3	4
y :	1	2.72	7.39	20.09	54.6

Simpson's $1/3^{rd}$ rule is $\int_{x_0}^{x_n} f(x) dx =$

$$\frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots) + 2(y_2 + y_4 + y_6 + \dots)]$$

$$\int_0^4 e^x dx = \frac{1}{3} [(1 + 54.6) + 4(2.72 + 20.09) + 2(7.39)]$$

$$= 53.8733$$

44. Compare Trapezoidal rule and Simpson's $1/3^{rd}$ rule for evaluating numerical integration.

Solution:

i) In Newton Cotes Quadrature formula, if we put $n = 1$ we get Trapezoidal rule whereas if we put $n = 2$, we get Simpson's $1/3^{rd}$ rule.

ii) In Trapezoidal rule, the interpolating polynomial is linear whereas in Simpson's $1/3^{rd}$ rule, the interpolating polynomial is of degree 2.

iii) In Trapezoidal rule, there is no restriction on the number of intervals whereas in Simpson's $1/3^{\text{rd}}$ rule, the number of intervals should be even.

45. What is the need of Newton's and Lagrange's interpolation formulae?. (DEC 2012)

46. Use Lagrange's formula to fit a polynomial to the data and find y at x=1

x :	-1	0	2	3
y :	-8	3	1	12

(MAY 2012)

47. Show that the divided difference of second order can be expressed as the quotient of two determinants of third order. (MAY 2012).

48. Find the area under the curve passing through the points (0,0), (1,2), (2,2.5), (3,2.3), (4,2), (5, 1.7) and (6, 1.5). (MAY 2012).

PART – B

1. Using Newton's Forward Interpolation formula find the polynomial f(x) satisfying the following data. Hence find f(2).

x :	0	5	10	15
f(x) :	14	379	1444	3584

(MAY 2009)

2. Construct Newton's forward interpolation polynomial for the following data.

x :	4	6	8	10
y :	1	3	8	16

Use it to find the value of y for x = 5. (MAY 2011)

3. Given

$$\sin 45^\circ = 0.7071, \sin 50^\circ = 0.7660, \sin 55^\circ = 0.8192, \sin 60^\circ = 0.8660$$

Find $\sin 52^\circ$ by Newton's formula.

4. Given

x :	0	0.1	0.2	0.3	0.4
e^x :	1	1.1052	1.2214	1.3499	1.4918

Find the value of $y = e^x$ when $x = 0.38$.

5. The following data are taken from the steam table

Temp $^\circ\text{C}$:	140	150	160	170	180
Pressure kg f/cm^2 :	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature $t = 175^\circ$.

6. From the following data, estimate the number of persons earning weekly wages between 60 and 70 rupees.

Wage (in Rs.)	Below 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of person (in thousands)	250	120	100	70	50

7. Use Newton Backward formula to construct an interpolating polynomial of degree 3 for the data : $f(-0.75) = -0.07181250$, $f(-0.5) = -0.024750$,

$f(-0.25) = 0.33493750$, $f(0) = 1.10100$ Hence find $f\left(-\frac{1}{3}\right)$. (DEC 2010)

8. Using Lagrange's interpolation formula calculate the profit in the year 2000 from the following data

Year :	1997	1999	2001	2002
Profit in lakhs of Rs.	43	65	159	248

9. Give the values $x : 14 \quad 17 \quad 31 \quad 35$
 $f(x) : 68.7 \quad 64.0 \quad 44.0 \quad 39.1$
 Find the value of $f(x)$ when $x = 27$. (MAY 2011)

10. Using Lagrange's interpolation formula fit a polynomial to the following data

$x :$	-1	0	2	3
$y :$	-8	3	1	12

and hence find y at $x = 1.5$ (MAY 2009)

11. Given $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$,
 $\log_{10} 659 = 2.8189$, $\log_{10} 661 = 2.8202$. Find $\log_{10} 656$ by using Lagrange's formula.

12. Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for
- | | | | | |
|----------|---|---|----|-----|
| $x :$ | 0 | 1 | 2 | 5 |
| $f(x) :$ | 2 | 3 | 12 | 147 |

13. Using Lagrange's formula, Prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8} \left[\frac{1}{2}(y_{-3} - y_{-1}) - \frac{1}{2}(y_1 - y_3) \right] \text{ (DEC 2010)}$$

14. Applying Lagrange's formula to find the roots of the equation $f(x) = 0$ when $f(30) = -30, f(34) = -13, f(38) = 3, f(42) = 18$. (MAY 2011)

15. Using Newton divided difference formula find $u(3)$ given $u(1) = -26, u(2) = 12, u(4) = 256, u(6) = 844$.

16. Using Newton divided difference method find $f(1.5)$ using the data $f(1.0) = 0.7651977, f(1.3) = 0.6200860, f(1.6) = 0.4554022, f(1.9) = 0.2818186, f(2.2) = 0.1103623$. (DEC 2010)

17. Given $u_0 = -4, u_1 = -2, u_4 = 220, u_5 = 546, u_6 = 1148$

Find u_2 and u_3 .

18. From the following table, find the value of $\tan 45^\circ 15'$ by Newton's Forward Interpolation formula.

x° :	45	46	47	48	49	50
$\tan x^\circ$:	1	1.03553	1.07237	1.11061	1.15037	1.19175

19. From the given table, the values of y are consecutive terms of a series of which 23.6 is the sixth term. Find the first and tenth terms of the series.

x :	3	4	5	6	7	8	9
y :	4.8	8.4	14.5	23.6	36.2	52.8	73.9

20. Find $f'(3)$ and $f''(3)$ for the following data:

x :	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$:	-14	-10.032	-5.296	-0.256	6.672	14

21. The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data

time (sec.):	0	5	10	15	20
velocity (m/sec.):	0	3	14	69	228

22. Find $\frac{d\theta}{dt}$ at $t = 3$ and $t = 8$ given

t :	1	3	5	7	9
θ :	85.3	74.5	67	60.5	54.3

(DEC 2010)

23. Find $y'(1)$ from the following data

x :	0	2	3	4	7	9
y :	4	26	58	112	466	922

(DEC 2012)

24. Find the value of $\sec 31^\circ$ from the following data :

θ (deg) :	31°	32°	33°	34°
$\tan \theta$:	0.6008	0.6249	0.6494	0.6745 (MAY 2009)

25. Consider the following table of data :

x :	0.2	0.4	0.6	0.8	1.0
f(x) :	0.9798652	0.9177710	0.8080348	0.6386093	0.3843735

Find $f'(0.25)$ using Newton Forward interpolation formula
and $f'(0.95)$ using Newton Backward interpolation formula.

26. Find the maximum and minimum value of y tabulated below

x :	-2	-1	0	1	2	3	4
y :	2	-0.25	0	-0.25	2	15.75	56 (MAY 2011)

27. Using Trapezoidal rule, evaluate $\int_{-1}^1 \frac{dx}{1+x^2}$ taking 8 intervals.

28. Evaluate $\int_0^1 e^{-x^2} dx$ by dividing the range of integration into four equal parts using Trapezoidal rule.

29. Find an approximate value of $\log_e 5$ by calculating to four decimal places by

Simpson's rule the integral $\int_0^5 \frac{dx}{4x+5}$ dividing the range into 10 equal parts.

30. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ take $h = 0.125$. Hence find π using Simpson's rule.

31. Evaluate $\int_0^\pi \sin x dx$ by trapezoidal and Simpsons's (1/3) rules by dividing the range into 10 equal parts. (DEC 2012)

32. Evaluate $\int_0^\pi \sin x dx$ by trapezoidal and Simpsons's rules by dividing the range into 10 equal parts Verify your answer with integration. (MAY 2013)

33. Using Newton divided difference method find the values of $f(2)$, $f(8)$ and $f(15)$ given the following table

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

(MAY 2013)

34. The population of a certain town is given below. Find the rate of growth of the population in 1931, 1941, 1951, 1961, and 1971.

Year x :	1931	1941	1951	1961	1971
Population in thousands (y):	40.62	60.80	79.95	103.56	132.65

(MAY 2013)

35. If $f(x)=1/x$, show that $f(x_0, x_1, \dots, x_n) = \frac{(-1)^r}{x_0 x_1 \dots x_r}$ where r is any positive integer.

