DOWNLOADED FROM STUCOR APP

MA3251 STATISTICS AND NUMERICAL METHODS L T P C 3 1 0 4

OBJECTIVES:

- This course aims at providing the necessary basic concepts of a few statistical and numerical methods and give procedures for solving numerically different kinds of problems occurring in engineering and technology.

UNIT I TESTING OF HYPOTHESIS 9+3

Large sample test based on Normal distribution for single mean and difference of means – Tests based on t^2 and F distributions for testing means and variances – Contingency table (Test for Independency) – Goodness of fit.

UNIT II DESIGN OF EXPERIMENTS 9+3

One way and two way classifications - Completely randomized design – Randomized block design –Latin square design - 2_2 factorial design.

UNIT III SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS 9+3

Newton Raphson method – Gauss elimination method – pivoting – Gauss Jordan methods – Iterative methods of Gauss Jacobi and Gauss Seidel – Matrix inversion by Gauss Jordan method – Eigen values of a matrix by power method.

UNIT IV INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION 9+3

Lagrange's and Newton's divided difference interpolations – Newton's forward and backward difference interpolation – Approximation of derivates using interpolation polynomials – Numerical single and double integrations using Trapezoidal and Simpson's 1/3 rules.

UNIT V NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS 9+3

Taylor's series method – Euler's method – Modified Euler's method – Fourth order Runge-Kutta method for solving first order equations – Milne's predictor corrector methods for solving first order equations – Finite difference methods for solving second order equations.

TOTAL (L:45+T:15): 60 PERIODS

OUTCOMES

- It helps the students to have a clear perception of the power of statistical and numerical techniques, ideas and would be able to demonstrate the applications of these techniques to problems drawn from industry, management and other engineering fields.

TEXT BOOKS

1. Johnson. R.A., and Gupta. C.B., "Miller and Freund's Probability and Statistics for Engineers",11th Edition, Pearson Education, , Asia, 2011.
2. Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition,Khanna Publishers, New Delhi, 2007.

REFERENCES

1. Walpole. R.E., Myers. R.H., Myers. S.L., and Ye. K., "Probability and Statistics for Engineers and Scientists", 8th Edition, Pearson Education, Asia, 2007.
2. Spiegel. M.R., Schiller. J., and Srinivasan. R.A., "Schaum's Outlines on Probability and Statistics", Tata McGraw Hill Edition, 2004.
3. Chapra. S.C., and Canale. R.P, "Numerical Methods for Engineers", 5th Edition, Tata McGrawHill, New Delhi, 2007.
4. Gerald. C.F., and Wheatley. P.O. "Applied Numerical Analysis" Pearson Education, Asia, NewDelhi, 2006.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

UNIT-I

SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

method of False position (or) Regula Falsi method (or) linear interpolation method.

If $f(x_1) f(a) < 0$, then x_2 lies between x_1 and a

$$x_2 = \frac{a f(x_1) - x_1 f(a)}{f(x_1) - f(a)}$$

① Find the positive root of $x^3 - 2x - 5 = 0$ by the Regula Falsi method.

Soln:

Let $f(x) = x^3 - 2x - 5 = 0$

There is only one poistive root by Descarte's rule of signs

$f(0) = -5 = -ve$

$f(1) = 1-2-5 = -6 = -ve$

$f(2) = 8-4-5 = -1 = -ve$

$f(3) = 27-6-5 = 16 = +ve$

Therefore the positive root lies between 2 and 3. It is closer to 2 also.

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2 f(3) - 3 f(2)}{f(3) - f(2)}$$

$$= \frac{2(16) - 3(-1)}{16 - (-1)}$$

$$= \frac{32 + 3}{17} = \frac{35}{17}$$

$= 2.0588$ [correct to 4 decimal places]

$f(x_1) = f(2.0588) = (2.0588)^3 - 2(2.0588) - 5$

$= 8.7265 - 4.1176 - 5$

$= -0.3911$

∴ The root lies between 2.0588 and 3

DOWNLOADED FROM STUCOR APP

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$x_2 = \frac{2.0588\, f(3) - 3\, f(2.0588)}{f(3) - f(2.0588)}$$

$$= \frac{2.0588(16) - 3(-0.3911)}{16 - (-0.3911)}$$

$$= \frac{32.9408 + 1.1733}{16.3911} = \frac{34.1141}{16.3911}$$

$$= 2.0813$$

$$f(x_2) = f(2.0813) = (2.0813)^3 - 2(2.0813) - 5$$

$$= 9.0158 - 4.1626 - 5$$

$$= -0.1468$$

∴ The root lies between 2.0813 and 3

$$x_3 = \frac{2.0813\, f(3) - 3\, f(2.0813)}{f(3) - f(2.0813)}$$

$$= \frac{2.0813(16) - 3(-0.1468)}{16 - (-0.1468)}$$

$$= \frac{33.3008 + 0.4404}{16.1468} = \frac{33.7412}{16.1468}$$

$$= 2.08965$$

$$= 2.0897 \text{ (four decimal places)}$$

$$f(x_3) = f(2.0897) = (2.0897)^3 - 2(2.0897) - 5$$

$$= -0.054 = -ve$$

∴ The root lies between 2.0897 and 3

$$x_4 = \frac{(2.0897)\, f(3) - 3\, f(2.0897)}{f(3) - f(2.0897)}$$

$$= \frac{(2.0897)(16) - 3(-0.054)}{16 - (-0.054)}$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= \frac{33.4352 + 0.162}{16.054}$$

$$= \frac{33.5972}{16.054} = 2.0928$$

$$f(x_4) = f(2.0928) = (2.0928)^3 - 2(2.0928) - 5$$

$$= 9.1661 - 4.1856 - 5$$

$$= -0.0195 = -ve$$

∴ The root lies between 2.0928 and 3

$$x_5 = \frac{2.0928\, f(3) - 3f(2.0928)}{f(3) - f(2.0928)}$$

$$= \frac{(2.0928)(16) - 3(-0.0195)}{16 - (-0.0195)}$$

$$= \frac{33.4848 + 0.0585}{16.0195}$$

$$= \frac{33.5433}{16.995}$$

$$= 2.0939$$

$$f(x_5) = f(2.0939) = (2.0939)^3 - 2(2.0939) - 5$$

$$= 9.1805 - 4.1878 - 5$$

$$= -0.0073$$

$$= -ve$$

∴ The root lies between 2.0939 and 3

$$x_6 = \frac{(2.0939)f(3) - 3f(2.0939)}{f(3) - f(2.0939)}$$

$$= \frac{(2.0939)(16) - 3(-0.0073)}{16 - (-0.0073)}$$

$$= \frac{33.5024 + 0.0219}{16.0073} = \frac{33.5243}{16.0073} = 2.0943$$

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$f(x_6) = f(2.0943) = (2.0943)^3 - 2(2.0943) - 5$

$= 9.1868 - 4.1886 - 5$

$= -0.0028$

$= -ve$

∴ The root lies between 2.0943 and 3

$$x_7 = \frac{(2.0943)f(3) - 3f(2.0943)}{f(3) - f(2.0943)}$$

$$= \frac{(2.0943)(16) - 3(-0.0028)}{16 - (-0.0028)}$$

$$= \frac{33.5088 + 0.0084}{16.0028}$$

$$= \frac{33.5172}{16.0028} = 2.0945$$

$f(x_7) = f(2.0945) = (2.0945)^3 - 2(2.0945) - 5$

$= 9.1884 - 4.189 - 5$

$= -0.0006 = -ve$

∴ The root lies between 2.0945 and 3

$$x_8 = \frac{2.0945\ f(3) - 3f(2.0945)}{f(3) - f(2.0945)}$$

$$= \frac{(2.0945)(16) - 3(-0.0006)}{16 - (-0.0006)}$$

$$= \frac{33.512 + 0.0018}{16.0006}$$

$$= \frac{33.5138}{16.0006}$$

$= 2.0945$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

We observe that $x_7 = x_8 = 2.0945$ correct to 4 places of decimals.

Hence the required root correct to four places of decimals is 2.0945

The results of the complete working are tabulated below.

Iteration(r)	a	b	x_r	sign of $f(x_r)$
1	2	3	2.0588	-0.3911
2	2.0588	3	2.0813	-0.1468
3	2.0813	3	2.0897	-0.054
4	2.0897	3	2.0928	-0.0195
5	2.0928	3	2.0939	-0.0073
6	2.0939	3	2.0943	-0.0028
7	2.0943	3	2.0945	-0.0006
8	2.0945	3	2.0945	

$$\text{Formula } x_r = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

we observe that $x_7 = x_8 = 2.0945$

Hence the required root is 2.0945

STUCOR APP

② using method of false position find a root of the equation $x^3 - 3x - 5 = 0$

Soln:

Given $f(x) = x^3 - 3x - 5$

$f(0) = 0 - 0 - 5 = -5 = -ve$

$f(1) = 1 - 3 - 5 = 1 - 8 = -7 = -ve$

$f(2) = 8 - 6 - 5 = 8 - 11 = -3 = -ve$

$f(3) = 27 - 9 - 5 = 27 - 14 = 13 = +ve$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$\therefore$ one root lies between 2 and 3

let $a = 2, \quad b = 3$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2(13) - 3(-3)}{13 - (-3)}$$

$$= \frac{26+9}{16} = \frac{35}{16} = 2.1875$$

$$f(x_1) = f(2.1875) = (2.1875)^3 - 3(2.1875) - 5$$

$$= 10.4675 - 6.5625 - 5$$

$$= -1.095$$

$\therefore$ one root lies between 2.1875 and 3

$$x_2 = \frac{(2.1875) f(3) - 3 f(2.1875)}{f(3) - f(2.1875)}$$

$$= \frac{(2.1875)(13) - 3(-1.095)}{13 - (-1.095)}$$

$$= \frac{28.4375 + 3.285}{14.095}$$

$$= \frac{31.7225}{14.095} = 2.2506$$

$$f(x_2) = f(2.2506) = (2.2506)^3 - 3(2.2506) - 5$$

$$= 11.3997 - 6.7518 - 5$$

$$= -0.3521 = -ve$$

$\therefore$ The root lies between 2.2506 and 3

$$x_3 = \frac{2.2506 f(3) - 3 f(2.2506)}{f(3) - f(2.2506)}$$

$$= \frac{(2.2506)(13) - 3(-0.3521)}{13 - (-0.3521)}$$

$$= \frac{29.2578 + 1.0563}{13.3521} = \frac{30.3141}{13.3521} = 2.2704$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$f(x_3) = f(2.2704) = (2.2704)^3 - 3(2.2704) - 5$$

$$= 11.7033 - 6.8112 - 5$$

$$= -0.1079 = -ve$$

∴ The root lies between 2.2704 and 3

$$x_4 = \frac{2.2704\, f(3) - 3\, f(2.2704)}{f(3) - f(2.2704)}$$

$$= \frac{(2.2704)(13) - 3(-0.1079)}{13 - (-0.1079)}$$

$$= \frac{29.5152 + 0.3237}{13.1079} = \frac{29.8389}{13.1079} = 2.2764$$

$$f(x_4) = f(2.2764) = (2.2764)^3 - 3(2.2764) - 5$$

$$= 11.7963 - 6.8292 - 5$$

$$= -0.0329 = -ve$$

∴ The root lies between 2.2764 and 3

$$x_5 = \frac{(2.2764)\, f(3) - 3\, f(2.2764)}{f(3) - f(2.2764)}$$

$$= \frac{(2.2764)(13) - 3(-0.03929)}{13 - (-0.0329)}$$

$$= \frac{25.5932 + 0.0987}{13.0329} = \frac{29.6919}{13.0329} = 2.2782$$

$$f(x_5) = f(2.2782) = (2.2782)^3 - 3(2.2782) - 5$$

$$= 11.8243 - 6.8346 - 5$$

$$= -0.0103 = -ve$$

∴ The root lies between 2.2782 and 3

$$x_6 = \frac{(2.2782)\, f(3) - 3 f(2.2782)}{f(3) - f(2.2782)}$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= \frac{(2.2782)(13) - 3(-0.0103)}{13 - (-0.0103)}$$

$$= \frac{29.6166 + 0.0309}{13.0103} = \frac{29.6475}{13.0103} = 2.2788$$

$$f(x_6) = f(2.2788) = (2.2788)^3 - 3(2.2788) - 5$$

$$= 11.8336 - 6.8304 - 5$$

$$= -0.0028 = -ve$$

∴ The root lies between 2.2788 and 3

$$x_7 = \frac{2.2788\, f(3) - 3 f(2.2788)}{f(3) - f(2.2788)}$$

$$= \frac{(2.2788)(13) - 3(-0.0028)}{13 - (-0.0028)}$$

$$= \frac{29.6244 + 0.0084}{13.0028}$$

$$= \frac{29.6328}{13.0028}$$

$$= 2.2790$$

$$f(x_7) = f(2.2790) = (2.2790)^3 - 3(2.2790) - 5$$

$$= 11.8367 - 6.837 - 5$$

$$= -0.0003 = -ve$$

∴ The root lies between 2.279 and 3

$$x_8 = \frac{2.279\, f(3) - 3 f(2.279)}{f(3) - f(2.279)}$$

$$= \frac{(2.279)(13) - 3(-0.0003)}{13 - (-0.0003)}$$

$$= \frac{29.627 + 0.0009}{13.0003}$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= \frac{29.6279}{13.0003}$$

$$= 2.2790$$

we observe that $x_7 = x_8 = 2.2790$ correct to four places of decimals.

Hence the required root is 2.2790.

TABLE

<table>
<tr><th colspan="5">$f(x) = x^3 - 3x - 5$ Formula $x_r = \frac{a f(b) - b f(a)}{f(b) - f(a)}$</th></tr>
<tr><th>Iteration(r)</th><th>a</th><th>b</th><th>x_r</th><th>$f(x_r)$</th></tr>
<tr><td>1</td><td>2</td><td>3</td><td>2.1875</td><td>-1.095</td></tr>
<tr><td>2</td><td>2.1875</td><td>3</td><td>2.2506</td><td>-0.3521</td></tr>
<tr><td>3</td><td>2.2506</td><td>3</td><td>2.2704</td><td>-0.1079</td></tr>
<tr><td>4</td><td>2.2704</td><td>3</td><td>2.2764</td><td>-0.0329</td></tr>
<tr><td>5</td><td>2.2764</td><td>3</td><td>2.2782</td><td>-0.0103</td></tr>
<tr><td>6</td><td>2.2782</td><td>3</td><td>2.2788</td><td>-0.0028</td></tr>
<tr><td>7</td><td>2.2788</td><td>3</td><td>2.2790</td><td>-0.0003</td></tr>
<tr><td>8</td><td>2.2790</td><td>3</td><td>2.2790</td><td></td></tr>
</table>

Hence the required root is 2.2790

③ Find an approximate root of $x \log_{10} x - 1.2 = 0$ by Regula Falsi method.

Soln:- Let $f(x) = x \log_{10}^{x} - 1.2$

$$f(1) = 0 - 1.2 = -1.2 = -ve$$

$$f(2) = 2(0.30103) - 1.2 = -0.5979 = -ve$$

$$f(3) = 3\cdot(0.47712) = 0.2314 = +ve$$

Hence a root lies between 2 and 3

$$x_1 = \frac{2 f(3) - 3 f(2)}{f(3) - f(2)}$$

$$= \frac{2(0.2314) - 3(-0.5979)}{0.2314 - (-0.5979)} = \frac{0.4628 + 1.7937}{0.8293}$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= \frac{2.2565}{0.8293} = 2.7210$$

$$f(x_1) = f(2.7210) = (2.7210)\log_{10} 2.7210 - 1.2$$

$$= -0.0171 = -ve$$

Therefore the root lies between 2.7210 and 3

$$x_2 = \frac{(2.7210)\, f(3) - 3f(2.7210)}{f(3) - f(2.7210)}$$

$$= \frac{0.68094}{0.2485} = 2.7402$$

$$f(x_2) = f(2.7402) = (2.7402)\log_{10} 2.7402 - 1.2$$

$$= -0.0004 = -ve$$

$\therefore$ The root lies between 2.7402 and 3

$$x_3 = \frac{2.7402\, f(3) - 3f(2.7402)}{f(3) - f(2.7402)}$$

$$= \frac{(2.7402)(0.2314) - 3(-0.0004)}{(0.2314) - (-0.0004)}$$

$$= \frac{(2.7407)(0.2314) - 3(-0.0004)}{(0.2314) - (-0.0004)}$$

$$= 2.7407$$

$$f(x_3) = f(2.7407) = 2.7407 \log_{10} 2.7407 - 1.2$$

$$= 0.0001 = +ve$$

$\therefore$ The root lies between 2.7402 and 2.7407

$$x_4 = \frac{2.7402\, f(2.7407) - (2.7407)\, f(2.7402)}{f(2.7407) - f(2.7402)}$$

$$= \frac{(2.7402)(0.0001) - (2.7407)(-0.0004)}{(0.0001) - (-0.0004)}$$

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= \frac{(2.7402)(0.0001) + (2.7407)(0.0004)}{0.0001 + 0.0004}$$

$$= 2.7406$$

$$f(x_4) = f(2.7406) = 2.7406 \log_{10} 2.7406 - 1.2$$

$$= -0.0004 = -ve$$

$$x_5 = \frac{2.7406\, f(2.7407) - 2.7407\, f(2.7406)}{f(2.7407) - f(2.7406)}$$

$$= \frac{(2.7406)(0.0001) - (2.7407)(-0.00004)}{0.0001 - (-0.00004)}$$

$$= \frac{(2.7406)(0.0001) + (2.7407)(0.00004)}{0.0001 + 0.00004} = 2.7406$$

we observe that $x_4 = x_5 = 2.7406$ correct to four places of decimals

Hence the required root is 2.7406

$f(x) = x \log_{10} x - 1.2$		Formula $x_r = \frac{a f(b) - b f(a)}{f(b) - f(a)}$		
Iteration(r)	a	b	x_r	$f(x_r)$
1	2	3	2.7210	−0.0171
2	2.7210	3	2.7402	−0.0004
3	2.7402	3	2.7407	0.0001
4	2.7402	2.7407	2.7406	−0.00004
5	2.7406	2.7407	2.7406	−

we find that f(2.7406) is approaching zero

Hence the required root is 2.7406.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

NEWTON'S METHOD [Newton - Raphson method]

Formula:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2 \ldots$$

① Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton - Raphson method.

Soln:-

Let $f(x) = x^4 - x - 10$

$f'(x) = 4x^3 - 1$

$f(1) = 1-1-10 = -10 = -ve$

$f(2) = 2^4 - 2 - 10 = 16-2-10 = 4 \quad +ve$

∴ a root lies between 1 and 2.

Take $x_0 = 2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)}$$

$$= 2 - \left[\frac{2^4 - 2 - 10}{4(2)^3 - 1}\right]$$

$$= 2 - \left[\frac{4}{31}\right]$$

$= 1.8709$

$= 1.871$ [correct to three decimal places]

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.871 - \frac{f(1.871)}{f'(1.871)}$$

$$= 1.871 - \frac{(1.871)^4 - 1.871 - 10}{4(1.871)^3 - 1}$$

$$= 1.871 - \frac{0.3835}{25.199} = 1.856$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.856 - \frac{f(1.856)}{f'(1.856)}$$

$$= 1.856 - \left[\frac{(1.856)^4 - 1.856 - 10}{4(1.856)^3 - 1}\right]$$

$$= 1.856 - \frac{0.010}{24.574}$$

$$= 1.856$$

The better approximate root is 1.856

② using Newton's iterative method find the root between 0 and 1 of $x^3 = 6x - 4$ correct to two decimal places.

Soln Let $f(x) = x^3 - 6x + 4$

$f'(x) = 3x^2 - 6$

$f(0) = 4 = +ve$

$f(1) = 1 - 6 + 4 = -1 = -ve$

∴ a root lies between 0 and 1

$|f(0)| > |f(1)|$

∴ This root is nearer to 1

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let $x_0 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{f(1)}{f'(1)} = 1 - \left[\frac{(1)^3 - 6(1) + 4}{3(1)^2 - 6}\right]$$

$$= 1 - \frac{(-1)}{-3} = 1 - 1/3 = 0.666$$

$= 0.67$ [correct to two decimal places]

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.67 - \frac{f(0.67)}{f'(0.67)}$$

$$= 0.67 - \left[\frac{(0.67)^3 - 6(0.67) + 4}{3(0.67)^2 - 6}\right]$$

$$= 0.67 - \frac{0.28}{-4.65}$$

$$= 0.67 + \frac{0.28}{4.65}$$

$$= 0.73$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.73 - \frac{f(0.73)}{f'(0.73)}$$

$$= 0.73 - \left[\frac{(0.73)^3 - 6(0.73) + 4}{3(0.73)^2 - 6}\right]$$

$$= 0.73 - \left[\frac{0.009}{-4.4013}\right]$$

$$= 0.73 + \frac{0.009}{4.4013}$$

$$= 0.7320$$

$= 0.73$ [correct to two decimal places]

Here $x_2 = x_3 = 0.73$

$\therefore$ The root is 0.73 correct to two decimal places

③ Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 6 decimal places.

Soln

let $f(x) = 3x - \cos x - 1$

$f(0) = 0 - 1 - 1 = -2 = -ve$

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$f(1) = 3 - \cos 1 - 1 = 2 - \cos 1 = 1.459698 = +ve$

$\therefore$ a root lies between 0 and 1

$|f(0)| > |f(1)|$

Hence the root is nearer to 1.

Let $x_0 = 0.6$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.6 - \frac{f(0.6)}{f'(0.6)}$$

$$= 0.6 - \left[\frac{3(0.6) - \cos(0.6) - 1}{3 + \sin(0.6)}\right]$$

$$= 0.6 - (-0.007101)$$

$$= 0.607108$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.607108 - \left[\frac{3(0.607108) - \cos(0.607108) - 1}{3 + \sin(0.607108)}\right]$$

$$= 0.607108 - (0.000006)$$

$$= 0.607102$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.607102 - \frac{f(0.607102)}{f'(0.607102)}$$

$$= 0.607102 - \left[\frac{3(0.607102) - \cos(0.607102) - 1}{3 + \sin(0.607102)}\right]$$

$$= 0.607102 - 0.0000004$$

$$= 0.607102$$

Here $x_2 = x_3 = 0.607102$

$\therefore$ The root is 0.607102 correct to six decimals

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

① Find by NR method, the root of $x \log_{10} x = 12.34$ start with $x_0 = 10$.

Soln:

Let $f(x) = x \log_{10}^{x} - 12.34$

$$f^{1}(x) = x \, \tfrac{1}{x} \log_{10}^{e} + \log_{10}^{x}$$

$$= \log_{10}^{e} + \log_{10}^{x}$$

Given $x_0 = 10$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f^{1}(x_n)}}$$

$$x_1 = x_0 - \frac{f(x_0)}{f^{1}(x_0)}$$

$$= 10 - \frac{f(10)}{f^{1}(10)}$$

$$= 10 - \left[\frac{10 \log_{10}^{10} - 12.34}{\log_{10}^{e} + \log_{10}^{10}}\right]$$

$$= 10 - \left[\frac{-2.34}{1.4343}\right]$$

$$= 10 + \frac{2.34}{1.4343}$$

$$= 11.6315$$

$$x_2 = x_1 - \frac{f(x_1)}{f^{1}(x_1)}$$

$$= 11.6315 - \frac{f(11.6315)}{f^{1}(11.6315)}$$

$$= 11.6315 - \left[\frac{11.6315 \log_{10}^{11.6315} - 12.34}{\log_{10}^{e} + \log_{10}^{11.6315}}\right]$$

$$= 11.6315 - \frac{0.0549}{1.5}$$

$$= 11.5949$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 11.5949 - \frac{f(11.5949)}{f'(11.5949)}$$

$$= 11.5949 - \left[\frac{11.5949 \log_{10} 11.5949 - 12.34}{\log_{10} e + \log_{10} 11.5949}\right]$$

$$= 11.5949 - \frac{0.00006}{1.4986}$$

$$= 11.5949$$

From x_2 and x_3 we find out the root is 11.5949.

FIXED POINT ITERATION $x = g(x)$ method.

① Solve the equation $x^2 - 2x - 3 = 0$ for the positive root by iteration method

Soln:

Let $f(x) = x^2 - 2x - 3 = 0$

$f(x)$ is easy to factor to show roots at $x = -1$ and $x = 3$

Rearrange equation (1)

$$x = g(x) = \sqrt{2x+3}$$

Let $x_0 = 4$

$$x_1 = g(x_0) = \sqrt{2x_0+3} = \sqrt{8+3} = \sqrt{11} = 3.31662$$

$$x_2 = g(x_1) = \sqrt{2x_1+3} = \sqrt{9.63325} = 3.10375$$

$$x_3 = g(x_2) = \sqrt{2x_2+3} = \sqrt{9.20750} = \sqrt{9.20750} = 3.03439$$

$$x_4 = g(x_3) = \sqrt{2x_3+3} = \sqrt{9.06877} = 3.01144$$

$$x_5 = g(x_4) = \sqrt{2x_4+3} = \sqrt{9.02288} = 3.00381$$

$$x_6 = g(x_6) = \sqrt{2x_6+3} = 3.00127$$

$$x_7 = g(x_7) = \sqrt{2x_7+3} = 3.00042$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$x_8 = g(x_7) = \sqrt{2x_7+3} = 3.00014$

$x_9 = g(x_8) = \sqrt{2x_8+3} = 3.00005$

$x_{10} = g(x_9) = \sqrt{2x_9+3} = 3.00002$

$x_{11} = g(x_{10}) = \sqrt{2x_{10}+3} = 3.00001$

$x_{12} = g(x_{11}) = \sqrt{2x_{11}+3} = 3.00000$

$x_{13} = g(x_{12}) = \sqrt{2x_{12}+3} = 3.00000$

Here $x_{12} = x_{13} = 3$ [correct 5 decimal places]

Hence the root is 3.

② Find a real root of the equation $x^3+x^2-100=0$

Soln:

Let $f(x) = x^3+x^2-100=0$

$f(0) = -100 = -ve$

$f(1) = 1+1-100 = -98 = -ve$

$f(2) = 8+4-100 = -88 = -ve$

$f(3) = 27+9-100 = -64 = -ve$

$f(4) = 64+16-100 = -20 = -ve$

$f(5) = 125+25-100 = 50 = +ve$

So there is a real root between 4 and 5

The given equation can be written as

$$x^2(x+1) = 100$$

$$x = \frac{10}{\sqrt{x+1}} = g(x)$$

$$g'(x) = \frac{10\left[-\frac{1}{2}\right]}{(x+1)^{3/2}} = \frac{-5}{(x+1)^{3/2}}$$

$$|g'(x)| = \frac{5}{(x+1)^{3/2}}$$

$$|g'(4)| = \frac{5}{5^{3/2}} < 1$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$|g'(5)| = \frac{5}{6^{3/2}} < 1$$

$\therefore$ $|g'(x)|$ is less than 1 in the interval $(4, 5)$

So the method can be applied

let $x_0 = 4.2$

$$x_1 = g(x_0) = \frac{10}{\sqrt{x_0+1}} = \frac{10}{\sqrt{4.2+1}} = 4.38529$$

$$x_2 = g(x_1) = \frac{10}{\sqrt{x_1+1}} = \frac{10}{\sqrt{4.38529+1}} = 4.30919$$

$$x_3 = g(x_2) = \frac{10}{\sqrt{x_2+1}} = \frac{10}{\sqrt{4.30919+1}} = 4.33996$$

$$x_4 = g(x_3) = \frac{10}{\sqrt{x_3+1}} = \frac{10}{\sqrt{4.33996+1}} = 4.32744$$

$$x_5 = g(x_4) = \frac{10}{\sqrt{x_4+1}} = 4.33252$$

$$x_6 = g(x_5) = \frac{10}{\sqrt{x_5+1}} = 4.33046$$

$$x_7 = g(x_6) = \frac{10}{\sqrt{x_6+1}} = 4.33129$$

$$x_8 = g(x_7) = \frac{10}{\sqrt{x_7+1}} = 4.33096$$

$$x_9 = g(x_8) = \frac{10}{\sqrt{x_8+1}} = 4.33109$$

$$x_{10} = g(x_9) = \frac{10}{\sqrt{x_9+1}} = 4.33104$$

$$x_{11} = g(x_{10}) = \frac{10}{\sqrt{x_{10}+1}} = 4.33106$$

$$x_{12} = g(x_{11}) = \frac{10}{\sqrt{x_{11}+1}} = 4.33105$$

$$x_{13} = g(x_{12}) = \frac{10}{\sqrt{x_{12}+1}} = 4.33105$$

Here $x_{12} = x_{13} = 4.33105$ correct to 5 decimal places.

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Gaussian Elimination method & Gauss-Jordan method

① Solve the system of equations by (i) Gauss elimination method (ii) Gauss-Jordan method.

$$10x - 2y + 3z = 23$$
$$2x + 10y - 5z = -33$$
$$3x - 4y + 10z = 41$$

Soln:

(i) Gauss elimination method.

The given system is equivalent to

$$\begin{bmatrix} 10 & -2 & 3 \\ 2 & 10 & -5 \\ 3 & -4 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23 \\ -33 \\ 41 \end{bmatrix}$$

Here $[A, B] = \begin{bmatrix} 10 & -2 & 3 & 23 \\ 2 & 10 & -5 & -33 \\ 3 & -4 & 10 & 41 \end{bmatrix}$

Now, we will make the matrix A as a upper triangular

Fix the first row, change 2 and 3 row with row 1

$$[A, B] \sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & -32 & 91 & 341 \end{bmatrix} \quad \begin{matrix} R_2 \leftrightarrow 5R_2 - R_1 \\ R_3 \leftrightarrow 10R_3 - 3R_1 \end{matrix}$$

Fix 1 and 2 row, change 3 row with 2nd row

$$\sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & 0 & 3780 & 11340 \end{bmatrix} \quad R_3 \leftrightarrow 52R_3 + 34R_2$$

This is an upper triangular matrix

From (1) we get [by back substitution]

$$3780z = 11340$$

$$\boxed{z = 3}$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$52y - 28z = -188$$

$$52y - 28(3) = -188$$

$$y = -2$$

$$10x - 2y + 3z = 23$$

$$10x - 2(-2) + 3(3) = 23$$

$$10x + 4 + 9 = 23$$

$$10x + 13 = 23$$

$$10x = 23 - 13$$

$$10x = 10$$

$$x = 1$$

Hence the solution is $x = 1, \ y = -2, \ z = 3$

(ii) Gauss - Jordan method.

Take the equation (1)

$$(A, B) \sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & 0 & 3780 & 11340 \end{bmatrix} \quad \begin{matrix} R_1 \leftrightarrow 1260R_1 - R_3 \\ R_2 \leftrightarrow 135R_2 + R_3 \end{matrix}$$

Now we will make the matrix A a diagonal matrix

Fix the third row and change 2nd row and first row

$$\sim \begin{bmatrix} 12600 & -2520 & 0 & 17640 \\ 0 & 7020 & 0 & -14040 \\ 0 & 0 & 3780 & 11340 \end{bmatrix}$$

Fix the 2 and 3 row change 1 row with 2nd row

$$\sim \begin{bmatrix} 88452000 & 0 & 0 & 88452000 \\ 0 & 7020 & 0 & -14040 \\ 0 & 0 & 3780 & 11340 \end{bmatrix} \quad R_1 \leftrightarrow 7020R_1 + 2520R_2$$

(ie) $$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$ $\therefore x = 1, \ y = -2, \ z = 3$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

② Solve the system of equations by Gauss - elimination method.

$$5x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 + 7x_2 + x_3 + x_4 = 12$$

$$x_1 + x_2 + 6x_3 + x_4 = -5$$

$$x_1 + x_2 + x_3 + 4x_4 = -6$$

Soln:

The given system is equivalent to

$$\begin{bmatrix} 5 & 1 & 1 & 1 \\ 1 & 7 & 1 & 1 \\ 1 & 1 & 6 & 1 \\ 1 & 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -5 \\ -6 \end{bmatrix}$$

$$[A, B] = \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 34 & 4 & 4 & 56 \\ 0 & 4 & 29 & 4 & -29 \\ 0 & 4 & 4 & 19 & -34 \end{array}\right] \quad \begin{array}{l} R_2 \leftrightarrow 5R_2 - R_1 \\ R_3 \leftrightarrow 5R_3 - R_1 \\ R_2 \leftrightarrow 5R_4 - R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 34 & 4 & 4 & 56 \\ 0 & 0 & 970 & 120 & -1210 \\ 0 & 0 & 120 & 630 & -1380 \end{array}\right] \quad \begin{array}{l} R_3 \rightarrow 34R_3 - 4R_2 \\ R_4 \rightarrow 34R_4 - 4R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 17 & 2 & 2 & 28 \\ 0 & 0 & 97 & 12 & -121 \\ 0 & 0 & 12 & 63 & -138 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow \frac{R_2}{2} \\ R_3 \rightarrow \frac{R_3}{10} \\ R_4 \rightarrow \frac{R_4}{10} \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 17 & 2 & 2 & 28 \\ 0 & 0 & 97 & 12 & -121 \\ 0 & 0 & 0 & 5967 & -11934 \end{array}\right] \quad R_4 \rightarrow 97R_4 - 12R_3$$

$$5967\, x_4 = -11934$$

$$x_4 = -2$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$97x_3 + 12x_4 = -121$

$97x_3 + 12(-2) = -121$

$97x_3 - 24 = -121$

$97x_3 = -121 + 24$

$97x_3 = -97$

$\boxed{x_3 = -1}$

$17x_2 + 2x_3 + 2x_4 = 28$

$17x_2 + 2(-1) + 2(-2) = 28$

$17x_2 - 2 - 4 = 28$

$17x_2 - 6 = 28$

$17x_2 = 28 + 6$

$17x_2 = 34$

$\boxed{x_2 = 2}$

$5x_1 + x_2 + x_3 + x_4 = 4$

$5x_1 + 2 + (-1) + (-2) = 4$

$5x_1 - 1 = 4$

$5x_1 = 5$

$\boxed{x_1 = 1}$

Hence the solution is $x_1 = 1, x_2 = 2, x_3 = -1, x_4 = -2$

③ using the Gauss-Jordan method solve the following equations

$10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 5z = 7$

Soln:

Interchanging the first and the last equation then

$$[A, B] = \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{array}\right]$$

Fix the pivot element row and make the other elements zero in the pivot element column.

