Agni College of Technology
Chennai - 130

B.E./ B.Tech. DEGREE EXAMINATION, NOV / DEC 2011
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA2266 –STATISTICS AND NUMERICAL METHODS
(Regulation 2008)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. The heights of college students in Chennai are normally distributed with standard deviation 6 cm and sample of 100 students had their mean height 158 cm. Test the hypothesis that the mean height of college students in Chennai is 160 cm at 1 % level of significance.

Ans :

Given $n = 100$, $\mu = 160$, $s = 6$, $\overline{x} = 158$, $\alpha = 1\%$

1. $H_0 : \mu = 160$
2. $H_1 : \mu \neq 160$ [use two tailed test]
3. $\alpha = 1\%$
4. critical region

$-Z_{\alpha/2} = -2.58$ $Z_{\alpha/2} = 2.58$

5. The test statistic $Z = \frac{\overline{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{158 - 160}{\left(\frac{6}{\sqrt{100}}\right)} = -3.33$

6. Conclusion : If $-Z_{\alpha/2} < Z < Z_{\alpha/2} \nless 1.96$ then accept H_0

2. A coin is tossed 400 times and it turns up head 216 items. Discuss whether the coin may be unbiased one at 5 % level of significance.

Ans : Given n = 400 , $P = \frac{1}{2}$, $Q = 1 - P = 1 - \frac{1}{2}$

X = Number of success = 216.

1. The parameter of interest is P.
2. H_0 : The coin is unbiased.

	3. H_1: The coin is biased. 4. $\alpha = 0.05$ 5. $Z = \dfrac{X - np}{\sqrt{npQ}}$ 6. Reject H_0 if $\|Z\| > 1.96$ 7. $Z = \dfrac{216 - (400)\left(\frac{1}{2}\right)}{\sqrt{(400)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}} = \dfrac{16}{10}$ 8. $\|Z\| = 1.6 < 1.96$ we accept H_0 at 5 % level of significance.
3.	Define Mean sum of squares. **Ans :** The sum of square divided by its degrees of freedom gives the corresponding variance or the mean sum of squares (M.S.S). Thus $\dfrac{S_i^2}{(k-1)} = \dfrac{S.S.T}{(k-1)} = s_t^2$ (say) is the M.S.S. due to treatments. And $\dfrac{S_E^2}{(N-k)} = \dfrac{S.S.E}{(N-k)} = s_E^2$ (say) is the M.S.S. due to error.
4.	What are the advantages of a CRD ? **Ans :** The following are the main advantages of this type of designs : 1. It is easy to lay out the design. 2. It allows for complete flexibility. Any number of factor classes and replications may be used. 3. The statistical analysis is relatively simple, even if we do not have the same number errors are not the same from class to class of this factor. 4. The method of analysis remains simple when data are missing or rejected and the loss of information due to missing data is smaller than with any other design.
5.	Find an iterative formula to find $\sqrt{N}$ where N is a positive number and hence find $\sqrt{5}$ **Ans :** If $x = \sqrt{N}$ then $x^2 - N = 0$ is the equation to be solved. Let $f(x) = x^2 - N$, $f'(x) = 2x$ By N.R rule, if x_n is the nth iterate,

	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - N}{2x_n} = \frac{2x_n^2 - x_n^2 + N}{2x_n}$ $= \frac{x_n^2 + N}{2x_n} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$, $n = 0,1,2,\ldots\ldots$ To find $\sqrt{5}$ Put N = 5 also x = $\sqrt{5}$ lies between 2 and 3 Let $x_0 = 2$ $x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$ $x_1 = \frac{1}{2}\left[x_0 + \frac{5}{x_0}\right] = 2.25$ $x_2 = \frac{1}{2}\left[x_1 + \frac{5}{x_1}\right] = 2.2361$ $x_3 = \frac{1}{2}\left[x_2 + \frac{5}{x_2}\right] = 2.2361$ Here $x_2 = x_3 = 2.2361$
6.	Solve by Gauss Jordan method $\begin{pmatrix}1 & 2 & 1\\ 0 & -1 & 1\\ 0 & 0 & -8\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}3\\ 4\\ -24\end{pmatrix}$ **Ans :** $[A, B] = \left[\begin{array}{ccc\|c}1 & 2 & 1 & 3\\ 0 & -1 & 1 & 4\\ 0 & 0 & -8 & -24\end{array}\right]$ $\sim \left[\begin{array}{ccc\|c}1 & 2 & 1 & 3\\ 0 & -1 & 1 & 4\\ 0 & 0 & 1 & 3\end{array}\right]$ $R_3 \Leftrightarrow \frac{R_3}{8}$ $\sim \left[\begin{array}{ccc\|c}1 & 2 & 0 & 0\\ 0 & -1 & 0 & 1\\ 0 & 0 & 1 & 3\end{array}\right]$ $R_2 \Leftrightarrow R_2 - R_3$ $R_1 \Leftrightarrow R_1 - R_3$

	$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \Leftrightarrow R_1 + 2R_2$ Here x = 2 , y = -1 , z = 3
7.	Find the parabola of the form $y = ax^2 + bx + c$ passing through the points (0, 0) , (1, 1) and (2, 20). **Ans :** We use Lagrange's interpolation formula $y = f(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)}(0) + \frac{(x-0)(x-2)}{(1-0)(1-2)}(1) + \frac{(x-0)(x-1)}{(2-0)(2-1)}(20)$ $= 0 - x(x-2) + 10x(x-1) = 9x^2 - 8x$
8.	Show that $\Delta^3_{bcd}\left(\frac{1}{a}\right) = -\frac{1}{abcd}$ **Ans :** Out of syllabus
9.	Using Taylor series method, find y (1.1) correct to four decimal places give $y' = xy^{1/3}$ and y(1) = 1 **Ans :** Given $x_0 = 1$; $y_0 = 1$, , $h = 0.1$ To find y(1.1) : $y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + ...$ $y' = xy^{1/3} \qquad y_0' = x_0 y_0^{1/3} = 1$ $y'' = \frac{1}{3}xy^{-2/3}y' + y^{1/3} \qquad y_0'' = \frac{1}{3}x_0 y_0^{-2/3}y_0' + y_0^{1/3} = \frac{4}{3}$ $y''' = \frac{x^2}{3}\left(\frac{1}{3}\right)y^{-4/3}y' + \frac{2x}{3}y^{-1/3} + \left(\frac{1}{3}\right)y^{-2/3}y'$ $y_0''' = \frac{x_0^2}{3}\left(\frac{1}{3}\right)y_0^{-4/3}y_0' + \frac{2x_0}{3}y_0^{-1/3} + \left(\frac{1}{3}\right)y_0^{-2/3}y_0' = \frac{8}{9}$ $y_1 = 1 + (0.1)(1) + \frac{(0.1)^2}{2!}\left(\frac{4}{3}\right) + \frac{(0.1)^3}{3!}\left(\frac{8}{9}\right) + ...$ $= 1.10681$
10.	Write down the finite difference for the following second order ODE with h = 1/n , $y'' = y + x$, y(0) = y(1) = 0. **Ans :**

	$y'' = y + x$, $h = \frac{1}{n}$ wkt $y_i + x_i = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$, $i = 1,2,3,....$ $\Rightarrow [\ y_{i+1} - 2y_i + y_{i-1}\ ] = -h^2(y_i + x_i)$ $\Rightarrow -y_{i+1} + (2+h^2)y_i - y_{i-1}\ ] = -h^2 x_i$ *with* $y_0 = 0, y_n = 0$

		PART - B (5 x 15 = 60)
11.	a(i)	Two sample polls of votes for two candidates A and B for a public office are taken one from among residents of rural areas. The results are given below. Examine whether the nature of the area is related to voting preference in this election.

Area/votes for	A	B	Total
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

Ans :

The parameter of interest is ψ^2

1. H_0: The nature area is independent of voiting prefernce in the electiion
2. H_1: dependent
3. $\alpha = 0.05$ $d.f. = (r-1)(s-1) = (2-1)(2-1) = 1$
4. Table value of $\psi^2 = 5.991$
5. The test statistic is $\psi^2 = \sum \frac{(O-E)^2}{E}$

expected frequency $= \frac{\text{(Corresponding Row total)(Column total)}}{\text{Grand total}}$

Expected frequency $620 = \frac{(1170)(1000)}{2000} = 585$

$380 = \frac{(830)(1000)}{2000} = 415$

$550 = \frac{(1170)(1000)}{2000} = 585$

$450 = \frac{(830)(1000)}{2000} = 415$

O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
620	585	35	1225	2
380	415	-35	1225	3
550	585	-35	1225	2
450	415	35	1225	3
				10

6. Conclusion : If calψ^2 < table ψ^2

Then we accept H_0 , here 10 > 5.5991

(ii) Fit a Poisson distribution to the following data and test the goodness of fit.

X	0	1	2	3	4	5	6
F(x)	275	72	30	7	5	2	1

Ans :

1. The parameter of interest is λ
2. H_0 : Poisson fit is a good fit.
3. H_1 : Poisson fit is a not good fit
4. $\alpha = 0.05$ $d.f. = 7-1-1-2 = 3$
5. The test statistic is $\psi^2 = \sum \frac{(O-E)^2}{E}$
6. Reject H_0 if ψ^2 > 7.82
7. Computation :

Mean of the distribution $= \frac{\sum f_i x_i}{\sum f_i} = \frac{189}{392} = 0.48$

x_i	f_i	$f_i x_i$
0	275	0
1	72	72
2	30	60
3	7	21
4	5	20
5	2	10
6	1	6
	N = 392	189

To fit a Poisson distribution we require 'm' parameter $m = \bar{x} = 0.48$. By

Poisson distribution the frequency of r successes is

Hence the frequency table is

X	0	1	2	3	4	5	6	Total
Observed Frequency	275	72	30	7	5	2	1	392
Expected frequency	243	116	28	4	1	0	0	392

Since the Four frequencies are small , so we pull then together.

O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
275	243	32	1024	4.21
72	116	-44	1936	16.69
30	28	2	4	0.14
15	5	10	100	20.00
				41.04

Conclusion :

Since $\psi^2 = 41.04 > 7.82$ so we reject H_0 at 5 % level of significance.

b(i) Sandal power is packed into packets by a machine. A random sample of 12 packets is drawn and their weight are found to be (in kg) 0.49 , 0.48 , 0.47 , 0.48 , 0.49, 0.50 , 0.51, 0.49, 0.48, 0.50, 0.51 and 0.48 . Test if the average weight of the packing can be taken as 0.5 kg at 5 % level of significance.

