MA3251 - SNM

ANNA UNIVERSITY, CHENNAI-25

MA8452 STATISTICS AND NUMERICAL METHODS L T P C 3 1 0 4

OBJECTIVES:

This course aims at providing the necessary basic concepts of a few statistical and numerical methods and give procedures for solving numerically different kinds of problems occurring inengineering and technology.

UNIT I TESTING OF HYPOTHESIS 9+3

Large sample test based on Normal distribution for single mean and difference of means – Tests based on t,λ^2 and F distributions for testing means and variances – Contingency table (Test for Independency) – Goodness of fit.

UNIT II DESIGN OF EXPERIMENTS 9+3

One way and two way classifications - Completely randomized design – Randomized block design – Latin square design - 22 factorial design.

UNIT III SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS 9+3

Newton Raphson method – Gauss elimination method – pivoting – Gauss Jordan methods – Iterative methods of Gauss Jacobi and Gauss Seidel – Matrix inversion by Gauss Jordan method – Eigen values of a matrix by power method.

UNIT IV INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION 9+3

Lagrange's and Newton's divided difference interpolations – Newton's forward and backward difference interpolation – Approximation of derivates using interpolation polynomials – Numerical single and double integrations using Trapezoidal and Simpson's 1/3 rules.

UNIT V NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS 9+3

Taylor's series method – Euler's method – Modified Euler's method – Fourth order Runge-Kutta method for solving first order equations – Milne's predictor corrector methods for solving first order equations – Finite difference methods for solving second order equations.

TOTAL (L:45+T:15):60 PERIODS

OUTCOMES

It helps the students to have a clear perception of the power of statistical and numerical techniques, ideas and would be able to demonstrate the applications of these techniques to problems drawn from industry, management and other engineering fields.

TEXT BOOKS

1. Johnson. R.A., and Gupta. C.B., "Miller and Freund's Probability and Statistics for Engineers", 11th Edition, Pearson Education, , Asia, 2011.
2. Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition, Khanna Publishers, New Delhi, 2007.

REFERENCES

1. Walpole. R.E., Myers. R.H., Myers. S.L., and Ye. K., "Probability and Statistics for Engineers and Scientists", 8th Edition, Pearson Education, Asia, 2007.
2. Spiegel. M.R., Schiller. J., and Srinivasan. R.A., "Schaum's Outlines on Probability and Statistics", Tata McGraw Hill Edition, 2004.
3. Chapra. S.C., and Canale. R.P, "Numerical Methods for Engineers", 5th Edition, Tata McGraw Hill, New Delhi, 2007.

TABLE OF CONTENTS

AIM AND OBJECTIVE OF THE SUBJECT

- To provide the necessary basic concepts of a few statistical and numerical methods
- To know about the Statistical Data
- Learn to solve problems in various type of Designs of Experiments
- To understand the concept of numerical methods
- To know the various type of Numerical Differentiation and Integration
- To know about the Interpolation problems
- Learn to solve problems Nnumerical solution of Ordinary Differential Equations
- To have a clear perception of the power of statistical and numerical techniques, and ideas
- Be able to demonstrate the applications of these techniques to solve the problems in industry, management and other engineering fields

DETAILED LESSON PLAN

TEXT BOOKS

1. Johnson. R.A., and Gupta. C.B., "Miller and Freund's Probability and Statistics for Engineers", 11th Edition, Pearson Education, , Asia, 2011.
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3. Chapra. S.C., and Canale. R.P, "Numerical Methods for Engineers", 5th Edition, Tata McGraw Hill, New Delhi, 2007.
4. Gerald. C.F., and Wheatley. P.O. "Applied Numerical Analysis" Pearson Education, Asia, New Delhi, 2006.

Sl. No	Unit	Topic / Portions to be Covered	Hours Required / Planned	Cumulative Hrs	Books Referred
UNIT I - SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS					
1	III	Gauss elimination method	1	1	TB2
2	III	Pivoting	1	2	TB2
3	III	Gauss Jordan methods	1	3	TB1
4	III	Iterative methods of Gauss Jacobi	2	5	TB1
5	III	Gauss Seidel	2	7	RB1
6	III	Matrix inversion by Gauss Jordan method	1	8	RB1
7	III	Eigen values of a matrix by power method	2	10	TB2
8	III	Newton Raphson method	2	12	TB2
9	III	REVISION	1	13	
UNIT II - DESIGN OF EXPERIMENTS					
10	II	Completely randomized design	3	16	RB1
11	II	Randomized block design	3	19	TB2
12	II	Latin square design	3	22	RB3
13	II	2^2 factorial design	3	25	TB2
14	II	REVISION	1	26	

Sl. No	Unit	Topic / Portions to be Covered	Hours Required / Planned	Cumulative Hrs	Books Referred
UNIT IV - INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION					
15	IV	Lagrange's interpolations	1	27	TB1
16	IV	Newton's divided difference	2	29	TB2
17	IV	Newton's forward and backward difference interpolation	3	32	TB1
18	IV	Approximation of derivates using interpolation polynomials	2	34	RB1
19	IV	Numerical single and double integrations using Trapezoidal rules	2	36	RB1
20	IV	Numerical single and double integrations using Simpson's 1/3 rules	2	38	TB2
21	IV	REVISION	1	39	
UNIT V - NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS					
22	V	Taylor's series method	2	41	TB2
23	V	Euler's method	1	42	TB1
24	V	Modified Euler's method	1	43	RB1
25	V	Fourth order Runge-Kuttamethod	3	46	RB1
26	V	Milne's predictor corrector methods	3	49	RB2
27	V	Finite difference methods for solving second order equations.	2	51	RB2
28	V	REVISION	1	52	
UNIT I - TESTING OF HYPOTHESIS					
29	I	Large sample test based on Normal distribution for single and diff. of means	2	54	TB1
30	I	Tests based on t	2	56	RB1
31	I	Tests based on chi square method	2	58	RB3
32	I	F Distributions for testing means	1	59	TB1
33	I	F Distributions for testing variances	2	61	TB1
34	I	Tests based on Contingency table	1	62	TB2
35	I	Goodness of fit.	2	64	TB2
36	I	REVISION	1	65	

Unit I

PART - A

1.) Mention the various steps involved in testing of hypothesis. (A/M2010)

Soln

(i.) Set up the null hypothesis

(ii.) Choose the appropriate level of significance

(iii.) compute the test statistic $z = \frac{t - E(t)}{SE(t)}$ under the null hypothesis

(iv.) We compare the computed value of z in step (iii.) with the significant value at the given level of significance.

2.) Define chi-square test for goodness of fit (A/M 2010)

Soln

Karl Pearson developed a test for testing the significance of discrepancy between experimental values and theoretical values obtained under some theory or hypothesis. This test is known as χ^2-test of goodness of fit.

3. The heights of college students in chennai are normally distributed with standard deviation 6cm and sample of 100 students had their mean height 158 cm. Test the hypothesis that the mean height of college students in chennai is 160 cm at 1% level of significance. (Nov/Dec: 2011)

Soln:

$n = 100,\ \sigma = 6,\ \bar{x} = 158,\ \mu = 160,\ H_0: \bar{x} = \mu,\ H_1: \bar{x} \neq \mu$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{158 - 160}{6/\sqrt{100}} = \frac{-2}{0.6} = -3.33$$

$\therefore |z| > z_\alpha$

H_0 is rejected

4. A coin is tossed 400 times and it turns up head 216 times. Discuss whether the coin may be unbiased one at 5% level of significance (Nov/Dec: 2011)

Soln:

H_0: The coin is unbiased, $p = 1/2$

H_1: $p \neq 1/2$

Los : 5%

$\therefore z_\alpha = 1.96$

$$z = \frac{x - nP}{\sqrt{nPQ}} = \frac{216 - 400 \times \frac{1}{2}}{\sqrt{400 \times \frac{1}{2} \times \frac{1}{2}}} = \frac{16}{10} = 1.6$$

$\therefore z < |z_\alpha|$

H_0 is accepted

5. Define Type-I error and Type-II error (Nov/Dec 2012)

Type-I error is the probability that we reject the null hypothesis when it is true. $\alpha = p$ [reject H_0 / H_0 is true]

③

Type - II error is the probability that we do not reject the null hypothesis when it is not true. $\beta = P$ [do not reject H_0 / H_0 is true]

6. State the applications of chi-square test (May/June: 2012)

(i) Test of Goodness of fit

(ii) To test Dependency of the Attributes

7. In a large city, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is difference between the proportions significant. (May/June 2012)

Soln:

$n_1 = 900,\ n_2 = 1600,\ p_1 = 0.2,\ p_2 = 0.185$

H_0: The difference between the two proportions are not significant

$$\text{Now, } p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{900(0.2) + 1600(0.185)}{900 + 1600} = 0.1904$$

$$q = 1 - p = 1 - 0.1904 = 0.8096$$

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.015}{0.016} = 0.9375$$

Since $|z| < 1.96$, we accept the hypothesis H_0 at 5% L.O.S

$p_1 = p_2$

The difference between the two proportions are not significant

8. Write the application of F-test and ψ^2-test

F-test (April/May 2014)

To test if the 2 samples have come from the same population.

ψ^2-test

To test the significance of discrepancy between experimental values and the theoretical values

9. Write the test of significance for difference of means (April/May 2014)

$$Z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$, $\bar{x}_1, \bar{x}_2$ are sample means

s_1^2 & s_2^2 sample means s_1^2 & s_2^2 sample variances n_1, n_2 are sample sizes

10. State the parameters and statistics in sampling? (Nov/Dec 2013)

To avoid the verbal confusion with the statistical constants of the population, namely mean μ, variance σ^2 which are usually referred to as parameters. Statistical measures are computed from sample observations alone. eg. mean $(\bar{x})$, variance (s^2) etc. are usually refered to as statistic

UNIT- II

1. Define Mean sum of squares (April / May 2013)

Mean squares are estimates of variance across groups. Mean squares are used in analysis of variance and are calculated as a sum of squares divided by its appropriate degrees of freedom.

2. What are the advantages of a CRD? (April / May 2013)

(i) It is completely flexible. Any number of treatments can be investigated

(ii) Each treatment can have any number of units although balance is desirable.

(iii) The statistical analysis is straight forward.

3. State the assumptions involved in ANOVA (May/June 2012)

There are several approaches to the analysis of variance (i) Independence of cases - this is an assumption of the model that simplifies the statistical analysis

(ii) Normality - the distributions of the residuals are normal

(iii) Equality (or homogeneity) of variances, called homoscedasticity

4. What are the advantages of a Latin square design?
(Nov/Dec: 2012)

(i) Controls more variation than CR or RCB designs because of 2-way stratification

(ii) Results in a smaller mean square for error

(iii) Simple analysis of data

(iv) Analysis is simple even with missing plots

5. Write down the ANOVA table for one way classfication
(Nov/Dec: 2012)

Sources of variation	Sum of Squares	Degree of freedom	Mean square	F-ratio
Between samples	SSC	$\nu_1 = k-1$	$MSC = \frac{SSC}{k-1}$	$F_c = \frac{MSC}{MSE}$
Within samples	SSE	$\nu_2 = N-k$	$MSE = \frac{SSE}{N-k}$	

6. What are the expected frequencies of 2x2 contigency table

a	b
c	d

? (April/May 2015)

Attributes	B	β	Total
A	a	b	a+b
α	c	d	c+d
Total	a+c	b+d	a+b+c+d

$$E(a) = E(A,B) = \frac{(a+b)(a+c)}{a+b+c+d}$$

$$P(b) = \frac{(a+b)(b+d)}{a+b+c+d}, \quad P(c) = \frac{(c+d)(a+c)}{a+b+c+d}$$

$$P(d) = \frac{(c+d)(b+d)}{a+b+c+d}$$

7. What are the basic principles of the design of experiment (April/May 2015)

Replication, Randomisation, local control

8. What do you understand by design of an experiment? (April/May 2011)

The design of experiment may be defined as its logical construction of the experiment in which the degree of uncertainity with which the inference is drawn may be well-defined

9. What are the advantages of completely randomised block design? (April/May 2011)

a) Easy to lay out
b) Allows flexibility
c) Simple statistical analysis
d) The lots of information due to missing data is smaller than with any other design

State the advantages of Latin square over other designs (April/May 2011)

a) Latin square controls more of the variation than the completely randomised block design with a two way stratification
b) The analysis is simple. Even with missing data the analysis remains relatively simple

UNIT - II

SOLUTION OF EQUATIONS & EIGEN VALUE PROBLEMS

1. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 6 decimal places (NOV/DEC: 2013)

Soln:

Let $f(x) = 3x - \cos x - 1$

$f'(x) = 3 + \sin x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(3x_n - \cos x_n - 1)}{3 + \sin x_n}$$

The root of $f(x) = 0$ lies between 0 & 1

choose $x_0 = 0.6$

$x_1 = 0.6071$, $x_2 = 0.6071$

$\therefore$ The required root is 0.6071

2. Solve the equation $x + y + z = 9$, $2x - 3y + 4z = 13$, $3x + 4y + 5z = 40$ by Gauss Elimination method (Nov/Dec 2013)

Soln:

$$(A/B) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 13 \\ 0 & -5 & 2 & -5 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 12 & 60 \end{array}\right] R_3 \to R_3 + 5R_2$$

The equivalent system is

$$x+y+z=9$$
$$y+2z=13$$
$$12z=60$$

By back substitution $z=5, y=3, x=1$

3. Find an iterative formula to find $\sqrt{N}$, where N is positive number (April /May 2014)

Soln:

$$x^2 - N = 0$$
$$f(x) = x^2 - N$$
$$f'(x) = 2x$$
$$x_{k+1} = x_k - \frac{x_k^2 - N}{2x_k}$$
$$x_{k+1} = \frac{x_k^2 + N}{2x_k}$$

4. Explain Gauss - Elimination method to solve $AX=B$ (April /May 2014)

Soln:

In this method, the given system is transformed into an equivalent system with upper-triangular coefficient matrix. (ie) a matrix in which all elements below the diagonal elements are zero which can be solved by back substitution.

5. Using power method, find the dominant eigenvalue of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (Nov/Dec 2012)

Soln:

Let $x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ be the initial eigenvector

Then $x_1 = Ax_0 = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$

$x_2 = Ax_1 = \begin{pmatrix} 17 \\ 37 \end{pmatrix}$

$x_3 = Ax_2 = \begin{pmatrix} 91 \\ 199 \end{pmatrix}$

$x_4 = Ax_3 = \begin{pmatrix} 489 \\ 1069 \end{pmatrix}$

$x_5 = Ax_4 = \begin{pmatrix} 2627 \\ 5743 \end{pmatrix}$

$x_6 = Ax_5 = \begin{pmatrix} 14113 \\ 30853 \end{pmatrix}$

Now the ratios of the components x_5 & x_6 are

5.3722

$\therefore$ The dominant eigen value is 5.3722 (approximately)

6. Solve the equations $x+2y+z=3$, $2x+3y+3z=10$, $3x-y+2z=13$. (Nov/Dec 2012)

Soln:

$$(A,B) \sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{bmatrix} \quad \begin{matrix} R_2 \to (-2)R_1 + R_2 \\ R_3 \to (-3)R_1 + R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{bmatrix} \quad R_3 \to -7R_2 + R_3$$

$$x + 2y + z = 3$$
$$-y + z = 4$$
$$-8z = -24$$

$\Rightarrow z = 3, \; y = -1, \; x = 2$.

7. Solve by Gauss Jordan method $\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -24 \end{pmatrix}$.

(Nov/Dec: 2011)

Soln:

$$(A, B) \sim \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 8 & -24 \end{pmatrix}$$

$$\sim \begin{pmatrix} 8 & 16 & 0 & 48 \\ 0 & -8 & 0 & 56 \\ 0 & 0 & 8 & -24 \end{pmatrix} \begin{array}{l} R_1 \to 8R_1 - R_3 \\ R_2 \to 8R_2 - R_3 \end{array}$$

$$\sim \begin{pmatrix} 8 & 0 & 0 & 160 \\ 0 & -8 & 0 & 56 \\ 0 & 0 & 8 & -24 \end{pmatrix} \; R_1 \to R_1 + 2R_2$$

$$8x = 160, \quad -8y = 56, \quad 8z = -24$$

$$x = 20, \quad y = -7, \quad z = -3$$

8. Solve the following system of equations using Gauss-Jordan elimination method $2x + y = 3$, $x - 2y = -1$ (May/June-2012)

Soln:

$$(A, B) \sim \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & -1 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 3 \\ 0 & 5 & 5 \end{pmatrix} \; R_2 \to -2R_2 + R_1$$

$$2x + y = 3$$
$$5y = 5 \Rightarrow \boxed{y = 1} \Rightarrow 2x = 3 - 1 = 2 \Rightarrow \boxed{x = 1}$$

9. Explain the power method to determine the eigen value of a matrix (April / May : 2010)

Soln:

(i) Find the dominant eigen value of A, say λ_1 by using power method

(ii) Consider the matrix $B = A - \lambda_1$

(iii) Again by applying power method, find the dominant eigen value of B. Then the smallest eigen value of A is equal to λ_1 + the dominant eigen value of B.

(iv) The remaining eigen value is found by using the relation, Sum of the eigen values of A = Trace of A

10. Use the power method to find the dominant eigenvalue and the corresponding eigen vector of the matrix $A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ (April / May 2010)

Soln

Let the initial eigen vector be $X = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Then $AX = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 0.25 \end{pmatrix}$

$AX_1 = 4.5 \begin{pmatrix} 1 \\ 0.39 \end{pmatrix}$

$AX_2 = 4.78 \begin{pmatrix} 1 \\ 0.45 \end{pmatrix}$

$AX_3 = 4.96 \begin{pmatrix} 1 \\ 0.48 \end{pmatrix}$

$AX_4 = 4.96 \begin{pmatrix} 1 \\ 0.49 \end{pmatrix}$

$AX_5 = 4.98 \begin{pmatrix} 1 \\ 0.49 \end{pmatrix}$

$AX_6 = 4.98 \begin{pmatrix} 1 \\ 0.49 \end{pmatrix}$

Dominant eigen value is 4.98 & Eigen vector is $\begin{pmatrix} 1 \\ 0.49 \end{pmatrix}$

UNIT - IV

INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

1. Given $f(2)=5$, $f(2.5)=5.5$ find the linear interpolating polynomial using Lagrange interpolation. (April / May 2010)

Soln:

$$l_0 = \frac{x-x_1}{x_0-x_1} = \frac{x-2.5}{-0.5}$$

$$l_1 = \frac{x-2}{0.5}$$

$$P_1 = l_0 f_0 + l_1 f_1$$

$$= -2(x-2.5)(5) + 2(x-2)(5.5)$$

$$= x+3$$

2. Construct the divided difference table for the data

x	0.5	1.5	3.0	5.0	6.5	8.0
f(x)	1.625	5.875	31	131	282.125	521

(April / May 2010)

Soln:

x	f(x)	Δx	$\Delta^2 x$	$\Delta^3 x$	$\Delta^4 x$	$\Delta^5 x$
0.5	1.625					
		4.25				
1.5	5.875		5			
		16.75		1		
3.0	31		9.5		0	
		50		1		0
5.0	131		14.8		0	
		100.75		1		
6.5	282.125		19.5			
		159.25				
8.0	521					

3. Write down the Lagrange's Interpolating formula (May / June 2012)

Soln :

The Lagrange's interpolation formula is given by

$$y(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} y_n$$

4. Write down the Simpson's 1/3 Rule in numerical Integration (May / June 2012)

Soln :

The Simpson's 1/3 Rule is given by

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3+\cdots) + 2(y_2+y_4+\cdots) + y_n\right]$$

5. Form the divided difference table for the following data

x	0	1	4	5
$f(x)$	8	11	78	123

(May / June 2013)

Soln.