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{array}\right] \quad \begin{array}{l} R_2 \leftrightarrow R_2 - 2R_1 \\ R_3 \leftrightarrow R_3 - 10R_1 \end{array}$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 1 & -1.125 & -0.125 \\ 0 & -9 & -49 & -58 \end{array}\right] \quad R_2 \leftrightarrow \frac{R_2}{8}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 6.125 & 7.125 \\ 0 & 1 & -1.125 & -0.125 \\ 0 & 0 & -59.125 & -59.125 \end{array}\right] \quad \begin{array}{l} R_1 \leftrightarrow R_1 - R_2 \\ R_3 \leftrightarrow R_3 + 9R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 6.125 & 7.125 \\ 0 & 1 & -1.125 & -0.125 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad R_3 \leftrightarrow \frac{R_3}{-59.125}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \leftrightarrow R_1 - 6.125R_3 \\ R_1 \leftrightarrow R_2 + 1.125R_3 \end{array}$$

$\therefore x_1 = 1, \ y = 1, \ z = 1$

ITERATIVE METHODS

(a) Gauss Jacobi method

(b) Gauss-seidal method.

① solve the following system of equations by Gauss-Jacobi method and Gauss-seidal method.

$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

Soln: As the coefficient matrix is not diagonally domiant we rewrite the equations.

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

diagonally dominant

$\left[\therefore |a_{11}| > |a_{12}| + |a_{13}|; \ |c_{33}| > |c_{31}| + |c_{32}| \right.$

$|b_{22}| > |b_{21}| + |b_{23}|$

Since the diagonal elements are dominant in the coefficient matrix we write x, y, z as follows:

$$x = \frac{1}{27}[85 - 6y + z]$$
$$y = \frac{1}{15}[72 - 6x - 2z]$$
$$z = \frac{1}{54}[110 - x - y]$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

(ii) Gauss Jacobi method

Let the initial values be $x=0,\ y=0,\ z=0$

First iteration:-

$$x^{(1)} = \frac{1}{27}[85] = 3.148$$

$$y^{(1)} = \frac{1}{15}[72] = 4.8$$

$$z^{(1)} = \frac{1}{54}[110] = 2.037$$

Second iteration:-

$$x^{(2)} = \frac{1}{27}\left[85 - 6y^{(1)} + z^{(1)}\right] = \frac{1}{27}\left[85 - 6.(4.8) + (2.037)\right] = 2.157$$

$$y^{(2)} = \frac{1}{15}\left[72 - 6x^{(1)} - 2z^{(1)}\right] = \frac{1}{15}\left[72 - 6(3.148) - 2(2.037)\right] = 3.269$$

$$z^{(2)} = \frac{1}{54}\left[110 - x^{(1)} - y^{(1)}\right] = \frac{1}{54}\left[110 - 3.148 - 4.8\right] = 1.890$$

Third iteration:-

$$x^{(3)} = \frac{1}{27}\left[85 - 6y^{(2)} + z^{(2)}\right] = \frac{1}{27}\left[85 - 6(3.269) + 1.890\right] = 2.492$$

$$y^{(3)} = \frac{1}{15}\left[72 - 6x^{(2)} + 2z^{(2)}\right] = \frac{1}{15}\left[72 - 6(2.157) - 2(1.890)\right] = 3.685$$

$$z^{(3)} = \frac{1}{54}\left[110 - x^{(2)} - y^{(2)}\right] = \frac{1}{54}\left[110 - 2.157 - 3.269\right] = 1.937$$

Fourth iteration:-

$$x^{(4)} = \frac{1}{27}\left[85 - 6y^{(3)} + z^{(3)}\right] = \frac{1}{27}\left[85 - 6(3.685) + 1.937\right] = 2.401$$

$$y^{(4)} = \frac{1}{15}\left[72 - 6x^{(3)} - 2z^{(3)}\right] = \frac{1}{15}\left[72 - 6(2.492) - 2(1.937)\right] = 3.545$$

$$z^{(4)} = \frac{1}{54}\left[110 - x^{(3)} - y^{(3)}\right] = \frac{1}{54}\left[110 - 2.492 - 3.685\right] = 1.923$$

Fifth iteration:-

$$x^{(5)} = \frac{1}{27}\left[85 - 6y^{(4)} + z^{(4)}\right] = \frac{1}{27}\left[85 - 6(3.545) + 1.923\right] = 2.432$$

$$y^{(5)} = \frac{1}{15}\left[72 - 6x^{(4)} - 2z^{(4)}\right] = \frac{1}{15}\left[72 - 6(2.401) - 2(1.923)\right] = 3.583$$

$$z^{(5)} = \frac{1}{54}\left[110 - x^{(4)} - y^{(4)}\right] = \frac{1}{54}\left[110 - 2.401 - 3.645\right] = 1.927$$

Sixth Iteration

$$x^{(6)} = \frac{1}{27}\left[85 - 6y^{(5)} + z^{(5)}\right] = \frac{1}{27}\left[85 - 6(3.583) + 1.927\right] = 2.423$$

$$y^{(6)} = \frac{1}{15}\left[72 - 6x^{(5)} - 2z^{(5)}\right] = \frac{1}{15}\left[72 - 6(2.4332) - 2(1.927)\right] = 3.570$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$z^{(6)} = \frac{1}{54}\left[110 - x^{(5)} - y^{(5)}\right] = \frac{1}{54}\left[110 - 2.432 - 3.583\right] = 1.926$$

Seventh iteration:

$$x^{(7)} = \frac{1}{27}\left[85 - 6y^{(6)} + z^{(6)}\right] = \frac{1}{27}\left[85 - 6(3.570) + 1.926\right] = 2.426$$

$$y^{(7)} = \frac{1}{15}\left[72 - 6x^{(6)} - 2z^{(6)}\right] = \frac{1}{15}\left[72 - 6(2.423) - 2(1.926)\right] = 3.574$$

$$z^{(7)} = \frac{1}{54}\left[110 - x^{(6)} - y^{(6)}\right] = \frac{1}{54}\left[110 - 2.423 - 3.570\right] = 1.926$$

Eighth Iteration:

$$x^{(8)} = \frac{1}{27}\left[85 - 6y^{(7)} + z^{(7)}\right] = \frac{1}{27}\left[85 - 6(3.574) + 1.926\right] = 2.425$$

$$y^{(8)} = \frac{1}{15}\left[72 - 6x^{(7)} - 2z^{(7)}\right] = \frac{1}{15}\left[72 - 6(2.426) - 2(1.926)\right] = 3.573$$

$$z^{(8)} = \frac{1}{54}\left[110 - x^{(7)} - y^{(7)}\right] = \frac{1}{54}\left[110 - 2.426 - 3.574\right] = 1.926$$

Nineth Iteration:

$$x^{(9)} = \frac{1}{27}\left[85 - 6y^{(8)} + z^{(8)}\right] = \frac{1}{27}\left[85 - 6(3.573) + 1.926\right] = 2.426$$

$$y^{(9)} = \frac{1}{15}\left[72 - 6x^{(8)} - 2z^{(8)}\right] = \frac{1}{15}\left[72 - 6(2.425) - 2(1.926)\right] = 3.573$$

$$z^{(9)} = \frac{1}{54}\left[110 - x^{(8)} - y^{(8)}\right] = \frac{1}{54}\left[110 - 2.425 - 3.573\right] = 1.926$$

Tenth Iteration:

$$x^{(10)} = \frac{1}{27}\left[85 - 6y^{(9)} + z^{(9)}\right] = \frac{1}{27}\left[85 - 6(3.573) + 1.926\right] = 2.426$$

$$y^{(10)} = \frac{1}{15}\left[72 - 6x^{(9)} - 2z^{(9)}\right] = \frac{1}{15}\left[72 - 6(2.426) - 2(1.926)\right] = 3.573$$

$$z^{(10)} = \frac{1}{54}\left[110 - x^{(9)} - y^{(9)}\right] = \frac{1}{54}\left[110 - 2.426 - 3.573\right] = 1.926$$

Hence $x = 2.426$, $y = 3.573$, $z = 1.926$

[Correct to three decimal places]

② Gauss - Seidel method.

Let the initial values be $y = 0$ $z = 0$.

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

First iteration

$$x^{(1)} = \frac{1}{27}[85 - 6y^{(0)} + z^{(0)}] = \frac{1}{27}[85 - 6(0) + 0] = 3.148$$

$$y^{(1)} = \frac{1}{15}[72 - 6x^{(1)} - 2z^{(0)}] = \frac{1}{15}[72 - 6(3.148) - 0] = 3.541$$

$$z^{(1)} = \frac{1}{54}[110 - x^{(1)} - y^{(1)}] = \frac{1}{54}[110 - 3.148 - 3.541] = 1.913$$

second iteration:-

$$x^{(2)} = \frac{1}{27}[85 - 6y^{(1)} + z^{(1)}] = \frac{1}{27}[85 - 6(3.541) + 1.913] = 2.432$$

$$y^{(2)} = \frac{1}{15}[72 - 6x^{(2)} - 2z^{(1)}] = \frac{1}{15}[72 - 6(2.432) - 2(1.913)] = 3.572$$

$$z^{(2)} = \frac{1}{54}[110 - x^{(2)} - y^{(2)}] = \frac{1}{54}[110 - 2.432 - 3.572] = 1.926$$

Third iteration:

$$x^{(3)} = \frac{1}{27}[85 - 6y^{(2)} + z^{(2)}] = \frac{1}{27}[85 - 6(3.572) + 1.926] = 2.426$$

$$y^{(3)} = \frac{1}{15}[72 - 6x^{(3)} - 2z^{(2)}] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^{(3)} = \frac{1}{54}[110 - x^{(3)} - y^{(3)}] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

Fourth iteration:-

$$x^{(4)} = \frac{1}{27}[85 - 6y^{(3)} + z^{(3)}] = \frac{1}{27}[85 - 6(3.573) + 1.926] = 2.426$$

$$y^{(4)} = \frac{1}{15}[72 - 6x^{(4)} - 2z^{(3)}] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^{(4)} = \frac{1}{54}[110 - x^{(4)} - y^{(4)}] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

Hence $x = 2.426$, $y = 3.573$, $z = 1.926$

This shows that the convergence is rapid in Gauss-Seidel method when compared to Gauss-Jacobi method.

③ Solve the following equations by Gauss-Seidel method.

$$4x + 2y + z = 14$$

$$x + 5y - z = 10$$

$$x + y + 8z = 20$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Soln:

as the coefficient matrix is diagonally dominant solving for x, y, z we get

$$x = \frac{1}{4}[14 - 2y - z]$$

$$y = \frac{1}{5}[10 - x + z]$$

$$z = \frac{1}{8}[20 - x - y]$$

let the initial values be $y=0$, $z=0$

First iteration:-

$$x^{(1)} = \frac{1}{4}[14 - 2(0) - (0)] = \frac{14}{4} = 3.5$$

$$y^{(1)} = \frac{1}{5}[10 - x^{(1)} + z^{(0)}] = \frac{1}{5}[10 - 3.5 + 0] = 1.3$$

$$z^{(1)} = \frac{1}{8}[20 - x^{(1)} - y^{(1)}] = \frac{1}{8}[20 - 3.5 - 1.3] = 1.9$$

Second iteration:-

$$x^{(2)} = \frac{1}{4}[14 - 2y^{(1)} - z^{(1)}] = \frac{1}{4}[14 - 2(1.3) - (1.9)] = 2.375$$

$$y^{(2)} = \frac{1}{5}[10 - x^{(2)} + z^{(1)}] = \frac{1}{5}[10 - 2.375 + 1.9] = 1.905$$

$$z^{(2)} = \frac{1}{8}[20 - x^{(2)} - y^{(2)}] = \frac{1}{8}[20 - 2.375 - 1.905] = 1.965$$

Third iteration:-

$$x^{(3)} = \frac{1}{4}[14 - 2y^{(2)} - z^{(2)}] = \frac{1}{4}[14 - 2(1.905) - 1.965] = 2.056$$

$$y^{(3)} = \frac{1}{5}[10 - x^{(3)} + z^{(2)}] = \frac{1}{5}[10 - 2.056 + 1.965] = 1.9818$$

$$z^{(3)} = \frac{1}{8}[20 - x^{(3)} - y^{(3)}] = \frac{1}{8}[20 - 2.056 + 1.9818] = 1.995$$

Fourth iteration:-

$$x^{(4)} = \frac{1}{4}[14 - 2y^{(3)} - z^{(3)}] = \frac{1}{4}[14 - 2(1.9818) - 1.965] = 2.510$$

$$y^{(4)} = \frac{1}{5}[10 - x^{(4)} + z^{(3)}] = \frac{1}{5}[10 - 2.510 + 1.995] = 1.897$$

$$z^{(4)} = \frac{1}{8}[20 - x^{(4)} - y^{(4)}] = \frac{1}{8}[20 - 2.510 - 1.897] = 1.949$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Fifth iteration:-

$x^{(5)} = \frac{1}{4}[14-2y^{(4)}-z^{(4)}] = \frac{1}{4}[14-2(1.897)-1.949] = 2.064$

$y^{(5)} = \frac{1}{5}[10-x^{(5)}+z^{(4)}] = \frac{1}{5}[10-2.064+1.949] = 1.977$

$z^{(5)} = \frac{1}{8}[20-x^{(5)}-y^{(5)}] = \frac{1}{8}[20-2.064-1.977] = 1.995$

Sixth iteration:-

$x^{(6)} = \frac{1}{4}[14-2y^{(5)}-z^{(5)}] = \frac{1}{4}[14-2(1.977)-1.995] = 2.013$

$y^{(6)} = \frac{1}{5}[10-x^{(6)}+z^{(5)}] = \frac{1}{5}[10-2.013+1.995] = 1.996$

$z^{(6)} = \frac{1}{8}[20-x^{(6)}-y^{(6)}] = \frac{1}{8}[20-2.013-1.996] = 1.999$

Seventh iteration:-

$x^{(7)} = \frac{1}{4}[14-2y^{(6)}-z^{(6)}] = \frac{1}{4}[14-2(1.996)-1.999] = 2.002$

$y^{(7)} = \frac{1}{5}[10-x^{(7)}+z^{(6)}] = \frac{1}{5}[10-2.002+1.999] = 1.999$

$z^{(7)} = \frac{1}{8}[20-x^{(7)}-y^{(7)}] = \frac{1}{8}[20-2.002-1.999] = 2.000$

Eighth iteration:-

$x^{(8)} = \frac{1}{4}[14-2y^{(7)}-z^{(7)}] = \frac{1}{4}[14-2(1.999)-2] = 2.001$

$y^{(8)} = \frac{1}{5}[10-x^{(8)}+z^{(7)}] = \frac{1}{5}[10-2.001+2] = 2.000$

$z^{(8)} = \frac{1}{8}[20-x^{(8)}-y^{(8)}] = \frac{1}{8}[20-2.001-2] = 2.000$

Ninth iteration:-

$x^{(9)} = \frac{1}{4}[14-2y^{(8)}-z^{(8)}] = \frac{1}{4}[14-2(2)-2] = 2$

$y^{(9)} = \frac{1}{5}[10-x^{(9)}+z^{(8)}] = \frac{1}{5}[10-2+2] = 2$

$z^{(9)} = \frac{1}{8}[20-x^{(9)}-y^{(9)}] = \frac{1}{8}[20-2-2] = 2$

Tenth iteration:-

$x^{(10)} = \frac{1}{4}[14-2y^{(9)}-z^{(9)}] = \frac{1}{4}[14-2(2)-2] = 2$

$y^{(10)} = \frac{1}{5}[10-x^{(10)}+z^{(9)}] = \frac{1}{5}[10-2+2] = 2$

$z^{(10)} = \frac{1}{8}[20-x^{(10)}-y^{(10)}] = \frac{1}{8}[20-2-2] = 2$

Hence $x=2$, $y=2$, $z=2$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

INVERSE OF A MATRIX BY GAUSS JORDON METHOD

Gauss - Jordan elimination method.

① using Gauss-Jordan method, find the inverse of the matrix.

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$$

Soln:-

$$[A, I] = \left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3/2 & 1/2 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 1 & 1 \end{array}\right] \quad R_1 \rightarrow \frac{R_1}{2}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3/2 & 1/2 & 0 & 0 \\ 0 & -1 & -2 & -1 & 1 & 0 \\ 0 & 2 & 7/2 & -1/2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \leftrightarrow R_2 - 2R_1 \\ R_3 \leftrightarrow R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3/2 & 1/2 & 0 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 2 & 7/2 & -1/2 & 0 & 1 \end{array}\right] \quad R_2 \leftrightarrow \frac{R_2}{-1}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & -1/2 & 1 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & -1/2 & -5/2 & 2 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \leftrightarrow R_1 - R_2 \\ R_3 \leftrightarrow R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1/2 & -1/2 & 1 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & 1 & 5 & -4 & -2 \end{array}\right] \quad R_3 \leftrightarrow R_3(-2)$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -9 & 7 & 4 \\ 0 & 0 & 1 & 5 & -4 & -2 \end{array}\right] \quad \begin{array}{l} R_1 \leftrightarrow R_1 + \frac{1}{2} R_3 \\ R_2 \leftrightarrow R_2 - 2R_3 \end{array}$$

Hence $A^{-1} = \begin{bmatrix} 2 & -1 & -1 \\ -9 & 7 & 4 \\ 5 & -4 & -2 \end{bmatrix}$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Verification:

$$AA^{-1} = I$$

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -9 & 7 & 4 \\ 5 & -4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

② using Gauss-Jordan method, find the inverse of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Soln:

$$\text{Let } [A, I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \leftrightarrow R_2 - R_1 \\ R_3 \leftrightarrow R_3 + 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \quad R_2 \leftrightarrow \frac{R_2}{2}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \leftrightarrow R_1 - R_2 \\ R_3 \leftrightarrow R_3 + 2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right] \quad R_3 \leftrightarrow \frac{R_3}{-4}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right] \quad \begin{array}{l} R_1 \leftrightarrow R_1 - 6R_3 \\ R_2 \leftrightarrow R_2 + 3R_3 \end{array}$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 12 & 4 & 6 \\ -5 & -1 & -3 \\ -1 & -1 & -1 \end{bmatrix}$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$AA^{-1} = I$$

$$= \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{\text{Ans.}}$$

EIGEN VALUE OF A MATRIX BY POWER METHOD

The power method

① Find the numerically largest eigenvalue of $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by power method.

Soln:

let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an arbitrary initial eigenvector.

$$AX_1 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} = 6 \begin{bmatrix} 0.167 \\ 0.667 \\ 1 \end{bmatrix} = 6X_2$$

$$AX_2 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.167 \\ 0.667 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.166 \\ 2.336 \\ 8.003 \end{bmatrix} = 8.003 \begin{bmatrix} 0.021 \\ 0.292 \\ 1 \end{bmatrix} = 8.003\,X_3$$

$$0.167 - 3(0.067) + 2(1) = 0.166$$

$$4(0.167) + 4(0.667) - 1 = 2.336$$

$$6(0.167) + 3(0.667) + 5 = 8.003$$

$$AX_3 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.021 \\ 0.292 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.145 \\ 0.252 \\ 6.002 \end{bmatrix} = 6.002 \begin{bmatrix} 0.191 \\ 0.042 \\ 1 \end{bmatrix} = 6.002\,X_4$$

$$1(0.021) - 3(0.292) + 2(1) = 1.145$$

$$4(0.021) + 4(0.292) - 1(1) = 0.252$$

$$6(0.021) + 3(0.292) + 5(1) = 6.002$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$AX_4 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.191 \\ 0.042 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.065 \\ -0.068 \\ 6.272 \end{bmatrix} = 6.272 \begin{bmatrix} 0.329 \\ -0.011 \\ 1 \end{bmatrix} = 6.272\, x_5$$

$$1(0.191) - 3(0.042) + 2(1) = 2.065$$

$$4(0.191) + 4(0.042) - 1(1) = -0.068$$

$$6(0.191) + 3(0.042) + 5(1) = 6.272$$

$$AX_5 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.329 \\ -0.011 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.362 \\ 0.272 \\ 6.941 \end{bmatrix} = 6.941 \begin{bmatrix} 0.34 \\ 0.039 \\ 1 \end{bmatrix} = 6.941\, X_6$$

$$AX_6 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.34 \\ 0.039 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.223 \\ 0.516 \\ 7.157 \end{bmatrix} = 7.157 \begin{bmatrix} 0.311 \\ 0.072 \\ 1 \end{bmatrix} = 7.157 X_7$$

$$AX_7 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.311 \\ 0.072 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.095 \\ 0.532 \\ 7.082 \end{bmatrix} = 7.082 \begin{bmatrix} 0.296 \\ 0.075 \\ 1 \end{bmatrix} = 7.082 X_8$$

$$AX_8 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.296 \\ 0.075 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.071 \\ 0.484 \\ 7.001 \end{bmatrix} = 7.001 \begin{bmatrix} 0.296 \\ 0.069 \\ 1 \end{bmatrix} = 7.001 X_9$$

This shows the largest eigenvalue $= 7$.

② Find the dominant eigen value and the corresponding eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ find also the least latent root and hence the third value also.

Soln:

Let $x_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be an approximate eigen value.

$$AX_1 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1\, X_2$$

$$AX_2 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 0.4286 \\ 0 \end{bmatrix} = 7 . X_3$$

$$AX_3 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.4286 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.5714 \\ 1.8572 \\ 0 \end{bmatrix} = 3.5714 \begin{bmatrix} 1 \\ 0.52 \\ 0 \end{bmatrix} = 3.5714\, X_4$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$Ax_4 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.52 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.12 \\ 2.04 \\ 0 \end{bmatrix} = 4.12 \begin{bmatrix} 1 \\ 0.4951 \\ 0 \end{bmatrix} = 4.12\ x_5$$

$$Ax_5 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.4951 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.9706 \\ 1.9902 \\ 0 \end{bmatrix} = 3.9706 \begin{bmatrix} 1 \\ 0.5012 \\ 0 \end{bmatrix} = 3.9706\ x_6$$

$$Ax_6 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.5012 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.0072 \\ 2.0024 \\ 0 \end{bmatrix} = 4.0072 \begin{bmatrix} 1 \\ 0.4997 \\ 0 \end{bmatrix} = 4.0072\ x_7$$

$$Ax_7 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.4997 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.9982 \\ 1.9994 \\ 0 \end{bmatrix} = 3.9982 \begin{bmatrix} 1 \\ 0.5000 \\ 0 \end{bmatrix} = 3.9982\ x_8$$

$$Ax_8 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix} = 4x_9$$

$\therefore$ Dominant eigen value = 4; corresponding eigen vector is $(1, 0.5, 0)$

To find the least eigen value, let $B = A - 4I$, since $\lambda_1 = 4$

$$\therefore B = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 1 \\ 1 & -2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

we will find the dominant eigen value of B.

let $Y_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be the initial vector.

$$BY_1 = \begin{bmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} = -3 \begin{bmatrix} 1 \\ -0.3333 \\ 0 \end{bmatrix} = -3Y_2$$

$$BY_2 = \begin{bmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -0.3333 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 1.6666 \\ 0 \end{bmatrix} = -5 \begin{bmatrix} 1 \\ -0.3333 \\ 0 \end{bmatrix} = -5Y_3$$

$$\therefore BY_3 = \begin{bmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -0.3333 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 1.6666 \\ 0 \end{bmatrix} = -5 \begin{bmatrix} 1 \\ -0.3333 \\ 0 \end{bmatrix}$$

$\therefore$ Dominant eigen value of B is -5

Adding 4, smallest eigen value of A $= -5 + 4 = -1$

Sum of eigen values = trace of A $= 1 + 2 + 3 = 6$

$4 + (-1) + \lambda_3 = 6, \quad \therefore \lambda_3 = 3$

All the three eigen values are, 4, 3, -1

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

UNIT – II

INTERPOLATION AND APPROXIMATION.

LAGRANGIAN POLYNOMIALS

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x)\cdots(x_n-x_{n-1})} y_n$$

①. Find the polynomial f(x) by using Lagrange's formula and hence find f(3) for

x	x_0 0	x_1 1	x_2 2	x_3 5
$f(x)$	2 y_0	3 y_1	12 y_2	147 y_3

Soln: By Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) + \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12)$$

$$+ \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$= \frac{(x-1)(x-2)(x-5)}{(-10)}(2) + \frac{x(x-2)(x-5)}{4}(3) + \frac{(x-1)(x-5)}{-6}(12)$$

$$+ \frac{x(x-1)(x-2)}{60}(147)$$

$$y = f(3) = \frac{(3-1)(3-2)(3-5)}{-10}(2) + \frac{3(3-2)(3-5)}{4}(3) + \frac{3(3-1)(3-5)}{-6}(12)$$

$$+ \frac{3(3-1)(3-2)}{60}(147)$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$y = \frac{(2)(1)(-2)}{(-10)}(2) + \frac{(3)(1)(-2)}{4}(3) + \frac{3(2)(-2)}{(-6)}(12) + \frac{(3)(2)(1)}{60}(147)$$

$$= \frac{4}{10}(2) - \frac{6}{4}(3) + 2(12) + \frac{1}{10}(147)$$

$$= \frac{8}{10} - \frac{18}{4} + 24 + \frac{147}{10}$$

$$= 35 \text{ Ans}$$

④ Find the third degree polynomial f(x) satisfying the following data.

x	1	3	5	7
y	24	120	336	720

Soln:

The Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-3)(x-5)(x-7)}{(1-3)(1-5)(1-7)}(24) + \frac{(x-1)(x-5)(x-7)}{(3-1)(3-5)(3-7)}(120) + \frac{(x-1)(x-3)(x-7)}{(5-1)(5-3)(5-7)}(336)$$

$$+ \frac{(x-1)(x-3)(x-5)}{(7-1)(7-3)(7-5)}(720)$$

$$= -\tfrac{1}{2}(x-3)(x-5)(x-7) + \frac{15}{2}(x-1)(x-5)(x-7) - 21(x-1)(x-3)(x-7)$$

$$+ 15(x-1)(x-3)(x-5)$$

$$= -\tfrac{1}{2}[x^3 - 15x^2 + 71x - 105] + \frac{15}{2}[x^3 - 13x^2 + 47x - 35] - 21[x^3 - 11x^2 + 31x - 21]$$

$$= +15[x^3 - 9x^2 + 23x - 15]$$

$$= \left[-\tfrac{1}{2} + \frac{15}{2} - 21 + 15\right]x^3 + \left[\frac{15}{2} - \frac{195}{2} + 231 - 135\right]x^2 + \left[\frac{-71}{2} + \frac{705}{2} - 605 + 345\right]x$$

$$+ \left[\frac{105}{2} - \frac{525}{2} + 441 - 225\right]$$

$$= x^3 + 6x^2 + 11x + 6$$

$$f(4) = 4^3 + 6(4^2) + 11(4) + 6$$

$$= 64 + 96 + 44 + 6$$

$$= 210$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

③ Using Lagrange's interpolation formula find $f(4)$ given that $f(0) = 2$, $f(1) = 3$, $f(2) = 12$, $f(15) = 3587$.

Soln:

Given

x	0	1	2	15
y	2	3	12	3587

By Lagrange's formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y = f(4) = \frac{(4-1)(4-2)(4-15)}{(0-1)(0-2)(0-15)}(2) + \frac{(4-0)(4-2)(4-15)}{(1-0)(1-2)(1-15)}(3)$$

$$+ \frac{(4-0)(4-1)(4-15)}{(2-0)(2-1)(2-15)}(12) + \frac{(4-0)(4-1)(4-2)}{(15-0)(15-1)(15-2)} 3587$$

$$= \frac{(3)(2)(-11)}{(-1)(-2)(-15)}(2) + \frac{(4)(2)(-11)}{(1)(-1)(-14)}(3) + \frac{(4)(3)(-11)}{(2)(1)(-13)}(12)$$

$$+ \frac{(4)(3)(2)}{(15)(14)(13)}(3587)$$

$$= \frac{132}{30} - \frac{264}{14} + \frac{1584}{26} + \frac{86088}{2730}$$

$$= 78$$

④ Find the missing term in the following table using Lagrange's interpolation.

x	0	1	2	3	4
y	1	3	9	–	81

Soln:

By Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Let $x_0 = 0$ $\quad y_0 = 1$

$x_1 = 1 \quad y_1 = 3$

$x_2 = 2 \quad y_2 = 9$

$x_3 = 4 \quad y_3 = 81$

$$y = f(x) = \frac{(x-1)(x-2)(x-4)}{(0-1)(0-2)(0-4)}(1) + \frac{(x-0)(x-2)(x-4)}{(1-0)(1-2)(1-4)}(3) + \frac{(x-0)(x-1)(x-4)}{(2-0)(2-1)(2-4)}(9) + \frac{(x-0)(x-1)(x-2)}{(4-0)(4-1)(4-2)}(81)$$

$$f(3) = \frac{(3-1)(3-2)(3-4)}{(-1)(-2)(-4)}(1) + \frac{(3-0)(3-2)(3-4)}{(1)(-1)(-3)}(3) + \frac{(3-0)(3-1)(3-4)}{(2)(-2)(1)}(9) + \frac{(3-0)(3-1)(3-2)}{(4)(3)(2)}(81)$$

$$= \frac{(2)(1)(-1)}{(-1)(-2)(-4)}(1) + \frac{(3)(1)(-1)}{(1)(-1)(-3)}(3) + \frac{(3)(2)(-1)}{(2)(1)(-2)}(9) + \frac{(3)(2)(1)}{(4)(3)(2)}(81)$$

$$= \frac{-2}{-8}(1) - 3 + \frac{27}{2} + \frac{81}{4}$$

$$= \frac{1}{4} - 3 + \frac{27}{2} + \frac{81}{4}$$

$$= 31$$

⑤ Find the parabola of the form $y = ax^2 + bx + c$ passing through the points $(0,0)$ $(1,1)$ and $(2,20)$

Soln:

We use Lagrange's interpolation formula

$$y = f(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)} . 0 + \frac{(x-0)(x-2)}{(1-0)(1-2)} . 1 + \frac{(x-0)(x-1)}{(2-0)(2-1)} . 20 .$$

$$= 0 - x(x-2) + 10x(x-1)$$

$$y = 9x^2 - 8x$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Inverse Interpolation

Taking y as independent variable

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)} x_0 + \frac{(y-y_0)(y-y_2)\cdots(y-y_n)}{(y_1-y_0)(y_1-y_2)\cdots(y_1-y_n)} x_1$$

$$+ \cdots + \frac{(y-y_0)(y-y_1)\cdots(y-y_{n-1})}{(y_n-0)(y_n-y_1)\cdots(y_n-y_{n-1})} x_n$$

This is called formula of inversion interpolation.

① Find the age corresponding to the annuity value 13.6 given the table.

Age (x):	30	35	40	45	50
Annuity Value (y)	15.9	14.9	14.1	13.3	12.5

Soln:-

$$x = \frac{(13.6-14.9)(13.6-14.1)(13.6-13.3)(13.6-12.5)}{(15.9-14.9)(15.9-14.1)(15.9-13.3)(15.9-12.5)} \times 30$$

$$+ \frac{(13.6-15.9)(13.6-14.1)(13.6-13.3)(13.6-12.5)}{(14.9-15.9)(14.9-14.1)(14.9-13.3)(14.9-12.5)} \times 35$$

$$+ \frac{(13.6-15.9)(13.6-14.9)(13.6-13.3)(13.6-12.5)}{(14.1-15.9)(14.1-14.9)(14.7-13.3)(14.1-12.5)} \times 40$$

$$+ \frac{(13.6-15.9)(13.6-14.9)(13.6-14.1)(13.6-12.5)}{(13.3-15.9)(13.3-14.9)(13.3-14.1)(13.3-12.5)} \times 45$$

$$+ \frac{(13.6-15.9)(13.6-14.9)(13.6-14.1)(13.6-13.3)}{(12.5-15.9)(12.5-14.9)(12.5-14.1)(12.5-13.3)} \times 50$$

$$\therefore x(y=13.6) = 43$$

② Find the value of θ given f(θ) = 0.3887 where $f(\theta) = \int_0^{\theta} \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}}$ using the table

θ	21°	23°	25°
f(θ)	0.3706	0.4068	0.4433

Soln: Now take f(θ) as independent and θ as dependent

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$y\,f(\theta)$:	0.3706	0.4068	0.4433
θ :	21	23	25

$$\theta = \frac{(y-0.4068)(y-0.4433)}{(0.3706-0.4068)(0.3706-0.4433)} \times 21 + \frac{(y-0.3706)(y-0.4433)}{(0.4068-0.3706)(0.4068-0.4433)} (23)$$

$$+ \frac{(y-0.3706)(y-0.4068)}{(0.4433-0.3706)(0.4433-0.4068)} \times 25$$

$$\theta(y=0.3887) = \frac{(0.3887-0.4068)(0.3887-0.4433)}{(0.3706-0.4068)(0.3706-0.4433)} \times 21$$

$$+ \frac{(0.3887-0.3706)(0.3887-0.4433)}{(0.4068-0.3706)(0.4068-0.4433)} \times 23$$

$$+ \frac{(0.3887-0.3706)(0.3887-0.4068)}{(0.4433-0.3706)(0.4433-0.4068)} \times 25$$

$$= 7.885832 + 17.202739 - 3.086525$$

$$= 22.0020$$

DIVDED DIFFERENCES

TABLE

Argument x	Entry $f(x)$	First divided difference $\Delta f(x)$	Second divided difference $\Delta_2 f(x)$	Third divided difference $\Delta_3 f(x)$
x_0	$f(x_0)$			
		$f(x_0, x_1)$		
x_1	$f(x_1)$		$f(x_0, x_1, x_2)$	
		$f(x_1, x_2)$		$f(x_0, x_1, x_2, x_3)$
x_2	$f(x_2)$		$f(x_1\ x_2\ x_3)$	
		$f(x_2, x_3)$		$f(x_1, x_2, x_3, x_4)$
x_3	$f(x_3)$		$f(x_2, x_3, x_4)$	
		$f(x_3, x_4)$		
x_4	$f(x_4)$			

① Form the divided difference table for the following data:-

x	1	2	3	4	7	12
$f(x)$	22	30	82	82	106	206

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Soln: The divided difference table is as follows:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	22				
		$\frac{30-22}{2-1}=8$			
2	30		$\frac{26-8}{4-1}=6$		
		$\frac{82-30}{4-2}=26$		$\frac{-3.6-6}{7-1}=-1$	
4	82		$\frac{8-26}{7-2}=-3.6$		$\frac{0.51+1.6}{12-1}=$ 0.19
		$\frac{106-82}{7-4}=8$		$\frac{1.5+36}{12-2}=0.51$	
7	106		$\frac{20-8}{12-4}=1.5$		
		$\frac{206-106}{12-7}=20$			
12	206				

③ Show that $\underset{bcd}{\Delta}^3\left(\frac{1}{a}\right) = -\frac{1}{abcd}$

Soln:

If $f(x) = 1/x$, $f(a) = 1/a$

$$f(a,b) = \underset{b}{\Delta}\left(1/a\right) = \frac{1/b - 1/a}{b-a} = -\frac{1}{ab}$$

$$f(a,b,c) = \frac{f(b,c)-f(a,b)}{c-a} = \frac{-\frac{1}{bc}+\frac{1}{ab}}{c-a} = \frac{1}{abc}\left(\frac{c-a}{c-a}\right) = \frac{1}{abc}$$

$$f(a,b,c,d) = \frac{f(b,c,d)-f(a,b,c)}{d-a} = \frac{\frac{1}{bcd}-\frac{1}{abc}}{d-a} = \frac{1}{abcd}\left(\frac{a-d}{d-a}\right) = -\frac{1}{abcd}$$

$$\therefore \underset{bcd}{\Delta}^3\left(1/a\right) = -\frac{1}{abcd}$$

Newton's divided difference formula (or) Newton's interpolation for un equal intervals

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + \dots + (x-x_0)(x-x_1)\cdots(x-x_{n-1})f(x_0,x_1\dots x_n)$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

① using Netwon's divided difference formula, find $u(3)$ given $u(1) = -26$, $u(2) = 12$, $u(4) = 256$, $u(6) = 844$.