(MAY 2013)

36. Find the cubic polynomial $y(x)$ for

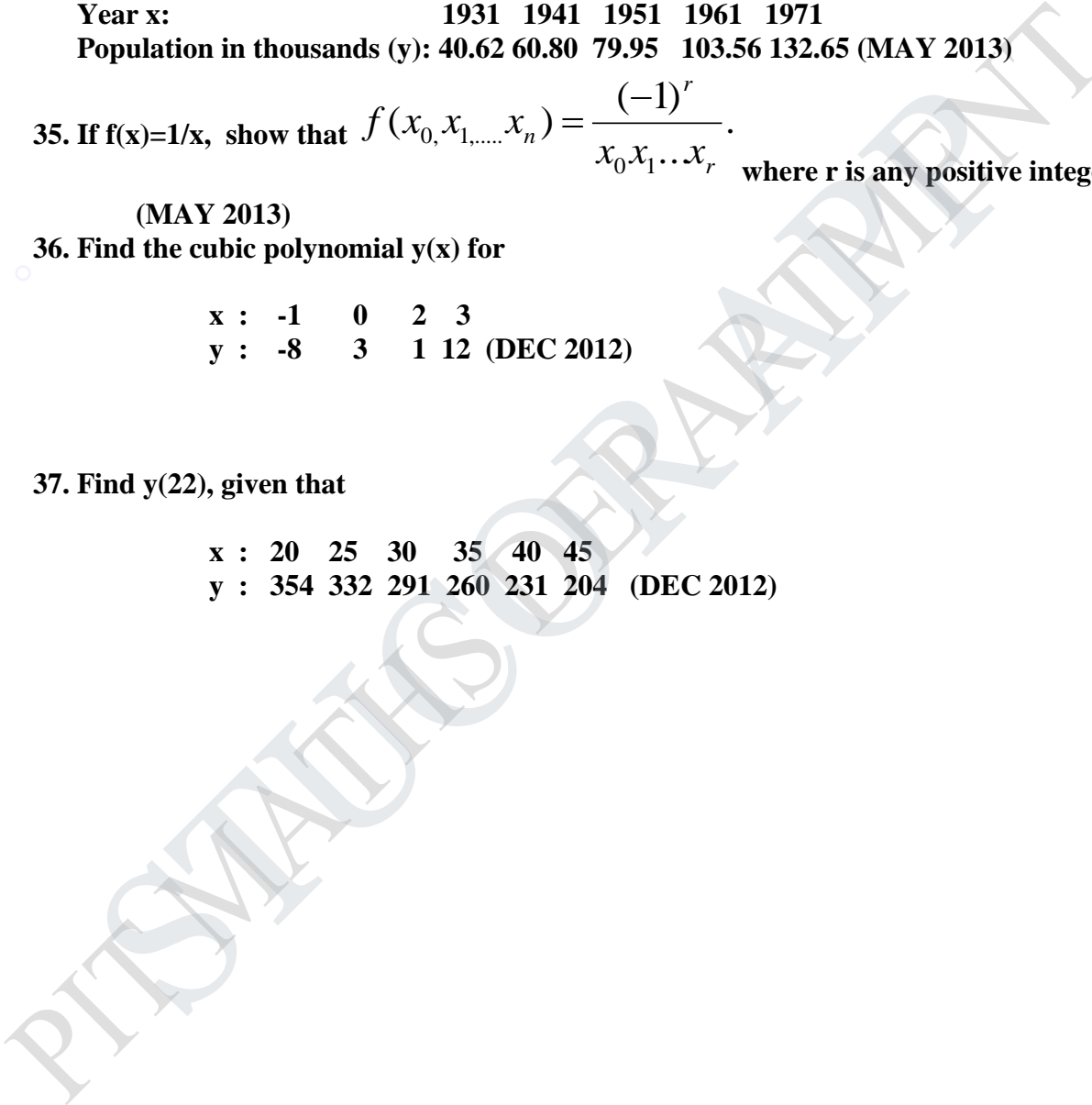
x :	-1	0	2	3
y :	-8	3	1	12

(DEC 2012)

37. Find $y(22)$, given that

x :	20	25	30	35	40	45
y :	354	332	291	260	231	204

(DEC 2012)



UNIT V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

PART – A

1. What are the merits and demerits of the Taylor series method. (DEC 2012).

Solution:

Taylor method is very powerful if we can calculate the successive derivative of y in an easy manner. If there is a simple expression for the higher derivatives in terms of the previous derivatives of y, Taylor’s method will work very well. But in the differential

equation $\frac{dy}{dx} = f(x, y)$, the function f(x,y) may have a complicated algebraical structure. Then the evaluation of higher order derivatives may become tedious. This is the demerit of this method.