Ans :

Given $n = 12 \quad \mu = 0.5$

X	0.49	0.48	0.47	0.48	0.49	0.50	0.51	0.49	0.48
X^2	0.49^2	0.48^2	0.47^2	0.48^2	0.49^2	0.50^2	0.51^2	0.49^2	0.48^2

0.50	0.51	0.48	Total = 5.88
0.50^2	0.51^2	0.48^2	2.883

$$\bar{x} = \frac{\sum x}{n} = \frac{5.88}{12} = 0.49$$

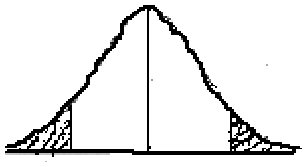
$$s^2 = \frac{\sum x^2}{n} - (\bar{x})^2 = \frac{2.883}{12} - (0.49)^2 = 0.00015 \quad \Rightarrow \quad s = 0.012$$

1. $H_0 : \mu = 0.5$

2. $H_1 : \mu \neq 0.5$ [*Two tailed test*]

3. $\alpha = 5\%$, $d.f = n - 1 = 12 - 1 = 11$

4. critical region



$-t_{\alpha/2} = -2.201$ $\quad\quad$ $-t_{\alpha/2} = 2.201$

5. The test statistic $t = \dfrac{\bar{x}-\mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)} = \dfrac{0.49-0.5}{\left(\dfrac{0.012}{\sqrt{12-1}}\right)} = -2.76$

6. Conclusion : If $-t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}}$ we accept H_0 , otherwise we reject H_0

Here $-2.201 \nless -2.76 < 2.201$

$\therefore$ we reject H_0 at 5% level of significance.

(ii) A group of 10 rats fed on diet A and another group of 8 rats fed on diet B recorded the following increase in weight

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	3		

Show that the estimates of the population variance from the samples are not significantly different.

Ans :

Given $n_1 = 10$, $n_2 = 8$

												Total
Sample I	x_1	5	6	8	1	12	4	3	9	6	10	64
	x_1^2	5^2	6^2	8^2	1^2	12^2	4^2	3^2	9^2	6^2	10^2	512
Sample II	x_2	2	3	6	8	10	1	2	8			400
	x_2^2	2^2	3^2	6^2	8^2	10^2	1^2	2^2	8^2			282

$\overline{x_1} = \frac{\sum x_1}{n_1} = \frac{64}{10} = 6.4$ & $\overline{x_2} = \frac{\sum x_2}{n_2} = \frac{40}{8} = 5$

$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x_1})^2 = \frac{512}{10} - (6.4)^2 = 10.24$ & $s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x_2})^2 = \frac{282}{8} - (5)^2$

$s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(10)(10.24)}{9} = 11.3777$ & $s_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{(8)(10.25)}{7} = 11.7143$

$s_2^2 > s_1^2$

The parameter of interest is σ_1^2 and σ_1^2

1. $H_0 : \sigma_1^2 = \sigma_2^2$ [The difference of a varaince is not significant]
2. $H_1 : \sigma_1^2 \neq \sigma_2^2$
3. $\alpha = 0.05$, $d.f(v_1) = 9$, $d.f(v_2) = 7$
4. Table value of $F = 3.29$
5. The test statistic is $F = \frac{s_2^2}{s_1^2} = \frac{11.7143}{11.3777} = 1.02958$

6.Conclusion :

If Cal F < table F , then we accept H_0 ; otherwise we reject H_0

Here , F = 1.02958 < 3.29 , we accept H_0 at 5 % level of significance.

We conclude that the two samples have come from populations with equal variances.

12 a(i) Using the 2^2 factorial design, draw the graphical presentation of the following table and the difference in mean without replication term and calculate $(\overline{Y_1} - \overline{Y_0})$, $(\overline{Y_{0.1}} - \overline{Y_{0.0}})$. Discuss when PH = 2 and 3.

	Temp.	PH	Rep 1	Rep 2	Total
1	300	2	10	14	24
a	350	3	21	19	40
b	300	3	17	15	32
ab	350	3	20	24	44

Ans : Here $r = 2$, $n = 4$

$A = \frac{1}{2n}[a + ab - b - (1)]$

$= \frac{1}{2(4)}[40 + 44 - 32 - (24)] = 3.5$ contrast A = 28 - - - -(1)

$$B = \frac{1}{2n}[b + ab - a - (1)]$$

$$= \frac{1}{2(4)}[32 + 44 - 40 - 24] = 1.5 \quad \text{contrast} \quad B = `12 - - - -(2)$$

$$AB = \frac{1}{2n}[ab + (1) - a - b]$$

$$= \frac{1}{2(4)}[44 + 24 - 40 - 32] = -0.5 \quad \text{contrast} \;\; AB = -4 - - - -(3)$$

$$SS_A = \frac{1}{4n}[a + ab - b - (1)]^2 = \frac{(28)^2}{16} = 49$$

$$SS_{\mathbf{B}} = \frac{1}{4n}[b + ab - a - (1)]^2 = \frac{(12)^2}{16} = 9$$

$$SS_{\mathbf{AB}} = \frac{1}{4n}[a\mathbf{b} + (1) - a - b]^2 = \frac{(-4)^2}{16} = 1$$

$$SS_T = 10^2 + 21^2 + 17^2 + 20^2 + 14^2 + 19^2 + 15^2 + 24^2 = 2588$$

$$SS_E = SS_T - SS_A - SS_{\mathbf{B}} - SS_{\mathbf{AB}} = 2588 - 49 - 9 - 1 = 2529$$

Analysing of variance

S.V	S.S	D.F	M.S.S	Variation ratio	Table value 5 %
A	49	1	49	$F_A = 4.30$	$F_A(12,1) = 243.9$
B	9	1	9	$F_B = 23.42$	$F_B(12,1) = 243.9$
AB	1	1	1	$F_{AB} = 210.75$	$F_{AB}(12,1) = 243.9$
Error	2529	210.75	210.75		

Here, Cal F_A < Table F_A

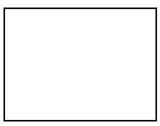
Cal F_B < Table F_B

Cal F_{AB} < Table F_{AB}

As in each of the cases, the compound value of F is less than the corresponding tabulated (critical) value, there are no significant main or interaction effects present in the experiment. The blocks, as well as treatments, do not differ significantly.

The graphical summary of a 2^2 factorial experiment

High $\overline{y_{01}} = 16$ $\overline{y_{11}} = 22$



PH

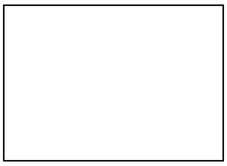
Low $\overline{y_{00}} = 12$ $\overline{y_{10}} = 20$

It is clear from the figure that changing temperature from low (300^0) to high (350^0) , increase the yield substantially, at both levels of PH. Changing PH has little or no effect.

$$\overline{y_1} - \overline{y_0} = \frac{1}{2}(\overline{y_{11}} + \overline{y_{10}}) - \frac{1}{2}(\overline{y_{01}} + \overline{y_{00}}) + \frac{1}{2}(22 + 20) - \frac{1}{2}(16 + 12) = 7$$

That is we take the average on the right side and subtract the average on the left side of the square, as indicated in figure.

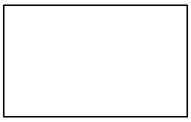
$-\frac{1}{2}(\overline{y_{01}}+\overline{y_{00}})$



$\frac{1}{2}(\overline{y_{11}}+\overline{y_{10}})$

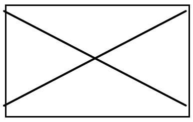
For the second factor, pH we estimate the difference in mean response between the low and high level by

$\frac{1}{2}(\overline{y_{01}}+\overline{y_{11}})$



$-\frac{1}{2}(\overline{y_{00}}+\overline{y_{10}})$

$\frac{1}{2}(\overline{y_{11}}+\overline{y_{00}})$



$-\frac{1}{2}(\overline{y_{10}}+\overline{y_{01}})$

$$\overline{y_1}-\overline{y_0} = \frac{1}{2}(\overline{y_{01}}+\overline{y_{11}})-\frac{1}{2}(\overline{y_{00}}+\overline{y_{10}}) \quad = \frac{1}{2}(16+22)-\frac{1}{2}(12+20) = 3 \text{ -----(4)}$$

To estimate the interaction, we mote that $\overline{y_{10}}-\overline{y_{00}}$ gives the increase in yield at pH = 2, whereas $\overline{y_{11}}-\overline{y_{01}}$ gives the increase in yield at pH = 3. The average of these two differences estimates the interaction.

$$\frac{1}{2}(\overline{y_{11}}-\overline{y_{01}})-\frac{1}{2}(\overline{y_{10}}-\overline{y_{00}}) = \frac{1}{2}(\overline{y_{11}}-\overline{y_{10}}-\overline{y_{01}}+\overline{y_{00}})$$

$$= \frac{1}{2}(22-20-16+12) = -1 \qquad -----(5)$$

From (2) & (4) we get $\overline{y_1}+\overline{y_0} = \frac{B}{2r}$

From (3) & (5) we get $\frac{1}{2}(\overline{y_{11}}+\overline{y_{01}})-\frac{1}{2}(\overline{y_{10}}-\overline{y_{00}}) = \frac{AB}{2r}$

(ii) Four types of health drinks A, B, C, D were tried on the school children. In order to study the effects of the age groups of the children and localities, four schools from four different localities, were selected and students were divided into four age groups . The Latin square design was arranged. The gain in weights in same units are recorded below. Test whether the localities are groups an d the varieties of food have any significant effect on the gain in weight.

	Age groups				
	5 – 8	8 - 11	11 - 14	14 - 17	Total
	A 2	B 1.8	C 2.1	D 1.5	7.4
	D 1.3	A 1.4	B 1	C 1.2	4.9

	B 1.7	C 1.6	D 1.1	A 1.9	6.3
Total	5.9	5.8	6.2	6.1	24

Ans :
Question wrong

b(i) The following table shows the live in hours of four brands of electric lamps brand.

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps.

Ans :

H_0 : There is no significant difference between the four brands.

H_1 : There is significant difference between the four brands.

Subtract 1600 and then divided by 10 we get

X_1 A	X_2 B	X_3 C	X_4 D	Total	X_1^2	X_2^2	X_3^2	X_4^2
1	-2	-14	-9	-24	1	4	196	81
1	4	-5	-8	-8	1	16	25	64
5	4	0	-7	2	25	16	0	49
8	10	2	-3	17	34	100	4	9
10	15	4	0	29	100	225	16	0
12	-	6	8	26	144	-	36	64
20	-	14	-	34	400	-	196	-
-	-	22	-	22	-	-	484	-
57	31	29	-19	98	735	361	957	267

Step 1 : N = 26

Step 2. T = 98

Step 3. C.F. = $\frac{T^2}{N} = \frac{9604}{26} = 369.39$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 735 + 361 + 957 + 267 - 369.39 = 1950.61$

Step 5. SSC = $\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(57)^2}{7} + \frac{(31)^2}{5} + \frac{(29)^2}{8} + \frac{(-19)^2}{6} - 369.39 = 452.25$

SSE = TSS – SSC = 1950.61 – 452.25 = 1498.336

Step 6. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 452.225	C –1 = 4 – 1 = 3	MSC = $\frac{SSC}{C-1}$ = $\frac{452.25}{3}$ = 150.75	$F_C = \frac{MSC}{MSE}$ = $\frac{150.75}{68.11}$ = 2.21 > 1 Since $\frac{MSE}{MSC}$ < 1	$F_C(3, 22)$ = 3.05
Error	SSE = 1498.36	N – C = 26 – 4 = 22	MSE = $\frac{SSE}{N-C}$ = $\frac{1498.36}{22}$		

Step 7 : Conclusion : Cal F_C < Table F_C . So we accept H_0.