Soln:

We form the divided difference table since the intervals are unequal.

x	$u(x)$	$\Delta u(x)$	$\Delta^2 u(x)$	$\Delta^3 u(x)$
1	-26			
		$\frac{12+26}{2-1} = 38$		
2	12		$\frac{122-38}{4-1} = 28$	
		$\frac{256-12}{4-2} = 122$		$\frac{43-28}{6-1} = 3$
4	256		$\frac{294-122}{6-2} = 43$	
		$\frac{844-256}{6-4} = 294$		
6	844			

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + \cdots$$

Here

$$u(x) = u(x_0) + (x-x_0) u(x_0, x_1) + (x-x_0)(x-x_1) u(x_0, x_1, x_2) + \cdots$$

$x_0 = 1$, $x_1 = 2$, $x_2 = 4$, $x_3 = 6$

$u(x_0) = -26$, $u(x_0, x_1) = 38$ $u(x_0, x_1, x_2) = 28$, $u(x_0, x_1, x_2, x_3) = 3$

$\therefore u(x) = -26 + (x-1)38 + (x-1)(x-2)28 + (x-1)(x-2)(x-4)3$

$\therefore u(3) = -26 + (2)(38) + (2)(1)(28) + (2)(1)(-1)(3)$

$= -26 + 76 + 56 - 6$

$u(3) = 100.$

② Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference formula.

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Soln:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		$\frac{33-1245}{(-1)-(-4)} = -404$			
			$\frac{-28-(-404)}{0-(-4)} = 94$		
-1	33				
		$\frac{5-33}{0-(-1)} = -28$			
				$\frac{10-94}{2-(-4)} = -14$	
0	5		$\frac{2-(-28)}{2-(-1)} = 10$		$\frac{13+14}{5-(-4)} = 3$
		$\frac{9-5}{2-0} = 2$			
				$\frac{88-10}{5-(-1)} = 13$	
2	9		$\frac{442-2}{5-0} = 88$		
		$\frac{1335-9}{5-2} = 442$			
5	1335				

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3) f(x_0, x_1, x_2, x_3, x_4)$$

Here $x_0 = -4 \quad x_1 = -1 \quad x_2 = 0 \quad x_3 = 2 \quad x_4 = 5$

$f(x_0) = 1245$

$f(x_0, x_1) = -404$

$f(x_0, x_1, x_2) = 94$

$f(x_0, x_1, x_2, x_3) = -14$

$f(x_0, x_1, x_2, x_3, x_4) = 3$

$$f(x) = 1245 + (x+4)(-404) + (x+4)(x+1)(94) + (x+4)(x+1)(x)(-14) + (x+4)(x+1)(x)(x-2)(3)$$

$$= 1245 - 404x - 1616 + 94[x^2+5x+4] - 14x[x^2+5x+4] + 3x[(x^2+5x+4)(x-2)]$$

$$= 1245 - 404x - 1616 + 94x^2 + 470x + 376 - 14x^3 - 70x^2 - 56x + 3x[x^3 - 2x + 5x^2 - 10x + 4x - 8]$$

$$= -14x^3 + 24x^2 + 10x + 5 + 3x[x^3 + 5x^2 - 8x - 8]$$

$$= -14x^3 + 24x^2 + 10x + 5 + 3x^4 + 15x^3 - 24x^2 - 24x. = 3x^4 + x^3 - 14x + 5$$ //

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

③ Using Newton's divided difference formula find the missing value from the table.

x	1	2	4	5	6
y	14	15	5	-	9

Soln:

x	f(x)	Δf(x)	Δ²f(x)	Δ³f(x)
1	14			
		$\frac{15-14}{2-1}=1$		
2	15		$\frac{-5-1}{4-1}=-2$	
		$\frac{5-15}{4-2}=-5$		$\frac{\frac{7}{4}+2}{6-1}=\frac{15/4}{5}=3/4$
4	5		$\frac{2+5}{6-2}=7/4$	
		$\frac{9-5}{6-4}=2$		
6	9			

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0,x_1,x_2,x_3) + \dots$$

$$= 14+(x-1)(1)+(x-1)(x-2)(-2)+(x-1)(x-2)(x-4)(3/4)$$

$$= 14+x-1-2(x-1)(x-2)+3/4\,(x-1)(x-2)(x-4)$$

$$f(5) = 13+5-2(4)(3)+3/4\,(4)(3)(1)$$

$$= 18-24+9 = 3$$

INTERPOLATION WITH A CUBIC SPLINE

Formula:

$$f_i(x) = \frac{f_i''(x_{i-1})}{6(x_i-x_{i-1})}(x_i-x)^3 + \frac{f_i''(x_i)}{6(x_i-x_{i-1})}(x-x_{i-1})^3 + \left[\frac{f(x_{i-1})}{x_i-x_{i-1}} - \frac{f''(x_{i-1})(x_i-x_{i-1})}{6}\right](x_i-x) + \left[\frac{f(x_i)}{x_i-x_{i-1}} - \frac{f''(x_i)(x_i-x_{i-1})}{6}\right](x-x_{i-1}) \quad -(1)$$

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

This equation contains only two unknowns - the second derivatives at the end of each interval.

These unknowns can be evaluated using the following equation.

$$(x_i - x_{i-1}) f''(x_i - 1) + 2(x_{i+1} - x_{i-1}) f''(x_i) + (x_{i+1} - x_i) f''(x_{i+1})$$

$$= \frac{6}{x_{i+1} - x_i}\left[f(x_{i+1}) - f(x_i)\right] + \frac{6}{x_i - x_{i-1}}\left[f(x_{i-1}) - f(x_i)\right] \quad \cdots ②$$

① From the following table.

x	x_{i-1} 1	x_i 2	x_{i+1} 3
y	-8	-1	18

Comput $y(1.5)$ and $y'(1)$ using cubic Spline

Soln:

$$(x_i - x_{i-1}) f''(x_i - 1) + 2(x_{i+1} - x_{i-1}) f''(x_i) + (x_{i+1} - x_i) f''(x_{i+1})$$

$$= \frac{6}{x_{i+1} - x_i}\left[f(x_{i+1}) - f(x_i)\right] + \frac{6}{x_i - x_{i-1}}\left[f(x_{i-1}) - f(x_i)\right] \quad \cdots (2)$$

$$(2-1) f''(1) + 2(3-1) f''(2) + (3-2) f''(3) = \frac{6}{(3-2)}(18+1) + \frac{6}{2-1}[-8+1]$$

$$f''(1) + 4 f''(2) + f''(3) = 6(19) + 6(-7)$$

$f''(1) = 0 \quad f''(3) = 0$ at the end points, $f'(3) = 0$, $4 f''(2) = 72$

$f''(2) = 18$

From (1) we get

$$f(x) = 0 + \frac{1}{6}\frac{f''(2)}{(2-1)}(x-1)^3 + \left[\frac{-8}{2-1} - 0\right](2-x) + \left[\frac{-1}{2-1} - \frac{18}{6}(2-1)\right](x-1)$$

$$= \frac{1}{6} 18 (x-1)^3 + (-8).(2-x) + [-1-3][x-1]$$

$$= 3(x-1)^3 - 8(2-x) - 4(x-1)$$

$$= 3(x-1)^3 - 16 + 8x - 4x + 4$$

$$= 3[x^3 - 3x^2 - 3 + 1] - 16 + 4x + 4 = 3x^3 - 9x^2 + 13 - 15$$

$$= 3x^3 - 9x^2 - 8$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$y(1.5) = f(1.5) = 3(0.5)^3 + 4(1.5) - 12 = -\frac{45}{8}$$

$$y' = f'(x) = 9(x-1)^2 + 4$$

$$y'(1) = f'(1) = 4 \text{ Ans.}$$

Another method

$$S(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right] + \frac{1}{h}(x_i - x)\left[y_{i-1} - \frac{h^2}{6}M_{i-1}\right]$$

$$+ \frac{1}{h}(x - x_{i-1})\left[y_i - \frac{h^2}{6}M_i\right] \cdots$$

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right] \text{ for } i = 1, 2, 3 \cdots (n-1)$$

① From the following table:

x	1	2	3
y	−8	−1	18

Compute $y(1.5)$ and $y'(1)$ using cubic spline

Soln:

Here $h = 1$, and $n = 2$. also assume $M_0 = 0$ and $M_2 = 0$

we have

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right] \text{ for } i = 1, 2 \ldots (n-1)$$

From this

$$M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$$

$$\therefore 4M_1 = 6[-8 - 2(-1) + 18] = 72$$

$$\therefore M_1 = 18$$

$$S(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right] + \frac{1}{h}(x_i - x)\left[y_{i-1} - \frac{h^2}{6}M_{i-1}\right]$$

$$+ \frac{1}{h}(x - x_{i-1})\left[y_i - \frac{h^2}{6}[M_i]\right] \cdots ⑥$$

For $1 \le x \le 2$ putting $i = 1$ we get

$$S(x) = \frac{1}{6}\left[18(x-1)^3\right] + (2-x)(-8) - 4(x-1)$$

$$= 3(x-1)^3 + 4x - 12 = 3x^3 - 9x^2 + 13x - 15$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$y(1.5) = s(1.5) = 3(0.5)^3 + 4(1.5) - 12 = -45/8$$

$$y' = s'(x) = 9(x-1)^2 + 4$$

$$y'(1) = 4.$$

② Given the points $(0, 0)$ $(\pi/2, 1)$ and $(\pi, 0)$ satisfying the function $y = \sin x$ $(0 \le x \le \pi)$ determine the value of $y(\pi/6)$ using the cubic spline approximation.

x	0	$\pi/2$	π
y	0	1	0

Soln:

Here $h = \pi/2$ $n = 2$, also assume $M_0 = 0$. $M_2 = 0$

we have

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right] \text{ for } i = 1, 2, \ldots (n-1)$$

From this

$$M_0 + 4M_1 + M_2 = \frac{6}{(\pi/2)^2}\left[0 - 2 + 0\right] = \frac{-48}{\pi^2}$$

$$4M_1 = \frac{-48}{\pi^2}$$

$$M_1 = \frac{-48}{12\pi^2} = \frac{-12}{\pi^2}$$

In the interval $[0, \pi/2]$, the national cubic spline is given by

$$S_1(x) = \frac{1}{6(\pi/2)}\left[(x-0)^3\left(\frac{-12}{\pi^2}\right) + \frac{1}{(\pi/2)}\left[\pi/2 - x\right]\left[0 - \frac{(\pi/2)^2}{6}0\right] + \frac{1}{(\pi/2)}\left[x - 0\right]\right.$$

$$\left.\left[1 - \frac{(\pi/2)^2}{6}\left(\frac{-12}{\pi^2}\right)\right]\right]$$

$$= \frac{1}{3\pi}\left[x^3\left(\frac{-12}{\pi^2}\right) + \frac{2}{\pi}(x)\left[1 + 1/2\right]\right]$$

$$= \frac{1}{3\pi}\left[\frac{-12}{\pi^2}x^3\right] + 3/\pi\, x$$

$$= \frac{1}{\pi}\left[\frac{-4}{\pi^2}x^3\right] + 3/\pi\, x. = 2/\pi\left[-2/\pi^2\, x^3 + 3/2\, x\right]$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$y(\pi/6) = 2/\pi \left[\frac{-\pi}{108} + \pi/4 \right] = 0.4815$$

NEWTON FORWARD AND BACKWARD DIFFERENCE FORMULA

Forward interpolation formula.

$$y(x) = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0$$

where $u = \frac{x - x_0}{h}$

① using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence evaluate y at $x=5$.

x	4	6	8	10
y	1	3	8	10

Soln:
we form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$(x_0) 4$	$(y_0) 1$			
		$3-1=2(\Delta y_0)$		
			$5-2=3(\Delta^2 y_0)$	
$(x_1) 6$	$(y_1) 3$			$-3-3=-6(\Delta^3 y_0)$
		$8-3=5(\Delta y_1)$		
			$2-5=-3(\Delta^2 y_1)$	
$(x_2) 8$	$(y_2) 8$			
		$10-8=2(\Delta y_2)$		
$(x_3) 10$	$(y_3) 10$			

There are only 4 data given. Hence the Polynomial will be degree 3.

$$y(x) = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 .$$

where $u = \frac{x - x_0}{h}$ Here $x_0 = 4$, $h = 6-4 = 2$ [difference]

$$y(x) = 1 + \frac{\left(\frac{x-4}{2}\right)}{1!}(2) + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)}{2!}(3) + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)\left(\frac{x-8}{2}\right)}{3!}(-6)$$

$$= 1 + x - 4 + \frac{(x-4)(x-6)}{8}(3) + \frac{(x-4)(x-6)(x-8)}{(8)(6)}(-6).$$

$$= x - 3 + 3/8 (x-4)(x-6) - 1/8 (x-4)(x-6)(x-8)$$

$$= x - 3 + 3/8 [x^2 - 10x + 24] - 1/8 [(x^2 - 10x + 24)(x-8)]$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= x - 3 + \frac{3}{8}\left[x^2 - 10x + 24\right] - \frac{1}{8}\left[x^3 - 10x^2 + 24x - 8x^2 + 80x - 192\right]$$

$$= \frac{1}{8}\left[8x - 24 + x^2 - 10x + 24 - x^3 + 10x^2 - 24x + 8x^2 - 80x + 192\right]$$

$$= \frac{1}{8}\left[-x^3 + 19x^2 - 106x + 192\right]$$

$$y(5) = \frac{1}{8}\left(-(5)^3 + 19(5)^2 - 106(5) + 192\right]$$

$$= \frac{1}{8}\left[-125 + 475 - 530 + 192\right]$$

$$= \frac{1}{8}\left[12\right]$$

$$y(5) = 1.5.$$

② A third degree polynomial passes through the points (0, -1) (1, 1) (2, 1) and (3, -2) using Newton's forward interpolation formula find the polynomial. Hence find the value at 1.5

Soln

we form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
(x_0) 0	(y_0) -1	$1+1=2(\Delta y_0)$		
			$0-2=-2(\Delta^2 y_0)$	
(x_1) 1	(y_1) 1			$-3+2=-1(\Delta^3 y_0)$
		$1-1=0(\Delta y_1)$		
(x_2) 2	(y_2) 1		$-3-0=-3(\Delta^2 y_1)$	
		$-2-1=-3(\Delta y_2)$		
(x_3) 3	(y_3) -2			

There are only 4 data given. Hence the polynomial of degree 3.

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

where $u = \frac{x - x_0}{h}$

$x_0 = 0$, $h = 1 - 0 = 1$ (difference)

$\therefore u = x$

$$y(x) = -1 + \frac{x}{1!}(2) + \frac{x(x-1)}{2!}(-2) + \frac{x(x-1)(x-2)}{3!}(-1)$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$= -1 + 2x - x(x-1) - \frac{1}{6} x(x-1)(x-2)$

$= -1 + 2x - x^2 + x - \frac{1}{6} x[x^2 - 3x + 2]$

$= -x^2 + 3x - 1 - \frac{1}{6}[x^3 - 3x^2 + 2x]$

$= \frac{1}{6}[-6x^2 + 18x - 6 - x^3 + 3x^2 - 2x]$

$= \frac{1}{6}[-x^3 - 3x^2 + 16x - 6]$

$= -\frac{1}{6}[x^3 + 3x^2 - 16x + 6]$

$y(1.5) = -\frac{1}{6}[(1.5)^3 + 3(1.5)^2 - 16(1.5) + 6]$

$= -\frac{1}{6}[3.375 + 6.75 - 24 + 6]$

$= -\frac{1}{6}[-7.875]$

$y(1.5) = 1.3125$

③ From the data given below, find the number of students whose weight is between 60 to 70.

Weight in lbs:	0-40	40-60	60-80	80-100	100-120
No. of students:	250	120	100	70	50

Soln:

Difference table

x weight	y (No. of students)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
Below 60	370		-20		
		100		-10	
Below 80	470		-30		20
		70		10	
Below 100	540		-20		
		50			
Below 120	590				

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Let us calculate the number of students whose weight is less than 70.

We will use forward difference formula.

$$u = \frac{x-x_0}{h} = \frac{70-40}{20} = 1.5$$

$$y(70) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_0 + \cdots$$

$$= 250 + (1.5)(120) + \frac{(1.5)(0.5)}{2}(-20) + \frac{(1.5)(0.5)(-0.5)}{6}(-10)$$

$$+ \frac{(1.5)(0.5)(-0.5)(-1.5)}{24}(20)$$

$$= 250 + 180 - 7.5 + 0.625 + 0.46875$$

$$= 423.59$$

$$\simeq 424.$$

Number of students whose weight is between 60 and 70

$$= y(70) - y(60) = 424 - 370 = 54.$$

STUCOR APP

Newton's backward interpolation formula

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}$$

$$\nabla^4 y_n + \cdots$$

where $v = \frac{x-x_n}{h}$

① Use Newton's backward difference formula to construct an interpolating polynomial of degree 3 for the data.

$f(-0.75) = -0.07181250$, $f(-0.5) = -0.024750$

$f(-0.25) = 0.33493750$, $f(0) = 1.10100$. Hence find $f(-\frac{1}{3})$

Soln:-

Newton's backward difference formula is

$$y(x) = y_3 + \frac{v}{1!}\nabla y_3 + \frac{v(v+1)}{2!}\nabla^2 y_3 + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_3$$

where $v = \frac{x-x_3}{h}$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Here we for the difference table.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
(x_0) (-0.75)	y_0 -0.07181250			
		0.0470625		
(x_1) -0.5	y_1 -0.024750		0.312625	
				$(\nabla^3 y_3)$ 0.09375
		0.3596875		
(x_2) -0.25	y_2 0.33493750			
			$(\nabla^2 y_3)$ 0.400375	
		(∇y_3) 0.7660625		
(x_3) 0	y_3 1.10100			

Here $x_3 = 0$ $h = 0.25$ $v = \frac{x}{0.25} = \frac{x}{(1/4)} = 4x$

$$y(x) = 1.10100 + 4x(0.7660625) + \frac{4x(4x+1)}{2}(0.406375)$$

$$+ \frac{4x(4x+1)(4x+2)}{6}(0.09375)$$

$$= 1.10100 + 3.06425 + 0.81275x(4x+1) + 0.0625x(4x+1)(4x+2)$$

$$= 1.101 + 3.06425x^2 + 0.81275x + 0.0625x[16x^2 + 12x + 2]$$

$$= 1.101 + 3.06425x + 3.251x^2 + 0.81275x + x^3 + 0.75x^2 + 0.125x.$$

$$= x^3 + 4.001x^2 + 4.002x + 1.101$$

To find $f(-1/3)$.

$$y(-1/3) = (-1/3)^3 + (4.001)(-1/3)^2 + 4.002(-1/3) + 1.101$$

$$= -\frac{1}{27} + 4.001(1/9) - 4.002(1/3) + 1.101$$

$$= 0.174518518_{Ans}$$

② From the following table find the value of tan (0.28)

x	0.10	0.15	0.20	0.25	0.30
$y = \tan x$	0.1003	0.1511	0.2027	0.2533	0.3093

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Soln: Let us form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0·10	0.1003				
		0·0508			
0·15	0.1511		0.0008		
		0.0516		0.0002	
0·20	0.2027				0.0002
			0.0010		
0·25	0.2553	0.0526		0.0004	
			0.0014		
		0.0540			
0·30	0.3093				

Since 0.28 lies in the end of the table, let us use Newton's backward interpolation formula.

$$f(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \cdots$$

where $v = \frac{x - x_n}{h} = \frac{0.28 - 0.30}{0.05} = -0.04$ $[\because x_n = 0.30]$

$$y = 0.3093 + \frac{(-0.4)}{1!}(0.0540) + \frac{(-0.4)(-0.4+1)}{2!}(0.0014)$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)}{3!}(0.0004)$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{4!}(0.0002)$$

$$y = 0.309 - 0.0216 - 0.000168 - 0.0000256 - 0.00000832$$

$$y = 0.28720.$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Numerical Differentiation and Integration

Derivatives from Difference tables – divided differences and Finite Differences

Newton's forward difference formula

Newton's forward difference interpolation formula is

$$y(x_0+uh) = y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

where $y(x)$ is a polynomial of degree n in x and $u = \frac{x-x_0}{h}$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots\right]$$

Newton's backward difference formula

Newton's backward difference formula is

$$y(x) = y(x_n+vh) = y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

where $v = \frac{x-x_0}{h}$

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \cdots\right]$$

Derivative using Stirling formula

The Stirling's formula is

$$y(x) = y_0 + \frac{u}{2}\left[\Delta y_0 + \Delta y_{n-1}\right] + \frac{u^2}{2}\Delta^2 y_{-1} + \frac{u^3-u}{12}\left[\Delta^3 y_{-1} + \Delta^3 y_{-2}\right].$$
$$+ \frac{u^4-u^2}{24}\Delta^4 y_{-2} + \cdots$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

where $u = \frac{x-x_0}{h}$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\frac{1}{2}(\Delta y_0 + \Delta y_{-1}) - \frac{1}{12}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \frac{1}{60}(\Delta^5 y_{-2} + \Delta^5 y_{-3}) + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{1}{12}\Delta^4 y_{-2} + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\frac{1}{2}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \cdots\right]$$

Derivative using Bessel's formula

$$y(x) = \frac{1}{2}[y_0 + y_1] + (u - \tfrac{1}{2})\Delta y_0 + \frac{u(u-1)}{4}(\Delta^2 y_{-1} + \Delta^2 y_0)$$
$$+ \frac{u(u-\frac{1}{2})(u-1)}{6}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{48}(\Delta^4 y_{-2} + \Delta^4 y_{-1})$$

$$y'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{4}(\Delta^2 y_{-1} + \Delta^2 y_0) + \frac{(3u^2 - 3u + \frac{1}{2})}{6}\Delta^3 y_{-1} + \cdots\right]$$

Maxima and minimal of a tabulated function

For maxima or minima $\frac{dy}{dx} = 0$ Hence equating the right hand side of (1) to zero and retaining only upto third differences we obtain

$$\Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{6}\Delta^3 y_0 = 0$$

(ie) $(\frac{1}{2}\Delta^3 y_0)u^2 + (\Delta^2 y_0 - \Delta^2 y_0)u + (\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{2}\Delta^3 y_0) = 0$

Substituting the values of $\Delta y_0, \Delta^2 y_0, \Delta^3 y_0$ from the difference table, we solve this quadratic form. Then the corresponding values of x are given by $x = x_0 + uh$ at which y is maximum (or) minimum.

Problems:-

① Find $f'(3)$ and $f''(3)$ for the following data:

x :	3.0	3.2	3.4	3.6	3.8	4.0
f(x) :	-14	-10.032	-5.296	-0.256	6.672	14

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Solution:

Since we require $f'(3)$ and $f''(3)$ we use Newton's forward formula.

Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
3.0	-14					
		3.968				
3.2	-10.032		0.768			
		4.736		-0.464		
3.4	-5.296		0.304		2.048	
		5.04		1.584		-5.12
3.6	-0.256		1.888		-3.072	
		6.928		-1.488		
3.8	6.672		0.4			
		7.328				
4.0	14					

By Newton's forward formula

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \cdots\right]$$

Here $h = 0.2$

$$= \frac{1}{0.2}\left[3.968 - \frac{1}{2}(0.768) + \frac{1}{3}(-0.464) - \frac{1}{4}(2.048) + \frac{1}{5}(-5.12)\right]$$

$$= \frac{1}{0.2}\left[3.968 - 0.384 - 0.1547 - 0.512 - 1.024\right]$$

$$= \frac{1}{0.2}\left[1.8933\right]$$

$$= 9.4665$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\right]$$

$$= \frac{1}{(0.2)^2}\left[0.768 - (-0.464) + \frac{11}{12}(2.048) - \frac{5}{6}(-5.12)\right]$$

$$= \frac{1}{0.04}\left[0.768 + 0.464 + 1.8773 + 4.267\right]$$

$$= \frac{1}{0.04}\left[7.3763\right]$$

$$= 184.4075$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

② Compute $f'(0)$ and $f''(4)$ from the data.

x	0	1	2	3	4
y	1	2.718	7.381	20.086	54.598

Solution:-

Since we require $f'(0.5)$ and $f''(3.5)$ we use Newton's forward formula and Newton's backward formula.

Difference table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		1.718			
1	2.718		2.945		
		4.663		5.097	
2	7.381		8.042		8.668
		12.705		13.765	
3	20.0826		21.807		
		34.512			
4	54.598				

By Newton's forward formula

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{1}\left[1.718 - \frac{1}{2}(2.945) + \frac{1}{3}(5.097) - \frac{1}{4}(8.668)\right]$$

$$= \left[1.718 - 1.4725 + 1.699 - 2.167\right]$$

$$= -0.2225$$

By Newton's backward difference formula

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \left(\frac{dy}{dx}\right)_{v=0} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$= \frac{1}{1}\left[(34.512) + \frac{1}{2}(21.807) + \frac{1}{3}(13.765) + \frac{1}{4}(8.668)\right]$$

$$= 34.512 + 10.9035 + 4.588 + 2.167$$

$$= 52.1705$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right]$$

$$= \frac{1}{1}\left[21.807 + 13.765 + \frac{11}{12}(8.668)\right]$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$= 21.807 + 13.765 + 7.9457$

$= 43.5177 /ms$

③ Find the maximum and minimum value of y tabulated below.

x	-2	-1	0	1	2	3	4
y	2	-0.25	0	-0.25	2	15.75	56

Solution:

Newton's forward difference formula is

$$y(x) = \frac{1}{h}\left[y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots \right]$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0 + \dots \right]$$

Forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
-2	2					
		-2.25				
-1	-0.25		2.5			
		0.25		-3		
0	0		-0.5		6	
		$\boxed{-0.25}$		3		0
1	-0.25		$\boxed{2.5}$		6	
		2.25		$\boxed{9}$		0
2	2		11.5		$\boxed{6}$	
		13.75		15		
3	15.75		26.5			
		40.25				
4	56					

Choosing $x_0 = 0$, $u = \frac{x-0}{1} = x$

$$\frac{dy}{dx} = \frac{1}{1}\left[-0.25 + \frac{(2x-1)}{2}(2.5) + \frac{3x^2-6x+2}{6}(9) + \frac{4x^3-18x^2+22x-6}{24}(6) \right]$$

$$= -0.25 + \frac{(2x-1)}{2}(2.5) + \frac{3x^2-6x+2}{6}(9) + \frac{4x^3-18x^2+22x-6}{24}(6)$$

$$= -0.25 + 2.5x - 1.25 + 4.5x^2 - 9x + 3 + x^3 - 4.5x^2 + 5.5x - 1.5$$

$$\frac{dy}{dx} = x^3 - x$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Now $\frac{dy}{dx} = 0 \Rightarrow x^3 - x = 0$

$$x(x^2-1) = 0$$

$$x=0, \quad x^2-1=0$$

$$x=0, \quad (x-1)\ (x+1) = 0$$

$$x=0, \quad x=1, \quad x=-1$$

$$\frac{d^2y}{dx^2} = 3x^2 - 1$$

at $x=0$, $\frac{d^2y}{dx^2} = -1 = -ve$

$x=1$ $\quad \frac{d^2y}{dx^2} = 3-1 = 2 = +ve$

$x=-1$ $\quad \frac{d^2y}{dx^2} = 3-1 = 2 = +ve$

$\therefore$ y is maximum at $x=0$ minimum at $x=1$ and -1

$\therefore y(x) = \frac{1}{h}\left[y_0 + x\Delta y_0 + \frac{x(x-1)}{2!}\Delta^2 y_0 + \cdots\right]$

$$y(0) = \frac{1}{1}\left[0+0\right] = 0$$

$\therefore$ maximum value $= 0$

$$y(1) = \frac{1}{1}\left[y_0 + \Delta y_0 + 0 + 0 + \cdots\right]$$

$$= \left[0 + (-0.25)\right]$$

$$= -0.25$$

$\therefore$ maximum at $x=1$, $y(1) = -0.25$.

④ Consider the following table of data:

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.9798652	0.9177710	0.8080348	0.6386093	0.3843735

Find $f'(0.25)$ using Newton's forward difference approximation $f'(0.6)$ using Stirling's approximation and $f'(0.95)$ using Newton's backward difference approximation.

Soln:

Here $h = 0.2$

Newton's forward interpolation formula for derivatives

STUCOR APP

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$y'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{4!}\Delta^4 y_0 + \dots\right]$$

where $u = \dfrac{x-x_0}{h}$ $\quad x = 0.25,\ x_0 = 0.2,\ h = 0.2\quad u = \dfrac{0.25-0.2}{0.2} = 0.25$

The difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.2	0.9798652 (y_{-2})				
		−0.0620942 Δy_{-2}			
0.4	0.9177710 (y_{-1})		−0.047642 $\Delta^2 y_{-2}$		
		−0.1097362 Δy_{-1}		−0.0120473 $\Delta^3 y_{-2}$	
0.6	0.8080348 (y_0)		−0.0596893 $\Delta^2 y_{-1}$		−0.01310985 $\Delta^4 y_{-2}$
		−0.1694255 Δy_{-0}		−0.02515715 $\Delta^3 y_{-1}$	
0.8	0.6386093 (y_1)		−0.08484645 $\Delta^2 y_0$		
		−0.25427195 Δy_1			
1.0	0.38433735 (y_2)				

$$y'(0.25) = \frac{1}{0.2}\Big[(-0.0620942) + \frac{(2(0.25)-1)}{2}(-0.047642)$$

$$+ \frac{3(0.25)^2 - 6(0.25)+2}{6}(-0.0120473)$$

$$+ \frac{4(0.25)^3 - 18(0.25)^2 + 22(0.25) - 6}{24}(-0.01310985)\Big]$$

$$= \frac{1}{0.2}\left[-0.0620942 + 0.0119105 - 0.001380419 + 0.000853505\right]$$

$$= \frac{1}{0.2}\left[-0.050710613\right]$$

$$= -0.253553065$$

$= -0.2536$ [correct to four decimal places]

Stirling's formula for derivative is

$u = \dfrac{x-x_0}{h}$, $\quad x = 0.6,\ x_0 = 0.2,\ h = 0.2$

$$u = \frac{0.6-0.2}{0.2} = \frac{0.4}{0.2} = 0.2$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$y'(x) = \frac{1}{h}\left[\frac{\Delta y_0 + \Delta y_{-1}}{2} + u\,\Delta^2 y_{-1} + \frac{3u^2-1}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{4u^3-2u}{4!}\Delta^4 y_{-2} + \cdots\right]$$

$$y'(0.6) = \frac{1}{0.2}\left[\frac{-0.1694255 - 0.1097362}{2} + (0.2)(-0.00596893) + \frac{3(0.2)^2-1}{6}\left[\frac{-0.02515715 - 0.0120473}{2}\right] + \frac{4(0.2)^3-2(0.2)}{24}(-0.01310985)\right]$$

$$= \frac{1}{0.2}\left[-0.13958085 - 0.01193786 + (-0.14667)(-0.01860225) + (-0.01533)(-0.01310985)\right]$$

$$= \frac{1}{0.2}\left[-0.13958085 - 0.01193786 + 0.002728388 + 0.000200974\right]$$

$$= \frac{1}{0.2}\left[-0.148589348\right]$$

$$= -0.74294674$$

$= -0.74295$ correct to five decimal places

Newton's backward difference formula

$$y'(x) = \frac{1}{h}\left[\nabla y_n + \frac{2u+1}{2}\nabla^2 y_n + \frac{2u^2+6u+2}{6}\nabla^3 y_n + \frac{2u^3+9u^2+11u+3}{12}\nabla^4 y_n + \cdots\right]$$

$$u = \frac{x-x_n}{h} = \frac{0.95-1}{0.2} = -0.25$$

$$y'(0.95) = \frac{1}{h}\left[-0.25427195 + \frac{2(-0.95)+1}{2}(-0.08484645) + \frac{2(-0.25)^2+6(-0.25)+2}{6}(-0.02515715) + \frac{2(-0.25)^3+9(-0.25)^2+11(-0.25)+3}{12}(-0.01310985)\right]$$

$$= \frac{1}{0.2}\left[-0.25427195 + (0.25)(-0.08484645) + (0.104167)(-0.02515715) + (0.065104166)(-0.01310985)\right]$$

$$= \frac{1}{0.2}\left[-0.343208273\right]$$

$$= -1.71604$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

③ Find the gradient of the road at the middle point of the elevation above a datum line of seven points of road which are given below:

x	0	300	600	900	1200	1500	1800
y	135	149	157	183	201	205	193

Solution:- we require $\left(\frac{dy}{dx}\right)_{x=900}$

Difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	135						
		14					
300	149		-6				
		8		24			
600	157		-18		-56		
		$\boxed{26}$		$\boxed{-26}$		70	
$\boxed{900}$	183 (y_0)		-8		20		-86
		$\boxed{18}$		$\boxed{-6}$		-16	
1200	201		-14		4		
		4		-2			
1500	205		-16				
		-12					
1800	193						

Since $x = 900$ is in the middle of the table we use Stirling's formula.