2. Taylor series method will be very useful to give some _____ for powerful numerical methods such as Runge kutta method, Milne’s method, etc.

Solution:

Initial starting values.

3. The use of Taylor series to solve numerically, differential equations is restricted by the labour involved in the determination of _____.

Solution:

Higher order derivatives.

4. Solve the differential equation $\frac{dy}{dx} = x + y + xy$, $y(0) = 1$ by Taylor series method to get the value of y at $x = h$.

Solution:

Given

$$y' = x + y + xy$$

$$x_0 = 0, y_0 = 1$$

$$y' = x + y + xy$$

$$y_0' = 0 + 1 + 0 = 1$$

$$y'' = 1 + y' + xy' + y$$

$$y_0'' = 1 + 1 + 0 + 1 = 3$$

$$y''' = y'' + xy'' + y' + y'$$

$$y_0''' = 3 + 1 + 0 + 1 = 5$$

$$y'''' = y''' + 2y'' + xy''' + y''$$

$$y_0'''' = 5 + 6 + 0 + 3 = 14$$

Taylor’s series is

$$y(x) = y_0 + (x - x_0)y'_0 + \frac{(x - x_0)^2}{2!} y''_0 + \frac{(x - x_0)^3}{3!} y'''_0 + \frac{(x - x_0)^4}{4!} y^{iv}_0 + \dots$$

$$y(h) = 1 + (h - 0)(1) + \frac{(h - 0)^2}{2} (3) + \frac{(h - 0)^3}{6} (5) + \frac{(h - 0)^4}{24} (14) + \dots$$

$$y(h) = 1 + h + \frac{3}{2} h^2 + \frac{5}{6} h^3 + \frac{7}{12} h^4 + \dots$$

5. State the disadvantage of Taylor series method?

Solution:

The disadvantage of Taylor series method is the evaluation of higher order derivative which may become tedious for the function which have a complicated algebraical structure.

6. Name the method which is Taylor's method of first order.

Solution:

Euler's method.

7. Taylor series and Runge kutta method are single step methods while _____ and _____ are multistep methods.

Solution:

Milne's predictor and corrector method and Adam-Bashforth predictor and corrector method.

8. Which formula is a particular case of Runge kutta formula of the second order.

Solution:

Euler's modified formula.

9. Compute y at x=0.25 by modified Euler's method given

$$\frac{dy}{dx} = x^2 + y^2, y(0) = 1.$$

(DEC 2012)

Solution:

Given $f(x,y) = x^2 + y^2$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$$y_1 = y_0 + \frac{h}{2} \{f(x_0, y_0) + f[x_0 + h, y_0 + hf(x_0, y_0)]\}$$

$$f(x_0, y_0) = x_0^2 + y_0^2 = 0 + 1 = 1$$

$$y_1 = 1 + \frac{0.1}{2} \{1 + f[0 + 0.1, 1 + 0.1(1)]\}$$

$$y_1 = 1 + \frac{0.1}{2} \{1 + f[0.1, 1.1]\}$$

$$y_1 = 1 + \frac{0.1}{2} \{1 + 1.22\}$$

$$y_1 = 1.111$$

10. Write down the formula to solve second order differential equation using Runge kutta method of 4th order.

Solution:

The second order differential equation is

$$\frac{d^2y}{dx^2} = g(x, y, \frac{dy}{dx})$$

put $\frac{dy}{dx} = z$ ----- (1)

then $\frac{dz}{dx} = g(x, y, z)$ ----- (2)

Equation (1) and (2) can be solved using the formula

$$k_1 = hf(x_0, y_0, z_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3, z_0 + l_3)$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \Delta y$$

$$l_1 = hg(x_0, y_0, z_0)$$

$$l_2 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$$

$$l_3 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$$

$$l_4 = hg(x_0 + h, y_0 + k_3, z_0 + l_3)$$

$$\Delta z = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$z_1 = z_0 + \Delta z$$

Similarly we can find the next increments by replacing x_0, y_0, z_0 and x_1, y_1, z_1 and so on.