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
0	8			
		3		
1	11		4.83	
		22.33		0.168
4	78		5.67	
		45		
5	123			

15

6. Evaluate $\int_0^1 e^{-x^2} dx$ by dividing the range of integration into 4 equal parts using trapezoidal rule (May / June 2013)

Soln:

x	0	0.25	0.5	0.75	1
e^{-x^2}	1	0.9394	0.7788	0.5698	0.3678

$$\int_0^1 e^{-x^2} dx = \frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[1.3678 + 2(2.2876)\right]$$

$$= 0.7428$$

7. Using Lagrange's formula, fit a polynomial to the data

x	0	1	3	4
y	-12	0	6	12

(April / May 2014)

Also find y at $x=2$

Soln:

$$y(x) = \frac{(x-1)(x-3)(x-4)}{-1\times-3\times-4}(-12) + \frac{x(x-1)(x-4)}{3\times2\times-1}(6) +$$

$$\frac{x(x-1)(x-3)}{4\times3\times1}(12)$$

$$= (x-1)\left[x^2-3x-4x+12-x^2+4x+x^2-3x\right]$$

$$= (x-1)(x^2-6x+12) = x^3-7x^2+18x-12$$

Substituting $x=2$ we get, $\boxed{y(2) = 4}$

8. Using Simpson's rule, find $\int_0^4 e^x\,dx$ given that $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.6$ (April / May 2014)

Soln:

By Simpson's rule, we have

$$\int_0^4 e^x\,dx = \frac{h}{3}\left[(y_0+y_4) + 2y_2 + 4(y_1+y_3)\right]$$

$$= \frac{1}{3}\left[(54.6+1) + 2(7.39) + 4(2.72+20.09)\right]$$

$$= 53.8733$$

9. What is the order of error in Trapezoidal rule? (May / June 2015)

Soln:

The order of error in Trapezoidal rule is h^2

$$E = -\frac{(b-a)}{12} h^2 y''(\xi)$$

10. Using Trapezoidal rule evaluate $\int_0^\pi \sin x\,dx$ by dividing the range into 6 equal parts (May / June 2015)

Soln:

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
y	0	0.5	0.8660	1	0.8660	0.5	0

$$\int_0^\pi \sin x\,dx = \frac{h}{6}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{\pi}{36}\left[0 + 2(0.5+0.8660+1+0.8660+0.5)\right]$$

$$= 0.65136$$

UNIT - V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

1. Define a difference quotient (April / May 2011)

A difference quotient, is the quotient obtained by dividing the difference between two values of a function, by the difference between the two corresponding values of the independent variable

2. Obtain the finite difference scheme for the difference equation $2\frac{d^2y}{dx^2} + y = 5$ (April / May 2011)

Soln:

The given equation is $2y''(x) + y(x) = 5$ — ①

$$2y'' = \frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} \quad \text{— ②}$$

Substitute ② in ①,

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} + y_k = 5$$

$$\Rightarrow y_{k-1} - 2y_k + y_{k+1} + h^2 y_k = 5h^2$$

3. Solve $\frac{dy}{dx} = x + y$, given $y(1) = 0$ and $y(1.1) = 0.11034$

Find $y(1.2)$ by Taylor's series method (May / June 2012)

Soln:

$y' = x + y$

$y'' = 1 + y'$, $y''' = y''$, $y^{iv} = y'''$

$\Rightarrow y_1' = 1.21034, \quad y_1'' = 2.21034, \quad y_1''' = 2.21034, \quad y_1^{iv} = 2.21034$

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1''' + \frac{h^4}{4!} y_1^{iv}$$

$y_2 = 0.24611.$

4. State the finite differences scheme of $u_{xx} + u_{yy} = 0$ (May / June : 2013)

Soln:

Given $u_{xx} + u_{yy} = 0.$

$$\frac{u_{i-1} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = 0$$

If $h = k = 1$, then $u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1}\right]$

5. Using Taylor's series find $y(0.1)$ for $\frac{dy}{dx} = 1 - y$, $y(0) = 0$. (May / June 2013)

Ans:

Here $x_0 = 0$, $y_0 = 0$

$y' = 1 - y \qquad y_0' = 1 - y_0 = 1$

$y'' = -y' \qquad y_0'' = -y_0' = -1$

$y''' = -y'' \qquad y_0''' = -y_0'' = 1$

$$y(0.1) = y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \cdots$$

$y(0.1) = 0.0952$

6. Solve $y_{x+2} - 4y_x = 0$ (May / June 2014)

Soln:

$$y_{x+2} - 4y_x = 0$$

$$(E^2 - 4) y_x = 0$$

$$m^2 - 4 = 0$$

$$m = \pm 2$$

$$y_x = A(2)^x + B(-2)^x$$

7. Using Euler's method, solve the following differential equation $y'' = -y$ subject to $y(0) = 1$. (May / June 2014)

Soln:

$$y_{n+1} = y_n + h[f(x_n, y_n)]$$

Given $\frac{dy}{dx} = f(x,y) = -y$

& $h = 0.01$

Put $n=0$ in the formula

$$\Rightarrow y_1 = y_0 + h f(x_0, y_0)$$

$$= 1 - 0.01 = 0.99$$

8. Write down the Milne's predictor - corrector formula for solving initial value problem in first order differential equation. (Nov / Dec 2010)

Ans: Milne's predictor formula

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

Milne's Corrector formula

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

9. State the merits of R.K method of order 4 over Taylor series method (Nov/Dec 2010)

Soln:

The R-K methods are designed to give greater accuracy and they possess the advantage of requiring only the function values at some selected points on the sub-interval

10. Write the finite difference equivalent of the differential equation $y''+2y'+y=x^2$ (May/June 2010)

Soln:

$$y'' = \frac{y_{k-1} - 2y_k + y_{k+1}}{h^2}$$

$$y' = \frac{y_{k+1} - y_{k-1}}{2h}$$

$$\therefore y''+2y'+y = x^2$$

$$\Rightarrow \frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} + \frac{2(y_{k+1} - y_{k-1})}{2h} + y_k = x_k^2$$

$$\Rightarrow (y_{k-1} - 2y_k + y_{k+1}) + h(y_{k+1} - y_{k-1}) + h^2 y_k = x_k^2 h^2$$

PART - B

UNIT - I

TESTING OF HYPOTHESIS.

1. A sample of 900 members has a mean 3.4 cm and standard deviation 2.61 cm. Is the sample from a large population of mean 3.25 cms and standard deviation of 2.61 cms? (Test at 5% L.O.S. The value of z at 5% level is $|z_\alpha| < 1.96$) (May / June 2015)

Soln:

Given $n = 900$, $\mu = 3.25$, $\bar{x} = 3.4$ cm, $\sigma = 2.61$

$s = 2.61$

Null hypothesis H_0: Assume that the sample has been drawn from the population with mean $\mu = 3.25$

Alternative hypothesis H_1: $\mu \neq 3.25$

The test statistic is $z = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

$$= \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$$z = 1.724 < 1.96$$

$\therefore$ We accept the null hypothesis H_0.

2. Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in duty, 800 people were tea drinkers in a sample of 1200 people. Using standard error of proportion, state whether there is a significant decrease in the assumption of tea after the increase in excise duty. (z_α at 5% level of 1.645, 1% level 2.33) (May / June 2015)

Soln:

Given $n_1 = 1000$ $n_2 = 1200$

$$p_1 = \frac{800}{1000} = 0.8 \qquad p_2 = \frac{800}{1200} = 0.667$$

Now, $$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{1000\left(\frac{800}{1000}\right) + 1200\left(\frac{800}{1200}\right)}{1000 + 1200}$$

$$= \frac{1600}{2200} = 0.727$$

$$\therefore q = 1 - 0.727 = 0.273.$$

Null hypothesis H_0: Assume that there is no significant difference in the consumption of tea before and after the increase in excise duty

$H_0 : p_1 = p_2$

Alternative hypothesis $H_1 : p_1 > p_2$ (right tailed test)

The test statistic is $z = \dfrac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

$= \dfrac{0.8 - 0.667}{\sqrt{0.727 \times 0.273\left(\frac{1}{1000}+\frac{1}{1200}\right)}}$

$= \dfrac{0.8-0.667}{0.019} = 7$

Since $Z > 1.64$ (one tail) we reject the null hypothesis at 5% L.O.S. (ie) there is a difference in the consumption of tea before and after the increase in excise duty.

Out of 8000 graduates in a town 800 are females, out of 1600 graduate employees 120 are females. Use χ^2 to determine if any distinction is made in appointment on the basis of sex. Value of χ^2 at 5% level of one degree of freedom is 3.84 (May/June 2014)

Soln:

The given information can be given in a tabular form

	Employment	Unemployment	Total
Males	1480	5720	7200
Females	120	680	800
Total	1600	6400	8000

Let us take the hypothesis that no distinction is made in appointment on the basis of sex.

Applying χ^2 - test

$$E_{11} = \frac{7200}{8000} \times 1600 = 1440$$

The table of expected frequencies is given below

			Total
	1440	5760	7200
	160	640	800
Total	1600	6400	8000

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1480	1440	1600	1.111
120	160	1600	10.000
5720	5760	1600	0.278
680	640	1600	2.500

$$\Sigma\left(\frac{(O-E)^2}{E}\right) = 13.889$$

$$\chi^2 = \Sigma\left(\frac{(O-E)^2}{E}\right) = 13.889$$

$$\nu = (r-1)(c-1) = (2-1)(2-1) = 1$$

$$\nu = 1, \quad \chi^2_{0.05} = 3.84$$

The calculated value of χ^2 is more than the table value. The hypothesis is rejected. Hence on the basis of the given information we can say that a distinction is made in appointment on the basis of sex.

4. An automobile company gives you the following information about age groups and the liking for particular model of car which it plans to introduce. On the basis of this data can it be concluded that the model appeal is independent of the age group. ($\chi^2_{0.05}(3) = 7.815$)

Persons who	Below 20	20-39	40-59	60 and above
Liked the car	140	80	40	20
Disliked the car	60	50	30	80

(May / June 2012)

Soln

Null hypothesis

Age.

	Below 20	20-39	40-59	>60	
Liked the car	140	80	40	20	280
Disliked the car	60	50	30	80	220
	200	130	70	100	500

Table of expected frequencies

$\frac{200 \times 280}{500} = 112$	$\frac{130 \times 280}{500} = 72.8$	$\frac{70 \times 280}{500} = 39.2$	$\frac{100 \times 280}{500} = 56$	280
$\frac{200 \times 220}{150} = 88$	$\frac{130 \times 220}{500} = 57.2$	$\frac{70 \times 220}{500} = 30.8$	$\frac{100 \times 220}{500} = 44$	220
200	130	70	100	500

Calculation of ψ^2

Observed Frequency (O)	Expected Frequency (E)	$(O-E)^2$	$\frac{(O-E)^2}{E}$
140	112	$(28)^2$	7
80	72.8	$(7.2)^2$	0.712
40	39.2	$(0.8)^2$	0.016
20	56	$(-36)^2$	23.14
60	88	$(-28)^2$	8.909
50	57.2	$(-7.2)^2$	0.906
30	30.8	$(-0.8)^2$	0.02
80	44	$(36)^2$	29.45

Calculated $\psi^2 = 70.154$

Tabulated ψ^2 for $(4-1)(2-1) = 3$ d.f at 5% L.O.S is 7.815.

Since calculated $\psi^2 >$ tabulated ψ^2, we reject the null hypothesis

5. Find if there is any association between extravagance in fathers and extravagance in sons for the following data

	Extravagant father	Miserly father
Extravagant son	327	741
Miserly son	545	234

Determine the coefficient of association also (Nov / Dec 2012)

Soln:

Null hypothesis H_0: Namely that the extravagance in sons and fathers are not significant

Alternative hypothesis H_1: Significant

$\alpha = 0.05$, $d.f = (r-1)(s-1) = (2-1)(2-1) = 1$

Table value of χ^2 is 3.841

$$\chi^2 = \frac{(ad-bc)^2(a+b+c+d)}{(a+b)(c+d)(a+c)(b+d)}$$

i.e) $$\chi^2 = \frac{[(327)(234)-(545)(741)]^2 \times (327+545+741+234)}{(872)(975)(1068)(779)}$$

$$= 230.24$$

Here $\chi^2 = 230.24 > 3.841$.

Hence we reject H_0 at 5% L.O.S.

$\therefore$ There is dependence between the attributes

Coefficient of attributes $= \dfrac{ad-bc}{ad+bc} = \dfrac{-327330}{480363}$

$$= -0.6814$$

b. 1000 students at college level were graded according to their I.Q and their economic conditions. What conclusion can you draw from the following data

Economic conditions	I.Q Level High	Low
Rich	460	140
Poor	240	160

(May | June 2013)

Soln:

Null hypothesis H_0: The given attributes are independent

Alternative hypothesis H_1: The given attributes are not independent

$\alpha = 0.05$, $d.f = (r-1)(s-1) = (2-1)(2-1) = 1$

Table value of χ^2 is 3.841

$$\chi^2 = \Sigma \frac{(O-E)^2}{E}$$

The expected frequencies are calculated using the following formulae.

$$\text{Expected frequency} = \frac{\text{Corresponding Row total} \times \text{Column Total}}{\text{Grand Total}}$$

(2)

$$\frac{600 \times 700}{1000} = 420 \qquad \frac{600 \times 300}{1000} = 180$$

$$\frac{700 \times 400}{1000} = 280 \qquad \frac{300 \times 400}{1000} = 120$$

Table

O	E	O-E	$(O-E)^2$	$(O-E)^2/E$
460	420	40	1600	3.81
140	180	-40	1600	8.88
240	280	-40	1600	5.714
160	120	40	1600	13.33
				31.7373

$\therefore \chi^2 = 31.7373$

Here, $\chi^2 = 31.7373 > 3.841$, So we reject H_0 at 5% L.O.S.

We conclude that the attributes I.Q as Economic conditions are not independent

7. 4 coins were tossed 160 times and the following results were obtained

No. of heads	0	1	2	3	4
Observed frequencies	17	52	54	31	6

Under the assumption that the coins are unbiased, find the expected frequencies of getting 0, 1, 2, 3, 4 heads and test the goodness of fit (May / June 2013)

Soln:

Null hypothesis H_0 : The coins are unbiased

Alternative hypothesis H_1 : The coins are biased

$\alpha = 0.05$, $d.f = n-1 = 4$

Table value, $\chi^2 = 9.488$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Probability of getting head $= p = 1/2$

Probability of getting tail $= q = 1/2$

Then the expected frequencies are

$p(x) = nC_x \, p^x q^{n-x}$, $x = 0, 1, 2, 3, \ldots$

$p(0 \text{ head}) = 4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 = 0.0625$

$p(1 \text{ head}) = 0.25$ $\quad p(2 \text{ heads}) = 0.375$

$p(3 \text{ head}) = 0.25$ $\quad p(4 \text{ heads}) = 0.0625$

$\Sigma O = 160$

Table

No. of heads (x_i)	O	$p(x_i)$	$E = 160 \times p(x_i)$	$\frac{(O-E)^2}{E}$
0	17	0.0625	10	4.9
1	52	0.25	40	3.6
2	54	0.375	60	0.6
3	31	0.25	40	2.025
4	6	0.0625	10	1.6
Total	160		160	12.725

Calculated $\chi^2 = 12.725$

Here $12.725 > 9.488$

So, we reject H_0 and accept H_1

$\therefore$ The coins are biased

8. The sales manager of a large company conducted a sample survey in states A and B taking 400 samples in each case. The results were in the following table. Test whether the average sales in the same in the 2 states at 1% level

	State A	State B
Average sales	Rs. 2,500	Rs. 2,200
S.D	Rs. 400	Rs. 550

(Nov/Dec 2012)

Soln:

Given $n_1 = 400$, $\bar{x}_1 = 2500$, $s_1 = 400$, $\alpha = 1\%$

$n_2 = 400$, $\bar{x}_2 = 2200$, $s_2 = 550$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (Use two-tailed test)

$\alpha = 1\%$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{2500 - 2000}{\sqrt{\frac{(400)^2}{400} + \frac{(550)^2}{400}}} = 8.82$$

Here $-2.58 < 8.82$

$8.82 > 2.58$

So we reject H_0.

9. A group of 10 rats fed on diet A and another group of 8 rats fed on diet B, recorded the following increase in weight (gms)

Diet A 5, 6, 8, 1, 12, 4, 3, 9, 6, 10

Diet B 2, 3, 6, 8, 10, 1, 2, 8

Does it show superiority of diet A over diet B. (Nov/Dec 2012)

Soln:

Given $n_1 = 10$, $n_2 = 8$

$$\Sigma x_1 = 5+6+8+1+12+4+3+9+6+10 = 64$$

$$\Sigma x_1^2 = 5^2+6^2+8^2+1^2+12^2+4^2+3^2+9^2+6^2+10^2 = 512$$

$$\Sigma x_2 = 2+3+6+8+10+1+2+8 = 40$$

$$\Sigma x_2^2 = 2^2+3^2+6^2+8^2+10^2+1^2+2^2+8^2 = 282$$

$$\bar{x}_1 = \frac{\Sigma x_1}{10} = \frac{64}{10} = 6.4$$

$$\bar{x}_2 = \frac{\Sigma x_2}{8} = \frac{40}{8} = 5$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - (x_1)^2 = \frac{512}{10} - (6.4)^2 = 10.24$$

$$s_2^2 = \frac{\Sigma x_2^2}{n_2} - (x_2)^2 = \frac{282}{8} - 25 = 10.25$$

$$S^2 = \left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right) = \frac{10(10.24) + 8(10.25)}{10+8-2} = 11.525$$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 > \mu_2$ [One tailed test (right)]

$\alpha = 5\%$, $d.f = n_1 + n_2 - 2 = 10+8-2 = 16$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{6.4 - 5}{\sqrt{11.525\left(\frac{1}{10}+\frac{1}{8}\right)}}$$

$$= \frac{1.4}{1.6103} = 0.869$$

Here $0.869 < 1.75$, so we accept H_0.

10. Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables

Sample I	18	13	12	15	12	14	16	14	15
Sample II	16	19	13	16	18	13	15		

Do the estimates of the population variance differ significantly at 5% level? (May / June 2011)

Soln:

Given $n_1 = 9$, $n_2 = 7$

Sample I		Sample II	
x_1	x_1^2	x_2	x_2^2
18	324	16	256
13	169	19	361
12	144	13	169
15	225	16	256
12	144	18	324
14	196	13	169
16	256	15	225
14	196		
15	225		
Total 129	1871	110	1760

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{129}{9} = 14.3333$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{110}{7} = 15.7143$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - (\bar{x}_1)^2 = \frac{1879}{9} - (14.33)^2$$

$$= 3.3342$$

$$s_2^2 = \frac{\Sigma x_2^2}{n_2} - (\bar{x}_2)^2 = \frac{1760}{7} - (15.7143)^2$$

$$= 4.4894$$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{9}{8}(3.3342) = 3.751$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{7}{6}(4.4894) = 5.2376$$

$$S_2^2 > S_1^2$$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.05$, d.f $\nu_1 = n_1 - 1 = 8$

$\nu_2 = n_2 - 1 = 6$

$\therefore$ d.f is $(6, 8)$

Table value of F is 3.58

$$F = \frac{s_2^2}{s_1^2} = \frac{5.2376}{3.7510}$$

$$= 1.3963$$

Here $F = 1.3963 < 3.58$, we accept H_0 at 5% L.O.S

Hence we conclude that the difference is not significant

11. Two groups of 100 people each were taken for testing the use of a vaccine, 15% contracted the disease out of inoculated persons, while 25 contracted the disease in the other group. Test the efficacy of the vaccine using χ^2 test

Soln:

Null hypothesis : H_0 : The vaccine is not effective

Alternative Hypothesis : H_1 : The vaccine is effective

Level of significance : $\alpha = 0.05$ (say)

	Affected	Not affected	Total
Inoculated	15	85	100
Not Inoculated	25	75	100
	40	160	200

Under the null hypothesis

$$\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

$$= \frac{200\left[(15\times 75)-(25\times 85)\right]^2}{100\times 100\times 40\times 160}$$

$$= 3.125$$

The expected value of χ^2 at 5% level of significance at 1 degree of freedom is given by $\chi^2_{0.05} = 3.841$

Conclusion

$\chi^2 < \chi^2_{0.05}$, hence we accept the null hypothesis and conclude that the vaccine is not effective

Excercises

1. In a random sample of 1000 people from city A, 400 are found to be consumers of wheat. In a sample of 800 from city B, 400 are found to be consumers of wheat. Does this data give a significant difference between the two cities as far as the proportion of wheat consumers is concerned? (Ans: $z = -4.243$)

2. A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that this sample is from a population of mean 165 cm

and SD 10 cm? Also find the 95% fiducial limits for the mean. (Ans. $|Z| = 5$)

3. A simple sample of heights of 6,400 Englishmen has a mean of 170 cm and a S.D of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D of 6.3 cm. Do the data indicate the Americans are, on the average, taller than Englishmen? (Ans: $Z = -11.3216$)

4. Two random samples gave the following results

Sample	Size	Sample mean	Sum of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population at 5% L.O.S? (Ans: $F = 1.02$)

5. Two independent samples of 8 & 7 items respectively had the following values

Sample I	9	11	13	11	15	9	12	14
Sample II	10	12	10	14	9	8	10	

Is the difference between the means of samples significant? (Ans: $t = 1.2151$)

UNIT- II

39

DESIGN OF EXPERIMENTS

1. The following table shows the lives in hours of four brands of electric lamps brand

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps (April/May 2015)

Soln:

Null hypothesis H_0: The lives of the 4 brands of lamps do not differ significantly. Code the data by subtracting 1640 from the given values.

Treatment A		Treatment B		Treatment C		Treatment D	
X_1^2	X_1^2	X_2	X_2^2	X_3	X_3^2	X_4	X_4^2
-30	900	-60	3600	-180	32400	-130	16900
-30	900	0	0	-90	8100	-120	14400
10	100	0	0	-40	1600	-110	12100
40	1600	60	3600	-20	400	-70	4900
60	3600	110	12100	0	0	-40	1600
80	6400	-	-	20	400	40	1600
160	25600	-	-	100	10000	-	-

-	-	-	-	180	32400	-	-
290	39100	110	19300	-30	85300	-430	51500
Σx_1	Σx_1^2	Σx_2	Σx_2^2	Σx_3	Σx_3^2	Σx_4	Σx_4^2

Step: 1

Sum of all the items (T) $= \Sigma x_1 + \Sigma x_2 + \Sigma x_3 + \Sigma x_4$

$= 290 + 110 - 30 - 430 = -60$

Step: 2

Correction factor (C.F) $= \frac{T^2}{N} = \frac{(-60)^2}{26} = 138.46$

Step: 3

TSS = Total sum of squares

= Sum of squares of all the items − C.F

$= \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - \frac{T^2}{N}$

$= 39100 + 19300 + 85300 + 51500 - 138.46$

$= 195061.54$

Step: 4

SSC = Sum of squares between samples

$= \frac{(\Sigma x_1)^2}{n} + \frac{(\Sigma x_2)^2}{n} + \frac{(\Sigma x_3)^2}{n} + \frac{(\Sigma x_4)^2}{n} - C.F$

$= \frac{(290)^2}{7} + \frac{(110)^2}{5} + \frac{(-30)^2}{8} + \frac{(-430)^2}{6} - 138.46$

$= (12014.29 + 2420 + 112.5 + 30816.66) - 138.46$

$= 45224.99$

Step : 5

MSC = Mean squares between samples

$= \dfrac{\text{Sum of squares between samples}}{\text{d.f}}$

$= \dfrac{45224.99}{3} = 15074.99$

Step : 6

SSE = Sum of squares within samples

= Total sum of squares − Sum of squares between samples

$= 195061.54 - 45224.99$

$= 149836.55$

Step : 7

MSE = Mean squares within samples

$= \dfrac{\text{Sum of squares within samples}}{\text{d.f}}$

$= \dfrac{149836.55}{22} = 6810.75$

ANOVA TABLE

Source of Variations	Sum of squares	Degrees of freedom	Mean Squares	F-ratio
Between samples	SSC = 45224.99	3	$MSC = \frac{SSC}{d.f} = 15074.99$	$F_c = \frac{MSC}{MSE} = 2.2134$
Within samples	SSE = 149836.55	22	$MSE = \frac{SSE}{d.f} = 6810.75$	

Tabulated value of F for $\nu_1 = 3$ and $\nu_2 = 22$ at 5% Level of significance is 3.05 ie) $F_{Tab} = 3.05$. Calculated value of F is $F_c = 2.2134$. Since $F_c < F_{TAB}$. We accept the null hypothesis H_0. The lives of the 4 brands of lamps do not differ significantly.

2. Analyze the variance in the Latin square of yields (in kgs) of paddy where P, Q, R, S denote the different methods of cultivation (April May 2015)

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether the different methods of cultivation have given significantly different yields

Soln:

Null hypothesis H_0: There is no significant difference between the different methods of cultivation

To simplify calculations, we subtract 120 from the given values

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Rows (i) / column (j)	1	2	3	4	Total
1	S2	P1	R3	Q2	8
2	Q4	R3	P2	S5	14
3	P0	Q-1	S0	R1	0
4	R2	S3	Q1	P2	8
Total	8	6	6	10	30

Treatment Total : $P = 5$, $Q = 6$, $R = 9$, $S = 10$, $G = 30$, $N = 16$

$$\text{Correction factor} = \frac{G^2}{N} = \frac{(30)^2}{16} = 56.25$$

$$\text{Total sum of squares} = \sum_i \sum_j y_{ij}^2 - CF$$

$$= SST = 92 - 56.25$$

$$= 35.75$$

Sum of squares between rows

$$R_1 = \frac{1}{4}(8^2 + 14^2 + 0^2 + 8^2) - 56.25 = 24.75$$

Sum of squares between columns

$$R_2 = \frac{1}{4}(8^2 + 6^2 + 6^2 + 10^2) - 56.25 = 2.75$$

Sum of squares between methods

$$R_3 = \frac{1}{4}(5^2 + 6^2 + 9^2 + 10^2) - 56.25 = 4.25$$

Error sum of squares

$$R_4 = SST - R_1 - R_2 - R_3$$
$$= 35.75 - 24.75 - 2.75 - 4.2 = 4$$

ANOVA TABLE

Source of Variation	d.f	Sum of Squares (SS)	Mean sum of square (MS)	Variance Ratio (F-ratio)
Between rows	3	24.75	8.25	
Between columns	3	2.75	0.917	$F = \frac{1.417}{0.667}$
Between methods	3	4.25	1.417	$= 2.124$
Error	6	4	0.667	
Total	15	35.75		

$$F_{0.05}(3,6) = 4.76$$

Conclusion

Since $F < 4.76$, we accept our null hypothesis and we may conclude that there is no significant difference between the different methods of cultivation.

5. Analyse the following results of a Latin Square experiments

Column Row	1	2	3	4
1	A(12)	D(20)	C(16)	B(10)
2	D(18)	A(14)	B(11)	C(14)
3	B(12)	C(15)	D(19)	A(13)
4	C(16)	B(11)	A(15)	D(20)

The letters A, B, C, D denote the treatments and the figures in brackets denote the observations (April/May 2012)

Soln:

Null hypothesis H_0 : There is no significant difference between rows, between columns and treatments.

Code the data by subtracting 15 from every value. we have

A(-3)	D(5)	C(1)	B(-5)
D(3)	A(-1)	B(-4)	C(-1)
B(-3)	C(0)	D(4)	A(-2)
C(1)	B(-4)	A(0)	D(5)

Table : I

	C_1	C_2	C_3	C_4	Row Total R_i	$R_i^2/4$
R_1	-3	5	1	-5	-2	1
R_2	3	-1	-4	-1	-3	-2.25
R_3	-3	0	4	-2	-1	-0.25
R_4	1	-4	0	5	2	1
Column Total C_j	-2	0	1	-3	-4 (T)	4.5 $\Sigma R_i^2/4$
$C_j^2/4$	1	0	0.25	2.25	3.5 $\Sigma C_j^2/4$	

Table - II

	1	2	3	4	Row Total T_i	$T_i^2/4$
A	-3	-1	0	-2	-6	9
B	-3	-4	-4	-5	16	64
C	1	0	1	-1	1	0.25
D	3	5	4	5	17	72.25
						145.5 $\Sigma(T_i^2/4)$

Step: 1

Grand Total $T = -4$

Step: 2

Correction factor $(C.F) = \frac{T^2}{N} = \frac{(-4)^2}{16} = 1$

Step : 3

Sum of squares of individual observations

$$= (-3)^2 + (-1)^2 + 0^2 + (-2)^2 + (-3)^2 + (-4)^2 + (-4)^2 + (-5)^2 + 1^2 + 0^2 + 1^2 + (-1)^2 + (3)^2 + (5)^2 + (4)^2 + (5)^2$$

$$= 158$$

Step : 4

TSS = Sum of squares of individual observations − C.F

$$= 158 - 1 = 157$$

Step : 5

SSR = Sum of squares of rows

$$= \Sigma\left(\frac{R_i^2}{4}\right) - C.F = 4.5 - 1 = 3.5$$

Step : 6

SSC = Sum of squares of columns

$$= \Sigma\left(\frac{C_j^2}{4}\right) - C.F = 3.5 - 1 = 2.5$$

Step : 7

SST = Sum of squares of treatments

$$= \Sigma\left(\frac{T_i^2}{4}\right) - C.F = 145.5 - 1 = 144.5$$

Step : 8

SSE = Residual

$$= TSS - (SSR + SSC + SST)$$

$= 157 - (3.5 + 2.5 + 144.5)$

$= 157 - 150.5 = 6.5$

ANOVA TABLE

Source of Variations	Sum of squares	Degrees of freedom	Mean Squares	F-ratio
Rows	$SSR = 3.5$	$4-1=3$	$MSR = \frac{SSR}{df} = 1.167$	$F_R = \frac{MSR}{MSE} = 1.081$
Columns	$SSC = 2.5$	$4-1=3$	$MSC = \frac{SSC}{df} = 0.87$	$F_c = \frac{MSE}{MSC} = 1.24$
Treatments	$SST = 144.5$	$4-1=3$	$MST = \frac{SST}{df} = 48.17$	$F_T = \frac{MST}{MSE} = 44.60$
Residual	$SSE = 6.5$	$(4-1)(4-2) = 6$	$MSE = \frac{SSE}{df} = 1.08$	–

Tabulated value of F for (3, 6) d.f at 5% level of significance is 4.76. $F_{tab} = 4.76$

$F_R < F_{tab}$. we accept the null hypothesis H_0.

Tabulated value of F for (6, 3) d.f at 5% level of significance is 8.94. Since $F_c < F_{tab}$ we accept the hypothesis ie) There is no significant difference between columns and also there is no significant difference between rows. Since $F_T > F_{tab}$ we reject the null hypothesis H_0. That is there is a significant difference between treatments

4) Carry out ANOVA (Analysis of variance) for the following (49)

		A	B	C	D
	1	44	38	47	36
Workers	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(April / May 2011)

Soln:

Null Hypothesis H_0:

(i) The mean productivity is the same for four different machines

(ii) The 5 men do not differ with respect to mean productivity.

Code the data by subtracting 40 from each valu

The coded data is

Workers	Machine Type A	B	C	D	Total
1	4	-2	7	-4	5
2	6	0	12	3	21
3	-6	-4	4	-8	-14
4	3	-2	6	-7	0
5	-2	2	9	-1	8
Total	5	6	38	-17	T = 20

Step: 1

Grand Total $(T) = 20$

Step: 2

Correction factor $(C.F) = \frac{T^2}{N} = \frac{400}{20} = 20$

Step: 3

SSC = Sum of squares between columns (machines)

$= \frac{5^2}{5} + \frac{(-6)^2}{5} + \frac{(38)^2}{5} + \frac{(-17)^2}{5} - C.F$

$= 338.8$

$d.f = \nu_1 = k-1 = 4-1 = 3$

Step: 4

SSR = Sum of squares between rows (workers)

$= \frac{5^2}{4} + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{0^2}{4} + \frac{8^2}{4} - C.F$

$= 161.5$

$d.f = \nu_2 = 8-1 = 5-1 = 4$

Step: 5

TSS = Total sum of squares

= Sum of squares of each value − Correction factor

$= 574$

Step: 6

SSE = Residual

= TSS − (SSC + SSR)

= 574 − 338.8 − 161.5 = 73.7

Step: 7

Degree of freedom for residual

$\nu_3 = (k-1)(r-1) = 3 \times 4 = 12$

ANOVA TABLE

Sources of Variation	Sum of Squares	Degrees of freedom	Mean Squares	F-ratio
Between columns (machinetypes)	SSC = 338.8	K−1 = 4−1 = 3	$MSC = \frac{SSC}{d.f}$ = 112.933	$F_c = \frac{MSC}{MSE}$ = 18.38
Between rows (workers)	SSR = 161.5	r − 1 = 4	$MSR = \frac{SSR}{d.f}$ = 40.375	$F_R = \frac{MSR}{MSE}$ = 6.574
Residual	SSE = 73.7	(K−1)(r−1) = 12	$MSE = \frac{SSE}{d.f}$ = 6.142	

(i) For $\nu_1 = 3$, $\nu_3 = 12$, $F_{\nu_1,\nu_3} = 3.49 = F_T$ at 5 %

Calculated value $F_c = 18.38 > F_T = 3.49$

∴ Mean productivity is not the same for the four different types of machines

(ii) For $\nu_2 = 4$, $\nu_3 = 12$, $F_{4,12} = 3.26 = F_T$ at 5%.

The computed value $F_c = 6.58$ is greater than the table value $F_T = 3.26$

∴ The workers differ w.r.to mean productivity

5. A set of data involving four "four tropical feed stuffs A, B, C, D" tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments and each feeding treatment is given to 5 chicks. Analyse the data. Weight gain of baby chicks fed on different feeding materials composed of tropical feed stuffs.

						Total T_i
A	55	49	42	21	52	219
B	61	112	30	89	63	355
C	42	97	81	95	92	407
D	169	137	169	85	154	719
						G = 1695

Grand Total

(April/May 2010)

Soln:

Null hypothesis H_0:

(i) There is no significant difference between treatments (columns)

(ii) There is no significant difference between stuffs (rows)

Code the data by subtracting 50 from each value

Stuffs	Treatments 1	2	3	4	5	Total
A	5	-1	-8	-29	2	-31 (V_1)
B	11	62	-20	39	13	105 (V_2)
C	-8	47	31	45	42	157 (V_3)
D	119	87	119	35	104	464 (V_4)
Total	127 (B_1)	195 (B_2)	122 (B_3)	90 (B_4)	161 (B_5)	695 (T)

Step: 1

Grand Total (T) = 695

Step: 2

Correction factor $(C.F) \frac{T^2}{N} = \frac{(695)^2}{20} = 24151.25$

Step: 3

SSC = Sum of squares between blocks (column)

$$= \frac{B_1^2}{4} + \frac{B_2^2}{4} + \frac{B_3^2}{4} + \frac{B_4^2}{4} + \frac{B_5^2}{4} - C.F$$

$= 1613.50$

$d.f = \nu_1 = 4$

Step: 4

SSR = Sum of squares between varieties (Rows)

$$= \frac{V_1^2}{5} + \frac{V_2^2}{5} + \frac{V_3^2}{5} + \frac{V_4^2}{5} - C.F$$

$= 26234.95$

$d.f = \nu_2 = 3$

Step: 3

TSS = Total Sum of squares

= Sum of squares of all the items - C.F

$= 37793.75$

Step: 6

SSE = Residual Sum of squares

$= TSS - (SSC + SSR)$

$= 9945.3$

Step: 7

$$d.f = \gamma_3 = (c-1)(r-1) = 12$$

ANOVA TABLE

Source of variations	Sum of squares	Degrees of freedom	Mean squares	F-ratio
Between Blocks (Columns)	$SSC = 1613.5$	$c-1 = 4$	$MSC = \frac{SSC}{df}$ $= 403.375$	$F_c = \frac{MSE}{MSC}$ $= 2.055$
Between Varieties (Rows)	$SSR = 26234.95$	$r-1 = 3$	$MSR = \frac{SSR}{df}$ $= \frac{26234.95}{3}$ $= 8744.98$	$F_R = \frac{MSR}{MSE}$ $= 10.55$
Residual	$SSE = 9945.3$	$(c-1)(r-1)$ $= 12$	$MSE = \frac{9945.3}{12}$ $= 828.775$	—

Tabulated value of F for (4, 12) d.f at 5% level of significance is 5.91. Since $F_c < F_{tab}$, we accept the null hypothesis H_0. That is there is no significant difference between blocks.