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\frac{1}{2}(\Delta y_0 + \Delta y_{-1}) - \frac{1}{12}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \frac{1}{60}(\Delta^5 y_{-2} + \Delta^5 y_{-3}) + \cdots\right]$$

$$= \frac{1}{300}\left[\frac{1}{2}(18+26) - \frac{1}{12}(-6-26) + \frac{1}{60}[70-16]\right]$$

$$= \frac{1}{300}\left[22 + 2.6666 + 0.9\right] = 0.085222$$

Hence the gradient of the road at the middle point is 0.084776.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

⑥ Obtain the value of $f'(0.04)$ using Bessel's formula given in the table below:

x	0.01	0.02	0.03	0.04	0.05	0.06
$f(x)$	0.1023	0.1047	0.1071	0.1096	0.1122	0.1148

Soln:
Since $x = 0.04$ is in the middle of the table we use central difference formula and in particular Bessel's formula.

The central difference table is

x	u	y	Δy	$\Delta^2 y$	$\Delta^3 y$	Δ^4	$\Delta^5 y$
0.01	−3	0.1023					
			0.0024				
0.02	−2	0.1047		0.0			
			0.0024		0.0001		
0.03	−1	0.1071		0.0001		−0.0001	
			0.0025				
0.04	0	0.1096 (y_0)			0.0		0.0
				0.0001		−0.0001	
			0.0026		−0.0001		
0.05	1	0.1122		0.0			
			0.0026	($\Delta^2 y_0$)			
0.06	2	0.1148					

Since $u = \dfrac{x - x_0}{h} = \dfrac{x - 0.04}{0.01}$

Taking $x_0 = 0.04$ as the origin

$y_0 = 0.1096 \quad \Delta y_0 = 0.0026 \quad \Delta y_{-1} = 0.0025, \quad \Delta y_{-2} = 0.0024$

By Bessel's formula

$$y(x_0+uh) = \frac{1}{2}(y_0+y_1)(u-\tfrac{1}{2})\Delta y_0 + \frac{u(u-1)}{4}(\Delta^2 y_{-1} + \Delta^2 y_0)$$
$$+ \frac{u(u-\tfrac{1}{2}(u-1))}{6}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{48}(\Delta^4 y_{-2} + \Delta^4 y_{-1})$$

$$y'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{4}(\Delta^2 y_{-1} + \Delta^2 y_0) + \frac{(3u^2 - 3u + \tfrac{1}{2})}{6}\Delta^3 y_{-1} + \cdots\right]$$

$$y'(x_0) = \frac{1}{0.01}\left[0.0026 - \frac{1}{4}(0+0.0001) + \frac{1}{12}(-0.0001) + \frac{1}{24}(-0.0001)\right]$$
$$= \frac{1}{0.24}\left[24 \times 0.0026 - 0.0006 - 0.0003\right]$$

$f'(0.04) = 0.25625$

DOWNLOADED FROM STUCOR APP
STUCOR APP

DOWNLOADED FROM STUCOR APP

Numerical integration by trapezoidal and simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rules, Romberg's method

Trapezoidal rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

This is known as the trapezoidal rule.

Simpson's one third rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\cdots+y_{n-1})+2(y_2+y_4+\cdots+y_{n-2})\right]$$

This is known as the simpson's one-third rule (or) simply simpson's rule and is most commonly used.

Note:- while applying (3), the given interval must be divided into even number of equal sub-intervals, since we find the area of two strips at a time.

Simpson's three - eight rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\cdots+y_{n-1}) + 2(y_3+y_6+\cdots+y_{n-3})\right]$$

which is known as simpson's three-eight rule.

Note:- while applying (4) the number of sub-intervals should be taken multiple of 3.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Formula.

AT A GLANCE				
Rule	Degree of $y(x)$	No. of intervals	Error	order
Trapezoidal rule	one	any	$\lvert E\rvert < \frac{(b-a)h^2}{12} M$	h^2
Simpson's 1/3 rule	Two	even	$\lvert E\rvert < \frac{(b-a)h^4}{180} M$	h^4
Simpson's 3/8 rule	Three	multiple of 3	$\lvert E\rvert = \frac{3}{8}h^5$	

Problems:-

① using trapezoidal rule evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

Soln:-

Here $y(x) = \frac{1}{1+x^2}$

Length of the interval $= 2$

So we divide 8 equal intervals with $h = 2/8 = 0.25$

we form a table

x :	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y :	0.5	0.64	0.8	0.9412	1	0.9412	0.8	0.64	0.5

Trapezoidal rule,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{h}{2}\left[\text{Sum of the first and last ordinates} + 2(\text{Sum of the remaining ordinates})\right]$$

$$= \frac{0.25}{2}\left[(0.5+0.5)+2(0.64+0.8+0.9412+0.9412+0.8+0.64)\right]$$

$$= \frac{0.25}{2}\left[1+2(5.7624)\right]$$

$$= \frac{0.25}{2}\left[12.5248\right]$$

$$= 1.5656$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

② Evaluate $\int_0^1 \frac{dx}{1+x^2}$ with $h=\frac{1}{6}$ by trapezoidal rule

Soln:-

Here $y(x) = \frac{1}{1+x^2}$, $h = 1/6$

x	0	$\frac{1}{6}$	$2/6$	$3/6$	$4/6$	$5/6$	1
y	1	$\frac{36}{37}$	$9/10$	$4/5$	$9/13$	$\frac{36}{61}$	$1/2$

By Trapezoidal Rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\Big[(\text{sum of the first and last oridinate}) + 2\,(\text{sum of the remaining ordinates})\Big]$$

$$= \frac{(1/6)}{2}\left[\left(1+1/2\right) + 2\left(\frac{36}{37} + 9/10 + 4/5 + 9/13 + \frac{36}{61}\right)\right]$$

$$= \frac{1}{12}\left[3/2 + 2[3.9554]\right]$$

$$= \frac{1}{12}\left[3/2 + 7.9108\right]$$

$$= 0.7842.$$

③ Evaluate the integral $\int_1^2 \frac{dx}{1+x^3}$ using Trapezoidal rule with two sub intervals.

Soln:-

Here $y(x) = \frac{1}{1+x^2}$, $h = 1/2 = 0.5$

x	1	1.5	2
y	0.5	0.3077	0.2

By Trapezoidal rule

$$\int_1^2 \frac{dx}{1+x^3} = \frac{h}{2}\Big[\text{sum of the first and last oridinates} + 2\,(\text{sum of the remaining ordinates})\Big]$$

$$= \frac{0.5}{2}\left[0.5 + 0.2 + 2\,(0.3077)\right]$$

$$= \frac{0.5}{2}\left[0.7 + 0.6154\right]$$

$$= \frac{0.5}{2}\left[1.3154\right] = 0.3289$$

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

④ dividing the range into 10 equal parts, find the value of

$\int_0^{\pi/2} \sin x \, dx$ by (i) Trapezoidal rule

(ii) Simpson's rule

Soln:

x	0	$\frac{\pi}{20}$	$\frac{2\pi}{20}$	$\frac{3\pi}{20}$	$\frac{4\pi}{20}$	$\frac{5\pi}{20}$	$\frac{6\pi}{20}$	$\frac{7\pi}{20}$	$\frac{8\pi}{20}$	$\frac{9\pi}{20}$	$\frac{10\pi}{20}$
$y=\sin x$	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511	0.9877	1

(i) By Trapezoidal rule

$$\int_0^{\pi/2} \sin x \, dx = \frac{h}{2}\left[y_0 + y_{11} + 2(y_1 + y_2 + \cdots + y_{10})\right]$$

$$h = \frac{\pi/2}{10} = \pi/20$$

$$\int_0^{\pi/2} \sin x \, dx = \frac{\pi}{40}(12.7062) = 0.9980$$

(ii) By Simpson's $\frac{1}{3}$ rule

$$\int_0^{\pi/2} \sin x \, dx = (h/3)\left[(y_0 + y_{11}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8 + y_{10})\right]$$

$$= \left(\frac{\pi/20}{3}\right)\left[(0+1) + 4(3.1962) + 2(2.6569)\right]$$

$$= \frac{\pi}{60}\left[1 + 12.7848 + 5.3138\right]$$

$$= \frac{\pi}{60}\left[19.0986\right]$$

$$= 1.0000$$

⑤ using Simpson's one third rule evaluate $\int_0^1 x e^x \, dx$ taking 4 intervals. Compare your result with actual value.

Soln:

x	0	0.25	0.5	0.75	1
$y = xe^x$	0	0.321	0.824	1.588	2.718

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\Big[\text{sum of the first and last even ordinates} + 2(\text{sum of remaining even ordinates}) + 4(\text{sum of odd ordinates})\Big]$$

$$= \frac{0.25}{3}\left[(0+2.718)+2(0.321+1.588)+4(0.824)\right]$$

$$= \frac{0.25}{3}\left[2.718+3.818+3.296\right]$$

$$= \frac{2.458}{3} = 0.819 = 1$$

$$\int_0^1 xe^x\,dx = \int_0^1 x\,d(e^x) = \left[xe^x\right]_0^1 - \int_0^1 e^x\,dx$$

$$= (e^1-0)-(e^x)_0^1$$

$$= e-[e-1] = 1$$

⑮ Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by (i) Trapezoidal rule (ii) Simpson's rule also check up the results by actual integration.

Soln:-

Here $b-a = 6-0 = 6$. Divide into 6 equal parts

$h = 6/6 = 1$. Hence, the table is

x	0	1	2	3	4	5	6
$\frac{1}{1+x^2} = f(x)$	1.00	0.500	0.200	0.100	0.058824	0.038462	0.27027

There are 7 ordinates $(n=6)$. we can use all the formula.

(i) By Trapezoidal rule,

$$I = \int_0^6 \frac{dx}{1+x^2} = \frac{1}{2}\left[(1+0.027027)+2(0.5+0.2+0.1+0.058824+0.038462)\right]$$

$$= 1.41079950$$

(ii) By Simpson's one-third rule,

$$I = \frac{1}{3}\left[(1+0.027027)+2(0.2+0.058824)+4(0.5+0.1+0.038462)\right]$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= \frac{1}{3}\left(1.027027 + 0.517648 + 2.553848\right)$$

$$= 1.36617433$$

(iii) By Simpson's 3/8 Rule.

$$I = \frac{3\times 1}{8}\left[(1+0.027027) + 3(0.5 + 0.2 + 0.058824 + 0.038462) + 2(0.1)\right]$$

$$= 1.35708188$$

(iv) By actual integration

$$I = \int_0^6 \frac{dx}{1+x^2} = \left(\tan^{-1} x\right)_0^6 = \tan^{-1} 6 = 1.40564765$$

Conclusion:- Here the value by trapezoidal rule is closer to the actual value than the value by Simpson's rule.

Romberg's method

① Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using Romberg's method. Hence obtain an approximate value for π.

Solution:

Let $y = \frac{1}{x^2+4}$ and Let $I = \int_0^2 \frac{dx}{x^2+4}$

Take $h = 1$

The tabulated value of y are

x	0	1	2
y	0.25	0.20	0.125

using trapezoidal rule,

$$I_1 = \int_0^2 \frac{dx}{x^2+4} = \frac{h}{2}\left[(y_0+y_2) + 2(y_1)\right]$$

$$= (0.5)\left[(0.25 + 0.125) + 2(0.20)\right]$$

$$= 0.3875$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Take h=0.5 The tabulated values of y are

x	0	0.5	1.0	1.5	2.0
y	0.25	0.2353	0.20	0.160	0.125

using Trapezoidal rule

$$I_2 = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3)\right]$$

$$=(0.25)\left[(0.25+0.125) + 2(0.2353+0.2+0.16)\right]$$

$$= 0.3914$$

Take h=0.25 The tabulated values of y are

x	0	0.50	0.75	1.0	1.25	1.50	1.75	2.00
y	0.25	0.2353	0.2192	0.20	0.1798	0.160	0.1416	0.125

By trapezoidal rule

$$I_3 = \frac{h}{2}\left[(y_0+y_8) + 2(y_1+y_2+\cdots+y_7)\right]$$

$$= \left(\frac{0.25}{2}\right)\left[(0.25+0.125) + 2(0.2462+0.2353+0.2192+0.20+0.1798+0.16+0.1416)\right]$$

$$=(0.125)\left[3.1392\right]$$

$$I_3 = 0.3924$$

using Romberg's formula for I_1 and I_2 we have

$$I = I_2 + \left(\frac{I_2-I_1}{3}\right)$$

$$= 0.3914 + \left(\frac{0.3914-0.3875}{3}\right)$$

$$I = 0.3953 \quad \text{——} ①$$

using Romberg's formula for I_2 and I_3 we have

$$I = I_3 + \left(\frac{I_3-I_2}{3}\right)$$

$$= 0.3924 + \left(\frac{0.3924-0.3914}{3}\right)$$

$$= 0.3927 \quad \text{——} ②$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Since (1) and (2) are not equal we go for one more application of Trapezoidal rule taking $h = 0.125$.

Take $h = 0.125$ The tabulated values are

x	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875
y	0.25	0.249	0.2462	0.2415	0.2353	0.2278	0.2192	0.2098

1.00	1.125	1.250	1.375	1.500	1.625	1.750	1.875	2.000
0.20	0.1899	0.1798	0.1698	0.160	0.1506	0.1416	0.1331	0.125

By Trapezoidal rule

$$I_4 = \frac{h}{2}\left[(y_0 + y_{16}) + 2(y_1 + y_2 + \cdots + y_{15})\right]$$

$$= \left(\frac{0.125}{2}\right)\left[(0.25 + 0.125) + 2(0.249 + 0.2462 + \cdots + 0.1331)\right]$$

$$I_4 = 0.3926$$

Using Romberg's formula for I_3 and I_4 we have

$$I = I_4 + \left(\frac{I_4 - I_3}{3}\right)$$

$$= 0.3926 + \left(\frac{0.3926 - 0.3924}{3}\right)$$

$$I = 0.3927 \quad \text{---} ③$$

Since (2) and (3) are almost equal we can take

$$I = \int_0^2 \frac{dx}{x^2+4} = 0.3927 \quad \text{---} ④$$

By actual integration

$$\int_0^2 \frac{dx}{x^2+4} = \int_0^2 \frac{dx}{x^2+2^2} = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\Big]_0^2$$

$$= \frac{1}{2}\left[\frac{\pi}{4}\right] = \frac{\pi}{8} \quad \text{---} ⑤$$

$\therefore$ From (4) and (5) we get $\frac{\pi}{8} = 0.3927$

$$\therefore \pi \approx 3.1416.$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

③ Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method Correct to 4 decimal places. Hence deduce an approximate value of π.

Solution:

Let $y = \frac{1}{1+x^2}$ and Let $I = \int_0^1 \frac{dx}{1+x^2}$

Take $h=0.5$ The tabulated values of y are

x	0	0.5	1
$y=\frac{1}{1+x^2}$	1	0.8	0.5

using Trapezoidal rule

$$I_1 = \int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_2)+2y_1\right]$$

$$= \frac{0.5}{2}\left[(1+0.5)+1.6\right]$$

$$= 0.775$$

Take $h=0.25$ The tabulated values of y are

x	0	0.25	0.50	0.75	1.00
$y=\frac{1}{1+x^2}$	1	0.9412	0.80	0.64	0.5

using Trapezoidal rule,

$$I_2 = \int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[(1+0.5)+2(0.9412+0.80+0.64)\right]$$

$$= 0.7828$$

Take $h=0.125$ The tabulated values of y are

x	0	0.125	0.25	0.375	0.50	0.625	0.750	0.875	1.0
$y=\frac{1}{1+x^2}$	1	0.9846	0.9412	0.8767	0.80	0.7191	0.64	0.5664	0.5

using Trapezoidal rule

$$I_3 = \int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_8)+2(y_1+y_2+\cdots+y_7)\right]$$

$$= \left(\frac{0.125}{2}\right)\left[(1+0.5)+2(0.9846+0.9412+0.8767+0.8+0.7191+0.64+0.5664)\right]$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= (0.0625)[1.5 + 2(5.528)]$$

$$= 0.78475$$

using Romberg's formula for I_1 and I_2 we have

$$I = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

$$= 0.7828 + \left(\frac{0.7828 - 0.775}{3}\right)$$

$$= 0.7828 + 0.0026$$

$$= 0.7854$$

using Romberg's formula for I_2 and I_3 we have

$$I = I_3 + \left(\frac{I_3 - I_2}{3}\right) = 0.78475 + \left(\frac{0.78475 - 0.7828}{3}\right)$$

$$= 0.78475 + 0.00065$$

$$= 0.7854$$

$$\therefore I = \int_0^1 \frac{dx}{1+x^2} = 0.7854 \quad \text{—①}$$

By actual evaluation of the definite integral we have

$$I = \int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^1 = \pi/4 \quad \text{—②}$$

From (1) and (2) we have $\pi/4 = 0.7854$.

Hence $\pi \approx 3.1416$.

TWO AND THREE POINT GAUSSIAN QUADRATURE FORMULAS

Two Points Gaussian Quadrature - Problems

Formula:

$$\int_{-1}^{1} f(x)\,dx = f\left(-1/\sqrt{3}\right) + f\left(\frac{1}{\sqrt{3}}\right).$$

This formula is exact for polynomials upto degree 3.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

① Apply Gauss two point formula to evaluate $\int_{-1}^{1} \frac{1}{1+x^2} dx$

Soln:

Given interval is -1 to 1 so we apply

$$\int_{-1}^{1} f(x)\,dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) \text{ formula.}$$

Here $f(x) = \frac{1}{1+x^2}$

$$f\left(\frac{-1}{\sqrt{3}}\right) = \frac{1}{1+\frac{1}{3}} = \frac{1}{\left(\frac{4}{3}\right)} = \frac{3}{4}$$

$$f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{1+\frac{1}{3}} = \frac{1}{\left(\frac{4}{3}\right)} = \frac{3}{4}$$

$$\therefore \int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{3}{4} + \frac{3}{4} = \frac{3}{2} = 1.5$$

But actual integration

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_{-1}^{1} = \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \tan^{-1}(1) + \tan^{-1}(1)$$

$$= 2\tan^{-1}(1)$$

$$= 2\,\frac{\pi}{4}$$

$$= \frac{\pi}{2} = 1.5708$$

Here the error due to two-point formula is 0.0708

②. Apply Gauss two-point formula to evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$

Soln:

Given interval is 0 to 1, to make them as -1 to 1

$$\int_{0}^{1} \frac{dx}{1+x^2} = \frac{1}{2}\int_{-1}^{1} \frac{dx}{1+x^2} \quad \left[\because \frac{1}{1+x^2} \text{ is an even function}\right]$$

$$= \frac{1}{2}[1.5] \quad [\text{by first problem}]$$

$$= 0.75$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

③ Using Gaussian two-point formula evaluate

(i) $\int_{-1}^{1} (3x^2 + 5x^4)\, dx$ (ii) $\int_{0}^{1} (3x^2 + 5x^4)\, dx$

Solution:-

(i) Given interval is -1 to 1

Hence we can apply the formula

$$\int_{-1}^{1} f(x)\, dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

Here $f(x) = 3x^2 + 5x^4$

$$f\left(-\frac{1}{\sqrt{3}}\right) = 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{9}\right) = 1 + \frac{5}{9} = \frac{14}{9} = 1.556$$

$$f\left(+\frac{1}{\sqrt{3}}\right) = 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{9}\right) = 1 + \frac{5}{9} = \frac{14}{9} = 1.556$$

$$\int_{-1}^{1} (3x^2 + 5x^4)\, dx = (1.556 + 1.556) = 3.112$$

(ii) Given interval is 0 to 1, so to make them as -1 to 1

Soln:- $$\int_{0}^{1} (3x^2 + 5x^4)\, dx = \frac{1}{2} \int_{-1}^{1} (3x^2 + 5x^4)\, dx$$

[∵ $3x^2 + 5x^4$ is an even function]

$$= \frac{1}{2} [3.112] = 1.556$$

④ Evaluate $\int_{-2}^{2} e^{-x/2}\, dx$ by Gauss two point formula.

Soln:-

Given The range is not $(-1, 1)$ so by using the formula to make them as $(-1, 1)$

$$x = \frac{b-a}{2} z + \frac{b+a}{2}.$$ Here $a = -2$; $b = 2$

$$x = \frac{2+2}{2} z + \frac{2-2}{2}$$

$$x = 2z \Rightarrow z = x/2$$

$$dx = 2dz$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$\int_{-2}^{2} e^{-x/2} dx = \int_{-1}^{1} e^{-z}(2dz)$$

$$= 2\int_{-1}^{1} e^{-z} dz$$

$$= 2\left[f(-1/\sqrt{3}) + f(1/\sqrt{3})\right]$$

Here $f(z) = e^{-z}$

$$f(-1/\sqrt{3}) = e^{1/\sqrt{3}} = 1.7813$$

$$f(1/\sqrt{3}) = e^{-1/\sqrt{3}} = 0.5614$$

$$= 2[0.5614 + 1.7813]$$

$$= 4.6854$$

Three points Gaussian quadrature

Formula $\int_{-1}^{1} f(x) dx = 5/9\left[f(-\sqrt{3/5}) + f(\sqrt{3/5})\right] + 8/9 f(0)$

This formula is exact for polynomials upto degree 5.

① using Gaussian three - point formula evaluate

(i) $\int_{-1}^{1} (3x^2 + 5x^4) dx$

(ii) $\int_{0}^{1} (3x^2 + 5x^4) dx$

also compare with exact values

Soln:-

Let $f(x) = 3x^2 + 5x^4$ [Range given is exact form]

$f(0) = 0$

$$f(-\sqrt{3/5}) = 3(3/5) + 5(3/5)^2 = 9/5 + 9/5 = 18/5$$

$$f(\sqrt{3/5}) = 3(3/5) + 5(3/5)^2 = 9/5 + 9/5 = 18/5$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$\therefore \int_{-1}^{1} f(x)\,dx = \frac{5}{9}\left[f\left(-\sqrt{\tfrac{3}{5}}\right) + f\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9} f(0)$$

$$= \frac{5}{9}\left[\frac{18}{5} + \frac{18}{5}\right] + 0$$

$$= \frac{5}{9}\,\frac{36}{5} = 4 \quad -①$$

Exact value

$$\int_{-1}^{1} (3x^2+5x^4)\,dx = 2\int_{0}^{1}(3x^2+5x^4)\,dx \quad [\because 3x^2+5x^4 \text{ is an even function}]$$

$$= 2\left[\frac{3x^3}{3} + \frac{5x^5}{5}\right]_0^1$$

$$= 2\left[x^3 + x^5\right]_0^1$$

$$= 2\left[(1+1) - (0+0)\right]$$

$$= 4$$

we get exact value by using Gaussian three-point formula

(ii) $\int_{0}^{1} (3x^2+5x^4)\,dx$ [The range is not exact form]

$$\int_{0}^{1} (3x^2+5x^4)\,dx = \frac{1}{2}\int_{-1}^{1}(3x^2+5x^4)\,dx \quad [\because 3x^2+5x^4 \text{ is an even function}]$$

$$= \frac{1}{2}[4] = 2 \quad [\text{by } (1)]$$

② using three-point Gaussian quadrature formula, evalute

(i) $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$

(ii) $\int_{0}^{1} \frac{1}{1+t^2}\,dt$

Soln: Let $f(x) = \frac{1}{1+x^2}$ [Range given is exact form]

$$f(0) = \frac{1}{1+0} = 1$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$f\left(-\sqrt{3/5}\right) = \frac{1}{1+3/5} = \frac{1}{8/5} = 5/8$$

$$f\left(-\sqrt{3/5}\right) = \frac{1}{1+3/5} = \frac{1}{8/5} = 5/8$$

Three-point Gaussian quadrature formula is

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = 5/9\left[f\left(-\sqrt{3/5}\right) + f\left(\sqrt{3/5}\right)\right] + 8/9\, f(0)$$

$$= 5/9\left[5/8 + 5/8\right] + 8/9\,(1)$$

$$= 5/9\left[2(5/8)\right] + 8/9$$

$$= \frac{50}{72} + 8/9 = \frac{19}{12} = 1.5833 \quad -①$$

Actual value

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = 2\int_{0}^{1} \frac{1}{1+x^2}\,dx \left[\frac{1}{1+x^2} \text{ is an even formula}\right]$$

$$= 2\left[\tan^{-1} x\right]_0^1$$

$$= 2\left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$= 2\left[\pi/4\right]$$

$$= \pi/2 = 1.5708$$

(ii) Range given is not exact form

$$\therefore \int_{0}^{1} \frac{1}{1+t^2}\,dt = 1/2 \int_{-1}^{1} \frac{1}{1+t^2}\,dt \left[\because \frac{1}{1+t^2} \text{ is an even function}\right]$$

$$= 1/2\left[1.5833\right] = 0.79165$$

③ Evaluate $\int_{0}^{2} \frac{x^2+2x+1}{1+(x+1)^4}\,dx$ by Gaussian three point formula

Soln:-

Let $f(x) = \frac{x^2+2x+1}{1+(x+1)^4}$ [Range given is not in exact form]

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Let $x = \frac{b-a}{2} z + \frac{b+a}{2}$ $[a=0, b=2]$

$$= \frac{2-0}{2} z + \frac{2+0}{2}$$

$x = z+1$ | $x = 0 \Rightarrow z = -1$

$dx = dz$ | $x = 2 \Rightarrow z = 1$

$$\int_0^2 \frac{x^2+2x+1}{1+(x+1)^4} dx = \int_{-1}^{1} \frac{(z+1)^2 + 2(z+1)+1}{1+[(z+1)+1]^4} dz$$

$$= \int_{-1}^{1} \frac{z^2+2z+1+2z+2+1}{1+(z+2)^4} dz$$

$$= \int_{-1}^{1} \frac{z^2+4z+4}{(z+2)^4+1} dz \quad \text{—} ①$$

[Range given is in exact form]

$$\therefore f(z) = \frac{z^2+4z+4}{(z+2)^4+1}$$

$$f(z) = \frac{(z+2)^2}{(z+2)^4+1}$$

$$f(0) = \frac{2^2}{2^4+1} = \frac{4}{17}$$

$$f\left[-\sqrt{3/5}\right] = \frac{\left[-\sqrt{3/5}+2\right]^2}{\left[-\sqrt{3/5}+2\right]^4+1} = \frac{1.50161}{3.2548} = 0.4614$$

$$f\left[+\sqrt{3/5}\right] = \frac{\left[\sqrt{3/5}+2\right]^2}{\left[\sqrt{3/5}+2\right]^4+1} = \frac{7.69839}{60.2652} = 0.12774$$

$$\therefore (1) \Rightarrow \int_{-1}^{1} f(z)\,dz = \frac{5}{9}\left[f\left(-\sqrt{3/5}\right) + f\left(\sqrt{3/5}\right)\right] + \frac{8}{9} f(0)$$

$$= \frac{5}{9}\left[0.4614 + 0.12774\right] + \frac{8}{9}\left[\frac{4}{17}\right]$$

$$= 0.3273 + 0.2092 = 0.5365$$

$$\therefore (1) \Rightarrow \int_0^2 \frac{x^2+2x+1}{1+(x+1)^4} dx = \int_{-1}^{1} \frac{z^2+4z+4}{(z+2)^2+1} dz = 0.5365$$

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOUBLE INTEGRALS USING TRAPEZOIDAL AND SIMPSON'S RULES

Trapezoidal rule

$$I = \frac{hk}{4}\left[\text{sum of the values of } f(x,y) \text{ at the four corner points}\right]$$

Simpson's rule for double integration

$I = \frac{hk}{9}\Big[$(sum of the values of f at the four corners)

+2 (sum of the values of f at the odd positions on the boundary except the corners)

+4 (sum of the values of f at the even positions on the boundary)

+{4 (sum of the values of f at odd poistions)

8 (sum of the values of f at even Poistions) on the odd rows of the matrix except boundary rows}

+ 8{(sum of the values of f at the odd poisitions)

+16 (sum of the values of f at the even poisitions) on the even rows of the matrix)}$\Big]$

problems:-

① Evaluate $\int_1^{1.4}\int_2^{2.4} \frac{1}{xy}\,dx\,dy$ using Trapezoidal and Simpson's rule verify your result by actual integration.

Solution:-

Divide the range of x and y into 4 equal parts.

$$h = \frac{2.4-2}{4} = 0.1 \quad \text{and} \quad k = \frac{1.4-1}{4} = 0.1$$

Get the values of $f(x,y) = \frac{1}{xy}$ at nodal points.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

y/x	2	2.1	2.2	2.3	2.4
1	0.5	0.4762	0.4545	0.4348	0.4167
1.1	0.4545	0.4329	0.4132	0.3953	0.3788
1.2	0.4167	0.3968	0.3788	0.3623	0.3472
1.3	0.3846	0.3663	0.3497	0.3344	0.3205
1.4	0.3571	0.3401	0.3247	0.3106	0.2976

Case (i): By Trapezoidal Rule, we get

$I = \frac{hk}{4}$ [(sum of values of f at the four corners

+ 2 (sum of values of f at the remaining nodes on the boundary) + 4 (sum of the values of f at the interior nodes)]

$$= \frac{(0.1)(0.1)}{4}\left[(0.5) + 0.4167 + 0.3571 + 0.2976\right]$$

$$+2\left[0.3846 + 0.4167 + 0.4545 + 0.4762 + 0.4545 + 0.4348 + 0.3788 + 0.3472 + 0.3205 + 0.3106 + 0.3247 + 0.3401\right]$$

$$+4\left[0.4329 + 0.4132 + 0.3953 + 0.3968 + 0.3788 + 0.3623 + 0.3663 + 0.3497 + 0.3344\right]$$

$$= \frac{0.01}{4}\left[1.5714 + 9.2864 + 13.7188\right]$$

$$= 0.0614$$

Case (ii) : By Simpson's rule

$I = \frac{hk}{9}$ [(sum of the values of f at the four corners)

+2(sum of the values of f at the odd positions on the boundary except the corners)

+4 (sum of the values of f at the even poisitions on the boundary)

+ {4 (sum of the values of f at odd positions)

+ 8 (sum of the values of f at even positions) on the odd row of the matrix except boundary rows}

+{8 (sum of values of f at the odd positions)

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

+16 (sum of values of f at the even positions) on the even rows of the matrix}]

$$= \frac{(0.1)(0.1)}{9}\Big[(0.5+0.4167+0.3571+0.2976)$$

$$+2(0.4167+0.4545+0.3472+0.3247)$$

$$+4(0.3846+0.4545+0.4762+0.4348+0.3788+0.3205+0.3106+0.3401)$$

$$+4(0.3788)$$

$$+8(0.3968+0.3623)$$

$$+8(0.3497+0.4132)$$

$$+16(0.3663+0.3344+0.4329+0.3953)\Big]$$

$$= \frac{0.01}{9}[55.2116] = 0.0657$$

Case (3) :- By actual integration

$$\int_1^{1.4}\int_2^{2.4}\frac{1}{xy}\,dx\,dy = \left(\int_1^{1.4}\frac{1}{y}\,dy\right)\left(\int_2^{2.4}\frac{1}{x}\,dx\right)$$

$$= (\log y)_1^{1.4}\,(\log x)_2^{2.4}$$

$$= (\log 1.4)[\log 2.4 - \log 2]$$

$$= \log(1.4)\,\log(1.2)$$

$$= 0.0613.$$

we get the actual value and the value by simpson's rule are equal while the value by trapezoidal rule differs only by 0.0001.

② Evaluate $\int_0^2\int_0^2 f(x,y)\,dx\,dy$ by Trapezoidal rule for the following data.

y/x	0	0.5	1	1.5	2
0	2	3	4	5	5
1	3	4	6	9	11
2	4	6	8	11	14

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Solution:

Here $h = 0.5$

$k = 1$

$$I = \int_0^2 \int_0^2 f(x,y)\, dx\, dy$$

$I = \frac{hk}{4}$[(sum of values of f at the four corners)

+ 2 (sum of the values of f at the remaining nodes on the boundary)

+ 4 (sum of the values of f at the interior nodes)]

$$= \frac{(0.5)(1)}{4}\left[(2+5+14+4)+2(3+3+4+5+11+11+8+6)+4(4+6+9)\right]$$

$$= \frac{(0.5)(1)}{4}\left[25+2(51)+4(19)\right]$$

$$=(0.125)[203]$$

$$= 25.375$$

③ using simpson's $\frac{1}{3}$ rule evaluate $\int_0^1 \int_0^1 \frac{1}{1+x+y}\, dx\, dy$ taking

$h = k = 0.5$

Soln:

y/x	0	0.5	1
0	1	0.6667	0.5
0.5	0.6667	0.5	0.4
1	0.5	0.4	0.3333

Simpson's rule:

$I = \frac{hk}{9}$[(sum of the values of f at the four corners)

+ 2 (sum of the values of f at the odd positions on the boundary except the corners)

+ 4 (sum of the values of f at the even positions on the boundary)

+{4 (sum of the values of f at odd positions)

+ 8 (sum of the values of f at even positions) on the odd row of the matrix except boundary rows}

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

+ {8 (sum of the values of f at the odd positions)

+16 (sum of the values of f at the even positions) on theeven rows of the matrix}]

$$I = \frac{(0.5)(0.5)}{9}\Big[(1+0.5+0.3333+0.5) + 2(0) + 4(0.6667+0.6667+0.4+0.4) + \{4(0)+8(0)\} + \{8(0)+16(0.5)\}\Big]$$

$$= (0.02778)\big[(2.3333) + 4(2.1334) + 8\big]$$

$$= (0.02778)(18.8669)$$

$$= 05241$$

④ Evaluate $\int_1^2\int_1^2 \frac{dx\,dy}{x^2+y^2}$ numerically with $h=0.2$ along x-direction and $k=0.25$ along y-direction.