(ii) Three varieties A, B and C of a crop are tested in a randomized block design with four replications. The plot yield in pounds are as follows :

A	6	C	5	A	8	B	9
C	8	A	4	B	6	C	9
B	7	B	6	C	10	A	6

Analyse the experimental yield and state your conclusions.

Ans :

Variety	Block 1	2	3	4	Total	X_1^2	X_2^2	X_3^2	X_4^2
A	6	4	8	6	24	36	16	64	36
B	7	6	6	9	28	49	36	36	81
C	8	5	10	9	32	64	25	100	81
Total	21	15	24	24	84	149	77	200	198

H_0 : The varieties are similar

H_1 : The varieties are not similar

Step 1 : N = 12

Step 2. T = 84

Step 3. C.F. = $\frac{T^2}{N} = \frac{(84)^2}{12} = 588$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

= 149 + 77 + 200 +198 –588 = 36

		Step 5. $SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$ [N_1 = number of elements in each column] $= \frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} - 588 = 18$ Step 6. $SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} - \frac{T^2}{N}$ [N_2 = number of elements in each row] $= \frac{(24)^2}{3} + \frac{(28)^2}{3} + \frac{(32)^2}{3} - 588 = 8$ SSE = TSS – SSC - SSR = 36 –18 - 8 = 10 Step 7. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Varieties	SSR = 8	r –1 = 3 – 1 = 2	MSR = $\frac{SSR}{r-1}$ = $\frac{8}{2} = 4$	$F_R = \frac{MSR}{MSE}$ $= \frac{4}{1.667}$ = 2.4	$F_R(2, 6)$ = 5.14
Between Blocks	SSC =18	C –1 = 4 – 1 = 3	MSC = $\frac{SSC}{C-1}$ = $\frac{18}{3} = 6$	$F_C = \frac{MSC}{MSE}$ $= \frac{6}{1.667}$ = 3.59	$F_C(3, 6)$ = 4.76
residual	SSE = 10	N – c – r + 1 = 6	MSE = $\frac{SSE}{N-c-r+1}$ = $\frac{10}{6}$ =1.667		
Total	36				

Step 7 : Conclusion : In both the cases, the calculated value is less than tabulated value. Therefore , null hypothesis is accepted. Hence , the three varieties are similar.

13	a(i)	Find the largest eigen value and the corresponding eigenvector of the matrix.

$$A = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}$$

Ans : Let $X_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ be an arbitrary initial eigen vector.

$$AX_1 = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 3 \\ 4 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 0.75 \\ 0.75 \\ 1 \end{pmatrix} = 4X_2$$

$$AX_2 = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0.75 \\ 0.75 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 2.75 \\ 2.75 \\ 3.5 \end{pmatrix} = 3.5\begin{pmatrix} 1 \\ 0.7857 \\ 0.7857 \\ 1 \end{pmatrix} = 3.5X_3$$

$$AX_3 = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0.7857 \\ 0.7857 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.5714 \\ 2.7857 \\ 2.7857 \\ 3.5714 \end{pmatrix} = 3.5714\begin{pmatrix} 1 \\ 0.78 \\ 0.78 \\ 1 \end{pmatrix} = 3.5714X_4$$

$$AX_4 = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0.78 \\ 0.78 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.56 \\ 2.78 \\ 2.78 \\ 3.56 \end{pmatrix} = 3.56\begin{pmatrix} 1 \\ 0.7809 \\ 0.7809 \\ 1 \end{pmatrix} = 3.56X_5$$

$$AX_5 = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0.7809 \\ 0.7809 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.5618 \\ 2.7809 \\ 2.7809 \\ 3.5618 \end{pmatrix} = 3.5618\begin{pmatrix} 1 \\ 0.7808 \\ 0.7808 \\ 1 \end{pmatrix} = 3.5618X_6$$

$$AX_6 = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0.7809 \\ 0.7809 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.5618 \\ 2.7809 \\ 2.7809 \\ 3.5618 \end{pmatrix} = 3.5618\begin{pmatrix} 1 \\ 0.7808 \\ 0.7808 \\ 1 \end{pmatrix} = 3.5618X_7$$

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		The largest eigen value = 3.56 eigen vector = $\begin{pmatrix} 1 \\ 0.7808 \\ 0.7808 \\ 1 \end{pmatrix}$
	(ii)	Find the inverse by Gauss Jordan method $A = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}$ **Ans :** $[A, I] = \left[\begin{array}{ccc\|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 3 & -1 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right]$ $\sim \left[\begin{array}{ccc\|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -5 & -4 & 1 & 0 \\ 0 & 2 & 0 & -3 & 0 & 1 \end{array}\right]$ $R_2 \Leftrightarrow 2R_2 - 4R_1$, $R_3 \Leftrightarrow R_3 - 3R_1$ $\sim \left[\begin{array}{ccc\|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -5 & -4 & 1 & 0 \\ 0 & 0 & 10 & -11 & 2 & 1 \end{array}\right]$ $R_3 \Leftrightarrow R_3 + 2R_2$ $\sim \left[\begin{array}{ccc\|ccc} 10 & 10 & 0 & -1 & 2 & 1 \\ 0 & 2 & 0 & -3 & 0 & 1 \\ 0 & 0 & -10 & -11 & 2 & 1 \end{array}\right]$ $R_2 \Leftrightarrow -2R_2 + R_3$, $R_1 \Leftrightarrow 10R_1 + R_3$ $\sim \left[\begin{array}{ccc\|ccc} 10 & 0 & 0 & 14 & 2 & -4 \\ 0 & 2 & 0 & -3 & 0 & 1 \\ 0 & 0 & -10 & -11 & 2 & 1 \end{array}\right]$ $R_1 \Leftrightarrow R_1 - 5R_2$ $\sim \left[\begin{array}{ccc\|ccc} 1 & 0 & 0 & 1.4 & 0.2 & -0.4 \\ 0 & 1 & 0 & -1.5 & 0 & 0.5 \\ 0 & 0 & 1 & 1.1 & -0.2 & -0.1 \end{array}\right]$ $R_1 \Leftrightarrow \frac{R_1}{10}$ $R_2 \Leftrightarrow \frac{R_2}{2}$ $R_3 \Leftrightarrow \frac{R_3}{-10}$ $A^{-1} = \begin{bmatrix} 1.4 & 0.2 & -0.4 \\ -1.5 & 0 & 0.5 \\ 1.1 & -0.2 & -0.1 \end{bmatrix}$
	b(i)	Solve the following system of equations by using Gauss Jacobi and Gauss Seidel method (correct to 3 decimal places)

		$8x-3y+2z=20$, $4x+11y-z=33$, $6x+3y+12z=35$ **Ans :** As the coefficient matrix is not diagonally dominant we rewrite the equations. $8x-3y+2z=20$ $4x+11y-z=33$ $6x+3y+12z=35$ Since, the diagonal elements are dominant in the coefficient matrix, we write x , y , z as follows: $x=\frac{1}{8}[20+3y-2z]$ $y=\frac{1}{11}[33-4x+z]$ $z=\frac{1}{12}[35-6x-3y]$ Let the initial values be $y=0$ $z=0$ First Iteration $x^{(1)}=\frac{1}{8}[20+3y^{(0)}-2z^{(0)}]=\frac{1}{8}[20+3(0)-2(0)]=2.5$ $y^{(1)}=\frac{1}{11}[33-4x^{(1)}+z^{(0)}]=\frac{1}{11}[33-4(2.5)+(0)]=2.091$ $z^{(1)}=\frac{1}{12}[35-6x^{(1)}-3y^{(1)}]=\frac{1}{12}[35-6(2.5)-3(2.091)]=1.144$

		Second Iteration $x^{(2)} = \frac{1}{8}\left[20+3y^{(1)}-2z^{(1)}\right] = \frac{1}{8}\left[20+3(2.091)-2(1.144)\right] = 2.998$ $y^{(2)} = \frac{1}{11}\left[33-4x^{(2)}+z^{(1)}\right] = \frac{1}{11}\left[33-4(2.998)+1.144\right] = 2.014$ $z^{(2)} = \frac{1}{12}\left[35-6x^{(2)}-3y^{(2)}\right] = \frac{1}{12}\left[35-6(2.998)-3(2.014)\right] = 0.914$ *Third Iteration* $x^{(3)} = \frac{1}{8}\left[20+3y^{(2)}-2z^{(2)}\right] = \frac{1}{8}\left[20+3(2.014)-2(0.914\right] = 3.027$ $y^{(3)} = \frac{1}{11}\left[33-4x^{(3)}+z^{(2)}\right] = \frac{1}{11}\left[33-4(3.027)+0.914\right] = 1.9824$ $z^{(3)} = \frac{1}{12}\left[35-6x^{(3)}-3y^{(3)}\right] = \frac{1}{15}\left[35-6(3.027)-3(1.9824)\right] = 0.908$ *Fourth Iteration* $x^{(4)} = \frac{1}{8}\left[20+3y^{(3)}-2z^{(3)}\right] = \frac{1}{8}\left[20+3(1.9824)-2(0.908)\right] = 3.016$ $y^{(4)} = \frac{1}{11}\left[33-4x^{(4)}+z^{(3)}\right] = \frac{1}{11}\left[33-4(3.016)+0.908\right] = 1.986$ $z^{(4)} = \frac{1}{12}\left[35-6x^{(4)}-3y^{(4)}\right] = \frac{1}{12}\left[35-6(3.016)-3(1.986)\right] = 0.912$ *Fourth Iteration* $x^{(5)} = \frac{1}{8}\left[20+3y^{(4)}-2z^{(4)}\right] = \frac{1}{8}\left[20+3(1.986)-2(0.912)\right] = 3.016$ $y^{(5)} = \frac{1}{11}\left[33-4x^{(5)}+z^{(4)}\right] = \frac{1}{11}\left[33-4(3.016)+0.912\right] = 1.986$ $z^{(5)} = \frac{1}{12}\left[35-6x^{(5)}-3y^{(5)}\right] = \frac{1}{12}\left[35-6(3.016)-3(1.986)\right] = 0.912$ *Sixth Iteration* $x^{(6)} = \frac{1}{8}\left[20+3y^{(5)}-2z^{(5)}\right] = \frac{1}{8}\left[20+3(1.986)-2(0.912)\right] = 3.016$ $y^{(6)} = \frac{1}{11}\left[33-4x^{(6)}+z^{(5)}\right] = \frac{1}{11}\left[33-4(3.016)+0.912\right] = 1.986$ $z^{(6)} = \frac{1}{12}\left[35-6x^{(6)}-3y^{(5)}\right] = \frac{1}{12}\left[35-6(3.016)-3(1.986)\right] = 0.912$ $x = 3.016 \quad y = 1.986 \quad z = 0.912$
	(ii)	Using Gauss Elimination method solve the system 3.15x - 1.96y + 3.85z = 12.95 , 2.13x – 5.12y – 2.89z = -8.61 , 5.92x + 3.05y + 2.15z = 6.88 **Ans :** The given system is equivalent to