Tabulated value of F for (3, 12) d.f at 5% level of significance is 8.74 is $F_R > F_{tab}$, we reject the null hypothesis H_0. That is there is a significant difference between strains

Excercises

1. The following data represent the number of units of production per day turned out by different workers using 4 different types of machines

		Machine type			
		A	B	C	D
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

1. Test whether the five men differ with respect to mean productivity and 2. Test whether the mean productivity is the same for the four different machine types

2. In a 5x5 Latin square experiment, the data collected is given in the matrix below. Yield per plot is given in quintals for the five different cultivation treatments A, B, C, D and E. Perform the analysis of variance.

A 48	E 66	D 56	C 52	B 61
D 64	B 62	A 50	E 64	C 63
B 69	A 53	C 60	D 61	E 67
C 57	D 58	E 67	B 65	A 55
E 67	C 57	B 66	A 60	D 57

Unit - III

Solutions of Equations and eigen value problems

part - B.

1. Find the dominant eigenvalue and its eigen vector of the matrix by power method

$$A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}.$$

[May/June 2013]
[Nov/Dec 2014]

Soln:

Let $X_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be an arbitrary initial eigen vector

$$AX_0 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} = 5\begin{pmatrix} 1 \\ 0 \\ 0.2 \end{pmatrix} = 5X_1$$

$$AX_1 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 5.2 \\ 0 \\ 2 \end{pmatrix} = 5.2\begin{pmatrix} 1 \\ 0 \\ 0.38 \end{pmatrix} = 5.2X_2.$$

$$AX_2 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0.38 \end{pmatrix} = \begin{pmatrix} 5.39 \\ 0 \\ 2.95 \end{pmatrix} = 5.39\begin{pmatrix} 1 \\ 0 \\ 0.55 \end{pmatrix} = 5.39X_3.$$

$$AX_3 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0.55 \end{pmatrix} = \begin{pmatrix} 5.55 \\ 0 \\ 3.75 \end{pmatrix} = 5.55\begin{pmatrix} 1 \\ 0 \\ 0.68 \end{pmatrix} = 5.55X_4.$$

$$AX_4 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0.68 \end{pmatrix} = \begin{pmatrix} 5.68 \\ 0 \\ 4.4 \end{pmatrix} = 5.68\begin{pmatrix} 1 \\ 0 \\ 0.78 \end{pmatrix} = 5.68X_5$$

$$AX_5 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0.78 \end{pmatrix} = \begin{pmatrix} 5.78 \\ 0 \\ 4.9 \end{pmatrix} = 5.78\begin{pmatrix} 1 \\ 0 \\ 0.85 \end{pmatrix} = 5.78X_6.$$

$$AX_6 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0.85 \end{pmatrix} = \begin{pmatrix} 5.85 \\ 0 \\ 5.25 \end{pmatrix} = 5.85\begin{pmatrix} 1 \\ 0 \\ 0.9 \end{pmatrix} = 5.85X_7$$

$$AX_7 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0.9 \end{pmatrix} = \begin{pmatrix} 5.9 \\ 0 \\ 5.5 \end{pmatrix} = 5.9\begin{pmatrix} 1 \\ 0 \\ 0.93 \end{pmatrix} = 5.8894X_8$$

$$AX_8 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0.93 \end{pmatrix} = \begin{pmatrix} 5.93 \\ 0 \\ 5.65 \end{pmatrix} = 5.93\begin{pmatrix} 1 \\ 0 \\ 0.95 \end{pmatrix} = 5.93X_9.$$

$$A x_9 = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0.95 \end{pmatrix} = \begin{pmatrix} 5.95 \\ 0 \\ 5.75 \end{pmatrix} = 5.95 \begin{pmatrix} 1 \\ 0 \\ 0.97 \end{pmatrix} = 5.95\, X_{10}$$

$$A X_{10} = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0.97 \end{pmatrix} = \begin{pmatrix} 5.97 \\ 0 \\ 5.85 \end{pmatrix} = 5.97 \begin{pmatrix} 1 \\ 0 \\ 0.98 \end{pmatrix} = 5.97\, X_{11}$$

$$A X_{11} = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0.98 \end{pmatrix} = \begin{pmatrix} 5.99 \\ 0 \\ 5.9 \end{pmatrix} = 5.99 \begin{pmatrix} 1 \\ 0 \\ 0.9 \end{pmatrix} = 5.99\, X_{12}$$

$$A X_{12} = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0.99 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} = 6 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 6\, X_{13}$$

$$A X_{13} = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} = 6 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 6\, X_{14}.$$

$\therefore$ dominant eigen value $= 6$.

corresponding eigen vector is $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

2. Find the inverse of the matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{pmatrix}$ using Gauss Jordan method. [Nov/Dec 2014]

Soln:

Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{pmatrix}$.

$$[A : I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & -2 & 4 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$$

Fix R_1 make the first elements of R_2 and R_3 zero using R_1

$$[A : I] \approx \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 1 & 0 \\ 0 & 3 & 1 & -1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

Fix R_2 make the second elements of R_1 and R_3 zero using R_2.

$$[A:I] \approx \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 2 & -1 & 0 \\ 0 & -1 & 3 & 2 & -1 & 0 \\ 0 & 0 & 10 & -4 & 3 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 + 3R_2 \end{array}$$

Fix R_3 make the third elements of R_1 and R_2 zero using R_3

$$[A:I] \approx \left[\begin{array}{ccc|ccc} 5 & 0 & 0 & 6 & -2 & 1 \\ 0 & -10 & 0 & 32 & -19 & -3 \\ 0 & 0 & 10 & -4 & 3 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to 5R_1 + R_3 \\ R_2 \to 10R_2 - 3R_3 \end{array}$$

$$[I:A^{-1}] \approx \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{6}{5} & -\frac{2}{5} & \frac{1}{5} \\ 0 & 1 & 0 & -32/10 & -19/10 & -3/10 \\ 0 & 0 & 1 & -4/10 & 3/10 & 1/10 \end{array}\right] \quad \begin{array}{l} R_1 \to \frac{R_1}{5} \\ R_2 \to \frac{R_2}{-10} \\ R_3 \to \frac{R_3}{10} \end{array}$$

$\therefore$ The inverse of the matrix is

$$A^{-1} = \begin{pmatrix} \frac{6}{5} & -\frac{2}{5} & \frac{1}{5} \\ -\frac{32}{10} & -\frac{19}{10} & -\frac{3}{10} \\ -\frac{4}{10} & \frac{3}{10} & \frac{1}{10} \end{pmatrix}.$$

3. Solve the system of equations using Gauss-elimination method $5x - 2y + z = 4$, $7x + y - 5z = 8$ and $3x + 7y + 4z = 10$. [Nov/Dec 2014]

Soln:

The augmented matrix is given by

$$[A, B] = \left[\begin{array}{ccc|c} 5 & -2 & +1 & 4 \\ 7 & 1 & -5 & 8 \\ 3 & 7 & 4 & 10 \end{array}\right].$$

Fix R_1 make the first elements of R_2 and R_3 zero using R_1

$$[A, B] \approx \left[\begin{array}{ccc|c} 5 & -2 & 1 & 4 \\ 0 & 19 & -32 & 12 \\ 0 & 41 & 17 & 38 \end{array}\right] \quad \begin{array}{l} R_2 \to 5R_2 - 7R_1 \\ R_3 \to 5R_3 - 3R_1 \end{array}$$

Fix R_1, R_2 make the second element of R_3 zero using R_2.

$$[A, B] \approx \left[\begin{array}{ccc|c} 5 & -2 & 1 & 4 \\ 0 & 19 & -32 & 12 \\ 0 & 0 & 1635 & 230 \end{array}\right] \quad R_3 \to 19R_3 - 41R_2.$$

using back substitution, $1635\,z = 230$

$$z = \frac{46}{327}$$

From the second row, we get

$$19y - 32z = 12$$

$$19y - 32\left(\frac{46}{327}\right) = 12.$$

$$19y = 12 + \frac{1472}{327}$$

$$19y = \frac{5396}{327}$$

$$y = \frac{284}{327}$$

From the first row, we get

$$5x - 2y + z = 4.$$

$$5x - \frac{568}{327} + \frac{46}{327} = 4.$$

$$5x = \frac{6408}{327}$$

$$x = \frac{6408}{1635}$$

The solution is $\boxed{x = \frac{6408}{1635},\ y = \frac{284}{327},\ z = \frac{46}{327}}$.

4. compute a real root of $x\log_{10}x - 1.2 = 0$, correct to three decim places by Newton Raphson method. [AU Nov/Dec 2004, May/Ju 200 May/June 2014].

Soln: Let $f(x) = x\log_{10}x - 1.2$

$$f'(x) = \log_{10}x + x \cdot \frac{1}{x}\log_{10}e$$

$$= \log_{10}x + \log e = \log_{10}x + 0.4343$$

$f(1) = \log_{10}1 - 1.2 = -1.2 = -ve$

$f(1.5) = 1.5\log_{10}1.5 - 1.2 = -0.9359 = -ve.$

$f(2) = 2\log_{10}2 - 1.2 = -0.5979 = -ve.$

$f(3) = 3\log_{10}3 - 1.2 = 0.2314 = +ve$

Since $f(2)$ and $f(3)$ are opposite in sign, a root lies betw 2 and 3.

Here $|f(2)| > |f(3)|$.

$\therefore$ The root is nearer to 3.

Let $x_0 = 2.7$.

By Newton-Raphson formula,

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.7 - \frac{f(2.7)}{f'(2.7)}$$

$$= 2.7 - \left[\frac{2.7\log_{10} 2.7 - 1.2}{0.4343 + \log_{10} 2.7}\right]$$

$$= 2.7 - \left(\frac{-0.035}{0.867}\right)$$

$$= 2.7 + \frac{0.035}{0.867}$$

$$= 2.740$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.740 - \frac{f(2.74)}{f'(2.74)}$$

$$= 2.74 - \left[\frac{2.74\log_{10}(2.74) - 1.2}{0.4343 + \log_{10} 2.74}\right]$$

$$= 2.74 - \left(\frac{-0.0006}{0.872}\right)$$

$$= 2.74 + \frac{0.0006}{0.872}$$

$$= 2.741.$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.741 - \frac{f(2.741)}{f'(2.741)}$$

$$= 2.741 - \left[\frac{(2.741)\log_{10} 2.741 - 1.2}{0.4343 + \log_{10} 2.741}\right]$$

$$= 2.741 - \frac{0.003}{0.872}$$

$$= 2.741.$$

Here $x_2 = x_3$.

$\therefore$ The better approximation is 2.741.

5. Using Gauss Jordan method, find the inverse of

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

[AU, NOV 2004, April 2010, Oct '96]
[AU M/J 2012]

Soln:

Let $[A, I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 + 2R_1$

$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$ $R_2 \to \frac{R_2}{2}$.

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right]$ $R_1 \to R_1 - R_2$, $R_3 \to R_3 + 2R_1$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$ $R_3 \to \frac{R_3}{-4}$.

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$ $R_1 \to R_1 - 6R_3$, $R_2 \to R_2 + 3R_3$.

$$\therefore A^{-1} = \begin{pmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{pmatrix}.$$

$$= \frac{1}{4}\begin{pmatrix} 12 & 4 & 6 \\ -5 & -1 & -3 \\ -1 & -1 & -1 \end{pmatrix}$$

6. Solve the following system of equations using Gauss Seidal Method.

$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

[May/June 2012]

Soln:

As the co-efficient matrix is not diagonally dominant, we rewrite the equations

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

Since the diagonal elements are dominant in the co-efficient matrix, we write x, y, z as follows:

$$x = \frac{1}{27}[85 - 6y + z]$$
$$y = \frac{1}{15}[72 - 6x - 2z]$$
$$z = \frac{1}{54}[110 - x - y]$$

Let the initial values be $y=0$, $z=0$

First iteration:

$$x^{(1)} = \frac{1}{27}[85 - 6y^{(0)} + z^{(0)}] = \frac{1}{27}[85 - 6(0) + 0] = 3.148$$
$$y^{(1)} = \frac{1}{15}[72 - 6x^{(1)} - 2z^{(0)}] = \frac{1}{15}[72 - 6(3.148) - 0] = 3.541$$
$$z^{(1)} = \frac{1}{54}[110 - x^{(1)} - y^{(1)}] = \frac{1}{54}[110 - 3.148 - 3.541] = 1.913.$$

Second iteration:

$$x^{(2)} = \frac{1}{27}[85 - 6y^{(1)} + z^{(1)}] = \frac{1}{27}[85 - 6(3.541) + 1.913] = 2.432.$$
$$y^{(2)} = \frac{1}{15}[72 - 6x^{(2)} - 2z^{(1)}] = \frac{1}{15}[72 - 6(2.432) - 2(1.913)] = 3.572$$
$$z^{(2)} = \frac{1}{54}[110 - x^{(2)} - y^{(2)}] = \frac{1}{54}[110 - 2.432 - 3.572] = 1.926.$$

Third iteration:

$$x^{(3)} = \frac{1}{27}[85 - 6y^{(2)} + z^{(2)}] = \frac{1}{27}[85 - 6(3.572) + 1.926] = 2.426$$

$y^{(3)} = \frac{1}{15}[72 - 6x^{(3)} - 2z^{(2)}] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.57$

$z^{(3)} = \frac{1}{54}[110 - x^{(3)} - y^{(3)}] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$

Fourth iteration:

$x^{(4)} = \frac{1}{27}[85 - 6y^{(3)} + z^{(3)}] = \frac{1}{27}[85 - 6(3.573) + 1.926] = 2.426$

$y^{(4)} = \frac{1}{15}[72 - 6x^{(4)} - 2z^{(3)}] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573.$

$z^{(4)} = \frac{1}{54}[110 - x^{(4)} - y^{(4)}] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926.$

Hence $x = 2.426$, $y = 3.573$, $z = 1.926$.

7. Solve by Gauss-Seidal iteration method the system [AU Apr/May '10, '13]

$5x - y + z = 10$; $2x + 4y = 12$; $x + y + 5z = -1$

Soln: The diagonal elements are dominant in the co-efficient matrix.

We write x, y and z as follows.

$x = \frac{1}{5}[10 + y - z]$

$y = \frac{1}{4}[12 - 2x]$

$z = \frac{1}{5}[-1 - x - y]$

Let the initial values be $y = 0$, $z = 0$.

First iteration:

$x^{(1)} = \frac{1}{5}[10 + y^{(0)} - z^{(0)}] = \frac{1}{5}[10] = 2$

$y^{(1)} = \frac{1}{4}[12 - 2x^{(1)}] = \frac{1}{4}[12 - 2(2)] = 2.$

$z^{(1)} = \frac{1}{5}[-1 - x^{(1)} - y^{(1)}] = \frac{1}{5}[-1 - 2 - 2] = -1.$

Second iteration:

$x^{(2)} = \frac{1}{5}[10 + y^{(1)} - z^{(1)}] = \frac{1}{5}[10+2+1] = 2.6$

$y^{(2)} = \frac{1}{4}[12 - 2x^{(2)}] = \frac{1}{4}[12 - 2(2.6)] = 1.7$

$z^{(2)} = \frac{1}{5}[-1 - x^{(2)} - y^{(2)}] = \frac{1}{5}[-1-2.6-1.7] = -1.06.$

Third iteration:

$x^{(3)} = \frac{1}{5}[10 + y^{(2)} - z^{(2)}] = \frac{1}{5}[10+1.7+1.06] = 2.552$

$y^{(3)} = \frac{1}{4}[12 - 2x^{(3)}] = \frac{1}{4}[12 - 2(2.552)] = 1.724.$

$z^{(3)} = \frac{1}{5}[-1 - x^{(3)} - y^{(3)}] = \frac{1}{5}[-1-2.552-1.724] = -1.055.$

Fourth iteration:

$x^{(4)} = \frac{1}{5}[10 + y^{(3)} - z^{(3)}] = \frac{1}{5}[10+1.724+1.055] = 2.556$

$y^{(4)} = \frac{1}{4}[12 - 2x^{(4)}] = \frac{1}{4}[12 - 2(2.556)] = 1.722$

$z^{(4)} = \frac{1}{5}[-1 - x^{(4)} - y^{(4)}] = \frac{1}{5}[-1-2.556-1.722] = -1.056.$

Fifth iteration:

$x^{(5)} = \frac{1}{5}[10 + y^{(4)} - z^{(4)}] = \frac{1}{5}[10+1.722+1.056] = 2.556$

$y^{(5)} = \frac{1}{4}[12 - 2x^{(5)}] = \frac{1}{4}[12 - 2(2.556)] = 1.722.$

$z^{(5)} = \frac{1}{5}[-1 - x^{(5)} - y^{(5)}] = \frac{1}{5}[-1-2.556-1.722] = -1.056.$

Hence $x = 2.556$, $y = 1.722$, $z = -1.056.$

8. Solve by Gauss Elimination Method

$x_1 + x_2 + x_3 + x_4 = 2$

$2x_1 - x_2 + 2x_3 - x_4 = -5$

$3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$ [Nov/Dec 2013]

$x_1 - 2x_2 - 3x_3 + 2x_4 = 5$

Soln:

The augmented matrix is given by

$$[A, B] = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & -1 & 2 & -1 & -5 \\ 3 & 2 & 3 & 4 & 7 \\ 1 & -2 & -3 & 2 & 5 \end{array}\right]$$

Fix R_1 make the first elements of R_2, R_3 and R_4 zero using R_1

$$[A, B] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -3 & 0 & -3 & -9 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & -3 & -4 & 1 & 3 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

Fix R_1 and R_2, make the second elements of R_3 and R_4 zero using R_2.

$$[A, B] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -3 & 0 & -3 & -9 \\ 0 & 0 & 0 & 6 & 12 \\ 0 & 0 & -4 & 4 & 12 \end{array}\right] \quad \begin{array}{l} R_3 \to 3R_3 - R_2 \\ R_4 \to R_4 - R_2. \end{array}$$

Interchange R_3 and R_4,

$$[A, B] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -3 & 0 & -3 & -9 \\ 0 & 0 & -4 & 4 & 12 \\ 0 & 0 & 0 & 6 & 12 \end{array}\right]$$

using back substitution, we get

$$6x_4 = 12$$

$$\boxed{x_4 = 2}$$

From the 3rd row,

$$-4x_3 + 4x_4 = 12.$$
$$-4x_3 + 4(2) = 12$$
$$-4x_3 = 12 - 8$$
$$-4x_3 = 4$$
$$\boxed{x_3 = -1}$$

From the 2nd row,

$$-3x_2 - 3x_4 = -9$$
$$-3x_2 - 3(2) = -9.$$
$$-3x_2 = -9 + 6$$
$$-3x_2 = -3$$
$$\boxed{x_2 = 1}$$

From the first row,

$$x_1 + x_2 + x_3 + x_4 = 2.$$
$$x_1 + 1 - 1 + 2 = 2.$$
$$\boxed{x_1 = 0}$$

The solution is $x_1 = 0$, $x_2 = 1$, $x_3 = -1$, $x_4 = 2$.