Solution:-

y/x	1	1.2	1.4	1.6	1.8	2
1	0.5	0.4098	0.3378	0.2809	0.2359	0.2
1.25	0.3902	0.3331	0.2839	0.2426	0.2082	0.1798
1.5	0.3077	0.2710	0.2375	0.2079	0.1821	0.16
1.75	0.2462	0.2221	0.1991	0.1779	0.1587	0.1416
2	0.2	0.1838	0.1679	0.1524	0.1381	0.125

By Trapezoidal rule

$$\int_1^2\int_1^2 \frac{1}{x^2+y^2}\,dx\,dy = \frac{hk}{4}\Big[\text{sum of values of } f \text{ at the four corners}$$

+ 2 (sum of the values of f at the remaining nodes on the boundary)

+4 (sum of the values of f at the interior nodes)]

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= \frac{(0.2)\,(0.25)}{4}\Big[(0.5 + 0.2 + 0.125 + 0.2) + 2\,(0.2462 + 0.3077$$

$$+ 0.3902 + 0.4098 + 0.3378 + 0.2809 + 0.2359 + 0.1798 + 0.16$$

$$+ 0.1416 + 0.1381 + 0.1524 + 0.1679 + 0.1838)$$

$$+ 4\,(0.3331 + 0.2839 + 0.2426 + 0.2082 + 0.2710 + 0.2375$$

$$+ 0.2079 + 0.1821 + 0.2221 + 0.1991 + 0.1779 + 0.1587$$

$$+ 0.1838 + 0.1679 + 0.1524 + 0.1381)\Big]$$

$$= (0.0125)\,[1.025 + 6.6566 + 13.4652]$$

$$= 0.2643$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Unit – IV

Designs of Experiments

Introduction:

The sequence of steps taken to ensure a scientific analysis leading to valid inferences about the hypothesis is called Design of Experiment.

For eg., to verify the claim that a particular manure causes increase in the yield of paddy, we may conduct an agricultural experiment. In this experiment the quantity of manure used and the quantity of yield are two variables involved directly. These variables are called **Experimental variables.** There may be other variables such as the fertility of soil, the amount of rainfall, the inherent quality of seed etc. which also effect the yield . These are called **extraneous variables** as far as the hypothesis is concerned.

The Prime objective of design of experiment is to control the extraneous variables so that the results could be attributed only to the experimental variables.

Basic Principles of Design of Experiment

There are three basic principles in designing of experiments.

1.Randomization
2.Replication
3.Local control

Randomization

Random assignment of treatment to the experimental units. Most effective way of eliminating any unknown bias in the experiment.

Replication

Replication is the process of repeating the same treatment on more than one of the experimental units. Two identically treated plots will not give identical results. The differences are attributed to uncontrollable random causes, Such differences are called experimental errors. As the number of replication increases this error is reduced. So,replication is necessary to increase the accuracy of estimates of the treatment effects.

Local Control

Another way of controlling the effects of extraneous variables is by employing the principle of **local control.** It consists of techniques of grouping blocking and balancing of the experimental units.

Grouping means combining sets of homogenous experimental units into groups so that different groups may be subjected to different treatments. Each group can have different number ofexperimental units.

Blocking :Blocking means assigning the same number of plots or experimental units to different groups, called blocks. The plots in the same block are relatively similar or homogeneous. We may use at random

STUCOR APP

DOWNLOADED FROM STUCOR APP

different manures to different plots in the block,

Balancing: Balancing means act of equalizing total effect of the extraneous variables on all the elements in the controlled group and in the experimental group.

Basic Designs of Experiments;

Depending on the number of extraneous variables whose effects are to be controlled ,various designs procedures are developed in the study of experimental design. We shall consider here three important designs.

1.Completely Randomized Designs(CRD)
2 Randomized Block design (RBD)
3.Latin Square Design. (LSD)

Completely Randomized Design

In a completely randomized design the treatments are given to the experimental units by a procedure of random allocation. It is used when the units are homogeneous. Suppose there are five manures and twenty plots we shall give a random allocation of treatments as below.

Write the numbers of the plots 1,2,3,4....20 in identical cards and shuffle well Name the manures or treatments as A,B,C,D,E.

Choose four cards at random. The plots bearing these numbers may be given manure A. Shuffle the remaining cards well and choose another four cards at random. The plots having these number may be given manure B and so on. This is called completely Randomized Design. In this design there is only one factor namely treatment.

Randomized Block Design

Suppose we want to test the effect of r fertilizers and the yield of paddy. We divide the plots into h blocks. Each block is relatively homogeneous and each block contains r plots. Within each block the plots are selected at random and the r treatments(fertilizers) are given. Thus in each block only one plot receives one fertilizers. This is repeated for all the H blocks. This design is called randomized block design. The basic idea in this design is to compare all treatments effects within a block of experimental unit, reducing the errors due to extraneous factors by the process of randomization. The data collected from experiments with randomized block design from a two way classification, classified according to two factors, blocks and treatments.

Latin square Design

In this design, the experimental units are divided accordingly to two factors and are arranged in rows and columns as a n X n square, known as latin square. Let there be n treatments each repeated n times so that each row and each column receives only one treatment. The treatment to a plot is decided randomly. Latin square design is used in a wide variety of fields. Latin square model is effective only if one of the factors has substantial

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

influence on the other. If it is not so, it is not an improvement of the randomized block design. Data from Latin square experiment formed a three way classification according to the factors rows,columns and treatment.

Analysis of Variance.(ANOVA)

Analysis of variance is a technique which enables us to test the significance of the differences among more than two samples means.

In order to use analysis of variance we make the following assumptions.

1. The samples are drawn from normal populations.
2. The samples are independently drawn from these populations.
3. All the populations have the same variance.

We consider the following types of ANOVA

1.One way classification for CRD.

2 Two way classification for RBD.

3.Three factor ANOVA for LSD.

One way classification –ANOVA Table.

Source of variation	Sum of Squares(SS)	Degress of Freesom(DF)	Mean Square(MS)	Variance ratio
Between samples	SSB	r-1	MSB=$\frac{SSB}{r-1}$	F=$\frac{MSB}{MSW}$ (or) F=$\frac{MSW}{MSB}$
Within samples	SSW	N-r	MSW=$\frac{SSW}{N-r}$	
Total	SST	N-1		

Since value of F should be greater than1,we take the larger of MSB or MSW to the numerator.

If the calculated value of F < the table value of F then H_0 is accepted at 5 % level of significance.

STEPS;

To construct the ANOVA table we adopt the following short cur procedure.

1.Find the total number of observations N.

2.Find the total value of all the observations T.

3.Find the Correction factor $\frac{T^2}{N}$

4.Calculate SST = $\sum_j \sum x_{ij}^2 - \frac{T^2}{N}$

5.Calculate SSB = $\frac{\sum {x_1}^2}{n_1} + \frac{\sum {x_2}^2}{n_2} + \cdots + \frac{\sum {x_r}^2}{n_r}$ -$\frac{T^2}{N}$

where $\sum x_1 = total\ of\ all\ values\ in\ sample\ 1$

$\sum x_2 = total\ of\ all\ values\ in\ sample\ 2\ and\ so\ on.$

6. SSW = SST- SSB

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

7.Find F=

$F=\frac{MSB}{MSW}$ if MSB>MSW

(or)

$F=\frac{MSW}{MSB}$ if msw>msb

8.Find the table value of F for (r-1,N-r) df or for (N-r,r-1)df at 5 % level of significance.

Conclusion: If the computed value of F< the table value of F we accept H_0 other wise reject H_0.

PROBLEMS

1. Define Mean sum of squares.

Ans :

The sum of square divided by its degrees of freedom gives the corresponding variance or the mean sum of squares (M.S.S). Thus

$$\frac{S_i^2}{(k-1)} = \frac{S.S.T}{(k-1)} = s_t^2 \text{ (say) is the M.S.S. due to treatments.}$$

$$\text{And } \frac{S_E^2}{(N-k)} = \frac{S.S.E}{(N-k)} = s_E^2 \text{ (say) is the M.S.S. due to error.}$$

2. What are the advantages of a CRD ?

Ans :

The following are the main advantages of this type of designs :

1. It is easy to lay out the design.
2. It allows for complete flexibility. Any number of factor classes and replications may be used.
3. The statistical analysis is relatively simple, even if we do not have the same number errors are not the same from class to class of this factor.
4. The method of analysis remains simple when data are missing or rejected and the loss of information due to missing data is smaller than with any other design.

3. State the assumptions involved in ANOVA.

Solution:

1.Normality 2. Homogeneity 3. Square Independence of error.

4. What are the advantages of a Latin square design?

Solution:

1.With a two way stratification or grouping, the Latin Square controls more of the variations than the completely randomized design or the randomized completely block design. The two way elimination of variations often results in small error mean square.

2. The analysis is simple , it is only slightly more complicated than that for the randomized complete block design.

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

5\. State the basic principles of design of experiments.

Ans :

There are three basic principles of Design of experiment. They are

(i) Randomisation (ii) Replication (iii) Local control (error control)

6\. Define :RBD

Ans :

Randomized block design is a simple design that controls the variability in the experimental units and gives the treatments equivalence to show their effects.

7\. What do you understand by "Design of an experiments" ?

Ans :

The design of experiment may be defined as " the logical construction of the experiment in which the degree of uncertainly with which the inference is drawn may be well defined.

8\. Write down the ANOVA table for one way classification.

Ans :

Analaysis of variance (ANOVA) table : one-way classification model

Sources of variation	Sum of squares	v d.f	Mean square	Variance ratio
Between sample	SSC	$v_1 = c - 1$	MSC = $\frac{SSC}{c-1}$	$F_C = \frac{MSC}{MSE}$
Within samples	SSE	$v_2 = n - c$	MSE = $\frac{SSE}{n-c}$	
Total	TSS	$n - 1$		

TSS = Total sum of squares of variations

SSC = Sum of squares between samples (columns)

SSE = Sum of squares within samples (rows)

MSC = Mean sum of squares between samples

MSE = Mean sum of squares within samples

9\. What is the aim of design of experiments?

Solution:

The design of experiment may be defined as the logical construction of the experiment in which the degree of uncertainty with which the inference is drawn may be well defined.

10\. Define 2^2 factorial design.

Solution:

When there are two factors A,B and two levels 'high' & 'low' for each factor we have a 2^2 factorial design. In spite of its simplicity, the 2^2 design is a powerful tool to improve products and process.

11\. Explain the situations in which randomized block design is considered an improvement over a completely randomized design.

Ans :

1. RBD is more efficient (or) accurate than CRD for most types of experiment.
2. In RBD, no restrictions are placed on number of treatments on the number of

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

replicates.

12. State the advantage of a factorial experiment over a simple experiment.

Ans :

Factorial experiment is the procedure of varying all factors simultaneously. A major conceptual advancement in experimental design is exemplified by factorial design.

In factorial designs, an assessment of each individual factor effect is based on the whole set of measurements so that a more efficient utilization of experimental resources is achieved in these designs.

13. Is a 2 x 2 Latin square design possible ? Why ?

Ans :

Consider, a n x n Latin square design, then the degrees of freedom for SSE is

$$= (n^2 - 1) - (n-1) - (n-1) - (n-1)$$

$$= n^2 - 1 - 3n + 3 \quad = n^2 - 3n + 2$$

$$= (n-1)(n-2)$$

For n=2, d.f. of SSE = 0 and hence, MSE is not defined.

$\therefore$ Comparisons are not possible. Hence, 2 x 2 Latin square design is not possible.

14. State the principles of Design of Experiments.

Solution:

Three important designs are the following:

(i)Completely Randomized Design(CRD)

(ii) Randomized Block Design(RBD)

(iii) Latin Square Design(LSD)

15. Using the 2^2 factorial design, draw the graphical presentation of the following table and the difference in mean without replication term and calculate $(\overline{Y_1} - \overline{Y_0})$, $(\overline{Y_{0.1}} - \overline{Y_{0.0}})$. Discuss when PH = 2 and 3.

	Temp.	PH	Rep 1	Rep 2	Total
1	300	2	10	14	24
a	350	3	21	19	40
b	300	3	17	15	32
ab	350	3	20	24	44

Ans : Here $r = 2$, $n = 4$

$$A = \frac{1}{2n}[a + ab - b - (1)]$$

$$= \frac{1}{2(4)}[40 + 44 - 32 - (24)] = 3.5 \qquad \text{contrast A} = 28 \text{----}(1)$$

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$B \;=\; \frac{1}{2n}[b+ab-a-(1)]$$

$$=\; \frac{1}{2(4)}[32+44-40-24] \;=\; 1.5 \quad \text{contrast} \quad B = \text{`}12\,\text{- - - -}(2)$$

$$AB \;=\; \frac{1}{2n}[ab+(1)-a-b]$$

$$=\; \frac{1}{2(4)}[44+24-40-32] \;=\; -0.5 \quad \text{contrast}\;\; AB = \text{-}\,4\,\text{- - - -}(3)$$

$$SS_A \;=\; \frac{1}{4n}[a+ab-b-(1)]^2 \;=\; \frac{(28)^2}{16} \;=\; 49$$

$$SS_{\mathbf{B}} \;=\; \frac{1}{4n}[b+ab-a-(1)]^2 \;=\; \frac{(12)^2}{16} \;=\; 9$$

$$SS_{\mathbf{AB}} \;=\; \frac{1}{4n}[a\mathbf{b}+(1)-a-b]^2 \;=\; \frac{(-4)^2}{16} \;=\; 1$$

$$SS_T = 10^2+21^2+17^2+20^2+14^2+19^2+15^2+24^2 \;=\; 2588$$

$$SS_E \;=\; SS_T - SS_A - SS_{\mathbf{B}} - SS_{\mathbf{AB}} \;=\; 2588-49-9-1 \;=\; 2529$$

Analysing of variance

S.V	S.S	D.F	M.S.S	Variation ratio	Table value 5 %
A	49	1	49	$F_A = 4.30$	$F_A(12,1) = 243.9$
B	9	1	9	$F_B = 23.42$	$F_B(12,1) = 243.9$
AB	1	1	1	$F_{AB} = 210.75$	$F_{AB}(12,1) = 243.9$
Error	2529	210.75	210.75		

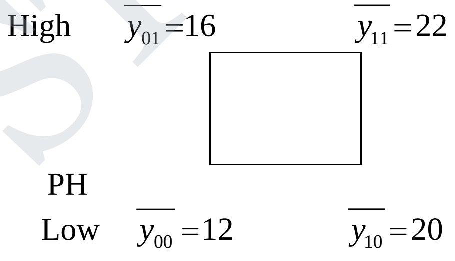
Here, Cal F_A < Table F_A

Cal F_B < Table F_B

Cal F_{AB} < Table F_{AB}

As in each of the cases, the compound value of F is less than the corresponding tabulated (critical) value, there are no significant main or interaction effects present in the experiment. The blocks, as well as treatments, do not differ significantly.

The graphical summary of a 2^2 factorial experiment



It is clear from the figure that changing temperature from low (300^0) to high (350^0) , increase the yield substantially, at both levels of PH. Changing PH has little or no effect.

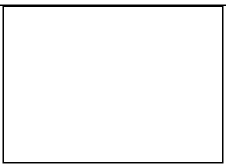
$$\overline{y}_1 - \overline{y}_0 \;=\; \frac{1}{2}(\overline{y}_{11}+\overline{y}_{10}) - \frac{1}{2}(\overline{y}_{01}+\overline{y}_{00}) + \frac{1}{2}(22+20) - \frac{1}{2}(16+12) = 7$$

That is we take the average on the right side and subtract the average on the left side of the square, as indicated in figure.

STUCOR APP

DOWNLOADED FROM STUCOR APP

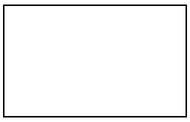
$-\frac{1}{2}(\overline{y_{01}}+\overline{y_{00}})$



$\frac{1}{2}(\overline{y_{11}}+\overline{y_{10}})$

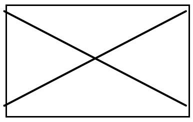
For the second factor, pH we estimate the difference in mean response between the low and high level by

$\frac{1}{2}(\overline{y_{01}}+\overline{y_{11}})$



$-\frac{1}{2}(\overline{y_{00}}+\overline{y_{10}})$

$\frac{1}{2}(\overline{y_{11}}+\overline{y_{00}})$



$-\frac{1}{2}(\overline{y_{10}}+\overline{y_{01}})$

$$\overline{y_1}-\overline{y_0} = \frac{1}{2}(\overline{y_{01}}+\overline{y_{11}})-\frac{1}{2}(\overline{y_{00}}+\overline{y_{10}}) \quad = \frac{1}{2}(16+22)-\frac{1}{2}(12+20)=3 \text{ -----(4)}$$

To estimate the interaction, we mote that $\overline{y_{10}}-\overline{y_{00}}$ gives the increase in yield at pH = 2, whereas $\overline{y_{11}}-\overline{y_{01}}$ gives the increase in yield at pH = 3. The average of these two differences estimates the interaction.

$$\frac{1}{2}(\overline{y_{11}}-\overline{y_{01}})-\frac{1}{2}(\overline{y_{10}}-\overline{y_{00}})=\frac{1}{2}(\overline{y_{11}}-\overline{y_{10}}-\overline{y_{01}}+\overline{y_{00}})$$

$$=\frac{1}{2}(22-20-16+12)=-1 \quad -----(5)$$

From (2) & (4) we get $\overline{y_1}+\overline{y_0} = \frac{B}{2r}$

From (3) & (5) we get $\frac{1}{2}(\overline{y_{11}}+\overline{y_{01}})-\frac{1}{2}(\overline{y_{10}}-\overline{y_{00}}) = \frac{AB}{2r}$

16. The following table shows the live in hours of four brands of electric lamps brand.

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps.

Ans :

H_0 : There is no significant difference between the four brands.

H_1 : There is significant difference between the four brands.

Subtract 1600 and then divided by 10 we get

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

X_1 A	X_2 B	X_3 C	X_4 D	Total	X_1^2	X_2^2	X_3^2	X_4^2
1	-2	-14	-9	-24	1	4	196	81
1	4	-5	-8	-8	1	16	25	64
5	4	0	-7	2	25	16	0	49
8	10	2	-3	17	34	100	4	9
10	15	4	0	29	100	225	16	0
12	-	6	8	26	144	-	36	64
20	-	14	-	34	400	-	196	-
-	-	22	-	22	-	-	484	-
57	31	29	-19	98	735	361	957	267

Step 1 : N = 26

Step 2. T = 98

Step 3. C.F. = $\frac{T^2}{N} = \frac{9604}{26} = 369.39$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 735 + 361 + 957 + 267 - 369.39 \quad = \quad 1950.61$

Step 5. SSC = $\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(57)^2}{7} + \frac{(31)^2}{5} + \frac{(29)^2}{8} + \frac{(-19)^2}{6} - 369.39 = 452.25$

SSE = TSS – SSC = 1950.61 – 452.25 = 1498.336

Step 6. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 452.225	C –1 = 4 – 1 = 3	MSC = $\frac{SSC}{C-1}$ = $\frac{452.25}{3}$ = 150.75	$F_C = \frac{MSC}{MSE}$ $= \frac{150.75}{68.11}$ = 2.21 > 1 Since $\frac{MSE}{MSC} < 1$	$F_C(3, 22)$ = 3.05
Error	SSE = 1498.36	N – C = 26 – 4 = 22	MSE = $\frac{SSE}{N - C}$ = $\frac{1498.36}{22}$		

STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 7 : Conclusion : Cal F_C < Table F_C . So we accept H_0.

17. Three varieties A, B and C of a crop are tested in a randomized block design with four replications. The plot yield in pounds are as follows :

A	6	C	5	A	8	B	9
C	8	A	4	B	6	C	9
B	7	B	6	C	10	A	6

Analyse the experimental yield and state your conclusions.

Ans :

Variety	Block				Total	X_1^2	X_2^2	X_3^2	X_4^2
	1	2	3	4					
A	6	4	8	6	24	36	16	64	36
B	7	6	6	9	28	49	36	36	81
C	8	5	10	9	32	64	25	100	81
Total	21	15	24	24	84	149	77	200	198

H_0 : The varieties are similar

H_1 : The varieties are not similar

Step 1 : N = 12

Step 2. T = 84

Step 3. $C.F. = \frac{T^2}{N} = \frac{(84)^2}{12} = 588$

Step 4. $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 149 + 77 + 200 + 198 - 588 = 36$

Step 5. $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} - 588 = 18$

Step 6. $SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{(24)^2}{3} + \frac{(28)^2}{3} + \frac{(32)^2}{3} - 588 = 8$

SSE = TSS – SSC - SSR = 36 –18 - 8 = 10

Step 7. ANOVA table

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Varieties	SSR = 8	r –1 = 3 – 1 = 2	MSR = $\frac{SSR}{r-1}$ = $\frac{8}{2} = 4$	$F_R = \frac{MSR}{MSE}$ $= \frac{4}{1.667}$ = 2.4	$F_R(2, 6)$ = 5.14
Between Blocks	SSC =18	C –1 = 4 – 1 = 3	MSC = $\frac{SSC}{C-1}$ = $\frac{18}{3} = 6$	$F_C = \frac{MSC}{MSE}$ $= \frac{6}{1.667}$ = 3.59	$F_C(3, 6)$ = 4.76
residual	SSE = 10	N – c – r + 1 = 6	MSE = $\frac{SSE}{N-c-r+1}$ = $\frac{10}{6}$ =1.667		
Total	36				

Step 7 : Conclusion : In both the cases, the calculated value is less than tabulated value. Therefore , null hypothesis is accepted. Hence , the three varieties are similar.

18. Four varieties A, B, C, D of a fertilizer are tested in a RBD with 4 replications. The plot yields in pounds are as follows:

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

Analyse the experimental yield.

Solution:

Let us take 12 as origin for simplifying the calculations

Row	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
(y_1) (1)	A 0	D 8	C 4	B -2	10	0	64	16	4
(y_2) (2)	D 6	A 2	B -1	C 2	9	36	4	1	4
(y_3) (3)	B 0	C 3	D 7	A 1	11	0	9	49	1
(y_4) (4)	C 4	B 1	A 3	D 8	14	16	1	9	64
Total	10	12	13	9	44	52	78	75	73

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = 44

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 3 : C.F $= \dfrac{T^2}{N} = \dfrac{(44)^2}{16} = 121$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \dfrac{T^2}{N}$

$= 52 + 78 + 75 + 73 - 121 = 157$

Step 5 : SSC $= \dfrac{(\Sigma X_1)^2}{N_1} + \dfrac{(\Sigma X_2)^2}{N_1} + \dfrac{(\Sigma X_3)^2}{N_1} + \dfrac{(\Sigma X_4)^2}{N_1} - \dfrac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(10)^2}{4} + \frac{(12)^2}{4} + \frac{(13)^2}{4} + \frac{(9)^2}{4} - 121 = 2.5$$

Step 6. SSR $= \dfrac{(\Sigma Y_1)^2}{N_2} + \dfrac{(\Sigma Y_2)^2}{N_2} + \dfrac{(\Sigma Y_3)^2}{N_2} + \dfrac{(\Sigma Y_4)^2}{N_2} - \dfrac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(10)^2}{4} + \frac{(9)^2}{4} + \frac{(11)^2}{4} + \frac{(14)^2}{4} - 121 = 3.5$$

To Find SSK

Treatment	1	2	3	4	Total
A	0	2	3	1	6
B	0	-1	-1	-2	- 4
C	4	3	4	2	13
D	6	8	7	8	29
					44

SSK $= \dfrac{(6)^2}{4} + \dfrac{(-4)^2}{4} + \dfrac{(13)^2}{4} + \dfrac{(29)^2}{4} - 121 = 144.5$

SSE = TSS – SSC – SSR = 157 – 2.5 – 3.5 – 144.5 = 6.5

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	F test 1%
Between Rows	SSR = 3.5	3	1.17	1.08	9.78
Between columns	SSC = 2.5	3	0.83	0.77	27.91
Variety	SSK = 144.5	3	48.17	44.60	9.78

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Error	SSE = 6.5	6	1.08		
Total	TSS = 157	11			

Step 8 . Conclusion :

The F ratios for rows and columns are not significant at 1 % level while that for varieties is very highly significant. The fact that there are no significant differences between rows and columns. Show that the Latin square arrangement has not been advantageous.

19. A variable trial was conducted on wheat with 4 varieties in a Latin Square design. The plan of the experiment and per plot yield are given below:

D25	B23	A20	D20
A19	D19	C21	B18
B19	A14	D17	C20
D17	C20	B21	A15

Analyse the data.

Solution:

Subtract 20 from all the items

Y_n	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
y_1	5	3	0	0	8	25	9	0	0
y_2	-1	-1	1	-2	-3	1	1	1	4
y_3	-1	-6	-3	0	-10	1	36	9	0
y_4	-3	0	1	-5	-7	9	0	1	25
Total	0	-4	-1	-7	-12	36	46	11	29

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = -12

Step 3 : C.F $= \frac{T^2}{N} = \frac{144}{16} = 9$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 36 + 46 + 11 + 29 - 9 = 113$

Step 5 : SSC $= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(0)^2}{4} + \frac{(4)^2}{4} + \frac{(1)^2}{4} + \frac{(7)^2}{4} - 9 = 7.5$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 6. $$SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$$

[N_2 = number of elements in each row]

$$= \frac{(8)^2}{4} + \frac{(-3)^2}{4} + \frac{(-10)^2}{4} + \frac{(-7)^2}{4} - 9 = 46.5$$

To Find SSK

Treatment	1	2	3	4	Total
A	0	-1	-6	-5	-12
B	3	-2	-1	1	1
C	5	1	0	0	6
D	0	-1	-3	-3	-7

$$SSK = \frac{(-12)^2}{4} + \frac{(1)^2}{4} + \frac{(6)^2}{4} + \frac{(-7)^2}{4} - 9 = 48.5$$

SSE = TSS – SSC – SSR = 113 – 7.5 – 46.5 – 48.5 = 10.5

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	Table value at 5% level
Between Rows	SSR = 46.5	k –1= 3	MSR = $\frac{SSR}{k-1}$ = 15.5	F_R = $\frac{MSR}{MSE}$ = 8.86	$F_R(3, 6)$ = 4.76
Between columns	SSC = 7.5	k – 2 = 3	MSC = $\frac{SSC}{k-1}$ = 2.5	F_C = $\frac{MSC}{MSE}$ = 1.43	$F_C(3, 6)$ = 4.76
Treatment	SSK = 48.5	k – 1 = 3	MSK = $\frac{SSK}{k-1}$ = 16.17	F_T = $\frac{MSK}{MSE}$ = 9.24	$F_T(3, 6)$ = 4.76
Error	SSE = 10.5	(k -1)(k – 2) = 6	MSE = $\frac{SSE}{(k-1)(k-2)}$ = 1.75		
Total	TSS = 113				

Step 8 . Conclusion :

Cal F_R > Table F_R

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Cal F_C < Table F_C

Cal F_T > Table F_T

There is significant difference between treatments and rows. But there is no significant difference between columns.

20. A farmer wishes to test the effect of 4 fertilizers A,B,C,D on the yield of wheat, The fertilizers are used in a LSD and the results are tabulated here perform an analysis of variance.

A18	C21	D25	B11
D22	B12	A15	C19
B15	A20	C23	D24
C22	D21	B10	A17

Solution:

Subtract 20 we get

A -2	C 1	D 5	B -9
D 2	B -8	A -5	C -1
B -5	A 0	C 3	D 4
C 2	D 1	B -10	A -3

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	-2	1	5	-9	-5	4	25	25	81
Y_2	2	-8	-5	-1	-12	4	25	25	1
Y_3	-5	0	3	4	2	25	9	9	16
Y_4	2	1	-10	-3	-10	4	100	100	9
TOTAL	-3	-6	-7	-9	-25	37	66	159	107

H_0: There is no significant difference between rows, columns and treatments.

H_1: There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = -25

Step 3 : $\frac{\boldsymbol{T^2}}{\boldsymbol{N}} = \frac{(-25)^2}{16} = \frac{625}{16} = 39.06$

Step 4 : TSS $= \sum X_1^2 + X_2^2 + X_3^2 + X_4^2 - \frac{T^2}{N}$

$= 37 + 66 + 159 + 107 - 39.06 = 329.94$

Step 5 : SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \frac{(\sum X_4)^2}{N_4} - \frac{T^2}{N}$

$= \frac{(-3)^2}{4} + \frac{(-6)^2}{4} + \frac{(-7)^2}{4} + \frac{(-9)^2}{4} - 39.06$

$(N_1, N\ _2 . N_3, N_4 \textit{ are element in each column})$

$= \frac{9}{4} + \frac{36}{4} + \frac{49}{4} + \frac{81}{4} - 39.06$

$= \frac{175}{4} - 39.06 = 43.75 - 39.06 = 4.69$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 6 : SSR $= \frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \frac{(\sum Y_4)^2}{N_4} - \frac{T^2}{N}$

$$= \frac{(-5)^2}{3} + \frac{(-12)^2}{3} + \frac{(2)^2}{3} + \frac{(-10)^2}{3} - \frac{T^2}{N}$$

$(N_1, N_2 . N_3, N_4 \text{ are element in each column})$

$$= \frac{25+144+4+100}{4} - 39.06 \quad = \frac{273}{4} - 39.06 \quad = \quad 68.25 - 39.06 \quad = \quad 29.19$$

To find SSK:

Arrange the elements in the order of treatment.

					Total
A	-2	-5	0	-3	-10
B	-9	-8	-5	-10	-32
C	1	-1	3	2	5
D	5	2	4	1	12

SSK $= \frac{(-10)^2}{4} + \frac{(-32)^2}{4} + \frac{(5)^2}{4} + \frac{(12)^2}{4} - \frac{T^2}{N}$

$$= \frac{100}{4} + \frac{1024}{4} + \frac{25}{4} + \frac{144}{4} - 39.06 \quad = \frac{1293}{4} - 39.06 \quad = \quad 284.19$$

SSE = TSS – SSC – SSR - SSK

= 329.94 – 4.69 – 29.19 - 284.19 = 11.87

Step 7 : ANOVA Table

Source of Variation	S. S	d.f	MSS	Variance Ratio	Table value 5% level
Between Rows	SSR = 29.19	k-1 = 3	MSR $= \frac{SSR}{K-1} =$ 9.73	$F_R = \frac{MSR}{MSE} = 4.91$	$F_R(3,6) = 4.76$
Between Columns	SSC = 4.69	k-1 = 3	MSC $= \frac{SSC}{K-1} =$ 1.56	$F_C = \frac{MSE}{MSC} = 1.26$	$F_C(6,3) = 8.94$
Between Treatments	SSK = 284.19	k-1 = 3	MSK $= \frac{SSK}{K-1} =$ 94.73	$F_T = \frac{MSK}{MSE} = 4.91$	$F_T(3,6) = 4.76$
Error	SSE = 11.87	(k-1)(k-2) = 6	MSE= $\frac{SSE}{(K-1)(K-2)} =$ 1.98		

Step 8 : Cal F_C < Tabulated F_C

Cal $F_T > Tabulated\ F_T$

Cal $F_R > Tabulated\ F_R$

Therefore We conclude that there is a significant difference between treatments

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

and rows. But, there is no significant difference between columns.

21. A sales of four salesmen in 3 seasons are tabulated here.

Salesmen

Seasons	A	B	C	D
Summer	36	36	21	35
Winter	28	29	31	32
Monsoon	26	28	29	29

Carry out an analysis of variance.

Solution:

The above data are classified according to criteria (i) salesmen and (ii) seasons. In order to simplify calculations, we code the data by subtracting 30 from each figure. The data in the coded form are given below:

SEASONS		Salesmen				Season Total	X_1^2	X_2^2	X_3^2	X_4^2
		A(X_1)	B(X_2)	C(X_3)	D(X_4)					
Y_1	Summer	+6	+6	-9	+5	+8	36	36	81	25
Y_2	Winter	-2	-1	+1	+2	0	4	1	1	4
Y_3	Monsoon	-4	-2	-1	-1	-8	16	4	1	1
Total		0	3	-9	6	Grand total T=0	56	41	83	30

$H_0: There\ is\ no\ significant\ difference\ between\ column\ means\ as\ well\ as\ row\ m$

$H_1: There\ is\ significant\ differnce\ between\ column\ means\ or\ the\ row\ means.$

Step 1 : N = 12

Step 2 : T = 0

Step 3 : $\frac{T^2}{N} = \frac{(0)^2}{N} = 0\ (Number\ of\ items\ or\ N\ is\ 12)$

Step 4 : TSS $= \sum X_1^2 + X_2^2 + X_3^2 + X_4^2 - \frac{T^2}{N}$ = 56 + 41 + 83 + 30 – 0 = 210

Step 5 : SSC $= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_2} + \frac{(\Sigma X_3)^2}{N_3} + \frac{(\Sigma X_4)^2}{N_4} - \frac{T^2}{N}$

$= \frac{(0)^2}{3} + \frac{(3)^2}{3} + \frac{(-9)^2}{3} + \frac{(6)^2}{3} - \frac{T^2}{N}$ = 0 + 3+ 27 + 12 - 0 = 42

Step 6 : SSR $= \frac{(\Sigma Y_1)^2}{N_1} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_3} + \frac{(\Sigma Y_4)^2}{N_4} - \frac{T^2}{N}$

$= \frac{(8)^2}{3} + \frac{(0)^2}{3} + \frac{(-8)^2}{3} + \frac{(6)^2}{3} - \frac{T^2}{N}$ = 16 + 0 + 16 - 0 = 32

SSE = TSS – SSC - SSR = 210 – 42 - 32 = 136

Step 7: Table of Analysis Variance:

Sources of Variation	Sum of squares	D.F	Mean Squares	Variance	Table value at 5%.
Between Columns	SSC = 42	c-1 = 4-1	MSC	$F_C = \frac{MSE}{MSC}$	$F_C(6,3)$ = 8.94

STUCOR APP

DOWNLOADED FROM STUCOR APP

(Salesmen)		= 3	$= \frac{SSC}{C-1} = 14$	$= \frac{22.67}{14} =$ 1.619	
Between rows(seasons)	SSR = 32	r-1 =3 -1 = 2	MSR = $\frac{SSR}{r-1}$ $= \frac{32}{2} = 16$	$F_R = \frac{MSE}{MSR} = \frac{22.67}{16}$	$F_R(6,2)$ = 19.33
Residual	SSE = 136	N-c-r+1 = 12-4-3+1 = 6	MSE = $\frac{SSE}{N-c-r+1}$ $= \frac{136}{6} = 22.67$		
	210	11			

Step 8: Conclusion : Cal F < Table F

Hence there is no significant difference in the seasons as far as the sales are concerned.