$$[A , B] = \left(\begin{array}{ccc|c} 3.15 & -1.96 & 3.85 & 12.95 \\ 2.13 & 5.12 & -2.89 & -8.61 \\ 5.92 & 3.05 & 2.15 & 6.88 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 3.15 & -1.96 & 3.85 & 12.95 \\ 0 & 20.3028 & -17.304 & -54.705 \\ 0 & 21.2107 & -16.0195 & -54.992 \end{array}\right) \quad \begin{array}{l} R_2 \Leftrightarrow 3.15R_2 - 2.13R_1 \\ R_3 \Leftrightarrow 3.15R_3 - 5.92R_1 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 3.15 & -1.96 & 3.85 & 12.95 \\ 0 & 20.3028 & -17.304 & -54.705 \\ 0 & 0 & 41.7892 & 43.8398 \end{array}\right) \quad R_3 \Leftrightarrow 20.3028R_3 - 21.2107R_2$$

By back substitution method

$$41.7892\, z = 43.8398 \Rightarrow z = 1.049$$

$$20.3028\, y - 17.304\, z = -54.705$$

$$20.3028\, y = -54.705 + 17.304(1.049) \Rightarrow y = -1.800$$

$$3.15x - 1.96y + 3.85z = 12.95$$

$$3.15x = 12.95 + 1.96(-1.800) - 3.85(1.049) \Rightarrow x = 1.709$$

14 a(i) Write the Newton's method formula and using it obtain f(x) as a polynomial in powers of (x - 5) from the given table.

X	0	2	3	4	7	9
Y(x)	4	26	58	112	466	922

Solution :

since the arguments are not equally spaced, we will use Newton's divided difference formula

Divided Difference Table

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	4				
		11			
2	26		7		
		32		1	
3	58		11		0
		54		1	
4	112		16		0
		118		1	
7	466		22		
		228			
9	922				

By Newton's divided difference formula

$y = f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$

$= 4 + (x - 0)11 + (x - 0)(x - 27)7 + (x - 0)(x - 2)(x - 3)1$

$= x^3 + 2x^2 + 3x + 4$

We have to express in powers of (x – 5)

5	1	2	3	4
	0	5	35	190
5	1	7	38	194 = constant term
	0	5	60	
5	1	12	98 = coefficient of (x – 5)	
	0	5		
5	1	17 = coefficient of $(x – 5)^2$		
	0			
	1 = coefficient of $(x – 5)^3$			

The given polynomial f(x) in powers of (x – 5) is

$$f(x) = (x-5)^3 + 17(x-5)^2 + 98(x-5) + 194$$

(ii) A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of time t (second). Calculate the angular velocity and angular acceleration of the rod at t = 0.6 seconds.

T	0	0.2	0.4	0.6	0.8	1.0
θ	0	0.12	0.49	1.12	2.02	3.20

Ans :

Now form the difference table

x	θ	$\nabla\theta$	$\nabla^2\theta$	$\nabla^3\theta$	$\nabla^4\theta$
0	0				
		0.12			
0.2	0.12		0.25		
		0.37		0.01	
0.4	0.49		0.26		0
		0.63		0.01	
0.6	1.12		0.27		0
		0.90		0.01	
0.8	2.02		0.28		
		1.18			
1	3.20				

Since x = 0.6 is towards the end, we will use backward difference formula

$$h = 0.2 \quad \therefore \quad p = \frac{x - x_n}{h} = \frac{0.6 - 1.0}{0.2} = -2$$

$$\left(\frac{dy}{dx}\right)_{x=x} = \frac{1}{h}\left[\nabla y_n + \left(\frac{2p+1}{2}\right)\nabla^2 y_n + \left(\frac{3p^2+6p+2}{6}\right)\nabla^3 y_n + \left(\frac{4p^3+18p^2+22p+6}{24}\right)\nabla^4 y_n + \ldots\ldots\right]$$

$$\left(\frac{d\theta}{dt}\right)_{t=0.6} = \frac{1}{0.2}\left[1.18 - \frac{3}{2}(0.28) + \frac{1}{3}(0.01)\right] = 3.81665$$

$$\left(\frac{d^2\theta}{dt^2}\right)_{t=0.6} = \frac{1}{h^2}\left[\nabla^2 y_n + (p+1)\nabla^3 y_n + \ldots..\right]$$

$$= \frac{1}{0.04}[0.28 - 0.01] = 6.75$$

b(i) Write the Trapezoidal rule and Simpson's rule for evaluation of $I = \int_{x_0}^{x_n} f(x)\, dx$

evaluate $I = \int_0^6 \frac{1}{1+x^2} dx$ using Trapezoidal rule, Simpson's rule. Also check up by direct integration.

Ans :

Here $b - a = 6 - 0 = 6$. Divide into 6 equal parts $h = \frac{6}{6} = 1$

x	0	1	2	3	4	5	6
$\frac{1}{1+x^2} = f(x)$	1.000	0.500	0.200	0.100	0.058824	0.038462	0.27027

There are 7 ordinates (n = 6). We can use all the formula

1. Trapezoidal rule

$$I = \int_0^6 \frac{dx}{1+x^2}$$

$$= \frac{1}{2}[(1 + 0.027027) + 2 (0.5 + 0.2 + 0.1 + 0.058824 +$$
$$0.038462)]$$

$$= 1.41079950$$

2. Simpson's 1/3 rule

y_0	y_1	y_2	y_3	y_4	y_5	y_6
1	4	1				
		1	4	1		
				1	4	1
1	4	2	4	2	4	1

$$\therefore \quad I = \frac{h}{3}[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

$$= \frac{1}{3}[(1 + 0.027027) + 2(0.2 + 0.58824) + 4(0.5 + 0.1 + 0.038462)]$$

$$= 1.36617433$$

3. Actual integration

$$I = \int_0^6 \frac{dx}{1+x^2} = (\tan^{-1} x)_0^6 = \tan^{-1} 6 = 1.40564765$$

(ii) Given the following data, find y'(6) and the maximum value of y.

X	0	2	3	4	7	9
Y(x)	4	26	58	112	466	922

ANS :

since the arguments are not equally spaced, we will use Newton's divided difference formula

Divided Difference Table

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$

0	4				
		11			
2	26		7		
		32		1	
3	58		11		0
		54		1	
4	112		16		0
		118		1	
7	466		22		
		228			
9	922				

By Newton's divided difference formula

$y = f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$

$= 4 + (x - 0)11 + (x - 0)(x - 27)7 + (x - 0)(x - 2)(x - 3)1$

$= x^3 + 2x^2 + 3x + 4$

Therefore, $y'(x) = 3x^2 + 4x + 3$

$y'(6) = 33(6)^2 + 4(6) + 3 = 135$ $\quad y'(5) = 33(5)^2 + 4(5) + 3 = 98$

y(x) is maximum if $y'(x) = 0$ $\quad 3x^2 + 4x + 3 = 0$. But the roots are imaginary. Therefore, there is no extremum value in the range. In fact, it is an increasing curve.

(iii) Using Lagrange's formula of interpolation find y(9.5) given.

X	7	8	9	10
Y	3	1	1	9

Ans : Given

X	(x_0) 7	(x_1) 8	(x_2) 9	(x_3) 10
Y	3 (y_0)	1 (y_1)	1 (y_2)	9 (y_3)

By Lagrange's interpolation formula we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-8)(x-9)(x-10)}{(7-8)(7-9)(7-10)}(3) + \frac{(x-7)(x-9)(x-10)}{(8-7)(8-9)(8-10)}(1)$$

$$+ \frac{(x-7)(x-8)(x-10)}{(9-7)(9-8)(9-10)}(1) + \frac{(x-7)(x-8)(x-9)}{(10-7)(10-8)(10-9)}(9)$$

When $x = 9.5$

$$y(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)}(3) + \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)}(1)$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)}(1) + \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)}(9)$$

		$= 0.1875 - 0.3125 + 0.9375 + 2.8125 = 3.625$
15	a(i)	Using Milne's predictor-corrector method, find y(0.4), given that $y' = \frac{(1+x^2)y^2}{2}$, $y(0)=1,\ y(0.1)=1.06,\ y(0.2)=1.12, y(0.3)=1.21$ **Solution:** $x_0=0,\ x_1=0.1,\ x_2=0.2,\ x_3=0.3,\ x_4=0.4$ $y_0=1,\ y_1=1.06, y_2=1.12,\ y_3=1.21,\ y_4=?\ h=0.1$ $y' = f(x,y)=\frac{1}{2}(1+x^2)y^2$ $y_0' = \frac{1}{2}(1+x_0^2)y_0^2 = \frac{1}{2}(1+0)1 = \frac{1}{2}$ $y_1' = \frac{1}{2}(1+x_1^2)y_1^2 = \frac{1}{2}[1+(0.1)^2][1.06]^2 = 0.5674$ $y_2' = \frac{1}{2}(1+x_2^2)y_2^2 = \frac{1}{2}[1+(0.2)^2][1.12]^2 = 0.6522$ $y_3' = \frac{1}{2}(1+x_3^2)y_3^2 = \frac{1}{2}[1+(0.3)^2][1.21]^2 = 0.7979$ Milne's method $y_4, p = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$ $= 1+\frac{4(0.1)}{3}[2(0.5674) - 0.6522 + 2(0.7979)]$ $=1.2771$ $y_4' = \frac{1}{2}(1+x_4^2)y_4^2 = [1+0.16][1.2771]^2 = 0.9460$ *By corrector method,* $y_4, c1 = y_2 + \frac{h}{3}\left[y_2' - 4y_3' + y_4'\right]$.........................(2) $= 1.12 + \frac{0.1}{3}[0.6522 + 4(0.7979) + 0.9460]$ *Now, use this on the R.H.S of* (2) *and we get* y_4c1 $y'_4c1 = \frac{1}{2}[1+0.16][1.2797]^2 = 0.9498$ Agani using (2), $y_4, c2 = 1.12 + \frac{0.1}{3}[0.6522 + 4(0.7979) + 0.9498] = 1.2798$ $y(0.4) = 1.2798$
	(ii)	Solve the BVP $u'' = xu$, $u(0)+u'(0)=1$, $u(1)=1$, $h=1/3$, use the second order method. **Ans :** Given

wkt $y_i'' = \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = x_i y_i$

$\Rightarrow \quad y_{i-1} - (2 + \frac{1}{9}x_i)y_i + y_{i+1} = 0 ------(1)$

since , $h = \frac{1}{3}$

putting $i = 0,1,2$ *in* (1)

put i=0 then $y_{-1} - 2y_0 + y_1 = 0 \quad ------(2)$

put i=1 then $y_0 - \frac{55}{27}y_1 + y_2 = 0 \ ------(3)$

put i=2 then $y_1 - \frac{56}{27}y_2 + y_3 = 0 \ ------(4)$

The first boundary condition is

wkt $y_0 + \dfrac{y_1 - y_{-1}}{2h} = 1$

(ie) $2y_0 + 3(y_1 - y_{-1}) = 2$ (ie) $y_{-1} = \dfrac{2y_0 + 3y_1 - 2}{3} \quad ------------(5)$