9. Find the dominant eigen value and the corresponding Eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Find also the least latent root and hence find the third eigen value.

[AU M/J 2007, 2008, 2010]

(or)

Find all the eigen values of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

Soln:

Let $X_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be an arbitrary initial eigen vector

$$AX_0 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1 X_1$$

$$AX_1 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = 7\begin{pmatrix} 1 \\ 0.4286 \\ 0 \end{pmatrix} = 7X_2$$

$$AX_2 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.4286 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.5714 \\ 1.8572 \\ 0 \end{pmatrix} = 3.5714\begin{pmatrix} 1 \\ 0.52 \\ 0 \end{pmatrix} = 3.5714 X_3.$$

$$AX_3 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.52 \\ 0 \end{pmatrix} = \begin{pmatrix} 4.12 \\ 2.04 \\ 0 \end{pmatrix} = 4.12\begin{pmatrix} 1 \\ 0.4951 \\ 0 \end{pmatrix} 4.12(X_4).$$

$$AX_4 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.4951 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.9706 \\ 1.9902 \\ 0 \end{pmatrix} = 3.9706\begin{pmatrix} 1 \\ 0.5012 \\ 0 \end{pmatrix}$$

$$= 3.9706 X_5$$

$$AX_5 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.5012 \\ 0 \end{pmatrix} = \begin{pmatrix} 4.0072 \\ 2.0024 \\ 0 \end{pmatrix} = 4.0072\begin{pmatrix} 1 \\ 0.4997 \\ 0 \end{pmatrix} = 4.0072X$$

$$AX_6 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.4997 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.9982 \\ 1.9994 \\ 0 \end{pmatrix} = 3.9982\begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} = 3.9982X_7$$

$$AX_7 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} = 4X_8$$

$$AX_8 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} = 4X_9.$$

$\therefore$ Dominant eigen value $= 4$.

corresponding eigen vector $= \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix}$.

To find least eigen value

Let $B = A - 4I$. Since $\lambda = 4$.

$$B = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 2 \\ 0 & 0 & -2 \end{pmatrix}.$$

Now, find the dominant eigen value of B.

Let $Y_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be an arbitrary initial eigen vector.

$$BY_0 = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} = -3\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = -3Y_1$$

$$BY_1 = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 1.6666 \\ 0 \end{pmatrix} = -5\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = -5Y_2$$

$$BY_2 = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 1.6666 \\ 0 \end{pmatrix} = -5\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix}$$

$\therefore$ Dominant eigen value of $B = -5$.

$\therefore$ Smallest eigen value of $A = -5 + 4 = -1$.

Sum of the eigen values = Sum of the diagonal elements

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 + 2 + 3 = 6$$

ie) $4 + (-1) + \lambda_3 = 6$

$$\lambda_3 = 3.$$

$\therefore$ All the three eigen values are $4, 3, -1$.

10. Solve by Jacobi's iteration method the system for $8x - 3y + 2z = 20$, $6x + 3y + 12z = 35$; $4x + 11y - z = 33$. [Nov/Dec '2010]

Soln:

Given
$$8x - 3y + 2z = 20$$
$$6x + 3y + 12z = 35$$
$$4x + 11y - z = 33.$$

Since the diagonal elements are not dominant in the iteration method. ~~we~~ interchanging the equations we get

$$8x-3y+2z=20$$
$$4x+11y-z=33$$
$$6x+3y+12z=35$$

$$\Rightarrow x=\frac{1}{8}[20+3y-2z]$$
$$y=\frac{1}{11}[33-4x+z]$$
$$z=\frac{1}{12}[35-6x-3y]$$

Let the initial approximation be $x_0=0$, $y_0=0$, $z_0=0$.

First iteration:

$$x^{(1)}=\frac{1}{8}[20+3y_0-2z_0]$$
$$=\frac{1}{8}[20+3(0)-2(0)]=2.5$$
$$y^{(1)}=\frac{1}{11}(33-4x_0+z_0)=\frac{1}{11}(33-4(0)+0)=3.$$
$$z^{(1)}=\frac{1}{12}[35-6x_0-3y_0]=\frac{1}{12}[35-6(0)-3(0)]=2.9167.$$

Second iteration:

$$x^{(2)}=\frac{1}{8}(20+3y^{(1)}-2z^{(1)})=\frac{1}{8}[20+3(3)-(2.9167)2]=2.89$$
$$y^{(2)}=\frac{1}{11}[33-4x^{(1)}+z^{(1)}]=\frac{1}{11}[33-4(2.5)+2.9167]=2.3561.$$
$$z^{(2)}=\frac{1}{12}[35-6x^{(1)}-3y^{(1)}]=\frac{1}{12}[35-6(2.5)-3(3)]=0.9167.$$

Third iteration:

$$x^{(3)}=\frac{1}{8}[20+3y^{(2)}-2z^{(2)}]=\frac{1}{8}[20+3(2.32561)-2(0.9167)]$$
$$=3.1544$$
$$y^{(3)}=\frac{1}{11}[33-4x^{(2)}+z^{(2)}]=\frac{1}{11}[33-4(2.8958)+0.9167]$$
$$=2.0303.$$
$$z^{(3)}=\frac{1}{12}[35-6(2.8958)-3(2.3561)]=0.8797.$$

Fourth iteration:

$x^{(4)} = \frac{1}{8}[20+3y^{(3)} - 2z^{(3)}] = \frac{1}{8}[20+3(2.0303)-2(0.8797)] = 3.0414$

$y^{(4)} = \frac{1}{11}[33-4x^{(3)}+z^{(3)}] = \frac{1}{11}[33-4(3.1544)+0.8797] = 1.9329$

$z^{(4)} = \frac{1}{12}[35-6x^{(3)}-3y^{(3)}] = \frac{1}{12}[35-6(3.1544)-3(2.0303)] = 0.8319$

Fifth iteration:

$x^{(5)} = \frac{1}{8}[20+3y^{(4)} - 2z^{(4)}] = \frac{1}{8}[20+3(1.9329)-2(0.8319)] = 3.0169$

$y^{(5)} = \frac{1}{11}[33-4x^{(4)}+z^{(4)}] = \frac{1}{11}[33-4(3.0414)+0.8319] = 1.9697$

$z^{(5)} = \frac{1}{12}[35-6x^{(4)}-3y^{(4)}] = \frac{1}{12}[35-6(3.0414)-3(1.9329)] = 0.9127$

Sixth iteration:

$x^{(6)} = \frac{1}{8}[20+3y^{(5)} - 2z^{(5)}] = \frac{1}{8}[20+3(1.9697)-2(0.9127)] = 3.010$

$y^{(6)} = \frac{1}{11}[33-4x^{(5)}+z^{(5)}] = \frac{1}{11}[33-4(3.0169)+0.9127] = 1.9859$

$z^{(6)} = \frac{1}{12}[35-6x^{(5)}-3y^{(5)}] = \frac{1}{12}[35-6(3.0169)-3(1.9697)]$
$= 0.9158$

Seventh iteration:

$x^{(7)} = \frac{1}{8}[20+3y^{(6)} - 2z^{(6)}] = \frac{1}{8}[20+3(1.9859)-2(0.9158)]$

$= 3.0158$

$y^{(7)} = \frac{1}{11}[33-4x^{(6)}-z^{(6)}]$

$= \frac{1}{11}[33-4(3.0105)+0.9158] = 1.9885$

$z^{(7)} = \frac{1}{12}[35-6x^{(6)}-3y^{(6)}]$

$= \frac{1}{12}[35-6(3.0105)-3(1.9859)] = 0.9149$

Eighth iteration:

$x^{(8)} = \frac{1}{8}[20+3y^{(7)} - 2z^{(7)}]$

$= \frac{1}{8}[20+3(1.9885)-2(0.9149)]$

$= 3.0170$

$y^{(8)} = \frac{1}{11}[33-4(3.0158)+0.9149] = 1.9865$

$z^{(8)} = \frac{1}{12}[35 - 6x^{(7)} - 3y^{(7)}]$.

$= \frac{1}{12}[35 - 6(3.0158) - 3(1.9885)] = 0.9116$.

Ninth iteration

$x^{(9)} = \frac{1}{8}[20 + 3y^{(8)} - 2z^{(8)}] = \frac{1}{8}[20 + 3(1.9865) - 2(0.9116)]$

$= 3.0170$

$y^{(9)} = \frac{1}{11}[33 - 4x^{(8)} + z^{(8)}]$

$= \frac{1}{11}[33 - 4(3.0170) + 0.9116]$

$= 1.9858$

$z^{(9)} = \frac{1}{12}[35 - 6x^{(8)} - 3y^{(8)}]$

$= \frac{1}{12}[35 - 6(3.0170) - 3(1.9865)]$

$= 0.9115$

Tenth iteration

$x^{(10)} = \frac{1}{8}[20 + 3y^{(9)} - 2z^{(9)}] = \frac{1}{8}[20 + 3(1.9858) - 2(0.9115)] = 3.0168$

$y^{(10)} = \frac{1}{11}[33 - 4x^{(9)} + z^{(9)}] = \frac{1}{11}[33 - 4(3.0170) + 0.9115] = 1.9858$

$z^{(10)} = \frac{1}{12}[35 - 6x^{(9)} - 3y^{(9)}] = \frac{1}{12}[35 - 6(3.0170) - 3(1.9858)] = 0.9117$

Eleventh iteration

$x^{(11)} = \frac{1}{8}[20 + 3y^{(10)} - 2z^{(10)}] = \frac{1}{8}[20 + 3(1.9858) - 2(0.9117)] = 3.0168$

$y^{(11)} = \frac{1}{11}[33 - 4x^{(10)} + z^{(10)}] = \frac{1}{11}[33 - 4(3.0168) + 0.9117] = 1.9859$

$z^{(11)} = \frac{1}{12}[35 - 6x^{(10)} - 3y^{(10)}] = \frac{1}{12}[35 - 6(3.0168) - 3(1.9858)]$

$= 0.9118$

Twelfth iteration

$x^{(12)} = \frac{1}{8}[20 + 3y^{(11)} - 2z^{(11)}] = \frac{1}{8}[20 + 3(1.9858) - 2(0.9118)]$

$= 3.0168$

$y^{(12)} = \frac{1}{11}[33 - 4x^{(11)} + z^{(11)}]$

$$= \frac{1}{11}[33 - 4(3.0168) + 0.9118] = 1.9859.$$

$$z^{(12)} = \frac{1}{12}[35 - 6x^{(11)} - 3y^{(11)}]$$

$$= \frac{1}{12}[35 - 6(3.0168) - 3(1.9858)] = 0.9118$$

From 11^{th} and 12^{th} iterations,

$x = 3.0168$, $y = 1.9858$, $z = 0.9118$ (correct to four decimal places)

Excercise

1. Using Newton Raphson method, establish the formula $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$ to calculate the square root of N. And find the square root of 5 correct to four places of decimals

2. Solve $10x + y + z = 12$, $2x + 10y + z = 13$, $x + y + 5z = 7$ by (i) Gauss elimination method (ii) Gauss Jordan method

3. Solve by Gauss-Seidel method. $2x + y + 6z = 9$, $8x + 3y + 2z = 13$, $x + 5y + z = 7$

4. Find the inverse of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{pmatrix}$ by Gauss Jordan method

5. Using power method, find the largest eigen values and the corresponding eigen vector of the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

UNIT - IV

INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION.

1. Evaluate $\int_1^2 \int_1^2 \frac{dx\,dy}{x+y}$ using Trapezoidal rule with $h = k = 0.25$.

Solution:-

y \ x	1	1.25	1.5	1.75	2	I
1	0.5	0.4444	0.4	0.3636	0.3333	0.40619
1.25	0.4444	0.4	0.3636	0.3333	0.30769	0.36826
1.5	0.4	0.3636	0.3333	0.30769	0.28571	0.33688
1.75	0.3636	0.3333	0.30769	0.28571	0.26667	0.31047
2	0.3333	0.30769	0.28571	0.26667	0.25	0.28783

We first form the table for $f(x,y) = \frac{1}{x+y}$ with $h = k = 0.25$.

$$\int_{x_0}^{x_4} y\,dx = \frac{h}{2}\left[(x_0 + x_4) + 2(x_1 + x_2 + x_3)\right]$$

Applying this rule to each row, We get.

$$I_0 = \frac{0.25}{2}\left[(0.5 + 0.3333) + 2(0.4444 + 0.4 + 0.3636)\right] = 0.40619$$

$$I_1 = \frac{0.25}{2}\left[(0.4444 + 0.30769) + 2(0.4 + 0.3636 + 0.3333)\right] = 0.36826$$

$$I_2 = \frac{0.25}{2}\left[(0.4 + 0.28571) + 2(0.3636 + 0.3333 + 0.30769)\right] = 0.33688$$

ASE

$$I_3 = \frac{0.25}{2}\left[(0.3636+0.26667)+2(0.3333+0.30769+0.28571)\right] = 0.31047.$$

$$I_4 = \frac{0.25}{2}\left[(0.3333+0.25)+2(0.30769+0.28571+0.26667)\right] = 0.28793$$

Now applying trapezoidal rule to I_0, I_1, I_2, I_3, I_4 We get

$$\int_1^2\int_1^2 \frac{dx\,dy}{x+y} = \frac{k}{2}\left[(I_0+I_4)+2(I_1+I_2+I_3)\right]$$

$$= \frac{0.25}{2}\left[(0.40619+0.28793)+2(0.36826+0.33688+0.31047)\right]$$

$$= 0.34067.$$

(2) The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earths' surface.

X :	100	150	200	250	300	350	400
Y :	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when $x = 218$ ft and 410 ft.

Solution.

The difference table is formed as follows.

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	10.63	2.4	-0.39	0.15	-0.07	0.02	0.02
150	13.03	2.01	-0.24	0.08	-0.05	0.04	
200	15.04	1.77	-0.16	0.03	-0.01		
250	16.81	1.61	-0.13	0.02			
300	18.42	1.48	-0.11				
350	19.90	1.37					
400	21.27						

② 95

To find $x = 218$ ft, We use Newton's forward interpolation formula.

$$f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

Where $u = \frac{218-100}{50} = 2.36$.

$$y(x) = 10.63 + \frac{(2.36)}{1}(2.4) + \frac{(2.36)(2.36-1)}{2}(-0.39) + \frac{(2.36)(2.36-1)(2.36-2)}{6}(0.15) + \frac{(2.36)(2.36-1)(2.36-2)(2.36-3)}{24}(0.07) + \cdots$$

$$= 15.6948.$$

To find $x = 410$ ft, We use Newton's Backward interpolation formula.

$$y(x) = y_6 + \frac{v}{1!}\nabla y_6 + \frac{v(v+1)}{2!}\nabla^2 y_6 + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_6 + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_6 + \cdots$$

Here $v = \frac{x - x_6}{h} = \frac{410-400}{50} = 0.2$.

$$\therefore y(x) = 21.27 + \frac{(0.2)}{1}(1.37) + \frac{(0.2)(0.2+1)}{2}(-0.11) + \frac{(0.2)(0.2+1)(0.2+2)}{6}(0.02) + \frac{(0.2)(0.2+1)(0.2+2)(0.2+3)}{24}(-0.01) + \cdots$$

$$= 21.525.$$

(3). Given that

x	1	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and y'' at $x=1.1$ and $x=1.6$

Soln:-

The difference table is formed as follows

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1	7.989						
		0.414					
1.1	8.403		−0.036				
		0.378		0.006			
1.2	8.781		−0.03		−0.002		
		0.348		0.004		0.001	
1.3	9.129		−0.026		−0.001		0.002
		0.322		0.003		0.003	
1.4	9.451		−0.023		0.002		
		0.299		0.005			
1.5	9.750		−0.018				
		0.281					
1.6	10.031						

To find $x=1.1$

Using Newton's forward difference formula for differentiation,

$$\frac{dy}{dt} = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2u-1}{2}\right)\Delta^2 y_0 + \left(\frac{3u^2-6u+2}{6}\right)\Delta^3 y_0 + \left(\frac{4u^3-18u^2+22u-6}{24}\right)\Delta^4 y_0 - - - -\right.$$

Here $h=0.1$ $u = \frac{1.1-1.0}{0.1} = 1$

$$= \frac{1}{0.1}\left[0.414 + \left(\frac{2-1}{2}\right)(-0.036) + \left(\frac{3-6-2}{6}\right)(0.006) + \left(\frac{4-18+22-6}{24}\right)(-0.002) + \cdots\right.$$

$$= 3.9483.$$

To find $x = 1.6$

Using Newton's Backward difference formula for differentiation,

$$\frac{dy}{dt} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\right]$$

$$= \frac{1}{0.1}\left[0.281 + \frac{1}{2}(-0.018) + \frac{1}{3}(0.005) + \frac{1}{4}(0.002) + \frac{1}{5}(0.003) + \frac{1}{6}(0.002)\right]$$

$$= 2.751$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.018 + 0.005 + \frac{11}{12}(0.002) + \cdots\right] = -1.1167$$

For $x = 1.1$,

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \left(\frac{6u^2-18u+11}{12}\right).\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.036 + (1-1)(0.006) + \left(\frac{6-18+11}{12}\right)(-0.002)\right]$$

$$= -3.5833.$$

④. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Trapezoidal rule, Simpson's $1/3$ rule and Simpson's $3/8$ rule and Compare the result with its actual value.