11. In the derivation of 4th order Runge kutta formula, why it is called fourth order.

Solution:

The number of parameters = 13. It is called fourth order formula since the parameters are determined such that y_{n+1} obtained by Runge kutta method agrees upto h^4 term in Taylor's method.

12. What are the values of k_1 and l_1 to solve $y''+xy'+y = 0, y(0) = 1, y'(0) = 0$ by Runge kutta method of 4th order.

Solution:

Given

$$y''+xy'+y = 0$$

$$\text{put } \frac{dy}{dx} = z \quad \text{then } \frac{dz}{dx} + xz + y = 0$$

$$\Rightarrow \frac{dz}{dx} = -xz - y$$

Also given $x_0 = 0, y_0 = 1, y'_0 = z_0 = 0$

$$k_1 = hf(x_0, y_0, z_0) = hz_0 = 0.$$

$$\begin{aligned} l_1 &= hg(x_0, y_0, z_0) = h(-x_0z_0 - y_0) \\ &= h(0 - 1) \\ &= -h. \end{aligned}$$

13. What do you mean by total error in error analysis?

Solution:

The difference between the computed value y_i and the true value $y(x_i)$ at any stage is known as the total error. It is comprised of truncation error and round off error.

14. State the special advantage of Runge kutta method over Taylor series method. (or) Compare Taylor series and Runge kutta method. (or) Which is better Taylor method or Runge kutta method.

Solution:

Runge kutta methods do not require prior calculation of higher derivatives of $y(x)$, as the Taylor method does. Also the Runge kutta formulas involve the computation of $f(x,y)$ at various positions, instead of derivatives and this function occurs in the given equation. Hence Runge kutta method is better method.

15. Runge kutta formulas involve the computation of $f(x,y)$ at various points instead of _____

Solution:

Calculation of higher order derivatives of $f(x,y)$.

16. Compare Taylor series and Runge kutta method.

Solution:

1. The use of Runge kutta method gives quick convergence to the solutions of the differential equation than Taylor's series method.

2. In Runge kutta method, the derivatives of higher order are not required for calculation as in Taylor's series method.

17. Write the formula to find k_2, k_4 in R-k method of fourth order.

Solution:

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

18. If $f(x,y) = f(x)$ i.e. only a function of x alone, then the fourth order Runge kutta method reduces to _____

Solution:

The area by Simpson's one-third rule.

19. Compare Runge kutta method and predictor - corrector method.

Solution:

Runge kutta methods are self starting whereas predictor and corrector method are not self starting. In Runge kutta method, it is not possible to get any information about truncation error. In predictor and corrector method it is possible to get easily a good estimate of the truncation error.

20. What will you do, if there is a considerable difference between predicted value and corrected value, in predictor and corrector methods?

Solution:

If there is a considerable difference between predicted value and corrected value, we take the corrected value as the predicted value and find out the new corrected value. This process is repeated till there is no great difference between two consecutive corrected values.

21. Write down the error in Milne's method.

Solution:

The truncation error in Milne's predictor formula is $\frac{14h^5}{45} y^5(\epsilon)$ The truncation

error in Milne's corrector formula is $\frac{-h^5}{90} y^5(\epsilon)$

22. What is a predictor – corrector method ?

Solution:

Predictor – corrector methods are methods which require the values of y at $x_n, x_{n-1}, x_{n-2}, \dots$ for computing the values of y at x_{n+1} . We first use a formula to find the value of y at x_{n+1} and this is known as a predictor formula. The value of y so got is improved or corrected by another formula is known as corrector formula.

23. What do you mean by saying that a method is self starting? Not self starting ?

Solution:

For computing the values of y at x_{n+1} , we do not require the values of y at $x_n, x_{n-1}, x_{n-2}, \dots$. This is called self starting. If we require the values of y at $x_n, x_{n-1}, x_{n-2}, \dots$ to find y at x_{n+1} then it is called not self starting.

24. Write down Milne's predictor and corrector formula.

Solution:

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3} [2y_{n-2}' - y_{n-1}' + 2y_n']$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3} [y_{n-1}' + 4y_n' + y_{n+1}']$$

25. What is the condition to apply Milne's method.

Solution:

It requires four starting values of y .