The second boundary condition is

$y_3 = 1$ -------(6)

Using (5) and (6) in equations (2) , (3) and (4) we have

$-2y_0 + 3y_1 = 1 \quad ----- \quad (7)$

$y_0 - \frac{55}{27}y_1 + y_2 = 0 \quad --------(8)$

$y_1 - \frac{56}{27}y_2 + 1 = 0 \quad ------(9)$

Solving equations (7) , (8) and (9) we get

$y_0 = -\frac{82}{83} = -0.9680$

$y_1 = -\frac{27}{83} = -0.3253$

$y_2 = \frac{27}{83} = 0.3253$

b(i) Given y''+ xy' + y =0, y(0)= 1, y'(0)= 0, find the value of y(0.1) by using Runge-Kutta method for fourth order .

Ans : Given

$$y'' = -xy' - y \quad , \quad y(0) = 1$$

$$y' = 0 \quad , y_0 = 1$$

Setting

$$y' = z \quad \text{and} \quad y'' = z'$$

The equations becomes

$$y'' = -xz - y$$

$$\frac{dy}{dx} = z = f_1(x,y,z) \qquad \frac{dz}{dx} = -xz - y = f_2(x,y,z)$$

$$(ie) \ y_0 = 1 , \ z_0 = y_0' = 0$$

By algorithm,

$$k_1 = h\, f_1(x_0, y_0, z_0) = h\ f_1(0,1,0) = h(0) = 0$$

$$l_1 = h\, f_2(x_0, y_0, z_0) = h\ f_2(0,1,0) = h(-0,-1) = -h$$

$$k_2 = h\ f_1\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\ f_1\left[0 + \frac{0.1}{2}, 1+0, 0 + \frac{(-0.1)}{2}\right] = 0.1\ f_1[0.05, 1, -0.05]$$

$$= (0.1)(-0.05) = -0.005$$

$$l_2 = h\ f_2\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\ f_2\left[0 + \frac{0.1}{2}, 1+0, 0 + \frac{(-0.1)}{2}\right] = 0.1\ f_2[0.05, 1, -0.05]$$

$$= (-0.1)[(0.05)(-0.05) + 1] = -0.0998$$

$$k_3 = h\ f_1\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)\ f_1\left[0 + \frac{0.1}{2}, 1 + \frac{(-0.005)}{2}, 0 + \frac{(-0.0998)}{2}\right]$$

$$= 0.1\ f_1[0.05, 0.9975, -0.0499]$$

$$= (0.1)(-0.0499) = -0.00499$$

		$l_3 = h\ f_2\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$ $= 0.1\ f_2[0.05, 0.9975, -0.0499]$ $= (-0.1)[(0.05)(-0.0499) + 0.9975] = -0.0995$ $k_4 = h\ f_1[x_0 + h, y_0 + k_3, z_0 + l_3]$ $= (0.1)\ f_1[0.1, 1 + (-0.00499), 0 + (-0.0995)]$ $= 0.1\ f_1[0.05, 0.995, -0.0995]$ $= (0.1)(-0.0995) = -0.00995$ $l_4 = h\ f_2[x_0 + h, y_0 + k_3, z_0 + l_3]$ $= 0.1\ f_2[0.1, 0.995, -0.0995]$ $= (0.1)[(0.1)(-0.0995) + 0.995] = -0.0985$ Now $\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}[0 + 2(-0.005) + 2(-0.00499) + (-0.00995)]$ $= -0.00498$ $\therefore\ y_1 = y_0 + \Delta y = 1 - 0.00498 = 0.9950$
	(ii)	If $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$, find $y(0.2), y(0.4)$, by Runge – Kutta method. **Ans :** $Given \frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, \quad y(0) = 1$ $Then\ f(x,y) = \frac{y^2 - x^2}{y^2 + x^2}, x_0 = 0, y_0 = 1, h = 0.2$ (since the valeus are required at $x = 0.2, 0.4$, given $x = 0$)

The fourth – order Runge – Kutta method is

$$k_1 = h\, f(x_n, y_n)$$

$$k_2 = h\, f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h\, f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h\, f(x_n + h, y_n + k_3),\ n = 0,1,2....$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

To find y_1

$$k_1 = h\, f(x_0, y_0) = h\left(\frac{y_0^{\,2} - x_0^{\,2}}{y_0^{\,2} + x_0^{\,2}}\right) = 0.2\left(\frac{1-0}{1+0}\right) = 0.2$$

$$k_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\ f[0.1,\ 1+0.1]$$

$$= 0.2\ f[0.1,\ \ 1.1] = 0.2\left(\frac{1.1^2 - 0.1^2}{1.1^2 - 0.1^2}\right) = 0.2\left(\frac{1.20}{1.22}\right) = 0.1967$$

$$k_3 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.2\ f\left[0.1,\ 1 + \frac{0.1967}{2}\right] = 0.2\ f[0.1,\ 1.09835]$$

$$= 0.2\left(\frac{1.09835^2 - 0.1^2}{1.09835^2 + 0.1^2}\right) = 0.2\left(\frac{1.2064 - 0.01}{1.2064 + 0.01}\right) = 0.2\left(\frac{1.1964}{1.2164}\right) = 0.1967$$

$$k_4 = h\, f(x_0 + h, y_0 + k_3) = 0.2\ f[0.2, 1 + 0.1967] = 0.2\left(\frac{1.1967^2 - 0.2^2}{1.1967^2 + 0.2^2}\right)$$

$$= 0.2\left(\frac{1.4321 - 0.04}{1.4321 + 0.04}\right) = 0.2\left(\frac{1.3921}{1.4721}\right) = 0.1891$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= \frac{1}{6}(0.2 + 2\times 0.1967 + 2\times 0.1967 + 0.1891) = 0.19598$$

$$\therefore\ y_1 = y_0 + \Delta y = 1 + 0.19598 = 1.196$$

Now $x_1 = 0.2, y_1 = 1.196, h = 0.2$

To find y_2

$$k_1 = h\, f(x_1, y_1) = h\left(\frac{y_1^{\,2} - x_1^{\,2}}{y_1^{\,2} + x_1^{\,2}}\right) = 0.2\left(\frac{1.4304 - 0.04}{1.4304 + 0.04}\right) = 0.2\frac{1.3904}{1.4704} = 0.1891$$

$$k_2 = h\, f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = 0.2\ f\left(0.2 + \frac{0.2}{2}, 1.196 + \frac{0.1891}{2}\right)$$

$$= \ 0.2\ f[0.3,\ 1.2906] = 0.1795$$

		$k_3 = h\,f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = h\,f\left(0.2 + \frac{0.2}{2}, 1.196 + \frac{0.1795}{2}\right)$ $= 0.2\,f[0.3, 1.2858] = 0.1793$ $k_4 = h\,f(x_1 + h, y_1 + k_3) = 0.2\,f[0.2 + 0.2, 0.1891 + 0.1793]$ $= 0.2\,f(0.4, 1.3753) = 0.1688$ $\Delta y = 0.2\,f(0.6, 1.5329) = 0.1469$ $\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}[0.1688 + 2\times 0.1579 + 2\times 15771 + 0.1469]$ $= 0.15782$ Hence y(0.4) = $y_2 = y_1 + \Delta y = 1 + 0.15782 = 1.15782$

Agni College of Technology

Chennai - 130

B.E./ B.Tech. DEGREE EXAMINATION, NOV/DEC 2012
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA2266 –STATISTICS AND NUMERICAL METHODS
(Regulation 2008)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. What are null and Alternate hypothesis?

Ans :

For applying the test of significance, we first set up a hypothesis which is a statement about the population parameter. This statement is usually a hypothesis of no difference and so it is called null hypothesis and is denoted by H_0.
Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis, denoted by H_1.

2. Give the formula for the chi square test of independence for

a	b
c	d

Ans :

	A	B	Total
A	a	b	a + b
B	c	d	c + d
Total	a + c	b + d	

$$\chi^2 = \frac{N(ad-bc)}{(a+b)(c+d)(a+c)(a+d)}$$

3. State the basic principles of design of experiments.

Ans :

There are three basic principles of Design of experiment. They are
(i) Randomisation (ii) Replication (iii) Local control (error control)

4. Define :RBD

Ans :

Randomized block design is a simple design that controls the variability in the experimental units and gives the treatments equivalence to show their effects.

5. Mention the order and condition for the convergence of Newton Raphson method.

Ans :

The order of Convergence is 2.

Condition of convergence is $|f(x)f''(x)| < |f'(x)|^2$

6. Compare Gauss elimination and Gauss-Jacobi methods.

Ans :

Gauss Elimination	Gauss Jacobi
Direct method	Indirect method
The coefficient matrix is transformed into upper triangular matrix.	The coefficient matrix should be diagonally dominant.

7. What is the need of Newton's and Lagrange's interpolation formulae?

Ans :

The forward and backward interpolation formulae of Newton can be used when the values of the independent variable x are equally spaced and also be used when the differences of the dependent variable y become smaller ultimately. Lagrange's interpolation formula can be used whether the values of x are equally spaced or not and whether the difference of y become smaller or not.

8. Find the area under the curve passing through the points (0, 0) , (1, 2) , (2 , 2.5) , (3, 2.3) , (4, 2) , (5, 1.7) and (6, 1.5)

Sol:

Given

x	0	1	2	3	4	5	6
y	0	2	2.5	2.3	2	1.7	1.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By using Trapezoidal rule

$$\text{Area} = \int_0^6 y\,dx = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + \cdots .. + y_5)]$$

$$= \frac{1}{2}[1.5 + 2(10.5)]$$

$$= \frac{22.5}{2} = 11.25.$$

9. Bring out the merits and demerits of Taylor series method.

Ans :

Merits	Demerits
Powerful single step method. Method will be very useful for finding the starting values for powerful methods like Runge-kutta method, Milne's method etc.	If f(x ,y) involves some complicated algebraic structures then the calculation of higher derivatives becomes tedious and the method fails.