Divide the interval $(0,6)$ into 6 equal parts each of $h=1$.

The Value of $f(x) = \dfrac{1}{1+x^2}$ are tabulated below.

x	0	1	2	3	4	5	6
$y=f(x)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027

(i) By Trapezoidal rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_2)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027)+2(0.5+0.2+0.1+0.0588+0.0385)\right]$$

$$= 1.4108$$

(ii) By Simpson's $1/3$ rule.

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(1+0.027)+4(0.5+0.1+0.0385)+2(0.2+0.0588)\right]$$

$$= 1.3662.$$

(iii) By Simpson's $3/8$ Rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3\right]$$

$$= \frac{3}{8}\left[(1+0.027)+3(0.5+0.2+0.0588+0.0385)+2(0.1)\right]$$

$$= 1.3571$$

By Actual Integration,

$$\int_0^6 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^6 \Rightarrow \tan^{-1}6 - \tan^{-1}0 \Rightarrow 1.4056.$$

5. Using Lagrange's Interpolation formula, find $y(10)$ given that $y(5)=12, y(6)=13, y(9)=14, y(11)=16$.

Solution:-

By Lagrange's Interpolation formula, We have.

$$y=f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\cdot y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\cdot y_1+$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\cdot y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\cdot y_3$$

Here $x_0=5,\ x_1=6,\ x_2=9,\ x_3=11,$

$y_0=12,\ y_1=13,\ y_2=14,\ y_3=16.$

$$=\frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)}(12)+\frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)}(13)+$$

$$\frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)}(14)+\frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)}\cdot 16.$$

To find $y(10)$ (ie) put $x=10$.

$$f(x)=\frac{(4)(1)(-1)}{(-1)(-4)(-6)}(12)+\frac{(5)(1)(-1)}{(1)(-3)(-5)}(13)+$$

$$\frac{(5)(4)(-1)}{(4)(3)(-2)}14+\frac{(5)(4)(1)}{(6)(5)(2)}(16)$$

$f(x)=14.6666.$

6. Using Newton's divided difference formula find $f(x)$ and $f(6)$ from the following data.

x	1	2	7	8
$f(x)$	1	5	5	4

Soln:-

The divided difference table is

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	1			
		$\frac{5-1}{2-1}=4$		
2	5		$\frac{0-4}{7-1}=\frac{-4}{6}$	
		$\frac{5-5}{7-2}=0$		$\frac{-1/6+4/6}{8-1}=\frac{1}{14}$.
7	5		$\frac{-1-0}{8-2}=-1/6$	
		$\frac{4-5}{7-8}=-1$		
8	4			

By Newton's divided difference interpolation formula, we have

$$f(x)=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)+$$
$$(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)+\cdots$$

$$=1+(x-1)4+(x-1)(x-2)\left(\frac{-4}{6}\right)+(x-1)(x-2)(x-7)\frac{1}{14}$$

$$=\frac{1}{42}\left[3x^3-58x^2+321x-224\right]$$

$$f(6)=\frac{1}{42}\left[3(6)^3-58(6)^2+321(6)-224\right]$$

$$=6.2381$$

(5).

(4). By dividing the range into ten equal parts, Evaluate $\int_0^{\pi} \sin x \, dx$ by Trapezoidal and Simpson's rule. Verify your answer with Integration.

Solu:-

Range $= \pi - 0 = \pi$ Hence $h = \frac{\pi}{10}$.

We tabulate the values of y at different x's.

x	0	$\pi/10$	$2\pi/10$	$3\pi/10$	$4\pi/10$	$5\pi/10$	$6\pi/10$	$7\pi/10$	$8\pi/10$	$9\pi/10$	π
$y = \sin x$	0	0.3090	0.5878	0.8090	0.9511	1.0	0.9511	0.8090	0.5878	0.3090	0

(i) By Trapezoidal rule,

$$I = \frac{\pi}{20}\left[(0+0) + 2(0.3090 + 0.5878 + 0.8090 + 0.9511 + 1.0 + 0.9511 + 0.8090 + 0.5878 + 0.3090)\right]$$

$$= 1.9843.$$

(ii) By Simpson's $1/3$ rule,

$$I = \frac{h}{3}\left[(y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right]$$

$$= \left(\frac{\pi/10}{3}\right)\left[(0+0) + 4(0.3090 + 0.8090 + 1 + 0.8090 + 0.3090) + 2(0.5878 + 0.9511 + 0.9511 + 0.5878)\right]$$

$$= 2.0001.$$

(8). Evaluate $\int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy} \, dx\, dy$ using Simpson's rule with $h = k = 1/4$.

Soln:-

Let $f(x,y) = \frac{\sin(xy)}{1+xy}$.

The values of $f(x,y)$ at the nodal points are given in the following table.

x \ y	0	1/4	1/2
0	0	0	0
1/4	0	0.0588	0.1108
1/2	0	0.1108	0.1979

By Simpson's Rule,

$$I = \left[\frac{1/4}{3}\right]\left[\frac{1/4}{3}\right]\left[(0+0+0.1979+0) + 4(0+0+0.1108+0.1108) + 16(0.0588)\right]$$

$$= \left(\frac{1}{144}\right) 2.0251$$

$$= 0.0141.$$

(6).

9. Find the first, Second and third derivative of $f(x)$ at $x=1.5$ if

x	1.5	2	2.5	3	3.5	4
$f(x)$	3.375	7.000	13.625	24.000	38.875	59.000

Solu:-

We have to find the derivative at the point $x=1.5$ which is the starting point of the given data.

$\therefore$ We use Newton's forward interpolation formula.

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.5	3.375				
		3.625			
2	7.000		3.000		
		6.625		0.750	
2.5	13.625		3.750		0
		10.375		0.750	
3	24.000		4.500		0
		14.875		0.750	
3.5	38.875		5.250		
		20.125			
4	59.000				

Here $x_0=1.5$, $y_0=3.375$, $\Delta y_0=3.625$, $\Delta^2 y_0=3$, $\Delta^3 y_0=0.75$, $h=0.5$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = f'(x_0) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 - \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

$$f'(1.5) = \frac{1}{0.5}\left[3.625 - \frac{1}{2}(3) + \frac{1}{3}(0.75)\right] = 4.75.$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = f''(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 \cdots\right]$$

$$f''(1.5) = \frac{1}{(0.5)^2}\left[3-0.75\right] = 9$$

$$\left[\frac{d^3y}{dx^3}\right]_{x=x_0} = f'''(x_0) = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots\right]$$

$$f'''(1.5) = \frac{1}{(0.5)^3}(0.75) = 6.$$

(10). Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x	0	1	2	5
$f(x)$	2	3	12	147.

Solu:-

By Lagrange's Interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}(y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}(y_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}(y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}(y_3).$$

Here $x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 5,\ y_0 = 2,\ y_1 = 3,\ y_2 = 12,\ y_3 = 147.$

Here $x = 3.$

$$y = f(3) = \frac{(3-1)(3-2)(3-5)}{-10}(2) + \frac{3(3-2)(3-5)}{4}(3) + \frac{3(3-1)(3-5)}{(-6)}(12) + \frac{3(3-1)(3-2)}{60}(147).$$

$$f(3) = 35.$$

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11. Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference formula. Also find $f(1)$

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

(May / June 2014)

Soln:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		$\frac{33-1245}{(-1)-(-4)} = -404$			
			$\frac{-28+404}{4} = 94$		
-1	33			$\frac{10-94}{2+4} = -14$	
		$\frac{5-33}{0-(-1)} = -28$			$\frac{13+14}{5+4} = 3$
			$\frac{2+28}{2+1} = 10$		
0	5			$\frac{88-10}{5+1} = 13$	
		$\frac{9-5}{2-0} = 2$			
			$\frac{442-2}{5} = 88$		
2	9				
		$\frac{1335-9}{5-2} = 442$			
5	1335				

$$f(x) = f(x_0) + (x-x_0)\, f(x_0, x_1) + (x-x_0)(x-x_1)\, f(x_0, x_1, x_2) +$$
$$(x-x_0)(x-x_1)(x-x_2)\, f(x_0, x_1, x_2, x_3) +$$
$$(x-x_0)(x-x_1)(x-x_2)(x-x_3)\, f(x_0, x_1, x_2, x_3, x_4)$$

Here $x_0 = -4$, $x_1 = -1$, $x_2 = 0$, $x_3 = 2$, $x_4 = 5$

$$\therefore f(x) = 1245 + (x+4)(-404) + (x+4)(x+1)(94) + (x+4)(x+1)(x)(-14) + (x+4)(x+1)(x)(x-2)(3)$$

$$= 1245 - 404x - 1616 + (94)(x^2+5x+4) - 14x(x^2+5x+4) + 3x(x^3-2x^2+5x^2-10x+4x-8)$$

$$= -14x^3 + 24x^2 + 10x + 5 + 3x(x^3+3x^2-6x-8)$$

$$= -14x^3 + 24x^2 + 10x + 5 + 3x^4 + 9x^3 - 18x^2 - 24x$$

$$= 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

$$f(1) = 3(1)^4 - 5(1)^3 + 6(1)^2 - 14(1) + 5$$

$$= 3 - 5 + 6 - 14 + 5 = -5$$

$$f(1) = -5$$

12. Use Lagrange's inverse interpolation formula to find the value of x when $y = 0.3$ from the following data

x	0.4	0.6	0.8
y	0.3683	0.3332	0.2987

Soln :

$$x = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} x_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} x_1 + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} x_2$$

$$= \frac{(0.3-0.3332)(0.3-0.2987)}{(0.3683-0.3332)(0.3683-0.2987)}(0.4) + \frac{(0.3-0.3683)(0.3-0.2987)}{(0.3332-0.3683)(0.3332-0.2987)}(0.6) + \frac{(0.3-0.3683)(0.3-0.3332)}{(0.2987-0.3683)(0.2987-0.3332)}(0.8)$$

$$x = 0.7924$$

13. From the following table estimate the number of students who obtained marks between 40 and 45

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	31				
		42			
Below 50	73		9		
		51		-25	
Below 60	124		-16		37
		35		12	
Below 70	159		-4		
		31			
Below 80	190				

$$u = \frac{x - x_0}{h} = \frac{45-40}{10} = 0.5$$

$$y(45) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 31 + (0.5)(42) + \frac{(0.5)(-0.5)}{2}(9) + \frac{(0.5)(-0.5)(-1.5)}{6}(-25$$

$$+ \frac{(0.5)(-0.5)(-1.5)(-2.5)}{24}(37)$$

$$= 48.84 \simeq 49$$

No. of students who obtained marks below 45 = 49

No. of students who obtained marks below 40 = 31

$\therefore$ No. of students who obtained marks between 40 & 45 $\}$ = 49 - 31 = 18

unit - V

Numerical Solutions of ordinary differential equations

part - B.

1. Using modified Euler's method, find $y(0.2)$ if $\frac{dy}{dx} = y - x^2 + 1$, $y(0) = 0.5$. [AU M/J'12, N/D'14] [AU A/M 2003]

Soln:

Given

x	$x_0 = 0$	$x_1 = 0.2$
$y(x)$	$y_0 = 0.5$	$y_1 = ?$

$\frac{dy}{dx} = y - x^2 + 1.$

$\therefore f(x, y) = y - x^2 + 1$, $h = 0.2$

Modified Euler's formula is

$$\boxed{y_{n+1} = y_n + h\, f\left[x_n + \frac{h}{2},\ y_n + \frac{h}{2} f(x_n, y_n)\right]}$$ $n = 0, 1, 2, \ldots$

$y_1 = y_0 + h f\left[x_0 + \frac{h}{2},\ y_0 + \frac{h}{2} f(x_0, y_0)\right]$

$= 0.5 + 0.2 f\left(0 + \frac{0.2}{2},\ 0.5 + \frac{0.2}{2}[(0.5) - 0 + 1]\right)$

$= 0.5 + 0.2 f(0.1,\ 0.5 + 0.1[1.5])$

$= 0.5 + 0.2 f(0.1, 0.65)$

$= 0.5 + 0.2 [0.65 - (0.1)^2 + 1]$

$= 0.5 + 0.2 [0.65 - 0.01 + 1]$

$= 0.5 + 0.2 (1.64)$

$= 0.5 + 0.328$

$= 0.828.$

$\therefore \boxed{y(0.2) = 0.828}$

2. Using Milne's Predictor - Corrector method, find $y(4.4)$ given $5xy'+y^2-2=0$; $y(4)=1$, $y(4.1)=1.0069$, $y(4.2)=1.0097$, $y(4.3)=1.014$ [AU M/J'12, M/J'1

Soln:

Milne's predictor - corrector formula is

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

$$y_{4,p} = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3) \quad \text{---}(1)$$

Given $5xy'+y^2-2=0$, $\quad x_0=4,\ x_1=4.1,\ x_2=4.2,\ x_3=4.3$

$5xy' = 2-y^2$ $\quad y_0=1,\ y_1=1.0049\ y_2=1.0097,$

$y' = \frac{2-y^2}{5x}$. $\quad y_3=1.0143$.

$$y'_1 = \frac{2-y_1^2}{5x_1} = \frac{2-(1.0049)^2}{5(4.1)} = 0.0493.$$

$$y'_2 = \frac{2-y_2^2}{5x_2} = \frac{2-(1.0097)^2}{5(4.2)} = 0.0467.$$

$$y'_3 = \frac{2-y_3^2}{5x_3} = \frac{2-(1.0143)}{5(4.3)} = 0.0452.$$

Sub all these values in (1),

$$y_{4,p} = 1+\frac{4(0.1)}{3}[2(0.0493)-0.0467+2(0.0452)]$$

$$= 1.01897.$$

$$y'_4 = \frac{2-y_4^2}{5(x_4)} = \frac{2-(1.01897)^2}{5(4.4)} = 0.0437.$$

Using Milne's corrector formula,

$$y_{4,c} = y_2 + \frac{h}{3}[y'_2 + 4y'_3 + y'_4]$$

$$= 1.0097 + \frac{0.1}{3}[(0.0467+4(0.0452)+0.0437]$$

$$y_{4,c} = 1.01874.$$

$$\therefore \boxed{y(4.4) = 1.01874}$$

3. Find $y(0.8)$ given that $y' = y - x^2$, $y(0.6) = 1.7379$, by using Runge-kutta method of fourth order. Take $h = 0.1$. [AU M/J '2012]

Soln:

Given $y' = f(x,y) = y - x^2$, $x_0 = 0.6$, $y_0 = 1.7379$.

$x_1 = 0.7$, $x_2 = 0.8$, $h = 0.1$.

To find $y(0.7)$

$K_1 = hf(x_0, y_0) = (0.1)[y_0 - x_0^2] = (0.1)[1.7379 - (0.6)^2] = 0.13779$.

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = 0.1\, f\left(0.6 + \frac{0.1}{2}, 1.7379 + \frac{0.13779}{2}\right)$

$= 0.1\, f(0.65, 1.8086795)$

$= (0.1)[1.806795 - (0.65)^2]$

$= 0.13843$.

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) = 0.1\, f\left(0.65, 1.7379 + \frac{0.13843}{2}\right]$

$= 0.1\, f(0.65, 1.807115)$

$= (0.1)(1.807115 - (0.65)^2) = 0.13846$

$K_4 = hf(x_0 + h, y_0 + K_3) = 0.1\, f[0.6 + 0.1, 1.7379 + 0.13846]$

$= 0.1\, f(0.7, 1.87636)$

$= (0.1)[1.87636 - (0.7)^2]$

$= 0.13864$.

$\Delta y = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.13779 + 2(0.13843) + 2(0.13846) + 0.13864]$

$= 0.13837$.

$\therefore y_1 = y(0.7) = y_0 + \Delta y = 1.7379 + 0.13837 = 1.87627$.

$\boxed{y_1 = 1.876}$ (app)

To find $y(0.8)$

Again apply Runge – Kutta Method,

$x_1 = 0.7, \quad y_1 = 1.876$

$K_1 = h f(x_1, y_1) = (0.1) f(0.7, \cancel{0.8}\ 1.876)$

$= (0.1)[1.876 - (0.7)^2] = 0.1386$

$K_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right) = (0.1) f\left[0.7 + \frac{0.1}{2}, 1.876 + \frac{0.1386}{2}\right]$

$= (0.1) f(0.75, 1.9\overset{45}{\cancel{543}}]$

$= (0.1)\ [1.9453 - (0.75)^2]$

$= 0.13828$

$K_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right).$

$= (0.1) f\left[0.75, 1.876 + \frac{0.13828}{2}\right]$

$= 0.1 f(0.75, 1.94514)$

$= (0.1).\ [1.94514 - (0.75)^2] = 0.138264.$

$K_4 = h f(x_1 + h, y_1 + K_3)$

$= (0.1) f(0.7 + 0.1, 1.876 + 0.138264)$

$= (0.1) f(0.8, 2.014264)$

$= (0.1)(2.014264 - (0.8)^2)$

$= 0.1374264.$

$\Delta y = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.1386 + 2(0.13828) + 2(0.138264) + 0.1374264]$

$= 0.138186$

$y_2 = y(0.8) = y_1 + \Delta y = 1.876 + 0.138186 = 2.014186$

$\boxed{y(0.8) = 2.0142}$ (correct to 4 decimal places).

4. Solve $\frac{d^2y}{dx^2} - y = 0$, $y(0) = 0$, $y(1) = 1$, $h = \frac{1}{4}$. [AU M/J '12]

Soln:

Given $x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 0.75$, $x_4 = 1$.

$y_0 = 0$, $y_1 = ?$, $y_2 = ?$, $y_3 = ?$, $y_4 = 1$.

$h = \frac{1}{4}$.