26. Mention the multistep methods available for solving ordinary differential equation

Solution:

- i) Milne's predictor - corrector method
- ii) Adam's Bashforth predictor – corrector method.

27. Explain the terms initial and boundary value problems.

Solution:

If the conditions are prescribed at only one point then the differential equation together with the conditions form an initial value problem and if the conditions are prescribed at two or more points, then the problem is termed as boundary value problems.

28. State the Taylor series formula to find $y(x_1)$ for solving

$$\frac{dy}{dx} = x^2 y - 1, y(x_0) = y_0$$

Solution:

$$y(x_1) = y_0 + (x - x_0)y'(x_0) + \frac{(x - x_0)^2}{2!} y''(x_0) + \frac{(x - x_0)^3}{3!} y'''(x_0) + \dots$$

29. Obtain the finite difference scheme for the differential

equation $2\frac{d^2y}{dx^2} + y = 5.$

Solution:

Given $2\frac{d^2y}{dx^2} + y = 5.$

$$2\left(\frac{y_{i+1} + y_{i-1} - 2y_i}{h^2}\right) + y_i = 5$$

$$2y_{i+1} + (h^2 - 4)y_i + 2y_{i-1} = 5h^2$$

30. Explain one-step methods and multi step methods.

Solution:

In one-step methods, we use the data of just one preceding step. In multi step methods, where in each step, we use data from more than one of the preceding steps.

31. Find $y(1.1)$ using Euler's method from $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.$

Solution:

$$y_1 = y_0 + h f(x_0, y_0) = 1 + (0.1)[1+1] = 1+0.2 = 1.2$$

32. Compute y at $x=0.25$ by Modified Euler's method given $\frac{dy}{dx} = 2xy$, $y(0)=1.$
(MAY 2012)

33. Using Taylor series method, find y at $x= 0.1,0.2$ given $\frac{dy}{dx} = x^2 - y$, $y(0)=1.$
(MAY 2012)

PART - B

1. Using Taylor's series find y at $x = 0.1$ if $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1.$

2. By means of Taylor series expansion, find y at $x = 0.1$ and $x = 0.2$ correct to three decimal places, given $\frac{dy}{dx} - 2y = 3e^x$, $y(0) = 0.$ (MAY 2011)

3. Use Taylor series solution to solve numerically $\frac{dy}{dx} = xy^{\frac{1}{3}}$, $y(1) = 1$. Tabulate y for $x = 1.1, 1.2$ (MAY 2011)
4. Find the value of $y(1.1)$ and $y(1.2)$ from $\frac{d^2y}{dx^2} + y^2 \frac{dy}{dx} = x^3$, $y(1) = 1$, $y'(1) = 1$ by using Taylor's series method. (MAY 2009)
5. Use Euler's method to approximate y when $x = 0.1$ given that $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y = 1$ for $x = 0$.
6. Solve $\frac{dy}{dx} = 1 - y$ with the initial condition $x = 0, y = 0$. Using Euler's algorithm, tabulate the solutions at $x = 0.1, 0.2, 0.3, 0.4$. Get the solutions by Euler's modified method also.
7. Given that $\frac{dy}{dx} = \log_{10}(x + y)$ with the initial condition that $y = 1$ when $x = 0$, use Euler's modified method to find y for $x = 0.2$ and $x = 0.5$ in more accurate form.
8. Using R-K method of fourth order, solve $y' = 3x + \frac{1}{2}y$ with $y(0) = 1$ at $x = 0.2$ taking $h = 0.1$
9. Use 4th order R-K method to solve $y' = xy$ for $x = 1.2, 1.4, 1.6$ Initially $x = 1, y = 2$ (take $h = 0.2$) (DEC 2010)
10. Given $y'' + xy' + y = 0$, $y(0) = 1, y'(0) = 0$, find the value of $y(0.1)$ by using R-K method of fourth order.
11. Consider the 2nd order initial value problem $y'' - 2y' + 2y = e^{2t} \sin t$ with $y(0) = -0.4, y'(0) = -0.6$
 - i) Using Taylor series approximation, find $y(0.1)$
 - ii) Using 4th order R-K method, find $y(0.2)$ (MAY 2009)