10. Find y (0.1) by Euler's, if $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$.

Solution:

Given f(x, y) $= x^2 + y^2$ $x_0 = 0,$ $y_0 = 1,$ $h = 0.1,$ $x_1 = 0.2$

By modified Euler Method

$$y_{n+1} = y_n + hf[x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)]$$

$$y_1 = y_0 + hf[x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)]$$

$$y_1 = 1 + 0.1 f[0 + \frac{0.1}{2}, 1 + \frac{0.1}{2} f(0,1)]$$

$$y(0.1) = 1 + (0.1) f(0.05 \;,\; 1 + 0.05(0 + 1))$$

$$= 1 + (0.1) f(0.05, 1.05)$$

$$= 1 + (0.1)[(0.05)^2 + (1.05)^2]$$

$$= 1 + (0.1)[0.0025 + 1.1025] = 1 + (0.1)[1.105]$$

$$= 1 + 0.1105 = 1.1105$$

PART - B (5 x 16 =80 marks)

11.a(i) A machine puts out 16 imperfect articles in a sample of 500.After it was overhauled, it puts out 3 imperfect articles in a sample of 100.Has the machine improved in its performance?

Ans : OUT OF SYLLABUS.

(ii) Test whether there is any significant difference between the variances of the populations from which the following samples are taken.

Sample : 1	20	16	26	27	23	22	
Sample : 2	27	33	42	35	32	34	38

Solution:

Given $n_1 = 6, \; n_2 = 7$

	x_1	x_1^2	x_2	x_2^2
	20	400	27	729
	16	256	33	1089
	26	676	42	1764
	27	729	35	1225
	23	529	32	1024
	22	484	34	1156
Total	134	3074	241	8431

$$\overline{x_1} = \frac{\sum x_1}{6} = \frac{134}{6} = 22.33$$

$$\overline{x_2} = \frac{\sum x_2}{7} = \frac{241}{7} = 34.33$$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x_1})^2 = \frac{3074}{6} - (22.33)^2 = 13.70$$

$$s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x_2})^2 = \frac{8431}{7} - (34.43)^2 = 19.00$$

$$s_2^2 > s_1^2$$

$H_0: \sigma_1^2 = \sigma_2^2$ & $H_1: \sigma_1^2 \neq \sigma_2^2$

$\alpha = 5\% = 0.05$ $d.f.\ v_1 = n_1 - 1 = 5$ & $v_2 = n_2 - 1 = 6$

$$F = \frac{s_2^2}{s_1^2} = \frac{19}{13.7} = 1.39 \qquad F(6,5) = 4.95$$

Cal F < Tabulated F. $\therefore$ Accept H_0

(b)(i) A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with the SD of 6,While boys made an average grade of 82 with the SD of 2.Test whether there is any significant difference between the performance of boys and girls.

Solution:

	No of cases	Mean	S.D
Sample I	50	76	6
Sample II	75	82	2

H_0: There is no significant difference.

H_1: There is significant difference.

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{76-82}{\sqrt{\frac{6^2}{50} + \frac{2^2}{75}}} = \frac{-6}{\sqrt{\frac{36}{50} + \frac{4}{75}}} = \frac{-6}{\sqrt{\frac{58}{75}}} = \frac{-6}{0.88} = -6.82$$

$|Z| = 6.82 > 1.96$ also $|Z| > 2.58$

So, we reject H_0.

(ii) A sample of 10 boys had the I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 100 and 107. Test whether the population mean I.Q may be 100.

Ans :

Given

$$\sum x_1 = 70 + 120 + 110 + 101 + 88 + 83 + 95 + 98 + 100 + 107 = 972$$

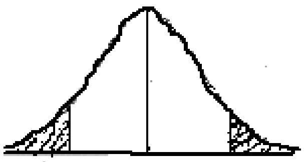
$$\sum x_1^2 = 70^2 + 120^2 + 110^2 + 101^2 + 88^2 + 83^2 + 95^2 + 98^2 + 100^2 + 107^2 = 96312$$

$$\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$$

$$s^2 = \frac{\sum x^2}{n} - (\bar{x})^2 = \frac{96312}{10} - (97.2)^2 = 9631.2 - 9447.84 = 183.36$$

1. $H_0 : \mu = 100$
2. $H_1 : \mu \neq 100$
3. $\alpha = 5\%$, $d.f = n - 1 = 10 - 1 = 9$

4. critical region



$-t_{\alpha/2}$ $= -2.26 \sim 5\%$ $-t_{\alpha/2}$ $= 2.26 \sim 5\%$

5. The test statistic $t = \dfrac{x-\mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)} = \dfrac{97.2-100}{\left(\dfrac{13.5}{\sqrt{9}}\right)} = \dfrac{-2.8}{4.5} = -0.62$

6. Conclusion : If $-t_{\alpha/2} < t < t_{\alpha/2}$

we accept H_0 , otherwise we reject H_0

Here -2.262 < - 0.62 < 2.262 ∴ we accept H_0

12(a) A sales of four salesmen in 3 seasons are tabulated here.

Salesmen

Seasons	A	B	C	D
Summer	36	36	21	35
Winter	28	29	31	32
Monsoon	26	28	29	29

Carry out an analysis of variance.

Solution:

The above data are classified according to criteria (i) salesmen and (ii) seasons. In order to simplify calculations, we code the data by subtracting 30 from each figure. The data in the coded form are given below:

SEASONS		Salesmen				Season Total	X_1^2	X_2^2	X_3^2	X_4^2
		A(X_1)	B(X_2)	C(X_3)	D(X_4)					
Y_1	Summer	+6	+6	-9	+5	+8	36	36	81	25
Y_2	Winter	-2	-1	+1	+2	0	4	1	1	4
Y_3	Monsoon	-4	-2	-1	-1	-8	16	4	1	1
Total		0	3	-9	6	Grand total T=0	56	41	83	30

H_0: *There is no significant difference between column means as well as row*

H_1: *There is significant differnce between column means or the row means.*

Step 1 : N = 12

Step 2 : T = 0

Step 3 : $\frac{T^2}{N} = \frac{(0)^2}{N} = 0$ (*Number of items or N is* 12)

Step 4 : TSS $= \sum X_1^2 + X_2^2 + X_3^2 + X_4^2 - \frac{T^2}{N}$ = 56 + 41 + 83 + 30 – 0 = 210

Step 5 : SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \frac{(\sum X_4)^2}{N_4} - \frac{T^2}{N}$

$= \frac{(0)^2}{3} + \frac{(3)^2}{3} + \frac{(-9)^2}{3} + \frac{(6)^2}{3} - \frac{T^2}{N}$ = 0 + 3+ 27 + 12 - 0 = 42

Step 6 : SSR $= \frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \frac{(\sum Y_4)^2}{N_4} - \frac{T^2}{N}$

$= \frac{(8)^2}{3} + \frac{(0)^2}{3} + \frac{(-8)^2}{3} + \frac{(6)^2}{3} - \frac{T^2}{N}$ = 16 + 0 + 16 - 0 = 32

SSE = TSS – SSC - SSR = 210 – 42 - 32 = 136

Step 7: Table of Analysis Variance:

Sources of Variation	Sum of squares	D.F	Mean Squares	Variance	Table value at 5%.
Between Columns (Salesmen)	SSC = 42	c-1 = 4-1 = 3	MSC $= \frac{SSC}{C-1}$ = 14	$F_C = \frac{MSE}{MSC} = \frac{22.67}{14} =$ 1.619	$F_C(6,3)$ = 8.94
Between rows(seasons)	SSR = 32	r-1 =3 -1 = 2	MSR $= \frac{SSR}{r-1} = \frac{32}{2}$ = 16	$F_R = \frac{MSE}{MSR} = \frac{22.67}{16}$	$F_R(6,2)$ = 19.33
Residual	SSE = 136	N-c-r+1 = 12-4-3+1 = 6	MSE $= \frac{SSE}{N-c-r+1} = \frac{136}{6}$ = 22.67		
	210	11			

Step 8: Conclusion : Cal F < Table F

Hence there is no significant difference in the seasons as far as the sales are concerned.

Thus the test shows that the salesmen and the seasons are alike, so far as the sales are concerned.

12(b) A farmer wishes to test the effect of 4 fertilizers A,B,C,D on the yield of wheat, The fertilizers are used in a LSD and the results are tabulated here perform an analysis of variance.

A18	C21	D25	B11
D22	B12	A15	C19
B15	A20	C23	D24
C22	D21	B10	A17

Solution:

Subtract 20 we get

A -2	C 1	D 5	B -9
D 2	B -8	A -5	C -1
B -5	A 0	C 3	D 4
C 2	D 1	B -10	A -3

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	-2	1	5	-9	-5	4	25	25	81
Y_2	2	-8	-5	-1	-12	4	25	25	1
Y_3	-5	0	3	4	2	25	9	9	16
Y_4	2	1	-10	-3	-10	4	100	100	9
TOTAL	-3	-6	-7	-9	-25	37	66	159	107

H_0: There is no significant difference between rows, columns and treatments.

H_1: There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = -25

Step 3 : $\frac{T^2}{N} = \frac{(-25)^2}{16} = \frac{625}{16} = 39.06$

Step 4 : TSS $= \sum X_1^2 + X_2^2 + X_3^2 + X_4^2 - \frac{T^2}{N}$

$= 37 + 66 + 159 + 107 - 39.06 = 329.94$

Step 5 : SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \frac{(\sum X_4)^2}{N_4} - \frac{T^2}{N}$

$= \frac{(-3)^2}{4} + \frac{(-6)^2}{4} + \frac{(-7)^2}{4} + \frac{(-9)^2}{4} - 39.06$

$(N_1, N_2. N_3, N_4$ *are element in each column*$)$

$= \frac{9}{4} + \frac{36}{4} + \frac{49}{4} + \frac{81}{4} - 39.06$

$= \frac{175}{4} - 39.06 = 43.75 - 39.06 = 4.69$

Step 6 : SSR $= \frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \frac{(\sum Y_4)^2}{N_4} - \frac{T^2}{N}$

$= \frac{(-5)^2}{3} + \frac{(-12)^2}{3} + \frac{(2)^2}{3} + \frac{(-10)^2}{3} - \frac{T^2}{N}$

$(N_1, N_2. N_3, N_4$ *are element in each column*$)$

$= \frac{25 + 144 + 4 + 100}{4} - 39.06 = \frac{273}{4} - 39.06 = 68.25 - 39.06 = 29.19$

To find SSK:

Arrange the elements in the order of treatment.