$\frac{d^2y}{dx^2} - y = 0$

ie) $y_i'' - y_i = 0$ — ① , $i = 1, 2, 3$.

we have $y_i'' = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$

$\therefore$ ① $\Rightarrow$ $\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} - y_i = 0$.

ie) $16(y_{i-1} - 2y_i + y_{i+1}) - y_i = 0$. $(\because h = 1/4)$

$16y_{i-1} - 32y_i + 16y_{i+1} - y_i = 0$ — ② $i = 1, 2, 3$.

when $i = 1$, ② becomes

$16y_0 - 32y_1 + 16y_2 - y_1 = 0$.

$16(0) - 32y_1 + 16y_2 - y_1 = 0$ $(\because y_0 = 0)$.

ie) $-33y_1 + 16y_2 = 0$ — ③

when $i = 2$, ② becomes.

$16y_1 - 32y_2 + 16y_3 - y_2 = 0$

$16y_1 - 32y_2 + 16y_3 - y_2 = 0$

$16y_1 - 33y_2 + 16y_3 = 0$ — ④

when $i = 3$, ② becomes

$16y_2 - 32y_3 + 16y_4 - y_3 = 0$

$16y_2 - 32y_3 + 16(1) - y_3 = 0$ $(\because y_4 = 1)$.

$16y_2 - 33y_3 = -16$ — ⑤

solving equations ③, ④ & ⑤.

(3) $\Rightarrow$ $-33y_1 = -16y_2$.

$$y_1 = \frac{16}{33} y_2 = 0.4848 y_2 \quad \text{--- (6)}$$

(5) $\Rightarrow$ $-33y_3 = -16 - 16y_2$.

$$y_3 = \frac{16 + 16y_2}{33}$$

$$= \frac{16}{33}(1 + y_2)$$

$$= 0.4848(1 + y_2) \quad \text{--- (7)}$$

Sub the values of y_1 and y_3 in (4), we get

$$16 \times (0.4848 y_2) - 31y_2 + 16 \times 0.4848(1 + y_2) = 0.$$

$$7.7568 y_2 - 31y_2 + 7.7568(1 + y_2) = 0$$

$$-23.2432 y_2 + 7.7568 + 7.7568 y_2 = 0.$$

$$-15.4864 y_2 = -7.7568$$

$$\boxed{y_2 = 0.5009.}$$

(6) $\Rightarrow$ $y_1 = 0.4848 \times 0.5009.$

$$\boxed{y_1 = 0.2428}$$

(7) $\Rightarrow$ $y_3 = 0.4848(1 + 0.2428)$

$$\boxed{y_3 = 0.6025.}$$

The Solution is $y_1 = 0.2428$, $y_2 = 0.5009$, $y_3 = 0.6025$.

5. Given $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$, find $y(0.2)$, $y(0.4)$, and $y(0.6)$ by R. Kutta method of fourth order. Find $y(0.8)$ by Milne's predictor - corrector method. [Nov/Dec 2012]

Soln:

Given $x_0 = 0$, $x_1 = 0.2$, $x_2 = 0.4$, $x_3 = 0.6$, $x_4 = 0.8$

$y_0 = 1.$

To find $y(0.2)$:

$x_0 = 0, \quad y_0 = 1, \quad h = 0.2$

$$\frac{dy}{dx} = f(x,y) = \frac{y^2 - x^2}{y^2 + x^2}$$

$$K_1 = h f(x_0, y_0) = 0.2 \; f(0,1) = 0.2 \left[\frac{1^2 - 0^2}{1+0} \right] = 0.2$$

$$K_2 = h f\left(x_0 + \frac{h}{2}, \; y_0 + \frac{K_1}{2}\right) = 0.2 \; f\left(0 + \frac{0.2}{2}, \; 1 + \frac{0.2}{2}\right)$$

$$= 0.2 \; f(0.1, 1.1) = (0.2) \left[\frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2} \right] = 0.1967$$

$$K_3 = h f\left(x_0 + \frac{h}{2}, \; y_0 + \frac{K_2}{2}\right)$$

$$= (0.2) f\left(0 + \frac{0.2}{2}, \; 1 + \frac{0.1967}{2}\right)$$

$$= (0.2) f(0.1, 1.0984)$$

$$= (0.2) \left[\frac{(1.0984)^2 - (0.1)^2}{(1.0984)^2 + (0.1)^2} \right] = 0.1967$$

$$K_4 = h f(x_0 + h, \; y_0 + K_3) = (0.2) f(0.2, 1.1967)$$

$$= (0.2) \left[\frac{(1.1967)^2 - (0.2)^2}{(1.1967)^2 + (0.2)^2} \right] = 0.1891$$

$$\Delta y = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$= \frac{1}{6}(0.2 + 2(0.1967) + 2(0.1967) + 0.1891)$$

$$= 0.1960$$

$$y(0.2) = y_1 = y_0 + \Delta y = 1 + 0.1960 = 1.1960$$

$$\boxed{y(0.2) = 1.1960}$$

To find $y(0.4)$.

$x_1 = 0.2, \quad y_1 = 1.1960, \quad h = 0.2$

$$K_1 = h f(x_1, y_1) = 0.2 \; f(0.2, 1.1960) = (0.2) \left[\frac{(1.1960)^2 - (0.2)^2}{(1.1960)^2 + (0.2)^2} \right]$$

$$= 0.1891$$

$$K_2 = h f\left(x_1 + \frac{h}{2}, \; y_1 + \frac{K_1}{2}\right)$$

$= 0.2\, f\left(0.2+\frac{0.2}{2},\ 1.196+\frac{0.1891}{2}\right)$

$= 0.2\, f(0.3,\ 1.2906)$

$= 0.2\left[\frac{(1.2906)^2-(0.3)^2}{(1.2906)^2+(0.3)^2}\right]$

$= 0.1795$

$K_3 = h\, f\left(x_1+\frac{h}{2},\ y_1+\frac{K_2}{2}\right)$

$= 0.2\, f\left(0.2+\frac{0.2}{2},\ 1.196+\frac{0.1795}{2}\right)$

$= 0.2\, f(0.3,\ 1.1858)$

$= 0.2\left[\frac{(1.1858)^2-(0.3)^2}{(1.1858)^2+(0.3)^2}\right] = 0.1793$

$K_4 = h\, f(x_1+h, y_1+K_3) = (0.2)\, f(0.4,\ 1.3753)$

$= (0.2)\left[\frac{(1.3753)^2-(0.4)^2}{(1.3753)^2+(0.4)^2}\right] = 0.1688$

$\Delta y = \frac{1}{6}\left[K_1+2K_2+2K_3+K_4\right]$

$= \frac{1}{6}(0.1891+2(0.1795)+2(0.1793)+0.1688)$

$= 0.1793$

$y(0.4) = y_2 = y_1+\Delta y = 1.196+0.1793$

$\boxed{y(0.4) = 1.3753}$

To find $y(0.6)$

$x_2 = 0.4,\quad y_2 = 1.3753,\quad h = 0.2$

$K_1 = (0.2)\, f(x_2, y_2) = 0.2\, f(0.4,\ 1.3753)$

$= (0.2)\left[\frac{(1.3753)^2-(0.4)^2}{(1.3753)^2+(0.4)^2}\right] = 0.1688$

$K_2 = h\, f\left(x_2+\frac{h}{2},\ y_2+\frac{K_1}{2}\right)$

$= (0.2)\, f\left(0.4+\frac{0.2}{2},\ 1.3753+\frac{0.1688}{2}\right)$

$= 0.2\, f(0.5,\ 1.4597)$

$= 0.2 \left[\dfrac{(1.4597)^2 - (0.5)^2}{(1.4597)^2 + (0.5)^2} \right] = 0.1580$

$K_3 = h\, f\left(x_2 + \frac{h}{2},\ y_2 + \frac{K_2}{2}\right)$

$= 0.2\, f\left(0.4 + \frac{0.2}{2},\ 1.3753 + \frac{0.1580}{2}\right)$

$= 0.2\, f(0.5,\ 1.4543)$

$= 0.2 \left[\dfrac{(1.4543)^2 - (0.5)^2}{(1.4543)^2 + (0.5)^2} \right] = 0.1577$

$K_4 = h\, f(x_2 + h,\ y_2 + K_3)$

$= (0.2)\, f(0.4 + 0.2,\ 1.3753 + 0.1577)$

$= (0.2)\, f(0.6,\ 1.5330)$

$= 0.2 \left[\dfrac{(1.5330)^2 - (0.6)^2}{(1.5330)^2 + (0.6)^2} \right] = 0.1469$

$\Delta y = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$

$= \frac{1}{6}\left[0.1688 + 2(0.1580) + 2(0.1577) + 0.1469\right]$

$= 0.1578$

$y(0.6) = y_3 = y_2 + \Delta y = 1.3753 + 0.1578$

$\boxed{y(0.6) = 1.5331}$

Now we find $y(0.8)$, by Milne's Predictor - Corrector method

Milne's predictor formula is

$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$

$\therefore y_{4,p} = y_0 + \frac{4h}{3}\left(2y'_1 - y'_2 + 2y'_3\right)$ — (1)

$y'_1 = \dfrac{y_1^2 - x_1^2}{y_1^2 + x_1^2} = \dfrac{(1.1960)^2 - (0.2)^2}{(1.1960)^2 + (0.2)^2} = 0.9456$

$y'_2 = \dfrac{y_2^2 - x_2^2}{y_2^2 + x_2^2} = \dfrac{(1.3753)^2 - (0.4)^2}{(1.3753)^2 + (0.4)^2} = 0.844$

$$y_3' = \frac{y_3^2 - x_3^2}{y_3^2 + x_3^2} = \frac{(1.5331)^2 - (0.6)^2}{(1.5331)^2 + (0.6)^2} = 0.7344.$$

$$① \Rightarrow y_{4,p} = 1 + \frac{4(0.2)}{3}[2(0.4956) - 0.844 + 2(0.7344)]$$
$$= 1.6709.$$

Milne's corrector formula is

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1})$$

$$y_{4,c} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4') \quad —②$$

$$y'_{4,p} = \frac{y_4^2 - x_4^2}{y_4^2 + x_4^2} = \frac{(1.6709)^2 - (0.8)^2}{(1.6709)^2 + (0.8)^2} = 0.6270.$$

$$② \Rightarrow y_{4,c} = 1.3753 + \frac{(0.2)}{3}[0.844 + 4(0.7344) + 0.6270] = 1.6692$$

$$\boxed{y(0.8) = 1.6692}$$

6. Using Taylor's method, find $y(0.1)$, $y(0.2)$ and $y(0.3)$ if $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. [Nov/Dec'12]

Soln:

Given $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$.

$y_0 = 1$, $h = 0.1$.

To find $y(0.1)$

Taylor's series expansion is

$$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0^{iv} + \cdots\cdots$$

$y' = x - y^2$ $\qquad$ $y_0' = x_0 - y_0^2 = -1$

$y'' = 1 - 2yy'$ $\qquad$ $y_0'' = 1 - 2y_0y_0' = 3$

$y''' = -2[yy'' + y'^2]$ $\qquad$ $y_0''' = -2[y_0y_0'' + y_0'^2] = -8$

$$\therefore y_1 = y(0.1) = 1 + \frac{(0.1)(-1)}{1!} + \frac{(0.1)^2}{2!}(3) + \frac{(0.1)^3}{3!}(-8) + \cdots\cdots$$

$= 1 - 0.1 + 0.0051 + 0.0113$

$= 0.9137$

To find $y(0.2)$.

$x_1 = 0.1, \quad y_1 = 0.9137$.

Taylor's series expansion is

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1'''$$

$$y_1' = x_1 - y_1^2 = 0.1 - (0.9137)^2 = -0.7348$$

$$y_1'' = 1 - 2y_1 y_1' = 1 - 2(0.9137)(-0.7348) = 2.3428.$$

$$y_1''' = -2[y_1 y_1'' + y_1'^2]$$

$$= -2[(0.9137)(2.3428) + (-0.7348)^2] = -5.3611$$

$$\therefore y_2 = y(0.2) = 0.9137 + (0.1)(-0.7348) + \frac{(0.1)^2}{2!}(2.3428) + \frac{(0.1)^3}{3!}(-5.3611)$$

$$= 0.9137 - 0.07348 + 0.0117 - 0.0009$$

$$\boxed{y(0.2) = 0.8510}$$

To find $y(0.3)$:

Here $x_2 = 0.2, \quad y_2 = 0.8510$.

Taylor's series is

$$y_3 = y(0.3) = y_2 + \frac{h}{1!} y_2' + \frac{h^2}{2!} y_2'' + \frac{h^3}{3!} y_2''' + \cdots$$

$$y_2' = x_2 - y_2^2 = 0.2 - (0.8510)^2 = -0.5242$$

$$y_2'' = 1 - 2y_2 y_2' = 1 - 2(0.851)(-0.5242) = 1.8922.$$

$$y_2''' = -2[y_2 y_2'' + y_2'^2] = -2[(0.851)(1.8922) + (0.5242)^2]$$

$$= -3.7701.$$

$$y_3 = y(0.3) = 0.8510 + (0.1)(-0.5242) + \frac{(0.1)^2}{2!}(1.8922) + \frac{(0.1)^3}{3!}(-3.7701)$$

$$= 0.8510 - 0.05242 + 0.0095 - 0.0006 = 0.8074.$$

$$\boxed{y(0.3) = 0.8074}$$

7. Consider the initial value problem $\frac{dy}{dx} = y - x^2 + 1$, $y(0) = 0.5$. Find $y(0.2)$ by Euler's method and modified Euler's method. [Nov/Dec '14]

Soln:

Given $x_0 = 0$, $x_1 = 0.2$

$y_0 = 0.5$ $h = 0.2$

$\frac{dy}{dx} = f(x, y) = y - x^2 + 1.$

Euler's formula is $y_{n+1} = y_n + h f(x_n, y_n)$; $n = 0, 1, 2, \ldots$

To find $y(0.2)$.

Here $x_0 = 0$, $y_0 = 0.5$, $h = 0.2$.

$$y_1 = y_0 + h f(x_0, y_0)$$
$$= 0.5 + (0.2) f(0, 0.5)$$
$$= 0.5 + (0.2)[0.5 - 0^2 + 1]$$
$$y(0.2) = 0.8$$

ie) $\boxed{y(0.2) = 0.8}$

Modified Euler's formula is

$$y_{n+1} = y_n + h f\left[x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right] ; n = 0, 1, 2, \ldots$$

To find $y(0.2)$.

$x_0 = 0$, $y_0 = 0.5$, $h = 0.2$

$$y_1 = y_0 + h f\left[x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right]$$
$$= 0.5 + (0.2) f\left[0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2} f(0, 0.5)\right]$$
$$= 0.5 + (0.2) f(0.1, 0.65).$$
$$= 0.5 + (0.2)[0.65 - (0.1)^2 + 1]$$
$$= 0.828$$

$\boxed{y(0.2) = 0.828}$

8. Given $\frac{dy}{dx} = x - y^2$, $y(0)=1$, $y(0.1) = 0.9052$, $y(0.2) = 0.8213$, find $y(0.3)$ using Taylor's method. (NOV/DEC 2013).

Soln: Here $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$

$y_0 = 1$, $y_1 = 0.9052$, $y_2 = 0.8213$, $h = 0.1$.

To find $y(0.3)$.

Here $x_2 = 0.2$, $y_2 = 0.8213$.

Taylor's series is

$$y_3 = y(0.3) = y_2 + \frac{h}{1!} y_2' + \frac{h^2}{2!} y_2'' + \frac{h^3}{3!} y_3''' + \cdots$$

$y' = x - y^2$

$y'' = 1 - 2yy'$

$y''' = -2[yy'' + y'^2]$

$y_2' = x_2 - y_2^2 = -0.47$

$y_2'' = 1 - 2y_2 y_2' = 1.77$

$y_2''' = -2[y_2 y_2'' + y_2'^2] = -3.35$

$$y_3 = y(0.3) = 0.8213 + (0.1)(-0.47) + \frac{(0.1)^2}{2!}(1.77) + \frac{(0.1)^3}{3!}(-3.35)$$

$$= 0.8213 - 0.047 + 0.01 - 0.0006$$

$$\boxed{y(0.3) = 0.7837.}$$

9. Using Taylor's series method, find $y(1.1)$ and $y(1.2)$ correct to four decimal places given $\frac{dy}{dx} = xy^{1/3}$ and $y(1) = 1$ [AU N/D 2006, M/J 2007, M/J 2012]

Soln: Take $x_0 = 1$, $y_0 = 1$, $h = 0.1$.

$y' = xy^{1/3}$

$y'' = \frac{1}{3} xy^{-2/3} y' + y^{1/3}$

$= \frac{1}{3} x^2 y^{-1/3} + y^{1/3}$.

$y''' = \left[\frac{-x^2}{3}\left(-\frac{1}{3}\right) y^{-4/3} y' + \frac{2x}{3} y^{-1/3} + \frac{1}{3} y^{-2/3} y'\right]$

$y_0' = 1(1)^{1/3} = 1$

$y_0'' = \frac{1}{3} x_0 y_0^{-2/3} y_0' + y_0^{1/3} = \frac{4}{3}$

$y_0''' = 8/9$.

By Taylor's series formula,

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!} y_0'' + \cdots$$

$$y_1 = y(1.1) = 1 + (0.1)(1) + \frac{(0.1)^2}{2}\left(\frac{4}{3}\right) + \frac{(0.1)^3}{6}\left(\frac{8}{9}\right) + \cdots$$

$$= 1 + 0.1 + 0.00666 + 0.000148 + \cdots$$

$$= 1.10681.$$

We start with (x_1, y_1) as the starting value

$$y_1 = 1.10681$$

$$y_1' = x_1 y_1^{1/3} = (1.1)(1.10681)^{1/3} = 1.13785$$

$$y_1'' = \frac{1}{3} x_1 y_1^{-2/3} y_1' + y_1^{1/3}$$

$$= \frac{1}{3}(1.1)(1.10681)^{-2/3}(1.13785) + (1.10681)^{1/3}$$

$$= 0.38992 + 1.03441$$

$$= 1.42433$$

$$y_1''' = 0.929787$$

$$\therefore y_2 = y + hy_1' + \frac{h^2}{2!} y_1'' + \cdots$$

$$y(1.2) = 1.10681 + (0.1)(1.13785) + \frac{(0.01)}{2}(1.42433) + \frac{0.001}{6}(0.92979$$

$$\boxed{y(1.2) = 1.22772}$$

10. Using Euler's method find $y(0.2)$ and $y(0.4)$ from $\frac{dy}{dx} = x+y$, $y(0) = 1$ with $h = 0.2$. [AU M/J 2000, A/M 2010, 2011] [May/June 2012]

Soln:

$$\frac{dy}{dx} = f(x,y) = x+y.$$

Here $x_0 = 0$, $y_0 = 1$, $x_1 = 0.2$, $x_2 = 0.4$.