12. Solve $y' = x - y^2, 0 \leq x \leq 1, y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762$ by Milne's method to find $y(0.8)$ and $y(1)$.
13. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2$
 i) Compute $y(0.2), y(0.4)$ and $y(0.6)$ by R-K method of 4th order.
 ii) Hence find $y(0.8)$ by Milne's predictor corrector method taking $h = 0.2$
14. Given that $y'' + xy' + y = 0, y(0) = 1, y'(0) = 0$. Obtain for $x = 0.1, 0.2, 0.3$ by Taylor's series method and find the solution for $y(0.4)$ by Milne's method. (DEC 2010)
15. Given $y' = x(x^2 + y^2)e^{-x}, y(0) = 1$ find y at $x = 0.1, 0.2$ and 0.3 by Taylor's series method and compute $y(0.4)$ by Milne's method. (DEC 2010)
16. Using fourth order Runge kutta method, solve the following equation taking each step of $h = 0.1$ Given $y(0) = 3, \frac{dy}{dt} = \frac{4t}{y} - t y$ Calculate y for $x = 0.1$ and 0.2 (MAY 2011)
17. Given $y' + x y^2 + y = 0, y(0) = 1$, find the value of $y(0.2)$ by using Runge-kutta method of 4th order.
18. Apply Runge-kutta method to find approximate value of y for $x = 0.2$ in steps of 0.1 if $\frac{dy}{dx} = x + y^2$ given that $y = 1$ when $x = 0$. (DEC 2010)
19. Using Taylor series method find correct to four decimal places, the value of $y(0.1)$ given $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$. (MAY 2009)
20. Find by Taylor series method, the values of y at $x = 0.1$ and $x = 0.2$ to four decimal places from $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$.
21. Using Euler's method, solve numerically the equation $y' = x + y, y(0) = 1$, for $x = 0.0 (0.2) (1.0)$. Check your answer with the exact solution.

22. Using the finite difference method, find $y(0.25)$, $y(0.5)$ and $y(0.75)$ satisfying the differential equation $\frac{d^2 y}{dx^2} + y = x$ subject to the boundary conditions $y(0) = 0$, $y(1) = 2$.
23. Solve $xy'' + y = 0$, $y(1) = 1$, $y(2) = 2$ with $h = 0.25$ by finite difference method.
24. Solve $y'' - xy = 0$ given $y(0) = -1$, $y(1) = 2$ by finite difference method taking $n = 2$.
25. Using finite difference method, solve for y given the differential equation $\frac{d^2 y}{dx^2} + y + 1 = 0$, $x \in (0,1)$ and the boundary conditions $y(0) = y(1) = 0$, taking
- i) $h = 1/2$ ii) $h = 1/4$ (MAY 2011)
22. Using finite difference method solve $\frac{d^2 y}{dx^2} = y$ in $(0,2)$ given $y(0) = 0$, $y(2) = 3.63$ subdividing the range of x into 4 equal parts.
27. Using finite difference method, solve for y given the differential equation $y'' - 64y + 10 = 0$, $x \in (0,1)$ given $y(0) = y(1) = 0$, subdividing the interval into
- i) 4 equal parts ii) 2 equal parts.
28. Solve $y'' - y = x$, $x \in (0,1)$ given $y(0) = y(1) = 0$ using finite differences dividing the interval into 4 equal parts.
29. Solve $y_{x+2} - 7y_{x+1} - 8y_x = x(x-1)2^x$. (MAY 2012)
30. Using 4th order R-K method to solve $y' = \frac{y^2 - x^2}{y^2 + x^2}$ given $y(0)=1$ at $x=0.2, 0.4$ (MAY 2012)
32. Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$. Find the value of $y(0.1)$ by using Runge kutta method of fourth order. (MAY 2012)
33. If $y' = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0)=1$, find $y(0.2)$, $y(0.4)$ and $y(0.6)$ by at $x=0.2, 0.4$ by R-K method of fourth order and hence find $y(0.8)$ by Milne's method. (DEC 2012)

34. By Taylor's series method find $y(0.1)$, $y(0.2)$ and $y(0.3)$ if $\frac{dy}{dx} = x - y^2$, $y(0)=1$.

(DEC 2012)

34. By Modified Euler's method find $y(0.1)$, $y(0.2)$ and $y(0.3)$ if $\frac{dy}{dx} = x + y$, $y(0)=1$.

(DEC 2012)

PITSMATHS DEPARTMENT