Thus the test shows that the salesmen and the seasons are alike, so far as the sales are concerned.

22. The following data represent the number of units of production per day turned out by 5 different workers using 4 different types of machine

		Machine Type			
		A	B	C	D
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(1) Test whether the mean production is the same for the different machine types

(2) Test whether the 5 men differ with mean productivity.

Ans :

The code data is

Row	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
y_1	4	-2	7	-4	5	16	4	49	16
y_2	6	0	12	3	21	36	0	144	9
y_3	-6	-4	4	-8	-14	36	16	16	64
y_4	3	-2	6	-7	0	9	4	36	49
Y_5	-2	2	9	-1	8	4	4	81	1
Total	5	-6	38	-17	20	101	28	326	139

H_0 : (i) the mean productivity is the same for four different machines and

H_1 : (ii) the 5 men do not differ with respect to mean productivity code the data by subtracting 40 from each value.

Step 1 : N = 20

Step 2 : T = 20

Step 3 : C.F $= \frac{T^2}{N} = \frac{(20)^2}{20} = 20$

STUCOR APP

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

= 101 + 28 + 326 + 139 - 20 = 574

Step 5 : SSC = $\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(5)^2}{5} + \frac{(-6)^2}{5} + \frac{(38)^2}{5} + \frac{(-17)^2}{5} - 20 = 338.8$$

Step 6. SSR $= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} + \frac{(\sum Y_5)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(5)^2}{4} + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{(0)^2}{4} + \frac{(8)^2}{4} - 20 = 161.5$$

SSE = TSS – SSC – SSR = 574 –338.8 – 161.5 = 73.7

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	Table value at 5% level
Between Rows	SSR = 161.5	r – 1 = 5-1 = 4	MSR = $\frac{SSR}{k-1}$ = 40.375	$F_R = \frac{MSR}{MSE}$ = 6.574	$F_R(4, 6)$ = 3.26
Between columns	SSC = 338.8	c –1 = 4-1 = 3	MSC = $\frac{SSC}{k-1}$ = 112.933	$F_C = \frac{MSC}{MSE}$ = 18.38	$F_C(3, 12)$ = 3.49
Residual	SSE = 73.7	N - c - r + 1 = 12	MSE = $\frac{SSE}{\text{N - c - r + 1}}$ = 6.142		

Step 8 . Conclusion :

(i) Table F_C (3, 12) at 5 % level = 3.49

Cal F_C > Table F_C . Reject H_0 .

(ii) Table F_R (4, 12) at 5 % level = 3.26

Cal F_R > Table F_R . Reject H_0

∴ The worker differ with respect to mean productivity.

23. The following is a Latin square of design when 4varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You can carry out suitable change of origin and scale.

STUCOR APP

STUCOR APP

A 105	B 95	C 125	D 115
C 115	D 125	A 105	B 105
D 115	C 95	B 105	A 115
B 95	A 135	D 95	C 115

Ans :

Subtract 100 and then divided by 5 we get

A 1	B -1	C 5	D 3
C 3	D 5	A 1	B 1
D 3	C -1	B 1	A 3
B -1	A 7	D -1	C 3

Y_n	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
y_1	1	-1	5	3	8	1	1	25	9
y_2	3	5	1	1	10	9	25	1	1
y_3	3	-1	1	3	6	9	1	1	9
y_4	-1	7	-1	3	8	1	49	1	9
Total	6	10	6	10	32	20	76	28	28

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = 32

Step 3 : C.F $= \frac{T^2}{N} = \frac{(32)^2}{16} = 64$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 20 + 76 + 28 + 28 - 64 = 88$

Step 5 : SSC $= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(6)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(10)^2}{4} - 64 = 4$$

Step 6. SSR $= \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(8)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(8)^2}{4} - 64 = 2$$

To Find SSK

STUCOR APP

DOWNLOADED FROM STUCOR APP

Treatment	1	2	3	4	Total
A	1	1	3	7	12
B	-1	1	1	-1	0
C	5	3	-1	3	10
D	3	5	3	-1	10

$$SSK = \frac{(12)^2}{4} + \frac{(0)^2}{4} + \frac{(10)^2}{4} + \frac{(10)^2}{4} - 64 = 22$$

SSE = TSS – SSC – SSR = 88 – 4 – 2 –22 = 60

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	Table value at 5% level
Between Rows	SSR = 2	k –1 = 3	$MSR = \frac{SSR}{k-1}$ = 0.67	$F_R = \frac{MSR}{MSE}$ = 14.9	$F_R(6, 3)$ = 8.94
Between columns	SSC = 4	k – 2 = 3	$MSC = \frac{SSC}{k-1}$ = 1.33	$F_C = \frac{MSC}{MSE}$ = 7.52	F_C6, 3) = 8.94
Treatment	SSK = 22	k – 1 = 3	$MSK = \frac{SSK}{k-1}$ = 7.33	$F_T = \frac{MSK}{MSE}$ $= \frac{10}{7.33}$ = 1.36	$F_T(6, 3)$ = 8.94
Error	SSE = 60	(k -1)(k – 2) = 6	MSE = $\frac{SSE}{(k-1)(k-2)}$ = 10		
Total	TSS = 88	15			

STUCOR APP

Step 8 . Conclusion :

Cal F_R > Table F_R
Cal F_C < Table F_C
Cal F_T > Table F_T

There is significant difference between treatments and rows. But there is no significant difference between columns.

24. Compare and contrast the Latin square design with the randomized block design.

Ans :

S.No	**LSD**	**RBD**
1.	It is suitable for small number of treatments, between 5 and	No such restrictions suitable for upto 24 treatments.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

	12.	
2.	The number of rows and columns are equal and hence the number of replication is equal to the number of treatments.	There is no such restriction. It can have any number replications and treatments.
3.	Experimental error is reduced to a large extent, because variation is controlled in two directions.	Variations is controlled in one directions only.
4.	LSD is preferred over RBD because of (3)	RBD is the most popular one for its simplicity , flexibility and validity.
5.	Experimental area must be a square.	Suitable if it is a rectangle or square.

25. Analyse the following RBD and find your conclusion.

		Treatments			
		T_1	T_2	T_3	T_4
	B_1	12	14	20	22
	B_2	17	27	19	15
Blocks	B_3	15	14	17	12
	B_4	18	16	22	12
	B_5	19	15	20	14

Solution:

1. H_0 : There is no significant difference between blocks and treatments.
2. H_1 : There is significant difference between blocks and treatments.

Subtract 15 from each number.

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	-3	-1	5	7	8	9	1	25	49
Y_2	2	12	4	0	18	4	144	16	0
Y_3	0	-1	2	-3	-2	0	1	4	9
Y_4	3	1	7	-3	8	9	1	49	9
Y_5	4	0	5	-1	8	16	0	25	1
Total	6	11	23	0	40	38	147	119	68

STUCOR APP

DOWNLOADED FROM STUCOR APP

$step\,1 : N = 20$

$step\,2 : T = 40$

$$step\,3 : \frac{T^2}{N} = \frac{(40)^2}{20} = 80$$

$$step\,4 : TSS = \sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \sum X_4{}^2 - \frac{T^2}{N}$$

$$= 38 + 147 + 119 + 68 - 80 = 292$$

$$step\,5 : SSC = \frac{\left(\sum X_1\right)^2}{N_1} + \frac{\left(\sum X_2\right)^2}{N_1} + \frac{\left(\sum X_3\right)^2}{N_1} + \frac{\left(\sum X_4\right)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{(6)^2}{5} + \frac{(11)^2}{5} + \frac{(23)^2}{5} - 0 - 80 = 57.2$$

$$step\,6 : SSR = \frac{\left(\sum Y_1\right)^2}{N_2} + \frac{\left(\sum Y_2\right)^2}{N_2} + \frac{\left(\sum Y_3\right)^2}{N_2} + \frac{\left(\sum Y_4\right)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{8^2}{4} + \frac{18^2}{4} + \frac{(-2)^2}{4} + \frac{8^2}{4} + \frac{8^2}{4} - 80 = 50$$

$$SSE = TSS - SSC - SSR$$

$$= 292 - 57.2 - 50 = 184.8$$

Step 7: ANOVA Table

Source of variance	Sum of squares	d.f	Mean square	Variance ratio	Table value at 5% level
Between rows	SSR = 50	r –1 =5 – 1 =4	$MSR = \frac{SSR}{r-1}$ $= \frac{57.2}{3}$ $= 19.1$	$F_R = \frac{MSR}{MSE}$ $= \frac{19.1}{15.4} = 1.24$	$F_R(3,12)$ $= 8.74$
Between column	SSC = 57.2	C-1 = 4 – 1 = 3	$MSC = \frac{SSC}{C-1}$ $= \frac{50}{4}$ $= 12.5$	$F_C = \frac{MSC}{MSE}$ $= 1\frac{12.5}{15.4} = 1.23$	$F_C(12,4)$ $= 3.26$
Residual	SSE = 184.8	N – C – r +1 = 20 - 4 -+1 =12	MSE $= \frac{SSE}{N-C-r+1}$ $= \frac{184.8}{12} = 15.4$		
Total	292				

Step 8: Conclusion: Cal F_C < Table F_C , so accept H_0
Cal F_R < Table F_R , so accept H_0

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

26. The following is a Latin square of a design when 4 varieties of seed being tested. Set up the analysis of variance table and state your conclusion. You can carry out the suitable charge of origin and scale.

A 110	B 100	C 130	D 120
C 120	D 130	A 110	B 110
D 120	C 100	B 110	A 120
B 100	A 140	D 100	C 120

Solution:

Subtract by 5 we get

A 105	B 95	C 125	D 115
C 115	D 125	A 105	B 105
D 115	C 95	B 105	A 115
B 95	A 135	D 95	C 115

Subtract 100 and then divided by 5 we get

A	1	B	-1	C	5	D	3
C	3	D	5	A	1	B	1
D	3	C	-1	B	1	A	3
B	-1	A	7	D	-1	C	3

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	1	-1	5	3	8	1	1	25	9
Y_2	3	5	1	1	10	9	25	1	1
Y_3	3	-1	1	3	6	9	1	1	9
Y_4	-1	7	-1	3	8	1	49	1	9
Total	6	10	6	10	32	20	76	28	28

H_0 : There is no significant difference between rows, columns and treatments.

H_1: There is significant difference between rows, columns and treatments.

$$\text{Step } 1: \text{N} = 16$$

$$\text{Step } 2: \text{T} = 32$$

$$\text{Sept } 3: \frac{\text{T}^2}{N} = \frac{(32)^2}{16} = 64$$

$$\text{step } 4: \text{TSS} = \sum X_1^{\ 2} + \sum X_2^{\ 2} + \sum X_3^{\ 2} + \sum X_4^{\ 2} - \frac{\text{T}^2}{N}$$

$$= 20 + 76 + 28 + 28 - 64 = 88$$

$$step\,5: SSC = \frac{\left(\sum X_1\right)^2}{N_1} + \frac{\left(\sum X_2\right)^2}{N_1} + \frac{\left(\sum X_3\right)^2}{N_1} + \frac{\left(\sum X_4\right)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{(6)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(10)^2}{4} - 64$$

$$= 9 + 25 + 9 + 25 - 64 = 4$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$step\,6: SSR = \frac{\left(\sum Y_1\right)^2}{N_2} + \frac{\left(\sum Y_2\right)^2}{N_2} + \frac{\left(\sum Y_3\right)^2}{N_2} + \frac{\left(\sum Y_4\right)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{8^2}{4} + \frac{10^2}{4} + \frac{(6)^2}{4} + \frac{8^2}{4} - 64 \quad = 16 + 25 + 9 + 16 - 64 = 2$$

To find SSK :

Arrange the elements in the order of treatment.

A	1	1	3	7	12
B	-1	1	1	-1	0
C	5	3	-1	3	10
D	3	5	3	-1	10

$$SSK = \frac{(12)^2}{4} + \frac{0^2}{4} + \frac{(10)^2}{4} + \frac{(10)^2}{4} - \frac{T^2}{N}$$

$$= 36 + 0 + 25 + 25 - 64 = 22$$

$$SSE = TSS - SSC - SSR - SSK$$

$$= 88 - 4 - 2 - 22 = 60$$

Step 7: ANOVA Table

Source of variance	Sum of square	d.f	Mean square	Variance ratio	Table value at 5% level
Between rows	SSR $= 2$	k – 1 $= 3$	$MSR = \frac{SSR}{k-1}$ $= 0.67$	$F_R = \frac{MSE}{MSR}$ $= \frac{10}{0.67}$ $= 14.9$	$F_R(6,3)$ $= 8.94$
Between columns	SSC $= 4$	k–1 $= 3$	$MSC = \frac{SSC}{k-1}$ $= 1.33$	$F_C = \frac{MSE}{MSC}$ $= \frac{10}{1.33}$ $= 7.52$	$F_C(6,3)$ $= 8.94$
Between Treatments	SSK $= 22$	k–1 $= 3$	MSK $= \frac{MSK}{K-1}$ $= 7.33$	$F_T = \frac{MSE}{MSK}$ $= \frac{10}{7.33}$ $= 1.36$	$F_T(6,3)$ $= 8.94$
Residual	SSE = 60	(k-1) (k–2) $= 6$	MSE $= \frac{SSE}{(K-1)(K-2)}$ $= 10$		
Total	88	15			

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 8: Conclusion: Cal F_C < Table F_C

Cal F_R > Table F_R

Cal F_T < Table F_T

There is a significant difference between rows as well as between columns. But, there is no significant difference between treatments.

27. Analyse the variance in the Latin square of yields (in kgs) of paddy where P, Q , R , S denote the different method of cultivation :

S122 P121 R123 Q122
Q124 R123 P122 S125
P120 Q119 S120 R121
R122 S123 Q121 P122

Estimate whether different method of cultivation have significantly different yields.

Ans :

Subtract 120 we get

S	2	P	1	R	3	Q	2
Q	4	R	3	P	2	S	5
P	0	Q	-1	S	0	R	1
R	2	S	3	Q	1	P	2

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	2	1	3	2	8	4	1	9	4
Y_2	4	3	2	5	14	16	9	4	25
Y_3	0	-1	0	1	0	0	1	0	1
Y_4	2	3	1	2	8	4	9	1	4
Total	8	6	6	10	30	24	20	14	34

Step 1. N = 16 [Total number of entries]

Step 2. T = 30

Step 3. $\frac{T^2}{N} = \frac{(30)^2}{16} = 56.25$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 24 + 20 + 14 + 34 - 56.25 = 35.75$

Step 5. SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{64}{4} + \frac{36}{4} + \frac{36}{4} + \frac{100}{4} - 56.25 = 2.75$

STUCOR APP

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 6. SSR $= \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{64}{4} + \frac{196}{4} + \frac{0}{4} + \frac{64}{4} - 56.25 = 24.75$$

To find SSK :

Arrange the elements in the order of treatment

					Total
P	0	1	2	2	5
Q	4	-1	2	2	6
R	2	3	1	1	9
S	2	3	5	5	10

$$SSK = \frac{(5)^2}{4} + \frac{(6)^2}{4} + \frac{(9)^2}{4} + \frac{(10)^2}{4} - \frac{T^2}{N}$$

$$= \frac{25}{4} + \frac{36}{4} + \frac{81}{4} + \frac{100}{4} - 56.25 = 4.25$$

SSE = TSS – SSC – SSR – SSK = 35.75 – 2.75 – 24.75 – 4.25 = 4

Step 7. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 1% level
Between Rows	SSR = 24.75	k –1 = 3	$MSR = \frac{SSR}{k-1}$ = 8.25	$F_R = \frac{MSR}{MSE}$ = 12.31	$F_R(3, 6)$ = 4.76
Between columns	SSC = 2.75	k –1 = 3	$MSC = \frac{SSC}{k-1}$ = 0.92	$F_C = \frac{MSC}{MSE}$ = 1.37	$F_C(3, 6)$ = 4.76
Between treatments	SSK = 4.25	k –1 = 3	$MSK = \frac{SSK}{k-1}$ = 1.42	$F_T = \frac{MSK}{MSE}$ = 2.12	$F_R(3, 6)$ = 4.76
Residual	SSE = 4	(k –1)(k - 2) = 6	MSE = $\frac{SSE}{(k-1)(k-2)}$ = 0.67		

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Total	TSS = 35.75	$n^2 - 1$ = 15			

Step 8 : Conclusion :

(i) Cal F (F_T) < Table F . There is no significant difference between treatments

(ii) Cal F (F_R) > Table F . There is significant difference rows

(iii) Cal F (F_C) < Table F . There is no significant difference columns.

28. A company wants to procedure cars for its own use. It has to select the make of the car out of the four makes A, B, C, D available in the market. For this he tries four cars of each make by assigning the cars to four drivers to run on four different routes. The efficiency of cars is measured in terms of time in hours. The layout and time consumed is as given below.

	Drivers			
Routes	1	2	3	4
1	18(C)	12(D)	16(A)	20(B)
2	26(D)	34(A)	25(B)	31(C)
3	15(B)	22(C)	10(D)	28(A)
4	30(A)	20(B)	15(C)	9(D)

Analyse the experimental data and draw conclusions. $(F_{0.05}(3,5) = 5.41)$

Ans:

H_0 : There is no significant difference between routes and drivers.

H_1 : There is significant difference between routes and drivers

Subtract 25 from all numbers

	X_1 A	X_2 B	X_3 C	X_4 D	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	-9	-5	-7	-13	-34	81	25	49	169
Y_2	9	0	6	6	21	81	0	36	36
Y_3	3	-10	-3	-15	-25	9	100	9	225
Y_4	5	-5	-10	-16	-26	25	25	100	256
	8	-20	-14	-38	-64	296	150	194	686

Step 1 : N = 16

Step 2. T = -64

Step 3. C.F. = $\dfrac{T^2}{N} = \dfrac{(-64)^2}{16} = 256$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \dfrac{T^2}{N}$

$= 196 + 150 + 194 + 686 - 256 = 970$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 5. $$SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$$

[N_1 = number of elements in each column]

$$= \frac{(8)^2}{4} + \frac{(-20)^2}{4} + \frac{(-14)^2}{4} + \frac{(-38)^2}{4} - 256 = 270$$

Step 6. $$SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$$

[N_2 = number of elements in each row]

$$= \frac{(-34)^2}{4} + \frac{(21)^2}{4} + \frac{(-25)^2}{4} + \frac{(-26)^2}{4} - 256 = 468.5$$

SSE = TSS – SSC – SSR = 970 - 270 – 468.5 = 231.5

Step 7. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 270	c –1 = 4 – 1 = 3	MSC $= \frac{SSC}{c-1}$ $= \frac{270}{3}$ = 90	$F_C = \frac{MSC}{MSE}$ = $\frac{90}{25.72} = 3.50$	F_C (3, 9) = 3.86
Between Rows	SSR = 468.5	r–1 = 4 – 1 = 3	MSR $= \frac{SSR}{r-1}$ $= \frac{468.5}{3}$ = 156.2	$F_R = \frac{MSR}{MSE}$ $= \frac{156.2}{25.72}$ = 6.07	F_R (5,15) = 4.5
Residual	SSE = 231.5	N – c –r + 1 = 16 – 4 – 4 + 1 = 9	MSE = $\frac{SSE}{N-c-r+1}$ $= \frac{231.5}{9} = 25.72$		

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 7 : Conclusion :

Cal F_C < Table F_C . So we accept H_0 .

Cal F_R > Table F_R . So we reject H_0

29. Consider the results given in the following table for an experiment involving six treatments in four randomized blocks. The treatments are indicated by numbers within parenthesis.

Blocks	Yield for a randomized block experiment treatment and yield					
1	(1) 24.7	(3) 27.7	(2) 20.6	(4) 16.2	(5) 16.2	(6) 24.9
2	(3) 22.7	(2) 28.8	(1) 27.3	(4) 15.0	(6) 22.5	(5) 17.0
3	(6) 26.3	(4) 19.6	(1) 38.5	(3) 36.8	(2) 39.5	(5) 15.4
4	(5) 17.7	(2) 31.0	(1) 28.5	(4) 14.1	(3) 34.9	(6) 22.6

Test whether the treatments differ significantly. $(F_{0.05}(3,15) = 5.42\ ;\ F_{0.05}(5,15) = 4.5)$

Ans:

H_0 : There is no significant difference between blocks and treatments.

H_1 : There is significant difference between blocks and treatments

Subtract 20 from all the numbers

	X_1 A	X_2 B	X_3 C	X_4 D	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	4.7	-7.3	18.5	8.5	39	22.09	53.29	342.25	72.25
Y_2	0.6	8.8	19.5	11.0	39.9	0.36	77.44	380.25	121
Y_3	7.7	2.7	16.8	14.9	42.1	59.29	7.29	282.24	222.01
Y_4	-3.8	-5	-0.4	-5.9	-15.1	14.44	25	0.16	34.81
Y_5	-3.8	-3	-4.6	-2.3	-13.7	14.44	9	21.16	5.29
Y_6	4.9	2.5	6.3	2.6	16.3	24.01	6.25	36.69	6.76
	10.3	13.3	56.1	28.8	108.5	134.63	178.27	1065.75	462.37

Step 1 : N = 24

Step 2. T = 108.5

Step 3. C.F. = $\frac{T^2}{N} = \frac{(108.5)^2}{24} = 490.5$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 134.63 + 178.27 + 1065.75 + 462.37 - 490.5 = 1350.52$

STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 5. SSC $= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(10.3)^2}{6} + \frac{(13.3)^2}{6} + \frac{(56.1)^2}{6} + \frac{(28.8)^2}{6} - 490.5 \quad = \ 219.44$$

Step 6. SSR $= \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} + \frac{(\Sigma Y_5)^2}{N_2} + \frac{(\Sigma Y_6)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(39)^2}{6} + \frac{(39.9)^2}{6} + \frac{(42.1)^2}{6} + \frac{(-15.1)^2}{6} + \frac{(-13.7)^2}{6} + \frac{(16.3)^2}{6} - 490.5$$

$= 901.2$

SSE = TSS – SSC – SSR = 1350.52 - 219.44 - 901.2 = 229.9

Step 6. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 219.44	C –1 = 4 – 1 = 3	MSC $= \frac{SSC}{C-1}$ $= \frac{219.44}{3}$ $= 73.15$	$F_C = \frac{MSC}{MSE}$ $= \frac{73.05}{15.33} = 4.8$	F_C (3, 15) = 5.42
Between rows	SSR =901.2	R–1 = 6 – 1 = 5	MSC $= \frac{SSC}{R-1}$ =180.24	$F_C = \frac{MSR}{MSE}$ $= \frac{180.24}{15.33}$ $= 11.76$	F_R (5,15) = 4.5
Residual	SSE =229.9	N – C-R+1 = 24 – 4 -6 +1 = 15	MSE = $\frac{SSE}{N-C-R+1}$ $= \frac{229.9}{15} = 15.3$		

STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 7 : Conclusion :

Cal F_C < Table F_C . So we accept H_0 .

Cal F_R < Table F_R . So we reject H_0

30. Find out the main effects and interactions in the following 2^2factorial experiment and write down the ANOVA table.

	I	A	b	ab
Block	00	10	01	11
I	64	25	30	6
II	75	14	50	33
III	76	12	41	17
IV	75	33	25	10

Ans :

Taking deviation from y = 37 we get

Treatment combination	Blocks				Total				
	I X_1	II X_2	III X_3	IV X_4		X_1^2	X_2^2	X_3^2	X_4^2
(y_1) (1)	27	38	39	38	142	729	1444	1521	1444
(y_2) a	-12	-23	-25	-4	-64	144	529	625	15
(y_3) b	-7	13	4	-12	-2	49	159	16	144
(y_4) ab	-31	-4	-20	-27	-82	961	16	400	729
Total	-23	24	-2	-5	-6	1883	2158	2562	2333

Step 1 : N = 16

Step 2 : T = - 6

Step 3 : C.F $= \frac{T^2}{N} = \frac{36}{16} = 2.25$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 1883 + 2158 + 2562 + 2333 + 2.25 = 8933.75$

Step 5 : SSC $= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(-23)^2}{4} + \frac{(24)^2}{4} + \frac{(-2)^2}{4} + \frac{(-5)^2}{4} - 2.25 = 281.25$

Step 6. SSR $= \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$= \frac{(142)^2}{4} + \frac{(-64)^2}{4} + \frac{(-2)^2}{4} + \frac{(-82)^2}{4} - 2.25 = 7744.75$$

Step 7 : SSE = TSS – SSC – SSR = 8933.75-281.25-7744.75 = 907.75

For 2^2 experiment
Contrast A = [a + ab – b-(1)] = - 64 – 82 + 2 -142 = -286
Contrast B = [b + ab – a-(1)] = - 2 – 82 + 64 -142 = -162
Contrast AB = [ab +(1) – a - b] = - 82+142 + 64 + 2 = 126

$$\text{Main effect A} = \frac{1}{2}[a + ab - b - (1)] = -143$$

$$B = \frac{1}{2}[b + ab - a - (1)] = -81$$

$$AB = \frac{1}{2}[ab + (1) - a - b] = 63$$

$$SS_A = \frac{[a+ab-b-(1)]^2}{16} = \frac{(-286)^2}{16} = 5112.25$$

$$SS_B = \frac{[b+ab-a-(1)]^2}{16} = \frac{(-162)^2}{16} = 1640.25$$

$$SS_{AB} = \frac{[ab+(1)-a-b]^2}{16} = \frac{(-126)^2}{16} = 992.25$$

Analysis of variance table for the 2^2 experiment

s.v	d.f	s.s	MSS	Variance ratio	Table value of F	
					5 %	1 %
Blocks	3	281.5	93.83	$\frac{100.86}{93.83} = 1.075$	F(9, 3) = 8.81	27.35
Treatments	3	7744.75	2581.58	$\frac{2581.88}{100.86} = 25.60$	F(3, 9) = 3.86	6.99
A	1	5112.25	5112.25	$\frac{5112.25}{100.86} = 50.69$	$F_A(1, 9) = 5.12$	6.99
B	1	1640.25	1640.25	$\frac{1640.25}{100.86} = 16.26$	$F_B(1, 9) = 5.12$	6.99
AB	1	992.25	992.25	$\frac{992.25}{100.86} = 9.84$	$F_{AB}(1, 9) = 5.12$	6.99
Error	9	907.75	100.86			

Error (d.f) = N – c – r +1 = 16 – 4 - 4 + 1 = 9

Cal F_A > Table F_A

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Cal F_B > Table F_B
Cal F_{AB} > Table F_{AB} .

31. Given

Detergent	Engine		
	1	2	3
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Perform ANOVA and test at 0.05 level of significance whether these are differences in the detergents or in the engines.

Ans :

Detergent	Engine			Total
	1	2	3	
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

The above data are classified according to criteria (i) Detergent (ii) Engine.

In order to simplify calculations, we code the data by subtracting 50 from each figure.

Detergent	Engine			Total			
	(X_1)	(X_2)	(X_3)		X_1^2	X_2^2	X_3^2
A(Y_1)	-5	-7	1	-11	25	49	1
B(Y_2)	-3	-4	2	-5	9	16	4
C(Y_3)	-2	0	5	3	4	0	25
D(Y_4)	-8	-13	-1	-22	64	1691	1
Total	-18	-24	7	-35	102	234	31

1. H_0 : There is no significant diference between columns means as well as row means.
2. H_1 : There is significant diference between columns means or the row means.

Step 1. N = 12 [Total number of entries]

Step 2. T = -35

Step 3. $\frac{T^2}{N} = \frac{(-35)^2}{12} = 102.08$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$

= (102) + (234) + (31) –(102.08)

= 264.92

Step 5. SSC = $\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{(7)^2}{4} - 102.08$$

$= 135.17$

Step 6. SSR = $\frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(-11)^2}{3} + \frac{(-5)^2}{3} + \frac{(3)^2}{3} + \frac{(-22)^2}{3} - 102.08$$

$= 110.91$

SSE = TSS – SSC – SSR

= 264.92 – 135.17 - 110.91

= 18.84

Step 7. Table of analysis of variance

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 1% level
Between columns	SSC = 135.17	c –1 = 3-1 = 2	MSC = $\frac{SSC}{c-1}$ $= \frac{135.17}{2} = 67.585$	F_C = $\frac{MSC}{MSE}$ = $\frac{67.585}{3.14} = 21.52$	$F_C(2, 6)$ =10.92
Between Rows	SSR = 110.91	r – 1 = 4-1 = 3	MSR = $\frac{SSR}{r-1}$ $\frac{110.91}{3} = 36.97$	F_R = $\frac{MSR}{MSE}$ = $\frac{36.9}{3.14} = 11.77$	$F_R(3, 6)$ = 9.78
Residual	SSE = 18.84	N – c-r +1 = 12-3-4+1 = 6	MSE = $\frac{SSE}{N-c-r+1}$ = $\frac{18.84}{6} = 3.14$		

STUCOR APP

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Total	TSS = 264.92	11			

Step 8 . Conclusion :

Cal F_C > Table F_C . So we reject H_0

Cal F_R> Table F_R . So we reject H_0

32. Three varieties of coal were analysed by 4 chemists and the ash content is tabulated here. Perform an analysis of variance.

		Chemists			
		A	B	C	D
Coal	I	8	5	5	7
	II	7	6	4	4
	III	3	6	5	4

Solution:

Coal	Chemists				Total of variance	X_1^2	X_2^2	X_3^2	X_4^2
	A	B	C	D					
I	8	5	5	7	25	64	25	25	49
II	7	6	4	4	21	49	36	16	16
III	3	6	5	4	18	9	36	25	16
Total	18	17	14	15	64	122	97	66	81

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

H_0 : There is no siginificant difference

H_1 : There is siginificant difference

Step 1: $N = 12$

Step 2: $T = 64$

Step 3: $\frac{T^2}{N} = \frac{(64)^2}{12} = 341.3$

Step 4: $TSS = \sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \sum X_4{}^2 - \frac{T^2}{N}$

$= 64 + 122 + 97 + 66 + 81 - 341.3$

$= 88.7$

Step 5: $SSC = \frac{\left(\sum X_1\right)^2}{N_1} + \frac{\left(\sum X_2\right)^2}{N_1} + \frac{\left(\sum X_3\right)^2}{N_1} + \frac{\left(\sum X_4\right)^2}{N_1} - \frac{T^2}{N}$

$= \frac{324}{3} + \frac{289}{3} + \frac{196}{3} + \frac{225}{3} - 341.31$

$= 108 + 96.33 + 65.33 + 75 - 341.3 = 3.36$

Step 6: $SSR = \frac{\left(\sum Y_1\right)^2}{N_2} + \frac{\left(\sum Y_2\right)^2}{N_2} + \frac{\left(\sum Y_3\right)^2}{N_2} - \frac{T^2}{N}$

$= \frac{625}{4} + \frac{441}{4} + \frac{324}{4} - 341.3$

$= 156.25 + 110.25 + 81 - 341.3 = 6.2$

$SSE = TSS - SSC - SSR = 88.7 - 3.36 - 6.2 = 79.14$

$SSE = 79.14$

Source of variance	Sum of squares	d.f.	Mean square	Variance ratio	Table value at 5% level
Between columns	SSC = 3.36	C-1 = 3 –1 = 2	$MSC = \frac{SSC}{C-1}$ $= \frac{3.36}{2}$ $= 1.68$	$F_C = \frac{MSC}{MSE}$ $= \frac{1.68}{13.19}$ $= 0.1274$	$F_C(2,6)$ $= 5.14$
Between rows	SSR = 6.2	r-1 = 4-1 = 3	$MSR = \frac{SSR}{r-1}$ $= \frac{6.2}{3}$ $= 2.0666$	$F_R = \frac{MSR}{MSE}$ $= \frac{2.0666}{13.19}$ $= 0.1566$	$F_R(3,6)$ $= 4.76$

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

Residual	SSE = 79.14	N – C-r+1 = 12 -4-3+1 = 6	MSE $= \frac{SSE}{N - C}$ $= \frac{79.14}{6}$ $= 13.19$		
Total	88.7				

Conclusion

Cal F_R < Tab F_R

Cal F_C < Tab F_C

In both the cases the calculated value is less than tabulated value Therefore null hypothesis is accepted.

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

UNIT – IV

Testing of Hypothesis

Population:

The group of individuals under study is called population.

Sample:

A finite subset of individuals in a population is called a sample and the number of individuals in a sample is called the sample size.

Sampling is only a tool which helps us to know the characteristics of the population or the universe by examining only a small part of it.

Large and small samples:

The number of elements in a sample is greater than or equal to 30,then the sample is called a large sample and if it is less than 30, then the sample is called a small sample.

Random Sample: A random sample of size n is a sample that is chosen in such a way that each subset of n elements of the population has the same probability of being selected.

Parameters: Statistical constants like mean µ,varianceσ^2, etc., computed from a population are called parameters of the population.

A population parameters is denoted by Ѳ.

Statistics: Statistical constants like mean $\bar{x}$, variance s^2 etc., computed from a sample are called sample statistics or statistics. A sample statistics is denoted by t.

Sample statistic t is an estimator of population parameter Ѳ.

If the population mean µ is unknown and if sample mean$\bar{x}$ is known ,we can estimate the value of µ using $\bar{x}$, so $\bar{x}$, is called an estimator of µ.

Sample Distribution:

The probability distribution of a statistic t is called the sampling distribution of t.