By Euler's Algorithm,

$$y_{n+1} = y_n + h f(x_n, y_n), \quad n = 0, 1, 2, \ldots$$

$y_1 = y_0 + h f(x_0, y_0)$

$= 1 + 0.2\ (x_0 + y_0) = 1 + 0.2\ (0+1) = 1.2$

ie) $y(0.2) = 1.2$

$y_2 = y_1 + h f(x_1, y_1)$

$= 1.2 + 0.2\ [x_1 + y_1]$

$= 1.2 + 0.2\ [0.2 + 1.2]$

$= 1.2 + 0.28$

$y_2(0.4) = 1.48$

$y_3 = y_2 + h f(x_2, y_2)$

$= 1.48 + (0.2)\ [x_2 + y_2]$

$= 1.48 + 0.2\ [0.4 + 1.48]$

$= 1.48 + 0.376$

$y(0.6) = 1.856$

11. Solve the equation $y'' = x + y$ with conditions $y(0) = y(1) = 0$ by finite difference method, taking $h = 0.25$

soln:

Given $y'' = x + y$

$x_0 = 0,\quad x_1 = 0.25,\quad x_2 = 0.5,\quad x_3 = 0.75,\quad x_4 = 1$

$y_0 = 0,\quad y_4 = 0,\quad h = 0.25$

The finite difference approximation of the given differential equation is

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = x_i + y_i$$

$$y_{i+1} - 2y_i + y_{i-1} = h^2(x_i + y_i) = 0.0625\ (x_i + y_i)$$

For $i=1$, $y_2 - 2y_1 + y_0 = 0.0625\,(x_1 + y_1)$

$$y_2 - 2y_1 + 0 = 0.0625\,(0.25 + y_1)$$

$$-2.0625\,y_1 + y_2 = 0.015625$$

For $i=2$, $y_3 - 2y_2 + y_1 = 0.0625\,(x_2 + y_2)$

$$y_3 - 2y_2 + y_1 = 0.0625\,(0.5 + y_2)$$

$$y_1 - 2.0625\,y_2 + y_3 = 0.03125$$

For $i=3$,

$$y_4 - 2y_3 + y_2 = 0.0625\,(x_3 + y_3)$$

$$0 - 2y_3 + y_2 = 0.0625\,(0.75 + y_3)$$

$$y_2 - 2.0625\,y_3 = 0.046875$$

Solving these equations we get

$$y_1 = -0.034885$$

$$y_2 = -0.056326$$

$$y_3 = -0.050037$$

The results are tabulated below

x	y
0	0
0.25	-0.03488
0.5	-0.05633
0.75	-0.05004

UNIT - IV

Excercise

1. Find $f(x)$ as a polynomial in x from the given data

x	3	7	9	10
$f(x)$	168	120	72	63

and find $f(8)$

2. The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data

Time (sec)	0	5	10	15	20
Velocity (m/sec)	0	3	14	69	228

3. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Trapezoidal rule, Simpson's $\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule and compare the result with its actual value.

4. Apply Simpson's rule to evaluate the integral

$$I = \int_2^{2.6} \int_4^{4.4} \frac{1}{xy}\, dx\, dy. \quad \text{Take } h = 0.2,\ k = 0.3.$$

5. Using Trapezoidal rule evaluate numerically

$$\int_1^2 \int_1^2 \frac{xy}{x+y}\, dx\, dy \text{ with } h = k = 0.25$$

Unit - V

Excercise

1. Find the Taylor series solution for $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ and hence find $y(0.1)$ and $y(0.2)$ correct to 5 decimal places

2. Using Euler's method solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ for $x = (0)(0.02)(0.1)$

3. Find $y(-0.1)$ and $y(-0.2)$ by using modified Euler's method with $h = -0.1$, given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$

4. Solve $\frac{d^2y}{dx^2} = y^3$, $y(0) = 10$, $y'(0) = 5$ using fourth order R-K method and evaluate $y(0.1)$, $y(0.2)$

5. Use Milne's method to find $y(4.4)$ given that $5xy' + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$.

6. Solve the boundary value problem $(1+x^2)y'' + 4xy' + 2y = 2$ $y(0) = 0$, $y(1) = 0.5$, $h = \frac{1}{3}$ using finite difference method

UNIT- III

SOLUTIONS OF EQUATIONS AND EIGEN VALUE PROBLEMS

IMPORTANT FORMULAE

NEWTON RAPHSON METHOD

$$\phi(x_n) = x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0,1,2,\ldots, \quad |\phi'(x)| < 1$$

UNIT- V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

TAYLOR'S SERIES

$$y_{n+1} = y_n + \frac{h}{1!}y_n' + \frac{h^2}{2!}y_n'' + \frac{h^3}{3!}y_n''' + \cdots$$

EULER'S METHOD

$$y_{n+1} = y_n + h f(x_n, y_n)$$

MODIFIED EULER'S METHOD

$$y_{n+1} = y_n + \frac{1}{2}h\left[f(x_n, y_n) + f(x_n+h, y_n + h f(x_n, y_n))\right]$$

RUNGE-KUTTA METHODS OF FOURTH ORDER

$$k_1 = h f(x_1, y_1)$$

$$k_2 = h f\left(x + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$k_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$k_4 = h f(x_1 + h, y_1 + k_3)$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

PREDICTOR - CORRECTOR METHODS

PREDICTOR FORMULA :

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

CORRECTOR FORMULA

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right)$$

BOUNDARY VALUE PROBLEMS

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y''_i = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

UNIT- IV IMPORTANT FORMULAS.

① Lagrange Interpolation formula.

$$y=f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3 .$$

② Newton's divided difference interpolations.

$$f(x) = f(x_0) + (x-x_0).f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2)$$

$$+ \cdots + (x-x_0)(x-x_1)\cdots(x-x_{n-1}).f(x_0,x_1,\cdots x_n).$$

③. Newton's forward Interpolation formula for equal intervals.

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}.\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

Here $u = \frac{x-x_0}{h}$

④ Newton's Backward interpolation formula.

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

Here $v = \frac{x-x_n}{h}$.

⑤. Newton's forward difference formula.

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots\right]$$

(6). Newton's Backward difference formula.

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \cdots\right]$$

(7). Trapezoidal rule.

$$I = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$

(8). Simpson's 1/3 rule,

$$I = \frac{h}{3}\left[(y_0+y_n) + 2(y_1+y_3+y_5+\cdots) + 4(y_2+y_4+\cdots)\right]$$

(9). Simpson's 3/8 rule.

$$I = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+\cdots+y_{n-1}) + 2(y_3+y_6+\cdots)\right].$$

Reg. No. :

Question Paper Code : 77194

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Mechanical Engineering

MA 6452 — STATISTICS AND NUMERICAL METHODS

(Common to Automobile Engineering, Mechatronics Engineering)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

Use of statistical tables is permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the expected frequencies of 2 × 2 contingency table $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$?

2. Write down the formula of test statistic t to test the significance of difference between the means of large samples.

3. What do you understand by design of an experiment?

4. What are the basic principles of the design of experiments?

5. Perform four iterations of the Newton-Raphson method to find the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$.

6. Solve the equations $10x - y + 2z = 4$; $x + 10y - z = 3$; $2x + 3y + 20z = 7$ using the Gauss elimination method.

7. Given $f(2) = 5$, $f(2.5) = 5.5$ find the linear interpolating polynomial using Lagrange interpolation.

8. Construct the divided difference table for the data.

X:	0.5	1.5	3.0	5.0	6.5	8.0
$F(X)$:	1.625	5.875	31	131	282.125	521

9. Given $y' = \frac{y-x}{x+y}$ with initial condition $y=1$ at $x=0$ find y for $x=0.1$ by Euler's method.

10. Given the initial value problem $u' = -2tu^2$, $u(0)=1$ estimate $u(0.4)$ using modified Euler-Cauchy method.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Fit a binomial distribution for the following data and also test the goodness of fit. (8)

X :	0	1	2	3	4	5	6	Total
$F(X)$:	5	18	28	12	7	6	4	80

(ii) The mean value of a random sample of 60 items was found to be 145, with a standard deviation of 40. Find the 95% confidence limits for the population mean. What size of the sample is required to estimate the population mean within 5 of its actual value with 95% or more confidence, using the sample mean? (8)

Or

(b) (i) Test made on the breaking strength of 10 pieces of a metal gave the following results 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg. (8)

(ii) A group of 10 rats fed on diet A and another group of 8 rats fed on diet B recorded the following increase in weight :

Diet A :	5	6	8	1	12	4	3	9	6	10
Diet B :	2	3	6	8	10	1	2	8		

Show that the estimates of the population variance from the samples are not significantly different. (8)

12. (a) The following table shows the lives in hours of four brands of electric lamps brand.

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps. (16)

(b) Analyze the variance in the following Latin square of yields of paddy where A, B, C, D denote the different methods of cultivation.

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C122	D123	B121	A122

Examine whether the different methods of cultivation have given significantly different yields. (16)

13. (a) (i) Find the inverse of the coefficient matrix of the system $\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$ by the Gauss Joardan method, also solve the system. (10)

(ii) Find the smallest eigenvalue in magnitude of the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ using four iterations of the inverse power method. (6)

Or

(b) Solve the equations $5x + 2y + z = 12$; $x + 4y + 2z = 15$; $x + 2y + 5z = 20$ by

(i) Jacobi's method and

(ii) Gauss Seidel method. (16)

14. (a) (i) Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ by Simpson's rule and Trapezoidal rule with $h = 0.5$ and $k = 0.25$. (8)

(ii) The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface. (8)

X:	100	150	200	250	300	350	400
Y:	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when $x = 218$ ft and 410 ft.

Or

(b) (i) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using trapezoidal rule and Simpson's 1/3 rule and compare with its exact solution. (8)

(ii) Given that : (8)

X:	1	1.1	1.2	1.3	1.4	1.5	1.6
Y:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

find $\frac{dy}{dx}$ and y'' at $x=1.1$ and $x=1.6$.

15. (a) (i) The deflection of a beam is governed by the equations $y'''' + 81y = \phi(x)$ where $\phi(x)$ is given by the table :

X:	1/3	2/3	1
$\phi(x)$:	81	162	243

And the boundary conditions $y(0) = y'(0) = y''(1) = y'''(1) = 0$. Evaluate the deflection at the pivotal points of the beam using three subintervals. (10)

(ii) Apply Taylor's method to obtain approximate value of y at $x = 0.2$ for the differential equation $y' = 2y + 3e^x$, $y(0) = 0$. Compare the numerical solution with its exact solution. (6)

Or

(b) Using R.K fourth order method to find y at $x = 0.1, 0.2, 0.3$ given that $y' = xy + y^2$, $y(0) = 1$. Continue the solution at $x = 4$ using Milne's P-C method. (16)

Reg. No. :

Question Paper Code : 27331

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Mechanical Engineering

MA 6452 — STATISTICS AND NUMERICAL METHODS

(Common to Automobile Engineering, Mechatronics Engineering)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Use of statistical tables is permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is random sampling?

2. Write about F – test.

3. Write two advantages of completely randomized experimental design.

4. Is a 2 × 2 Latin square design possible? Why?

5. Compare Gauss elimination with Gauss seidel.

6. Obtain the iterative formula to find $\frac{1}{N}$ using Newton-Raphson method.

7. Give the Newton's backward difference table for

x :	0	1	2	3
y :	–1	–2	–1	2

8. Compare Trapezoidal rule with Simpson's $\frac{1}{3}$ rule.

9. If $y' = -y$, $y(0) = 1$ then find $y(.1)$ by Euler method.

10. What are single step and multistep methods? Give an example.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Test if the variances are significantly different for (8)

$X1$:	24	27	26	21	25	
$X2$:	27	30	32	36	28	23

(ii) The number of automobile accidents in a certain locality was 12, 8, 20, 2, 14 10, 15, 6, 9, 4. Are these frequencies in agreement with the belief that accident conditions were the same during this 10 week period. (8)

Or

(b) (i) A certain pesticide is packed into bags by a machine. A random sample of 10 bags is chosen and the contents of the bags is found to have the following weights (in kgs) 50, 49, 52, 44, 45, 48, 46, 45, 49 and 45. Test if the average quantity packed be taken as 50 kg. (8)

(ii) Given :

$\overline{X}_1 = 72$, $\overline{X}_2 = 74$

$s_1 = 8$, $s_2 = 6$

$n_1 = 32$, $n_2 = 36$

Test if the means are significant. (8)

12. (a) Given

Detergent	Engine 1	Engine 2	Engine 3
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Perform ANOVA and test at .05 level of significance whether these are differences in the detergents or in the engines. (16)

Or

(b) Find out the main effects and interactions in the following 2^2 – factorial experiment and write down the ANOVA table. (16)

	I	a	b	ab
Block	00	10	01	11
I	64	25	30	6
II	75	14	50	33
III	76	12	41	17
IV	75	33	25	10

13. (a) (i) Find the +ve root of $x^4 - x - 9 = 0$ using Newton method. (8)

(ii) Find the largest eigen value and its corresponding eigen vector using Power method, for

$$A = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}.$$ (8)

Or

(b) (i) Solve by Gauss Seidel : (8)

$$5x - 2y + z = -4$$
$$x + 6y - 2z = -1$$
$$3x + y + 5z = 13.$$

(ii) Find the inverse of $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ by Gauss Jordan method. (8)

14. (a) (i) Given :

x :	0	2	3	4	7	9
y :	4	26	58	112	466	922

Find $y\,(10)$, $y'(6)$ using Newton's divided difference formula. (8)

(ii) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x^2}$ using Simpson's $1/3$ rule by taking $h = 1/4$. (8)

Or

(b) (i) Evaluate $\int_1^2 \frac{dx}{1+x^2}$ taking $h = .2$ using trapezoidal rule. (8)

(ii) Given :

x :	140	150	160	170	180
y :	3.685	4.854	6.302	8.076	10.225

Find $y(175)$. (8)

15. (a) Using Runge-Kutta method of fourth order, solve $y' = \frac{y^2 - x^2}{x^2 + y^2}$ given $y(0) = 1$. Find y at $x = .2, .4, .6$. (16)

Or

(b) Compute $y(.5)$, $y(1)$ and $y(1.5)$ using Taylor's series for $y' = \frac{x+y}{2}$ with $y(0) = 2$ and hence find $y(2)$ using Milne's method. (16)

Table values for relevant problems

Table Values : (at 5% Los)

$|Z| = 1.96$, $t_8 = 2.31$, $t_9 = 2.26$, $F_{2,6} = 5.14$, $F_{3,6} = 4.76$, $F_{9,3} = 8.81$, $F_{3,9} = 3.86$, $F_{1,9} = 5.12$, $F_{4,5} = 5.19$, $F_{5,4} = 6.26$, $\psi_9^2 = 16.9$, $\psi_{10}^2 = 18.3$.

Reg. No. | | | | | | | | | | | | |

Question Paper Code : 57506

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fourth Semester

Mechanical Engineering

MA 6452 – STATISTICS AND NUMERICAL METHODS

(Common to Fourth Semester Automobile Engineering and Mechatronics Engineering)

[Also common to Fifth Semester for Mechanical Engineering (Sandwich)]

Time : Three Hours **Maximum : 100 Marks**

Use of statistical tables is permitted.

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. What are Type – I and Type – II errors ?

2. Give the formula for the χ^2 – test of independence for

a	b
c	d

3. State the principles of Design of Experiments.

4. Is 2 × 2 Latin square Design possible ? Why ?

5. Mention the order and condition for the convergence of Newton-Raphson method.

6. What is the procedure of Gauss-Jordan method ?

7. Specify the Newton's backward difference formulae for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

8. Write down the errors in Trapezoidal and Simpson's rules of numerical integration.

9. Find y(0.1) by Euler's method, if $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 0.1$

10. Give the central difference approximations for $y'(x)$, $y''(x)$.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with an SD of 6 and the boys made an average grade of 82 with an SD of 2. Test whether there is any difference between the performance of boys and girls.

(ii) Theory predicts the proportion of beans in the groups A, B, C, D as 9 : 3 : 3 : 1. In an experiment among beans the numbers in the groups were 882, 313, 287 and 118. Does the experiment support the theory ?

OR

(b) (i) 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test whether these two proportions are same.

(ii) The IQ's of 10 girls are respectively 120, 110, 70, 88, 101, 100, 83, 98, 95, 107. Test whether the population mean IQ is 100.

12. (a) Three varieties of coal were analysed by 4 chemists and the ash content is tabulated here. Perform an analysis of variance. **(16)**

		Chemists			
		A	B	C	D
Coal	I	8	5	5	7
	II	7	6	4	4
	III	3	6	5	4

OR

(b) The result of an RBD experiment on 3 blocks with 4 treatments A, B, C, D are tabulated here. Carry out an analysis of variance.

Blocks	Treatment effects			
I	A36	D35	C21	B36
II	D32	B29	A28	C31
III	B28	C29	D29	A26

13. (a) (i) Solve the following equations by Gauss elimination method :

$2x + y + 4z = 12,$

$8x - 3y + 2z = 20,$

$4x + 11y - z = 33,$

(ii) Using power method find the dominant eigen value of the matrix

$$\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

OR

(b) (i) If $A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$, find A^{-1} by Gauss-Jordan method.

(ii) Solve the following equations by Gauss-Seidel method

$x + y + 9z = 15,$

$x + 17y - 2z = 48,$

$30x - 2y + 3z = 75$

14. (a) (i) Interpolate y(12), if

x :	10	15	20	25	30	35
$y(x)$:	35	33	29	27	22	14

(ii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's (1/3) rule, dividing the range into four equal parts.

OR

(b) (i) Find y'(1), if

x :	−1	0	2	3
$y(x)$:	−8	3	1	12

(ii) Using Trapezoidal rule, evaluate $\int_1^2 \int_1^2 \frac{dx \cdot dy}{x + y}$ with h = K = 0.5.

15. (a) If $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$, find $y(0.1)$, $y(0.2)$ and $y(0.3)$ by Taylor series method. Hence find $y(0.4)$ by Milne's Predictor-Corrector method.

OR

(b) If $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$, find $y(0.2)$, $y(0.4)$, $y(0.6)$ by Runge-Kutta method. Hence find $y(0.8)$ by Milne's method.