					Total
A	-2	-5	0	-3	-10
B	-9	-8	-5	-10	-32
C	1	-1	3	2	5
D	5	2	4	1	12

$$SSK = \frac{(-10)^2}{4} + \frac{(-32)^2}{4} + \frac{(5)^2}{4} + \frac{(12)^2}{4} - \frac{T^2}{N}$$

$$= \frac{100}{4} + \frac{1024}{4} + \frac{25}{4} + \frac{144}{4} - 39.06 = \frac{1293}{4} - 39.06 = 284.19$$

SSE = TSS – SSC – SSR - SSK

= 329.94 – 4.69 – 29.19 - 284.19 = 11.87

Step 7 : ANOVA Table

Source of Variation	S. S	d.f	MSS	Variance Ratio	Table value 5% level
Between Rows	SSR = 29.19	k-1 = 3	MSR $= \frac{SSR}{K-1} =$ 9.73	$F_R = \frac{MSR}{MSE} = 4.91$	$F_R(3,6) = 4.76$
Between Columns	SSC = 4.69	k-1 = 3	MSC $= \frac{SSC}{K-1} =$ 1.56	$F_C = \frac{MSE}{MSC} = 1.26$	$F_C(6,3) = 8.94$
Between Treatments	SSK = 284.19	k-1 = 3	MSK $= \frac{SSK}{K-1} =$ 94.73	$F_T = \frac{MSK}{MSE} = 4.91$	$F_T(3,6) = 4.76$
Error	SSE = 11.87	(k-1)(k-2) = 6	MSE= $\frac{SSE}{(K-1)(K-2)} =$ 1.98		

Step 8 : Cal F_C < Tabulated F_C

Cal $F_T > Tabulated\ F_T$

Cal $F_R > Tabulated\ F_R$

Therefore We conclude that there is a significant difference between treatments and rows. But, there is no significant difference between columns.

13.a(i) Solve the following equations by Gauss Elimination Method.

$x + y + z = 9,\qquad 2x - 3y + 4z = 13,\qquad 3x + 4y + 5z = 40.$

Solution:

Given $x + y + z = 9,$

$2x - 3y + 4z = 13$

$3x + 4y + 5z = 40$

$$|A, B| = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -6 \\ 0 & 1 & 2 & 13 \end{array}\right) \qquad R_2 \Leftrightarrow R_2 - 2R_1, \quad R_3 \Leftrightarrow R_3 - 3R_1$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & -5 & 2 & -6 \\ 3 & 0 & 12 & 59 \end{array}\right) \qquad R_3 \Leftrightarrow 5R_3 + R_2$$

$$= \left(\begin{array}{ccc|c} 12 & 12 & 0 & 49 \\ 0 & -30 & 0 & -95 \\ 0 & 0 & 12 & 59 \end{array}\right) \qquad R_1 \Leftrightarrow 12R_1 - R_3, \quad R_2 \Leftrightarrow 6R_2 - R_3$$

$$= \left(\begin{array}{ccc|c} 360 & 0 & 0 & 330 \\ 0 & -30 & 0 & -95 \\ 0 & 0 & 12 & 59 \end{array}\right) \qquad R_1 \Leftrightarrow 30R_1 + 12R_2,$$

$360\,X = 330$ -30 Y = - 95 12 Z = 59

$X = \frac{11}{12}$ $Y = \frac{-95}{30} = \frac{19}{6}$ $Z = \frac{59}{12}$

(ii) Find the dominant eigen values of $\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$ by power method.

Solution:

Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an arbitrary initial eigen vector.

$$AX_1 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = 3\begin{bmatrix} 0.3333 \\ 1 \\ -0.3333 \end{bmatrix} = 3X_2$$

$$AX_1 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = 3\begin{bmatrix} 0.3333 \\ 1 \\ -0.3333 \end{bmatrix} = 3X_2$$

$$AX_2 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.3333 \\ 1 \\ -0.3333 \end{bmatrix} = \begin{bmatrix} 3.6666 \\ 1.6667 \\ -0.3337 \end{bmatrix} = 3.6666\begin{bmatrix} 1 \\ 0.4546 \\ 0.0910 \end{bmatrix} = 3.6666X_3$$

$$AX_3 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 0.4546 \\ 0.0910 \end{bmatrix} = \begin{bmatrix} 2.2728 \\ 4.2732 \\ 1.7284 \end{bmatrix} = 4.2732\begin{bmatrix} 0.5319 \\ 1 \\ 0.4045 \end{bmatrix} = 4.2732X_4$$

$$AX_4 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 1.5319 \\ 1 \\ 0.4045 \end{bmatrix} = \begin{bmatrix} 3.1274 \\ 5.2137 \\ 7.5131 \end{bmatrix} = 7.5131\begin{bmatrix} 0.4163 \\ 0.6939 \\ 1 \end{bmatrix} = 7.5131X_5$$

$$AX_5 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.1212 \\ 0.5370 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.7322 \\ 5.4376 \\ 12.0268 \end{bmatrix} = 12.0268\begin{bmatrix} 0.0609 \\ 0.4521 \\ 1 \end{bmatrix} = 12.0268X_7$$

$$AX_7 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.0609 \\ 0.4521 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.4172 \\ 5.0869 \\ 11.7475 \end{bmatrix} = 11.7475\begin{bmatrix} 0.0355 \\ 0.4330 \\ 1 \end{bmatrix} = 11.7475X_8$$

$$AX_8 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.0355 \\ 0.4330 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3345 \\ 4.9725 \\ 11.6965 \end{bmatrix} = 11.6965\begin{bmatrix} 0.0289 \\ 0.4251 \\ 1 \end{bmatrix} = 11.6965X_9$$

$$AX_9 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.0289 \\ 0.4251 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3042 \\ 4.9369 \\ 11.6715 \end{bmatrix} = 11.6715\begin{bmatrix} 0.0261 \\ 0.4230 \\ 1 \end{bmatrix} = 11.6715X_{10}$$

$$AX_{10} = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.0261 \\ 0.4230 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2951 \\ 4.9243 \\ 11.6659 \end{bmatrix} = 11.6659\begin{bmatrix} 0.0253 \\ 0.4221 \\ 1 \end{bmatrix} = 11.6659X_{11}$$

$$AX_{11} = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.0253 \\ 0.4221 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2916 \\ 4.9201 \\ 11.6631 \end{bmatrix} = 11.6631\begin{bmatrix} 0.025 \\ 0.4219 \\ 1 \end{bmatrix} = 11.6631X_{12}$$

$$AX_{12} = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.025 \\ 0.4219 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2907 \\ 4.9188 \\ 11.626 \end{bmatrix} = 11.6626\begin{bmatrix} 0.0249 \\ 0.4218 \\ 1 \end{bmatrix} = 11.6626X_{13}$$

$$AX_{13} = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.0249 \\ 0.4218 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2903 \\ 4.9183 \\ 11.6623 \end{bmatrix} = 11.6623\begin{bmatrix} 0.0249 \\ 0.4217 \\ 1 \end{bmatrix} = 11.6623X_{14}$$

$$AX_{14} = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.0249 \\ 0.4217 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.29 \\ 4.9181 \\ 11.6619 \end{bmatrix} = 11.6619\begin{bmatrix} 0.0249 \\ 0.4217 \\ 1 \end{bmatrix}$$

The dominant eigen value is 11.6619 and the corresponding eigen vector is $\begin{bmatrix} 0.0249 \\ 0.4217 \\ 1 \end{bmatrix}$

(b)(i) If A = $\begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ Find A^{-1} *by Gauss Jordan method.*

Solution:

$$[A, I] = \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 7 & 17 & -1 & 0 & 2 \end{array}\right) \qquad R_2 \Leftrightarrow 2R_2 - 3R_1, \qquad R_3 \Leftrightarrow 2R_3 - R_1,$$

$$= \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & -4 & 20 & -14 & 2 \end{array}\right) \qquad R_3 \Leftrightarrow R_3 - 7R_2,$$

$$= \left(\begin{array}{ccc|ccc} 8 & 4 & 0 & 24 & -14 & 2 \\ 0 & 4 & 0 & 48 & -34 & 6 \\ 0 & 0 & -4 & 20 & -14 & 2 \end{array}\right) \quad R_2 \Leftrightarrow 4R_2 + 3R_3, \quad R_1 \Leftrightarrow 4R_1 + R_3$$

$$= \left(\begin{array}{ccc|ccc} 8 & 0 & 0 & -24 & 20 & -4 \\ 0 & 4 & 0 & 48 & -34 & 6 \\ 0 & 0 & -4 & 20 & -14 & 2 \end{array}\right) \quad R_1 \Leftrightarrow R_1 - R_2,$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & \frac{5}{2} & \frac{-1}{2} \\ 0 & 1 & 0 & 12 & \frac{-17}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -5 & \frac{7}{2} & \frac{-1}{2} \end{array}\right)$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -6 & 5 & -1 \\ 24 & -17 & 3 \\ -10 & 7 & -1 \end{bmatrix}$$

(ii) Solve the following equation by Gauss Seidal method:

$$x + y + 54z = 110, \quad 27x + 6y - z = 85, \quad 6x + 15y + 2z = 72.$$

Solution :

Write x, y, z as follows:

$$x = \frac{1}{27}[85 - 6y + z] \qquad y = \frac{1}{15}[72 - 6x - 2z] \quad z = \frac{1}{54}[110 - x - y]$$

Let the initial values be y = 0, z = 0

First Iteration

$$x^1 = \frac{1}{27}[85 - 6y^0 + z^0] = \frac{1}{27}[85 - 6(0) + 0] = 3.148$$

$$y^1 = \frac{1}{15}[72 - 6x^1 - 2z^0] = \frac{1}{15}[72 - 6(3.148) - 0] = 3.541$$

$$z^1 = \frac{1}{54}[110 - x^1 - y^1] = \frac{1}{54}[110 - 3.148 - 3.541] = 1.913$$

Second iteration

$$x^2 = \frac{1}{27}[85 - 6y^1 + z^1] = \frac{1}{27}[85 - 6(3.541) + 1.913] = 2.432$$

$$y^2 = \frac{1}{15}[72 - 6x^2 - 2z^1] = \frac{1}{15}[72 - 6(2.432) - 2(1.913)] = 3.572$$

$$z^2 = \frac{1}{54}[110 - x^2 - y^2] = \frac{1}{54}[110 - 2.432 - 3.572] = 1.926$$

Third iteration

$$x^3 = \frac{1}{27}[85 - 6y^2 + z^2] = \frac{1}{27}[85 - 6(3.572)+1.926] = 2.426$$

$$y^3 = \frac{1}{15}[72 - 6x^3 - 2z^2] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^3 = \frac{1}{54}[110 - x^3 - y^3] = \frac{1}{54}[110 - 2.426 - 3.573] = 2.426$$

Fourth iteration

$$x^4 = \frac{1}{27}[85 - 6y^3 + z^3] = \frac{1}{27}[85 - 6(3.573) - 1.926] = 2.426$$

$$y^4 = \frac{1}{15}[72 - 6x^4 - 2z^3] = \frac{1}{15}[72 - 6(2.426) - 2(10926)] = 3.573$$

$$z^4 = \frac{1}{54}[110 - x^4 - y^4] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

Hence $x = 2.426, \quad y = 3.573, \quad z = 1.926$

14.a(i) Find y(22),given that

X:	20	25	30	35	40	45
Y(x):	354	332	291	260	231	204

Ans :

To find y (22) , Use Newton forward interpolation formula

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots \text{-----}(1)$$

$$u = \frac{x - x_0}{h} = \frac{22-20}{5} = 0.4 \quad Here \;\; x_0 = 20 \;\; , \;\; h = 5$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
20	354					
		-22				
25	332		-19			
		-41		29		
30	291		10		-37	
		-31		-8		45
35	260		2		8	
		-29		0		
40	231		2			
		-27				
45	204					

Substituting in (1)

$$y(22) = 354 + \frac{0.4}{1!}(-22) + \frac{0.4\,(0.4-1)}{2!}(-19) + \frac{(0.4)(0.4-1)(0.4-2)}{3!}(29)$$

$$+ \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{4!}(-37)$$

$$+ \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)(0.4-4)}{5!}(29)$$

$= 354 - 8.8 + 2.28 + 1.856 + 1.5392 + 1.34784$

$= 352.22$

(ii) Evaluate $\int_0^{\pi} \sin x \, dx$, by Trapezoidal and Simpson's (1/3) rules by dividing the range into 10 equal parts.