For eg. If we take k samples each of size $n_2, n_3, n_4, \dots n_k$ *we can find their means* $\bar{x}_1, \bar{x}_2 \bar{x}_3 \bar{x}_4 \dots \bar{x}_k$. This set of sample means is called a sampling distribution of the sample statistics $\bar{x}$

Similarly we can have sampling distribution of sample variance s^2, sample median m etc.

Standard error: the standard deviation of the sampling distribution of a statistic t is called the standard error of t.

If the sample is large (n≥ 30) and σ is the standard deviation of the population then standard error of $\bar{x}$ is $\frac{\sigma}{\sqrt{n}}$

Test of Significance (OR) Hypothesis Testing.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Statistical Hypothesis: In making statistical decisions we make assumptions or guesses about populations involved.Such assumptions which may be true or false are called statistical hypothesis.

Null Hypothesis H_0:

For applying the test of significance, we first set up a hypothesis which is a statement about the population parameter. This statement is usually a hypothesis of no difference and so it is called null hypothesis and is denoted by H_0.

Alternate Hypothesis H_1

Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis, denoted byH_1.

Errors in Sampling:

After applying a test of significance a decision is to be taken to accept or reject the null hypothesis. There is always some possibility of committing an error in taking a decision.These errors are of two types.

1. Type I error.
2. Type II error.

Type I error: If H_0 is rejected while it should have been accepted.

Type II error: If $H_0$is accepted while it should have been rejected.

Level of significance:

The probability of Type I error is called the level of significance of the test and is denoted by α .

We usually take either α=5% or 1%.We test the hypothesis at 5% level of significance (or) at 1% level of significance.

Critical region:

For a test statistic the area under the probability curve, which is normal is divided into two regions namely the region of acceptance of H_0 and the region of rejection of H_0.The region in which H_0 is rejected is called the critical region.The area of the critical region is α, α-level of significance. The region in which H_0 is accepted is called the acceptance region.

Critical Ratio:

For the statistic t,we find the ratio $Z = \frac{t-E(t)}{S.E(t)}$ which is called the critical ratio. This number separates the acceptance and rejection regions.

One Tailed and Two Tailed Test:

If the alternate hypothesis is one sided then it is called one tailed test. It may be right tailed or left tailed test.

For eg: H_0:μ= 300.

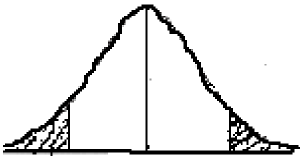
H_1: μ> 300 Right tail test

μ<300 left tail test.

Two Tailed Test: If the alternate hypothesis is two sided then it is called a

STUCOR APP

DOWNLOADED FROM STUCOR APP



Two tailed test.

Large Sample Test of Significance:

1. Test for the significant difference between sample mean and population mean.
2. Test for the significant difference between two sample means.
3. Test for the significant difference between sample SD and population SD.
4.Test for the significant difference between two sample(SD).
5.Test for the significant difference between sample proportion and population proportion.
6.Test for the significant difference between two sample proportions..

Procedure for testing of hypothesis:

Step 1: Set Null Hypothesis.
Step 2: Decide alternate hypothesis.[One tailed or Two tailed]
Step 3: Choose α (Level of Significance)
Step 4: Compute test statistics $z = \frac{t-E(t)}{S.E\ of\ (t)}$
Step 5: Compare calculated value and Tabulated value.
Step 6: Inferecne.

Note:

1. 5% level fo significance means we are 95% confident that we have made the right decision.
2. 1% level of significance means we are 99% confident that we have made the correct decision.

Table value of Z:

Level of significance	5%	1%
1.Two tailed test	$\|Z\| = 1.96$	$\|Z\| = 2.58$
1.Right tailed test	Z=1.645	Z = 2.33
1.Left tailed test	Z=− 1.645	Z =−2.33

Test of Significance difference betweenSample Mean and Population Mean.

Let N(μ,σ^2) be the normal population from which a large sample of size n be taken.to test the significance of the difference between $\bar{x}$ and μ.Let μ_0 be the hypothesis value of the population mean.

STUCOR APP

DOWNLOADED FROM STUCOR APP

Large sample : Size n
Sample mean ; $\bar{x}$
Population mean :µ
Population variance :$\frac{\sigma^2}{n}$
Population standard deviation:$\frac{\sigma}{\sqrt{n}}$

H_0:µ=µ$_0$,
$H_1 = µ \neq µ_0$
The test Statistic $z = \frac{\bar{x}-µ}{\frac{\sigma}{\sqrt{n}}}$
$|Z| < 1.96 H_0$is accepted at 5 % level of significance.
$|Z| < 2.58 H_0$is accepted at 1 % level of significance.

Test for the significant difference between sample standard deviation and population standard deviation or test for the equality of sample standard deviation and population standard deviation.

Procedure

Let a large sample of size n be taken a normal population with standard deviation σ. Let s be standard deviation of the sample.

Let σ_0 be the hypothetical value of the standard deviation of the population.

$H_0 \; : \; \sigma = \sigma_0$

$H_1 \; : \; \sigma \neq \sigma_0$

∴ Two tailed test

∴ Under H_0 , $Z = \frac{s-\sigma}{\sigma / \sqrt{2n}} \sim N(0,1)$

Inference

If $|Z| < 1.96$, H_0 is accepted at 5 % level of significance.

If $|Z| > 1.96$, H_0 is rejected at 5 % level of significance.

If $|Z| < 2.58$, H_0 is accepted at 1 % level of significance.

If $|Z| > 2.58$, H_0 is rejected at 5 % level of significance.

Test for the significant difference between thestandard deviationsof two large samples or Test for equality of two standard deviations.

Procedure

Let two samples of sizes n$_1$ and n$_2$ be drawn from two populations with

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

standard deviation σ_1 *and* σ_2 respectively. Let s_1 *and* s_2 bethe standard deviations of the two samples. We have to test whether the difference between the standard deviations is significant or not.

$H_0 : \sigma_1 = \sigma_2$ (*ie*) *there is no different*

$H_1 : \sigma_1 \neq \sigma_2$

Case (i) : σ_1 , σ_2 are unknown.

Under H_0 , the test statistic is $Z = \dfrac{s_1 - s_2}{\sqrt{\dfrac{\sigma_1^2}{2n_1} + \dfrac{\sigma_{21}^2}{2n_2}}} \sim N(0,1)$

Inference

If $|Z| < 1.96$, H_0 is accepted at 5 % level of significance.

If $|Z| > 1.96$, H_0 is rejected at 5 % level of significance.

If $|Z| < 2.58$, H_0 is accepted at 1 % level of significance.

If $|Z| > 2.58$, H_0 is rejected at 5 % level of significance.

Case (ii) : σ_1 , σ_2 are not unknown.

Then S.E. of $s_1 = \dfrac{\sigma_1}{\sqrt{2n_1}} = = \dfrac{s_1}{\sqrt{2n_1}}$ &S.E. of $s_2 = \dfrac{\sigma_2}{\sqrt{2n_2}} = = \dfrac{s_2}{\sqrt{2n_2}}$

$\therefore$ S.E. of $(s_1 - s_2) = \sqrt{\dfrac{s_1^2}{2n_1} + \dfrac{s_2^2}{2n_2}}$

$$\therefore Z = \frac{s_1 - s_2}{\sqrt{\dfrac{\sigma_1^2}{2n_1} + \dfrac{\sigma_2^2}{2n_2}}} \sim N(0,1)$$

**Inference : **Aslike the case of case 1

Case : (iii)

If the two samples are drawn from the same normal population or populations with same standard deviation

(ie) $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and σ is known

$\Rightarrow \sigma_1 = \sigma_2 = \sigma$

$$Z = \frac{s_1 - s_2}{\sigma\sqrt{\dfrac{1}{2n_1} + \dfrac{1}{2n_2}}} \sim N(0,1)$$

**Inference : **Aslike the case of case 1

Test for the significant difference between sample proportionand

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

population proportion.

Procedure

A large sample of size n be taken from a population. Let P be the proportion of some attribute in the population. Let p be the proportion of the same attribute in the sample. We have to test whether p and P differ significantly or not. Let P_0 be the hypothetical value of the proportion in the population.

Null hypothesis $H_0 : P = P_0$

Alternate hypothesis $H_1 : P \neq P_0 \ (two\ tailed)$

$$Q = 1 - P$$

Standard error of $p = \sqrt{\frac{PQ}{n}}$

Under H_0 is large n , the sampling distribution of p is approximately normal with mean P and variance $\sqrt{\frac{PQ}{n}}$

∴The test statistic is $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} \sim N(0,1)$

Inference

If $|Z| < 1.96$, H_0 is accepted at 5 % level of significance. (ie) difference is not significant.

If $|Z| > 1.96$, H_0 is rejected at 5 % level of significance. (ie) difference is significant.

If $|Z| < 2.58$, H_0 is accepted at 1 % level of significance.

If $|Z| > 2.58$, H_0 is rejected at 5 % level of significance. (ie) difference is highly significant.

Test for the significant difference between two proportions in two samples or Test for equality of twoproportions.

Procedure

Let two large samples of sizes n_1 and n_2 be drawn from a population with proportion P for some attribute A. Let p_1 , p_2 be the proportions of the same attribute A in the samples respectively. We want to test the significant difference between p_1 , p_2 and Q = 1 - P

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

H_0 : $P_1 = P_2$ *(ie) there is no significant difference between sample proportions.*

H_1 : $P_1 \neq P_2$

Two tailed test

Under H_0, the sampling distribution of $p_1 - p_2$ is approximately normal with mean P variance PQ$\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$

Case (i) : P is known ie. The population proportion is known.

Under H_0, the test statistic is $Z = \dfrac{p_1 - p_2}{PQ\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$

Inference

If $|Z| < 1.96$, H_0 is accepted at 5 % level of significance.

If $|Z| > 1.96$, H_0 is rejected at 5 % level of significance.

If $|Z| < 2.58$, H_0 is accepted at 1 % level of significance.

If $|Z| > 2.58$, H_0 is rejected at 5 % level of significance.

Case (ii) :P is not known ie. The population proportion is not known.

If P is not known , then the unbiased estimate of P using p_1, p_2 is

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$Z = \frac{p_1 - p_2}{PQ\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$$

Inference : Aslike the case of case 1

Exact Sampling Distribution (t , F , χ^2)

Small sampling theory is a study of sampling distributions of statistics of small samples.

For small samples tests we having three distributions

1. Student's t – distribution
2. Snedecor's F – distribution
3. χ^2 - (Chi – Square) distribution

1. Student's t – distribution

A random variable T is said to follow Student's t – distribution if its probability density function is given by

STUCOR APP

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$f(t) = y_0\left(1+\frac{t^2}{v}\right)^{-\frac{(v+1)}{2}} \quad , -\infty < t < \infty$$

Where y_0 is a constant depending on n such that the total area under the curve is 1 and $v = n-1$ is called the number of degrees of freedom.

Properties :

1. The probability curve of the t – distribution is symmetric about t = 0, bell shaped and asymptotic to t – axis.
2. For large values of v or n the t – distribution approaches the standard normal distribution.
3. The mean of the t – distribution is 0.
4. The variance of t – distribution is $\frac{v}{v-2}$ $if \ v > 2$, but tends to 1 as $v \to -\infty$
5. It has greater dispersion than the normal.

Uses :

The t – distribution is used extensively in testing the significance of the difference between

1. The mean of a small sample and the mean of the population.
2. The means of two small samples.
3. The means of paired observations.
4. Sample correlation so-efficient and population correlation coefficient
5. Regression coefficients.

Degrees of freedom

The number of degrees of freedom of statistic is usually denoted by v and is defined by $v = n-k$, where n is the size of the sample, and k is the number of population parameters.

t – testfor single mean or Test the significant difference between sample mean and population mean.

Let a small sample of size n be drawn from a normal population with mean μ and unknown variance. Let $\bar{x}$ be the mean and s be the standard deviation of the sample. We want to test the significance of the difference between the population mean μ and sample mean $\bar{x}$.

Let μ_0 be the hypothetical value of μ

$H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$ $(two\ tailed\ test)$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

Under H_0 , $t = \frac{\overline{x} - \mu}{s / \sqrt{n-1}}$ follows student's t – distribution with $v = n - 1$ degrees of freedom.

For $v = n - 1$ let $t_{0.05}$ be the table value of t at 5 % significance.

Inference :

If $|t| < t_{0.05}$, H_0 is accepted at 5 % level of significance.ie the difference between $\overline{x}$ and μ is not significance.

If $|t| > t_{0.05}$, H_0 is rejected at 5 % level of significance.ie the difference is significant.

If $|t| < t_{0.01}$, H_0 is accepted at 1 % level of significance.

If $|t| > t_{0.01}$, H_0 is rejected at 1 % level of significance.ie the difference is highly significant.

Test of significance of the difference between the means of the two samples

Procedure

Let two independent small random samples of sizes n_1, n_2 be drawn from two normal populations with mean μ_1, μ_2. Let $\overline{x_1}, \overline{x_2}$ be the means of the samples and s_1, s_2 be their standard deviations.

The best estimate is given by $S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

Assume

H_0 : $\mu_1 = \mu_2$ *(ie) there is no significant difference between means of the two population*

H_1 : $\mu_1 \neq \mu_2$ *(two tailed)*

The standard error of $\overline{x_1} - \overline{x_2}$ is $\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

Under H_0 , the test statistic is $t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ with degrees of freedom $v = n_1 + n_2 - 2$

Inference :

If $|t| < t_{0.05}$, H_0 is accepted at 5 % level of significance.ie the difference between mean is not significant.

If $|t| > t_{0.05}$, H_0 is rejected at 5 % level of significance.ie the difference between mean is significant.

STUCOR APP

DOWNLOADED FROM STUCOR APP

If $|t| < t_{0.01}$, H_0 is accepted at 1 % level of significance.ie the difference is not significant.

If $|t| > t_{0.01}$, H_0 is rejected at 1 % level of significance.ie the difference is highly significant.

t – test for paired observations or dependent samples.

Let μ be the difference between the mean of the population before and after training.

$H_0 : \mu = 0$ (*there is no difference in the mean*)

$H_1 : \mu \neq 0$ (*there is difference*)

Under H_0 , the test statistic is $t = \frac{\bar{d}}{s/\sqrt{n-1}}$ where s^2 is variance of d *and* $v = n-1$ is degrees of freedom.

Inference :

If $|t| < t_{0.05}$, H_0 is accepted at 5 % level of significance.

If $|t| > t_{0.05}$, H_0 is rejected at 5 % level of significance.

Snedecor's F – distribution

Def : If χ_1^2 *and* χ_2^2 are two independent Chi – square variates with (v_1, v_2) degrees of freedom then the F – random variable is defined by $F = \frac{\chi_1^2/v_1}{\chi_2^2/v_2}$

Def. : A random variable f is said to follow F-distribution with (v_1, v_2) degrees of freedom if its probability density function is given by

$f(F) = K F^{\frac{v_1}{2}-1}\left(1+\frac{v_1 F}{v_2}\right)^{-\frac{(v_1+v_2)}{2}}$ where F > 0 and k is constant to be determined such that $\int_0^\infty f(F)\, dF = 1$

Properties of F-distribution

1.The shape of the curve depends on v_1 *and* v_2 and also their order.

2.Mean of the F-distribution is $\frac{v_1}{v_2 - 2}$, $v_2 > 2$

STUCOR APP

3.Variance of the F- distribution is $\frac{2\upsilon_1(\upsilon_1+\upsilon_1-2)}{\upsilon_1(\upsilon_2-2)^2(\upsilon_2-4)}$, $\upsilon_2 > 4$

Chi Square Distribution

Definition: Chi square variate.

The square of a standard normal variate is known as a chi-square variate with one degree of freedom and it is denoted by χ^2.

Definition: The probability density function of χ^2

The probability density function of$\chi^2$is given by

f(χ^2) = **k** $(\chi^2)^{\frac{\upsilon}{2}-1} e^{-\frac{x^2}{2}}, \chi^2 > 0$

where the parameter $\boldsymbol{\upsilon}$ I called degree of freedom and k is a constant.

Condition for the application of χ^2 *test*.

1. The experimental data must b independent of each other.
2. The sample size should be reasonably large,≥ 50.
3. The theoretical cell frequency should be atleast 5.If it is less than 5,it is combined with the adjacent frequencies so that the pooled frequency is > 5.
4. The constraints on the cell frequencies should be linear. Eg. $\sum O_i = \sum E_i = N \geq 50$.

χ^2 –test of goodness of fit

Procedure.

Let $o_1, o_2, o_3, \ldots\ldots o_n$ be the observed frequencies and $E_1, E_2, \ldots. E_n$
Be the corresponding expected frequencies. We want to test the significance of the difference between observed and expected frequencies.

H_0:
There is no significant difference between observed and the expected frequencies.

(i.e.) $\sum O_i = \sum E_i$i=1,2,3,…n

H_1:

There is a significant difference between the observed and expected frequencies.

The test statistics

χ^2=$\sum_{i=1}^{n} \frac{(O_i-E_i)^2}{E_i}$ follows a χ^2 distribution with $\boldsymbol{\upsilon}$=n-1 degree of freedom.

STUCOR APP

DOWNLOADED FROM STUCOR APP

Inference: If the calculated value of $\chi^2 < \chi^2_{0.05}$ H_0 is accepted at 5% level of significance.

(ie) the difference between observed and expected frequencies is not significant.

If $\chi^2 > \chi^2_{0.05}$, H_0 is rejected at 5% level of significance.

(ie) the difference is significant.

If $\chi^2 > \chi^2_{0.01}$, H_0 is acceptedat 1% level of significance.

(ie) the difference is highly significant.

Attributes

An attribute is a characteristic or a quality which may be present amongst the members of a population.

Contingency Table

The 2 x 2 contingency table is

	A	B	Total
A	a	B	a + b
B	c	d	c + d
Total	a + c	b + d	N = a +b+c+d

$$\chi^2 = \frac{N(ad - bc)}{(a+b)(c+d)(a+c)(a+d)}$$

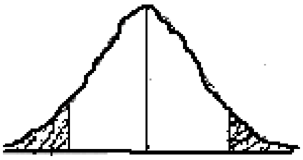
Problems

1. The heights of college students in Chennai are normally distributed with standard deviation 6 cm and sample of 100 students had their mean height 158 cm. Test the hypothesis that the mean height of college students in Chennai is 160 cm at 1 % level of significance.

Ans :

Given $n = 100$, $\mu = 160$, $s = 6$, $\overline{x} = 158$, $\alpha = 1\%$

1. $H_0 : \mu = 160$
2. $H_1 : \mu \neq 160$ [use two tailed test]
3. $\alpha = 1\%$
4. critical region



STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

	$-Z_{\alpha/2}\ Z_{\alpha/2}$ $= -2.58 = 2.58$ 5. The test statistic $Z = \frac{\bar{x}-\mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{158-160}{\left(\frac{6}{\sqrt{100}}\right)} = -3.33$ 6. Conclusion : If $-Z_{\alpha/2} < Z < Z_{\alpha/2} \nless$ 1. 96 then accept H_0
2.	A coin is tossed 400 times and it turns up head 216 items. Discuss whether the coin may be unbiased one at 5 % level of significance. **Ans :**Given n = 400 , P = $\frac{1}{2}$, Q = 1 – P = 1 - $\frac{1}{2}$ X = Number of success = 216. 1. The parameter of interest is P. 2. H_0: The coin is unbiased. 3. H_1: The coin is biased. 4. α = 0.05 5. $Z = \frac{X-np}{\sqrt{npQ}}$ 6. Reject H_0 if $\lvert Z\rvert > 1.96$ 7. $Z = \frac{216-(400)\left(\frac{1}{2}\right)}{\sqrt{(400)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}} = \frac{16}{10}$ 8. $\lvert Z\rvert = 1.6 < 1.96$ we accept H_0 at 5 % level of significance.
3.	Define Type – I error and Type – II error. **Solution:** Type I error : If H_0 is rejected while it should have been accepted. Type II error: If $H_0$is accepted while it should have been rejected.
4.	State the application of Chi- test **Solution:** 1. To test the goodness of fit. 2. To test the independent of attributes. 3. To test the homogeneous of independent estimations.
5.	What are null and Alternate hypothesis? **Ans :**

STUCOR APP

DOWNLOADED FROM STUCOR APP

For applying the test of significance, we first set up a hypothesis which is a statement about the population parameter. This statement is usually a hypothesis of no difference and so it is called null hypothesis and is denoted by H_0.
Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis, denoted by H_1.

6. Give the formula for the chi square test of independence for

A	b
C	d

Ans :

	A	B	Total
A	a	B	a + b
B	c	D	c + d
Total	a + c	b + d	

$$\chi^2 = \frac{N(ad-bc)}{(a+b)(c+d)(a+c)(a+d)}$$

7. A random sample of 200 tins of coconut oil gave an average weight of 4.95. With a standard deviation of 0.21kg. Do we accept that the net weight is 5 kgs per tin at 5% level?

Ans :

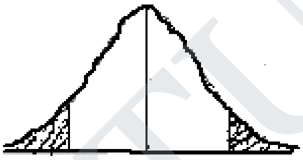
Given $n = 200$, $\mu = 5$, $s = 0.21$, $\overline{x} = 4.95$, $\alpha = 5\ \%$

1. $H_0 : \mu = 5$

2. $H_1 : \mu \neq 5$ [use two tailed test]

3. $\alpha = 5\%$

4. critical region



$-Z_{\alpha/2}$ $Z_{\alpha/2}$
$= -1.96$ $= 1.96$

5. The test statistic $Z = \dfrac{\overline{x}-\mu}{\left(\dfrac{s}{\sqrt{n}}\right)} = \dfrac{4.95-5}{\left(\dfrac{0.21}{\sqrt{200}}\right)} = -3.36$

$|\ Z\ | = 3.36$

6. Conclusion : If $|\ Z\ | = 3.36 \not< 1.96$ we accept H_0

8. State level of significance.

Solution:

The probability that the value of the statistic lies in the critical region is called the level of significance.

In general, these levels are chosen as 0.01 or 0.05, called 1% level and 5% level of

STUCOR APP

DOWNLOADED FROM STUCOR APP

	significance respectively.
9.	State the conditions for applying χ^2 test. **Ans:**(i) The Sample observations should be independent. (ii) Constraints on the cell frequencies, if any, must be linear. (iii) N, the total frequency, should be atleast 50. (iv) No theoretical cell frequency should be less than 5.
10	Write the formula for the Chi-square test of goodness of fit of a random sample to a hypothetical distribution. **Ans :** Chi square test of goodness of fit is a test to find if the derivation of the experiment from theory is just by chance or it is due to the inadequacy of the theory to fit the observed data. By this test, we test whether differences between observed and expected frequencies are significant or not. Chi square test statistic of goodness of fit is defined by $\chi^2 = \sum \frac{(O-E)^2}{E}$, where O is the observed frequency and E the expected frequency.
11.	Write down the formula of test statistic t to test the significance of difference between the means of large samples. **Ans :** $Z = \frac{\left\lvert \overline{x}_1 - \overline{x}_2 \right\rvert}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ where $\overline{x}_1$, $\overline{x}_2$ sample means ; s_1^2 & s_2^2 sample variances ; n_1 & n_1 sample sizes.
12.	What is random sampling ? **Ans :** A random sampling is one in which each number of population has an equal chance of being included in it. There are NC_n different samples of size n that can be picked up from a population size N.
13.	Write about F-test. **Ans :** A random variable F is said to follow F-distribution with (v_1, v_2) degrees of freedom if its density function given by $f(F) = K F^{\frac{v_1}{2}-1} \left(1 + \frac{v_1 F}{v_2}\right)_{\infty}^{-\frac{(v_1+v_2)}{2}}$ where F > 0 and k is constant to be determined such that $\int_0^\infty f(F)\, dF = 1$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

14. Two sample polls of votes for two candidates A and B for a public office are taken one from among residents of rural areas. The results are given below. Examine whether the nature of the area is related to voting preference in this election.

Area/votes for	A	B	Total
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

Ans :

The parameter of interest is ψ^2

1. H_0: The nature area is independent of voiting preferncе in the electiion
2. H_1: dependent
3. $\alpha = 0.05 \quad d.f.=(r-1)(s-1)=(2-1)(2-1)=1$
4. Table value of $\psi^2 = 5.991$
5. The test statistic is $\psi^2 = \sum \frac{(O-E)^2}{E}$

$$\text{expected frequency} = \frac{(\text{Corresponding Row total})(\text{Column total})}{\text{Grand total}}$$

$$\text{Expected frequency } 620 = \frac{(1170)(1000)}{2000} = 585$$

$$380 = \frac{(830)(1000)}{2000} = 415$$

$$550 = \frac{(1170)(1000)}{2000} = 585$$

$$450 = \frac{(830)(1000)}{2000} = 415$$

O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
620	585	35	1225	2
380	415	-35	1225	3
550	585	-35	1225	2
450	415	35	1225	3
				10

6. Conclusion : If cal $\psi^2 <$ table ψ^2

Then we accept H_0 , here 10 > 5.5991

15. Fit a Poisson distribution to the following data and test the goodness of fit.

X	0	1	2	3	4	5	6
F(x)	275	72	30	7	5	2	1

Ans :

1. The parameter of interest is λ
2. H_0 : Poisson fit is a good fit.

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

3. H_1 : Poisson fit is a not good fit
4. $\alpha = 0.05 \quad d.f. = 7-1-1-2 = 3$
5. The test statistic is $\psi^2 = \sum \frac{(O-E)^2}{E}$
6. Reject H_0 if $\psi^2 > 7.82$
7. Computation :

Mean of the distribution $= \frac{\sum f_i x_i}{\sum f_i} = \frac{189}{392} = 0.48$

x_i	f_i	$f_i x_i$
0	275	0
1	72	72
2	30	60
3	7	21
4	5	20
5	2	10
6	1	6
	N = 392	189

To fit a Poisson distribution we require 'm' parameter $m = \bar{x} = 0.48$. By Poisson distribution the frequency of r successes is

Hence the frequency table is

X	0	1	2	3	4	5	6	Total
Observed Frequency	275	72	30	7	5	2	1	392
Expected frequency	243	116	28	4	1	0	0	392

Since the Four frequencies are small , so we pull then together.

O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
275 72 30 15	243 116 28 5	32 -44 2 10	1024 1936 4 100	4.21 16.69 0.14 20.00
				41.04

Conclusion :

Since $\psi^2 = 41.04 > 7.82$ so we reject H_0 at 5 % level of significance.

16. Sandal power is packed into packets by a machine. A random sample of 12 packets is drawn and their weight are found to be (in kg) 0.49 , 0.48 , 0.47 , 0.48 , 0.49, 0.50 , 0.51, 0.49, 0.48, 0.50, 0.51 and 0.48 . Test if the average weight of the packing can be taken as 0.5 kg at 5 % level of significance.

Ans :

Given $n = 12 \quad \mu = 0.5$

X	0.49	0.48	0.47	0.48	0.49	0.50	0.51	0.49	0.48

STUCOR APP

DOWNLOADED FROM STUCOR APP

X^2	0.49^2	0.48^2	0.47^2	0.48^2	0.49^2	0.50^2	0.51^2	0.49^2	0.48^2

0.50	0.51	0.48	Total = 5.88
0.50^2	0.51^2	0.48^2	2.883

$$\bar{x} = \frac{\sum x}{n} = \frac{5.88}{12} = 0.49$$

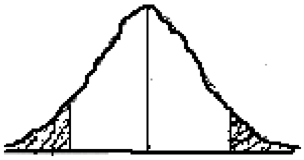
$$s^2 = \frac{\sum x^2}{n} - (\bar{x})^2 = \frac{2.883}{12} - (0.49)^2 = 0.00015 \quad \Rightarrow \ s = 0.012$$

1. $H_0 : \mu = 0.5$

2. $H_1 : \mu \neq 0.5$ [*Two tailed test*]

3. $\alpha = 5\%$, $d.f = n-1 = 12-1 = 11$

4. critical region



$-t_{\alpha/2}$ $-t_{\alpha/2}$

$= -2.201$ $= 2.201$

5. The test statistic $t = \dfrac{\bar{x} - \mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)} = \dfrac{0.49 - 0.5}{\left(\dfrac{0.012}{\sqrt{12-1}}\right)} = -2.76$

6. Conclusion : If $-t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}}$ we accept H_0 , otherwise we reject H_0

Here $-2.201 \nless -2.76 < 2.201$

$\therefore$ we reject H_0 at 5% level of significance.

17. A group of 10 rats fed on diet A and another group of 8 rats fed on diet B recorded the following increase in weight

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	3		

Show that the estimates of the population variance from the samples are not significantly different.

Ans :

Given $n_1 = 10$, $n_2 = 8$

												Total
Sample I	x_1	5	6	8	1	12	4	3	9	6	10	64
	x_1^2	5^2	6^2	8^2	1^2	12^2	4^2	3^2	9^2	6^2	10^2	512
Sample II	x_2	2	3	6	8	10	1	2	8			400
	x_2^2	2^2	3^2	6^2	8^2	10^2	1^2	2^2	8^2			282

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$\overline{x_1} = \frac{\sum x_1}{n_1} = \frac{64}{10} = 6.4 \qquad \& \qquad \overline{x_2} = \frac{\sum x_2}{n_2} = \frac{40}{8} = 5$$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x_1})^2 = \frac{512}{10} - (6.4)^2 = 10.24 \qquad \& \quad s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x_2})^2 = \frac{282}{8} - (5)^2 = 1$$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(10)(10.24)}{9} = 11.3777 \qquad \& \quad S_2^2 = \frac{n_2 1 s_2^2}{n_2 - 1} = \frac{(8)(10.25)}{7} = 11.7143$$

$$S_2^2 > S_1^2$$

The parameter of interest is σ_1^2 and σ_1^2

1. $H_0 : \sigma_1^2 = \sigma_2^2$ [The difference of a varaince is not significant]
2. $H_1 : \sigma_1^2 \neq \sigma_2^2$
3. $\alpha = 0.05$, $d.f(v_1) = 9$, $d.f(v_2) = 7$
4. Table value of $F = 3.29$
5. The test statistic is $F = \frac{S_2^2}{S_1^2} = \frac{11.7143}{11.3777} = 1.02958$

6.Conclusion :

If Cal F < table F , then we accept H_0 ; otherwise we reject H_0

Here , F = 1.02958 < 3.29 , we accept H_0 at 5 % level of significance.

We conclude that the two samples have come from populations with equal variances.

18. A dice is thrown 400 times and a throw of 3 or 4 is observed 150 times. Test the hypothesis that the dice is fair.

Solution:

Given n = 400, X = Number of successes = 150

P = probability of getting 3 or 4. $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

$Q = 1 - P = 1 - \frac{1}{3} = \frac{2}{3}$

$$H_0 = P = \frac{1}{3}(unbaised)$$

$$H_1 : P \neq \frac{1}{3}$$

$$Z = \frac{X - nP}{\sqrt{nPQ}} = \frac{150 - 400\frac{1}{3}}{\sqrt{(400)(\frac{1}{3})(\frac{2}{3})}} = \frac{16.6667}{9.4281} = 1.7678$$

Cal $|z| = 1.7678$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

For two tailed test = $|z| = 1.96$ $(5\%\ level)$

$\text{Cal}|z| < table|z|$

Accept $H_{0.}$ Thus the dice is fair.

19. Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory?

Solution:

Given : A,B,C,D in the ratio 9:3:3:1

					Total
E_i	900	300	300	100	1600
O_i	882	313	287	118	1600

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{18^2}{900} + \frac{13^2}{300} + \frac{13^2}{300} + \frac{18^2}{100} = 4.73$$

$\sum E_i = \sum O_{i,}$ degree of freedom $= 4 - 1 = 3$

$H_0 = Theexperimentsupportsthetheory$

Cal $\chi^2 = 4.73$

Table value for 3 degree of freedom = 7.82 $\Rightarrow$ $\text{Cal}cal\chi^2 < tab\chi^2$

Accept H_0.

20. The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0inches respectively. Can the samples be regarded as drawn from the same populations of standard deviation 2.5 inches?

Solution:

Given n_1=1000, $n_2 = 2000, \overline{x_2} = 67.5,$ $\overline{x_1} = 68$ $\sigma_1 = \sigma_2 = 2.5$

The parameter of interests is $\mu_1 and \mu_2$

1. $H_0 : \mu_1 = \mu_2$ [No significant difference]

2. $H_1 : \mu_1 \neq \mu_2$

3. α = 0.05, (Two Tailed Test)

the test statistic is $$z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{67.5 - 68}{\sqrt{\frac{2.5^2}{1000} + \frac{2.5^2}{2000}}} = \frac{-0.5}{\sqrt{0.00625 + 0.003125}} = -5.16$$

Conclusion: -1.96 < - 5.16 < 1.96 So we reject $H_{0.}$

21. Two random samples gave the following results:

Sample	Size	Sample Mean	Sum of squares of deviations from the

STUCOR APP

DOWNLOADED FROM STUCOR APP

			mean
1	10	15	90
2	12	14	108

Test whether the samples have come from the same normal population.

Solution:

A normal population has two parameters namely the mean μ and the variance $\boldsymbol{\sigma}^2$. If we want to test the samples from the same normal population,we have to test

(i) The equality of population variance (Using F-Test)

(ii) The equality of population means (using t-test).

Since t-test assumes$\boldsymbol{\sigma}_1^2 = \boldsymbol{\sigma}_2^2$ we shall first apply F-test and then t-test.

(i) F-test:

Given: $n_{1=}10,\ n_2 = 12, \overline{x_2}$ =14, $\overline{x_1}$ =15

$$S_1^2 = \frac{\sum(x_1-\overline{x_1})^2}{n_1-1} = \frac{90}{9} = 10S_2^2 = \frac{\sum(x_2-\overline{x_2})^2}{n_2-1} = \frac{108}{11} = 9.8181$$

$$S_1^2 > S_2^2$$

The parameters of interest is $\boldsymbol{\sigma}_1^2 \quad and \quad \boldsymbol{\sigma}_2^2$

1. $H_0 \ : \boldsymbol{\sigma}_1^2 = \boldsymbol{\sigma}_2^2$
2. $H_1 \ : \boldsymbol{\sigma}_1^2 \neq \boldsymbol{\sigma}_2^2$
3. α = 0.05, d.f.$(v_1) = n_1 - 1 = 9$d.f.$(v_2) = n_2 - 1 = 11$
4. Table value of F :2.90
5. The test statistic is F = $\frac{S_1^2}{S_2^2} = \frac{10}{9.8182} = 1.019$
6. Conclusion:

If Cal F < table F, then we accept H_0; Otherwise we reject H_0.

Here, F = 1.019 < 2.90, we accept H_0 at 5% level of significance.

(Note: If F-test failed, then t-test should not be used)

(ii) t-test:

Given n_1=10, $n_2 = 12, S_1^2 = 10,\ S_2^2 = 9.8181$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{100 + 117.82}{10 + 12 - 2} = 10.9$$

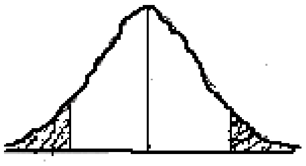
The parameter of interests is $\mu_1 \quad and \quad \mu_2$

1. $H_0: \mu_1 = \mu_2$
2. $H_1: \mu_1 \neq \mu_2$
3. α = 0.05, d.f = $n_1 + n_2 - 2 = 20$ (Two Tailed Test)
4. Critical region

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP



$-t_{\alpha/2}$ $-t_{\alpha/2}$

$= -2.086 = 2.086$

5.the test statistic is $t = \frac{\overline{x_1}-\overline{x_2}}{\sqrt{S^2(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{15-14}{\sqrt{(10.9)(\frac{1}{10}+\frac{1}{12})}} = 0.707$

6. Conclusions:

Calculated $|t|=$ 0.707< 2.086, we accept H_0 at 5% level of significance.

22. Test whether there is any significant difference between the variances of the populations from which the following samples are taken.

Sample : 1	20	16	26	27	23	22	
Sample : 2	27	33	42	35	32	34	38

Solution:

Given$n_1 = 6,\ n_2 = 7$

	x_1	x_1^2	x_2	x_2^2
	20	400	27	729
	16	256	33	1089
	26	676	42	1764
	27	729	35	1225
	23	529	32	1024
	22	484	34	1156
Total	134	3074	241	8431

$\overline{x_1} = \frac{\Sigma x_1}{6} = \frac{134}{6} = 22.33$

$\overline{x_2} = \frac{\Sigma x_2}{7} = \frac{241}{7} = 34.33$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x}_1)^2 = \frac{3074}{6} - (22.33)^2 = 13.70$$

$$s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x}_2)^2 = \frac{8431}{7} - (34.43)^2 = 19.00$$

$$s_2^2 > s_1^2$$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

$$H_0: \sigma_1^2 = \sigma_2^2 \quad \& \quad H_1: \sigma_1^2 \neq \sigma_2^2$$

$$\alpha = 5\% = 0.05 \qquad d.f.\ v_1 = n_1 - 1 = 5 \ \ \& \ \ v_2 = n_2 - 1 = 6$$

$$F = \frac{s_2^2}{s_1^2} = \frac{19}{13.7} = 1.39 \qquad F(6,5) = 4.95$$

Cal F < Tabulated F. $\therefore$ Accept H_0

23. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with the SD of 6,While boys made an average grade of 82 with the SD of 2.Test whether there is any significant difference between the performance of boys and girls.

Solution:

	No of cases	Mean	S.D
Sample I	50	76	6
Sample II	75	82	2

H_0: There is no significant difference.

H_1: There is significant difference.

$$Z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{76-82}{\sqrt{\frac{6^2}{50} + \frac{2^2}{75}}} = \frac{-6}{\sqrt{\frac{36}{50} + \frac{4}{75}}} = \frac{-6}{\sqrt{\frac{58}{75}}} = \frac{-6}{0.88} = -6.82$$

$|Z| = 6.82 > 1.96$ also $|Z| > 2.58$

So, we reject H_0.

24. A sample of 10 boys had the I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 100 and 107. Test whether the population mean I.Q may be 100.

Ans :

Given

$$\sum x_1 = 70 + 120 + 110 + 101 + 88 + 83 + 95 + 98 + 100 + 107 = 972$$

$$\sum x_1^2 = 70^2 + 120^2 + 110^2 + 101^2 + 88^2 + 83^2 + 95^2 + 98^2 + 100^2 + 107^2 = 96312$$

$$\overline{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$$

$$s^2 = \frac{\sum x^2}{n} - (\overline{x})^2 = \frac{96312}{10} - (97.2)^2 = 9631.2 - 9447.84 = 183.36$$

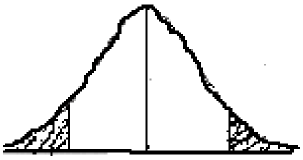
1. $H_0 : \mu = 100$
2. $H_1 : \mu \neq 100$
3. $\alpha = 5\%$, $d.f = n - 1 = 10 - 1 = 9$

4. critical region

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP



$-t_{\alpha/2}$ $-t_{\alpha/2}$

$= -2.26 \sim 5\% = 2.26 \sim 5\%$

5. The test statistic $t = \dfrac{x-\mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)} = \dfrac{97.2-100}{\left(\dfrac{13.5}{\sqrt{9}}\right)} = \dfrac{-2.8}{4.5} = -0.62$

6. Conclusion : If $-t_{\alpha/2} < t < t_{\alpha/2}$

we accept H_0 , otherwise we reject H_0

Here -2.262 < - 0.62 < 2.262 $\therefore$ we accept H_0

25. Random samples drawn from two countries gave the following data relating to the heights of adult males. Is the difference between standard deviation significant?

	Country A	Country B
Mean height (in inches)	67.42	67.25
S.D (in inches)	2.58	2.50
Number in samples	1000	1200

Ans :

Given

$n_1 = 1000$, $\overline{x_1} = 67.42$, $s_1 = 2.58$

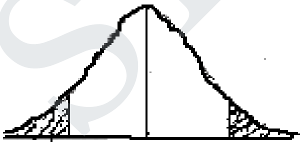
$n_2 = 1200$, $\overline{x_2} = 67.25$, $s_2 = 2.50$

1. $H_0 : \mu_1 = \mu_2$

2. $H_1 : \mu_1 \neq \mu_2$ [use two tailed test]

3. $\alpha = 5\%$

4. critical region



$-Z_{\alpha/2}$ $Z_{\alpha/2}$

$= -1.96 = 1.96$

5. The test statistic $Z = \dfrac{(\overline{x_1} - \overline{x_2})}{\sqrt{\dfrac{s_1^{\,2}}{n_1} + \dfrac{s_2^{\,2}}{n_2}}} = \dfrac{67.42 - 67.25}{\sqrt{\dfrac{(2.58)^2}{1000} + \dfrac{(2.50)^2}{1200}}} = 1.03$

6. Conclusion : If $-Z_{\alpha/2} < Z < Z_{\alpha/2}$ then accept H_0, otherwise we

STUCOR APP

DOWNLOADED FROM STUCOR APP

reject H_0 . Here $-1.96 \not< -5.16 < 1.96$ So we reject $H_{0.}$

26. 1000 students at college level were graded according to their I.Q. and their economic conditions. What conclusion can you draw from the following data:

Economic conditions	I.Q. level	
	High	Low
Rich	460	140
Poor	240	160

Ans :

The parameter of interest is χ^2

1. H_0 : The given attributes are independent
2. H_1 : The given attributes are not independent le.
3. $\alpha = 0.05$, d. f = (r-1)(s-1) = (2-1)(2-1) = 1

4. Table value of χ^2 = 3.841

5. The test statistic is $\chi^2 = \sum\frac{(O-E)^2}{E}$

The expected frequencies are calculated using the following formulae

Expected frequency for 460 $= \frac{600 \times 700}{1000} = 420$

Expected frequency for 140 $= \frac{600 \times 300}{1000} = 80$

Expected frequency for 240 $= \frac{700 \times 400}{1000} = 280$

Expected frequency for 160 $= \frac{300 \times 400}{1000} = 120$

O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
460	400	40	1600	3.81
140	180	- 40	1600	8.88
240	100	-40	1600	5.714
160	100	-40	1600	13.33
				31.7373

$$\therefore \quad \chi^2 = \sum\frac{(O-E)^2}{E} = 31.7373$$

8. Conclusion:

If Cal $\chi^2 <$ table χ^2, then we accept H_0 . Otherwise , we reject H_0

Here, $\chi^2 = 31.7373 > 3.841$ .16 . so reject H_0 at 5 % level of significance.
We conclude that the attributes I.Q as economic conditions are not independent.

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

27.

The sales manager of a large company conducted a sample survey in states A and B taking 400 samples in each case. The results were in the following table. Test whether the average sales in the same in the 2 states at 1 % level.

Average sales S.D.	State A	State B
	Rs. 2,500	Rs. 2,200
	Rs. 400	Rs. 550

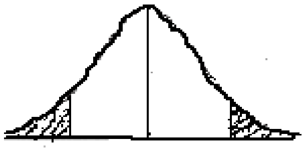
Ans :

Given

$n_1 = 400$, $\overline{x_1} = 2500$, $s_1 = 400$

$n_2 = 400$, $\overline{x_2} = 2200$, $s_2 = 550$ $\alpha = 1\%$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 \neq \mu_2$ [use two tailed test]
3. $\alpha = 1\%$
4. critical region



$-Z_{\alpha/2}$ $Z_{\alpha/2}$

$= -2.58$ $= 2.58$

5. The test statistic $Z = \dfrac{(\overline{x_1} - \overline{x_2})}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{2500 - 2000}{\sqrt{\dfrac{(400)^2}{400} + \dfrac{(550)^2}{400}}} = 8.82$

6. Conclusion :

If $-Z_{\alpha/2} < Z < Z_{\alpha/2}$ then accept H_0, otherwise we reject H_0 .

Here - 2.58 < 8.82 $\nless$ 2.58 So we reject H_0.

28.

Find if there is any association between extravagance in fathers and extravagance in sons from the following data. Determine the coefficient of association also

	Extravagant father	Miserly father
Extrav. sons	Under 327	741
Misser. Sons	545	234

Ans :

1. The parameter interest is χ^2
2. H_0 : Namely that the extravagance in sons and fathers are not significant.
3. H_1 : Significant
4. $\alpha = 0.05$, $d.f = (r-1)(s-1) = (2-1)(2-1) = 1$
5. The test statistic is $\chi^2 = \dfrac{(ad-bc)^2(a+b+c+d)}{(a+b)(c+d)(a+c)(b+d)}$
6. Reject H_0 if $\chi^2 > 3.84$

STUCOR APP

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

7. Computations :

$$\chi^2 = \frac{[(327)(234)-(545)(741)]^2(327+545+741+234)}{(872)(975)(1068)(779)} = 279.77$$

8. Conclusion :

Since $\chi^2 = 230.24 > 3.841$ solve reject H_0 at 5 % level of significance

∴ There is dependence between the attributes

Coefficient of attributes $= \dfrac{ad-bc}{ad+bc} = \dfrac{-327330}{480363} = -0.6814$

29. Time taken by workers in performing a job are given below:

Type I :	21	17	27	28	24	23	--
Type II :	28	34	43	36	33	35	39

Test whether there is any significant difference between the variance of time distribution.

Solution:

$Here, n_1 = 6, \& n_2 = 7$

								Total
X_1 :	21	17	27	28	24	23	--	140
X_1^2 :	441	289	729	784	576	529	--	3348
X_2 :	28	34	43	36	33	35	39	248
X_2^2 :	784	1156	1849	1296	1089	1225	1521	8920

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{140}{6} = 23.3$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{248}{7} = 35.43$$

$$s_1{}^2 = \frac{\sum x_1{}^2}{n_1} - (\bar{x}_1)^2 = \frac{3348}{6} - (23.3)^2 = 15.11$$

$$s_2{}^2 = \frac{\sum x_2{}^2}{n_2} - (\bar{x}_2)^2 = \frac{8920}{7} - (35.43)^2 = 19$$

$$S_1{}^2 = \frac{n_1 s_1{}^2}{n_1 - 1} = \frac{(6)(15.11)}{6-1} = \frac{90.66}{5} = 18.132$$

$$S_2{}^2 = \frac{n_2 s_2{}^2}{n_2 - 1} = \frac{(7)(19)}{7-1} = \frac{133}{6} = 22.17$$

$Here,\ S_2{}^2 > S_1{}^2$

1. The parameter of interest is $\sigma_1{}^2$ $and\ \sigma_2{}^2$

2. $H_0 : \sigma_1{}^2 = \sigma_2{}^2$ [The difference of variance is not significant]

3. $\mathrm{H}_1 : \sigma_1{}^2 \neq \sigma_2{}^2$

4. $\alpha = 0.05, d.f(v_1) = 5, d.f(v_2) = 6$

5. $F = \frac{S_2{}^2}{S_1{}^2}$

6. Reject H_0 if $\mathrm{F} > 3.29$ [From F table]

7. Computation:

$$F = \frac{S_2{}^2}{S_1{}^2} = \frac{22.17}{18.132}$$

8. Conclusion: $\mathrm{F} = 1.22 < 3.29$, we accept H_0 at 5% level of significance.

30. The sales manager of a large company conducted a sample survey in two places A and B taking 200 samples in each case. The results were the following table. Test whether the average sales in the same in the two areas at 5% level.

Solution:

	Place A	Place B
Average sales	Rs.2,000	Rs.1,700
S.D	Rs.200	Rs.450

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$Given: \quad n_1 = 200, \quad \bar{x}_1 = 2000, \quad s_1 = 200$

$\quad n_2 = 200, \quad \bar{x}_2 = 1700, \quad s_2 = 450$

1. $H_0 : \mu_1 = \mu_2$

2. $H_1 : \mu_1 \neq \mu_2$ [Use two – tailed test]

3. $\alpha = 5\%$

5. The test statistic

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{2000 - 1700}{\sqrt{\frac{(200)^2}{200} + \frac{(450)^2}{200}}} = \frac{300}{34.8} = 8.62$$

6. Conclusion :

If $-Z_{\alpha/2} < Z < Z_{\alpha/2}$, then accept H_0; otherwise, we reject H_0.

Here, $-.196 < 8.62 < 1.96$

So, we reject H_0

31. A survey of 320 families with 5 children each revealed the following distribution :

No. of boys	5	4	3	2	1	0
No. of girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

Is this result consistent with the hypothesis that male and female births are equally probable ?

Ans:

1. H_0: Male and Female births are equally probable.
2. H_1: Male and Female births are not equally probable.
3. $\alpha =$ 0.05, d.f = n-1 = 6-1 = 5
4. Table value of χ^2 =11.07
5. The test statistic is $\chi^2 = \Sigma \frac{(O-E)^2}{E}$

On the assumption H_0,

the expected frequencies frequencies are given by the terms of $N(q+p)^n$

$$= 320\left(\frac{1}{2}+\frac{1}{2}\right)^5 = 320\left[5C_0 + 5C_1 + 5C_2 + 5C_3 + 5C_4 + 5C_5\right]$$

$$= 10\ [1+5+10+10+5+1]$$

The expected frequencies are 10,50,100,100,50,10

No. of boys	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
5	14	10	4	16	1.6
4	56	50	6	36	0.72
3	110	100	10	100	1
2	88	100	-12	144	1.44
1	40	50	-10	100	2

STUCOR APP

DOWNLOADED FROM STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

0	12	10	2	4	0.4
Total	320				7.16

$$\therefore \quad \chi^2 = \sum \frac{(O-E)^2}{E} = 7.16$$

8. Conclusion:

If Cal $\chi^2 <$ table χ^2, then we accept H_0 . Otherwise , we reject H_0

Here, 7.16 < 11.06 , so we accept H_0

32. Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables.

Sample I	18	13	12	15	12	14	16	14	15
Sample II	16	19	13	16	18	13	15		

Do the estimates of population variance differ significantly at 5% level of significance ?

Ans:

Given $n_1 = 9$, $n_2 = 7$

											Total
Sample I	x_1	18	13	12	15	12	14	16	14	15	129
	x_1^2	18^2	13^2	12^2	15^2	12^2	14^2	16^2	14^2	15^2	1871
Sample II	x_2	16	19	13	16	18	13	15			110
	x_2^2	16^2	19^2	13^2	16^2	18^2	13^2	15^2			1760

$$\overline{x_1} = \frac{\sum x_1}{n_1} = \frac{129}{9} = 14.333 \quad \& \quad \overline{x_2} = \frac{\sum x_2}{n_2} = \frac{110}{7} = 15.7143$$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x_1})^2 = \frac{1879}{9} - (14.3333)^2 = 3.3342$$

$$s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x_2})^2 = \frac{1760}{7} - (15.7143)^2 = 4.4894$$

$$s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(9)(3.3342)}{8} = 3.751$$

$$s_2^2 = \frac{n_{21} s_2^2}{n_2 - 1} = \frac{(7)(4.4894)}{6} = 5.2376$$

$$s_2^2 > s_1^2$$

The parameter of interest is σ_1^2 and σ_1^2

1. $H_0 : \sigma_1^2 = \sigma_2^2$

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

2. $H_1 : \sigma_1^2 \neq \sigma_2^2$

3. $\alpha = 0.05$, $d.f\,(v_1) = n_1 - 1 = 9 - 1 = 8$, $d.f\,(v_2) = n_2 - 1 = 7 - 1 = 6$

4. Table value of $F = 3.58$

5. The test statistic is $F = \dfrac{s_2^2}{s_1^2} = \dfrac{5.2376}{3.7510} = 1.3963$

6.Conclusion :

If Cal F < table F , then we accept H_0 ; otherwise we reject H_0

Here , F = 1.3963 < 3.58 , we accept H_0 at 5 % level of significance.

We conclude that the difference is not significant.

33. Explain clearly the procedure generally followed in testing of a hypothesis.

Ans :

1. Set up null hypothesis H_0: It is a statement of no difference about the population parameters.
2. Set up alternative hypothesis H_1. It is a statement, complement to H_0
3. Compute the test statistic.
4. Obtain the tabulated value of the statistic at given level of significance (α)
5. Write the conclusion. If tabulated value of the statistic is less than the calculated value, the reject H_0.

34. The demand for a particular spare part in a factory was found to vary for day-to-day. In a sample study the following information was obtained.

Days	Mon	Tues	Wed	Thu	Fri	Sat
No. of spare parts demanded	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week $(\chi^2_{0.05}(5) = 11.07)$

Ans :

Null hypothesis H_0: The number of parts demanded does not depend on the day of the week.

Attractive hypothesis H_1: The number of parts demanded depend on the day of the week.

Statistic : $\chi^2 = \sum \dfrac{(O-E)^2}{E}$

$$E = \frac{1124+1125+1110+1120+1126+1115}{6} \Rightarrow E = 1120$$

O	1124	1125	1110	1120	1126	1115	
E	1120	1120	1120	1120	1120	1120	

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$\frac{(O-E)^2}{E}$	0.0143	0.0223	0.0893	0	0.0321	0.0223	0.1803

Calculated $\chi^2 = 0.1803$

Table $\chi^2(5) = 11.07$

Conclusion : Table $\chi^2 >$ Cal χ^2 . Accept H_0 .

35. Explain briefly the procedure involved in testing the significance for difference of proportions in the case of large samples.

Ans :

1. Null hypothesis H_0: There is no significant difference.
2. Alternative hypothesis $H_1 : p_1 \neq p_2$
3. Statistic : $Z = \left| \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \right|$ where $H_1 : p_1 \neq p_2$

   Where $P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ and Q= 1 - P

   n_1 , n_2 are sample sizes.
4. Obtain the tabulated Z for given LOS (α)
5. If tab Z < Cal Z , Reject H_0

36. The height of six randomly chosen sailors are (in inches) : 63, 65, 68, 69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Discuss, the height that these data thrown on the suggestion that sailors are on the average taller than soldiers $(t_{0.01}(14) = 1.76)$

Ans:

$$\sum x_1 = 63 + 65 + 68 + 69 + 71 + 72 = 408$$

$$\sum x_1^2 = 63^2 + 65^2 + 68^2 + 69^2 + 71^2 + 72^2 = 27804$$

$$\sum x_2 = 61 + 62 + 65 + 66 + 69 + 69 + 70 + 71 + 72 + 73 = 678$$

$$\sum x_2^2 = 61^2 + 62^2 + 65^2 + 66^2 + 69^2 + 69^2 + 70^2 + 71^2 + 72^2 + 73^2 = 46122$$

STUCOR APP

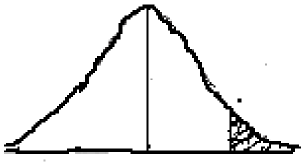
$\overline{x_1} = \frac{\sum x_1}{n_1} = \frac{408}{6} = 68$ & $\overline{x_2} = \frac{\sum x_2}{n_2} = \frac{678}{10} = 67.8$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x_1})^2 = \frac{27804}{6} - (68)^2 = 10$$

$$s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x_2})^2 = \frac{46122}{10} - (67.8)^2 = 15.26$$

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{(6)(10) + (10)(15.36)}{6 + 10 - 2} = 15.26$$

1. H_0 ; $\mu_1 = \mu_2$
2. H_1 ; $\mu_1 > \mu_2$ $= 50$
3. $\alpha = 5\ \%$, $d.f = n - 1 = 5 - 1 = 4$
4. critical region



$-t_\alpha = 7.76$

$= 2.26$

5. The test statistic $t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{68 - 67.8}{\sqrt{15.26\left(\frac{1}{6} + \frac{1}{10}\right)}} = 0.099$

6. Conclusion : If $t < t_\alpha$ we accept H_0 , otherwise we reject H_0

Here $0.099 < 1.76$ ∴ we accept H_0

The sailors are not an the average taller than the soldiers

37. Fit a binomial distribution for the following data and also test the goodness of fit.

X	0	1	2	3	4	5	6	Total
F(X)	5	18	28	12	7	6	4	80

Ans :

1. H_0 : Binomial is good fit
2. H_1 : Binomial is not good fit

STUCOR APP

DOWNLOADED FROM STUCOR APP

3. $\alpha = 0.05 \quad d.f. = n-1 = 7-1 = 6$

4. Table value of $\psi^2 = 12.592$

5. The test statistic is $\psi^2 = \sum \frac{(O-E)^2}{E}$

On the assumption H_0 , the expected frequencies frequencies are given by the terms of

$$= 80\left(\frac{1}{2}+\frac{1}{2}\right)^6 \quad = 80\left[6C_0 + 6C_1 + 6C_2 + 6C_3 + 6C_4 + 6C_5 + 6C_6\right]$$

$$= \frac{80}{64}\left[1+6+15+20+15+6+1\right] \; = \; 1.25\,[1+6+15+20+15+6+1]$$

x	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
0	5	1.25	3.75	14.1	11.28
1	18	7.5	10.5	110.3	14.71
2	28	18.75	9.25	85.6	4.57
3	12	25	-13	169	6.76
4	7	18.75	-11.75	138.1	7.37
5	6	7.5	-1.5	2.3	0.31
6	4	1.25	2.75	7.6	6.08
	N = 80				51.08

$\therefore \psi^2 = 51.08$

6. Conclusion :

If cal $\psi^2 <$ table ψ^2

Then we accept H_0 , here 12.592 < 51.08

$\therefore$ we accept H_0

38. The mean value of a random sample of 60 items was found to be 145, with a standard deviation of 40. Find the 95% confidence limits for the population mean. What size of the sample is required to estimate the population mean within 5 of its actual value 95% or more confidence, using the sample mean ?

Ans :

S.D. is not given

$\therefore$ limits of μ is $\left|\frac{\mu - \bar{x}}{s/\sqrt{n}}\right| \leq 1.96$

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$\bar{x}-1.96\frac{s}{\sqrt{n}} \le \mu \le \bar{x}+1.96\frac{s}{\sqrt{n}}$$

$$134.9 \le \mu \le 155.1$$

$$p\left(\bar{x}-5 \le \mu \le \bar{x}+5\right) \ge 0.95 \quad \Rightarrow \quad p\left(\left|\bar{x}-\mu\right| \le 5\right) \ge 0.95$$

$$(ie)\ p\left(|z| \le \frac{5\sqrt{n}}{\sigma}\right) \ge 0.95$$

$$wkt \quad p\left(|z| \le 1.96\right) = 0.95$$

$$\therefore \quad n = n\frac{5\sqrt{n_1}}{\sigma} = 1.96 \quad \Rightarrow \quad \sqrt{n_1} = \frac{1.95}{5} \quad \Rightarrow \quad n_1 = \left(\frac{1.96\times 40}{5}\right)^2 = 245.86$$

$\therefore$ Least sample since is 246

39. Test made on the breaking strength of 10 pieces of a metal gave the following results 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg.

Ans :

Find $\bar{x}$ and sample S. D. s and then test if $\bar{x}$ differs from $\mu = 577$

Assume A = 582

$$d_i = x_i - A \quad , \quad x_i = d_i + A$$

$$\bar{x} = \frac{\sum d_i}{n} + A = 575.2 \quad , \quad s^2 = \frac{1}{n}\sum d_i^2 - \left(\frac{\sum d_i}{n}\right)^2 = 8.26$$

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = -0.65 \quad , \quad v = n-1 = 9$$

n_i :	578	572	570	568	572	570	570	572	596	584
d_i :	-4	-10	-12	-14	-10	-12	-12	-10	14	2
d_i^2 :	16	100	144	196	100	144	144	100	196	4

$H_0 : \bar{x} = \mu \ H_1 : \bar{x} \ne \mu$

$t_{0.05} = 2.26 \ |t| < t_{0.05} \ \therefore$ H_0 is accepted.

$\therefore$ we can assumed 577 kg at 5% loss.

40. Test if the variances are significantly different for

X_1 24 27 26 21 25

X_2 27 30 32 36 28 23

Ans :

Given $n_1 = 5$, $n_2 = 6$

STUCOR APP

DOWNLOADED FROM STUCOR APP

x_1	x_1^2	x_2	x_2^2
24	576	27	729
27	729	30	900
26	676	32	1024
21	441	36	1296
25	625	28	784
		23	529
123	3047	176	5262

$$\overline{x}_1 = \frac{123}{5} = 24.6 \quad \& \quad \overline{x}_2 = \frac{176}{6} = 29.33$$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x}_1)^2 = \frac{3047}{5} - (24.6)^2 = 4.24 \quad \& \quad s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x}_2)^2 = \frac{5262}{6} - (29.33)^2 = 16.7511$$

$$s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(5)(4.24)}{4} = 5.3 \quad \& \quad s_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{(6)(16.7511)}{5} = 20.10$$

$$s_2^2 > s_1^2$$

$$H_0 : \sigma_1^2 = \sigma_2^2 \quad \& \quad H_1 : \sigma_1^2 \neq \sigma_2^2$$

$$\alpha = 5\% = 0.05 \quad d.f.\ v_1 = n_1 - 1 = 4 \quad \& \quad v_2 = n_2 - 1 = 5$$

$$F = \frac{s_2^2}{s_1^2} = \frac{20.10}{5.3} = 3.79 \qquad F(5,4) = 6.26$$

$\therefore$ Cal F < Table F , $\therefore H_0$ is accepted.

41. The number of automobile accidents in a certain locality was 12, 8, 20, 2, 14, 10, 15, 6, 9, 4. Are these frequencies in agreement with the belief that accident conditions were the same during this 10 week period.

Ans : Given $n = 10$

x	12	8	20	2	14	10	15	6	9	4	100
x^2	12^2	8^2	20^2	2^2	14^2	10^2	15^2	6^2	9^2	4^2	1266

$$\overline{x} = \frac{\sum x}{n} = \frac{100}{10} = 10$$

$$s^2 = \frac{\sum x^2}{n} - (\overline{x})^2 = \frac{1266}{10} - 100 = 26 \quad \Rightarrow \quad s = 5.16$$

1. $H_0 : \mu = 0$ [*No change in during this 10 week period*]

2. $H_1 : \mu \neq 0$ [*There is a change in this 10 week period*]

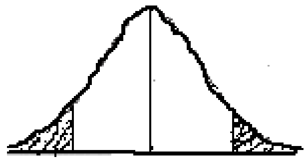
3. $\alpha = 5\%$, $d.f = n - 1 = 10 - 1 = 9$

4. critical region

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP



$-t_{\alpha/2}$ $-t_{\alpha/2}$

$= -2.26 \sim 5\%$ $= 2.26 \sim 5\%$

5. The test statistic $t = \dfrac{\bar{x}-\mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)} = \dfrac{10-0}{\left(\dfrac{5.16}{\sqrt{10-1}}\right)} = 5.81$

6. Conclusion : If $-t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}}$ we accept H_0 , otherwise we reject H_0

Here -2.26 < 5.81 ≮ 2.26

∴ H_0 at 5% level of significance.

42. A certain pesticide is packed into bags by a machine. A random sample of 10 bags is chosen and the contents of the bags is found to have the following weights (in kgs) 50, 49, 52, 44, 45, 48, 46, 45, 49 and 45. Test if the average quantity packed be taken as 50 kg.

Ans :

Given $n = 10$, μ =50 kg

x	50	49	52	44	45	48	46	45	49	45	473
x^2	50^2	49^2	52^2	44^2	45^2	48^2	46^2	45^2	49^2	45^2	22437

$$\bar{x} = \frac{\sum x}{n} = \frac{473}{10} = 47.3$$

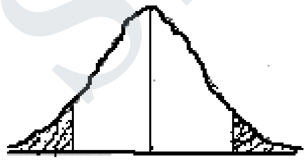
$$s^2 = \frac{\sum x^2}{n} - (\bar{x})^2 = \frac{22437}{10} - (47.3)^2 = 6.41 \quad \Rightarrow \quad s = 2.53$$

1. $H_0 \ : \ \mu = 50$

2. $H_1 \ : \ \mu \neq 50$

3. $\alpha = 5\%$, $d.f = n-1 = 10-1 = 9$

4. critical region



$-t_{\alpha/2}$ $-t_{\alpha/2}$

$= -2.26$ $= 2.26$

5. The test statistic $t = \dfrac{\bar{x}-\mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)} = \dfrac{47.3-50}{\left(\dfrac{2.53}{\sqrt{10-1}}\right)} = -3.2$

STUCOR APP

DOWNLOADED FROM STUCOR APP

DOWNLOADED FROM STUCOR APP

	6. Conclusion : If $-t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}}$ we accept H_0 , otherwise we reject H_0 Here $-2.26 \nless -3.2 < 2.26$ $\therefore$ H_0 at 5% level of significance.
43.	Given $\overline{X_1} = 72, \overline{X_2} = 74, s_1 = 8, s_2 = 6, n_1 = 32, n_2 = 36$. Test if the means are significant. **Ans :** $s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(32)(8^2)}{32-1} = 66.07$ & $s_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{(36)(6^2)}{36-1} = 37.03 \quad \Rightarrow \quad s_1^2 > s_2^2$ 1. $H_0 : \sigma_1^2 = \sigma_2^2$ 2. $H_1 : \sigma_1^2 \neq \sigma_2^2$ 3. $\alpha = 5\%$ $d.f.$ $v_1 = 31$ & $v_2 = 35$ 4. Table value of F is < 1.80 5. The test statistic is $F = \frac{s_1^2}{s_2^2} = \frac{66.07}{37.03} = 1.80$ 6. Conclusion : If Cal F<table F , then we accept H_0; otherwise , we reject H_0 $\therefore$ Cal F $\nless$ table F , we reject H_0.
44.	400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favor of the proposal. Test whether these two propositions are same. **Solution:** Let p_1, p_2 be the proportions favorable to have over among men and women respectively.

STUCOR APP

DOWNLOADED FROM STUCOR APP

STUCOR APP

$$\therefore p_1 = \frac{200}{400}; \quad p_2 = \frac{325}{600}$$

p is not known $\quad \therefore$ The best eliminate of $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

$$= \frac{400 \times \frac{200}{400} + 600 \times \frac{325}{600}}{400 + 600} = \frac{525}{100} = 0.525$$

$$Q = 1 - P = 1 - 0.525 = 0.475$$

$H_0 : p_1 = p_2$

$H_1 : p_1 \neq p_2 \quad [two - tailed\ test]$

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.5 - 0.542}{\sqrt{0.525 \times 0.475\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= \frac{0.5 - 0.542}{\sqrt{0.525 \times 0.475(0.0025 + 0.00167)}}$$

$$= \frac{-0.042}{0.0324} = -1.30$$

$$|Z| = 1.30$$

The table value of Z at 5% level is 1.96

Inference : Since $|Z| < 1.96$, H_0 is accepted at 5% level of significance.

The difference is not significant i.e. Men and women are equally favourable for fly over near their residence.

45. The IQ’s of 10 girls are respectively 120, 110, 70, 88, 101, 100, 83, 98, 95, 107. Test whether the population mean IQ is 100.

Solution:

$Given \quad n = 10 \quad \mu = 100$

X	120	110	70	88	101	100	83	98	95	107	972
X^2	14400	12100	4900	7744	10201	10000	6889	9604	9025	11449	96312

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP

$$\overline{X} = \frac{\sum x}{n} = \frac{972}{10} = 97.2 \quad i.e\ \overline{X} = 97.2$$

$$s^2 = \frac{\sum x^2}{n} - (\overline{x}^2) \quad = \frac{96312}{10} - (97.2)^2 = 9631.2 - 9447.84$$

$$s^2 = 183.96 \quad \Rightarrow \quad s = 13.5$$

$H_0 : \mu = 100$

$H_1 : \mu_1 \neq 100$ [Two – tailed test]

$\alpha = 5\%$ degree of freedom $= n - 1 = 10 - 1 = 9$

The test statistic

$$t = \frac{\overline{X} - \mu}{\left(\frac{s}{\sqrt{n-1}}\right)} = \frac{97.2 - 100}{\left(\frac{13.5}{\sqrt{9}}\right)} = \frac{-2.8}{4.5} = -0.62$$

Conclusion

If $-t_{\alpha/2} < t < t_{\alpha/2}$ then we accept H_0; otherwise we reject H_0

Here $-2.262 < -0.62 < 2.262$ So we accept H_0

We may conclude that the data are consistent with the assumption of mean I.Q. of 100 in the population.

STUCOR APP

STUCOR APP

DOWNLOADED FROM STUCOR APP