Solution:

Range $= \pi - 0 = \pi$ Hence $h = \frac{\pi}{10}$

We tabulate below the values of y at different x 's.

X :	0	$\frac{\pi}{10}$	$\frac{2\pi}{10}$	$\frac{3\pi}{10}$	$\frac{4\pi}{10}$	$\frac{5\pi}{10}$	$\frac{6\pi}{10}$
Y= sin x	0	0.3090	0.5878	0.8090	0.9511	1	0.9511
			$\frac{7\pi}{10}$	$\frac{8\pi}{10}$	$\frac{9\pi}{10}$	π	
			0.8090	0.5878	0.3090	0	

Note that the values are symmetrical about x $= \frac{\pi}{2}$

Trapezoidal rule:

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots .. + y_{n-1})]$$

$$I = \left(\frac{\pi}{20}\right)[(0 + 0) + 2(0.3090 + 0.5878 + 0.8090 + 0.9511 + 1 + 0.9511 + 0.8090 + 0.5878 + 0.3090] = 1.9843 \text{ nearly.}$$

Simpson's one third rule

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_2 + y_4 + \cdots .) + 4(y_1 + y_3 + \cdots .)]$$

$$= \frac{1}{3}\left(\frac{\pi}{10}\right)[(0 + 0) + 2(0.5878 + .9511 + 0.9511 + 0.5878) + 4(0.3090 + 0.8090 + 1 + 0.8090 + 0.3090)]$$

$$= 2.00091.$$

(b) (i) Find the cubic polynomial y(x) for

X:	-1	0	2	3
Y(x):	-8	-3	1	12

Solution:

	x_0	x_1	x_2	x_3
x	-1	0	2	3
y	-8	1	1	12

By Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}(-8) + \frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)}(3)$$

$$+\frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)} + \frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)}(12)$$

$$= \frac{x(x^2-5x+6)}{(-1)(-3)(-4)}(-8) + \frac{(x+1)(x^2-5x+6)}{(-2)(-3)}(3) + \frac{(x+1)(x^2-3x)}{(3)(2)(-1)}(1) + \frac{(x+1)(x^2-2x)}{(4)(3)(1)}(12)$$

$$= \frac{2}{3}(x^3-5x^2+6x) + \frac{1}{2}(x^3-5x^2+6x+x^2-5x+6) - \frac{1}{6}(x^3-3x^2+x^2-3x) + x^3 - 2x^2 + x^2 - 2x$$

$$= \frac{(4+3-1+6)}{6}x^3 + (-6)x^2 + 3x + 3$$

$$y(x) = 2x^3 - 6x^2 + 3x + 3$$

(ii) Find y'(1) if

X	0	2	3	4	7	9
Y(x)	4	26	58	112	466	922

Solution :

since the arguments are not equally spaced, we will use Newton's divided difference formula

Divided Difference Table

x	$y=f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	4				
		11			
2	26		7		
		32		1	
3	58		11		0
		54		1	
4	112		16		0
		118		1	
7	466		22		
		228			
9	922				

By Newton's divided difference formula

$y = f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$

$= 4 + (x - 0)11 + (x - 0)(x - 27)7 + (x - 0)(x - 2)(x - 3)1$

$= \ x^3 + 2x^2 + 3x + 4$

Therefore, $y'(x) = 3x^2 + 4x + 3$

$y'(1) = 3(1)^2 + 4(1) + 3 = 3 + 4 + 3 = 10$

15.a(i) By Taylor series method find y (0.1) , y (0.2) and y(0.3) $\frac{dy}{dx} = x - y^2$, $y(0) = 1.$

Solution :

Given $y' = x - y^2$, $y(0) = 1$

i.e. $y' = x - y^2$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$y' = x - y^2 \quad \Rightarrow \quad y_0' = x_0 - {y_0}^2 = 0 - 1^2 = -1$

$y'' = 1 - 2yy' \quad \Rightarrow \quad y_0'' = 1 - 2y_0y_0' = 1 - 2(1)(-1) = 3$

$y''' = -2[yy'' + y'y'] \quad \Rightarrow \quad y_0''' = -2[y_0y_0'' + (y_0')^2]$

$= -2[[yy'' + (y')^2] \quad\quad = -2[(1)(3) + (-1)^2] = -8$

By Taylor's series, we have

$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \ldots\ldots\ldots$

$y(0.1) = 1 + (0.1)(-1) + \frac{(0.1)^2}{2!}(3) + \frac{(0.1)^3}{3!}(-8) + \cdots$

$= 1 - 0.1 + 0.015 - 0.0133 = 0.91367 = 0.91$

$y(0.2) = 1 + (0.2)(-1) + \frac{(0.2)^2}{2!}(3) + \frac{(0.2)^3}{3!}(-8) + \cdots$

$= 1 - 0.2 + 0.06 - 0.01067 = 0.8493 = 0.85$

$y(0.3) = 1 + (0.3)(-1) + \frac{(0.3)^2}{2!}(3) + \frac{(0.3)^3}{3!}(-8) + \cdots$

$= 1 - 0.3 + 0.135 - 0.036 = 0.799$

(ii) By modified Euler's method, find y(0.1), y(0.2) and y(0.3), if $\frac{dy}{dx} = x + y$, $y(0) = 1.$

Solution:

Given $y' = x + y$, $y(0) = 1$

$f(x, y) = x + y$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2} \ , \ y_n + \frac{h}{2}f(x_n \ , \ y_n)\right]$

$$y_1 = y_0 + hf\left[x_0 + \frac{h}{2}\ ,\ y_0 + \frac{h}{2} f(x_0\ ,y_0)\right]$$

$$= 1 + (0.1)f[1 + \tfrac{0.1}{2}\ ,\ 1 + \tfrac{0.1}{2} f(0.1)]$$

$$= 1 + (0.1)f[0.05\ ,\ 1 + 0.05[0 + 1]]$$

$$= 1 + (0.1)f[0.05, 1.05]$$

$$= 1 + (0.1)[0.05 + 1.05]$$

$$= 1 + (0.1)(1.1) = 1.11$$

$$\boldsymbol{y_1 = y(0.1) = 1.11}$$

$$y_2 = y_1 + hf\left[x_1 + \frac{h}{2},\ y_1 + \frac{h}{2} f(x_1\ ,\ y_1)\right]$$

$$= 1.11 + (0.1)f[0.1 + \tfrac{0.1}{2}, 1.11 + \tfrac{0.1}{2} f(0.1, 1.11)]$$

$$= 1.11 + (0.1)f[0.15\ ,\ 1.11 + \tfrac{0.1}{2}[0.1 + 1.11]]$$

$$= 1.11 + (0.1)f[0.15, 1.1705]$$

$$= 1.11 + (0.1)[0.15 + 1.1705]$$

$$\boldsymbol{y_2 = y(0.2) = 1.2421}$$

$$y_3 = y_2 + hf\left[x_2 + \frac{h}{2}\ ,\ y_2 + \frac{h}{2} f(x_2, y_2)\right]$$

$$= 1.2421 + (0.1)f[0.2 + \frac{0.1}{2}\ ,\ 1.2421 + \frac{0.1}{2} f(0.2, 1.2421)]$$

$$= 1.2421 + (0.1)f[0.25, 1.2421 + \tfrac{0.1}{2}[0.2 + 1.2421]]$$

$$= 1.2421 + (0.1)f[0.25, 1.314205]$$

$$= 1.2421 + (0.1)[0.25 + 1.314205]$$

$$\boldsymbol{y_3 = y(0.3) = 1.3985}$$

(b) If $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, y(0) = 1 , find y(0.2) , y(0.4) and y(0.6)by Runge - Kutta method of fourth order and hence find y(0.8) by Milne Thomson method.

Ans :

Given $y' = f(x,y) = \dfrac{y^2 - x^2}{y^2 + x^2}$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.2$, $x_2 = 0.4$, $x_3 = 0.6$, $h = 0.2$

To find y(0.2) :

$$k_1 = h f(x_0\ ,y_0) = (0.2)\left[\frac{y_0^{\,2} - x_0^{\,2}}{y_0^{\,2} + x_0^{\,2}}\right] = (0.2)\left[\frac{1-0}{1+0}\right] = 0.2$$

$$k_2 = h\, f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right] = (0.2) f\left[0 + \frac{0.2}{2}\ ,\ 1 + \frac{0.2}{2}\right]$$

$$= (0.2)\ f\ [0.1, 1.1] = (0.2)\left[\frac{(1.1)^2-(1.1)^2}{(1.1)^2+(0.1)^2}\right]$$

$$= (0.2)\left(0.9836\right) = 0.19672$$

$$k_3 = h\ f\left[x_0+\frac{h}{2},\ y_0+\frac{k_2}{2}\right] = (0.2) f\left[0+\frac{0.2}{2},\ 1+\frac{0.19672}{2}\right]$$

$$= (0.2)\ f\ [0.1,\ 1.0983606]$$

$$= (0.2)\left[\frac{(1.0983606)^2-(0.1)^2}{(1.0983606)^2+(0.1)^2}\right] = 0.1967$$

$$k_4 = h\ f\ (x_0+h,\ y_0+k_3) = (0.2)\ f\ [0.2,\ 1.1967]$$

$$= (0.2)\left[\frac{(1.1967)^2-(0.2)^2}{(1.1967)^2+(0.2)^2}\right] = 0.1891$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.2 + 2(0.19672) + 2(1.1967) + 0.1891] = 0.19598$$

$$y(0.2) = y_1 = y_0 + \Delta y = 1 + 0.19598 = 1.19598$$

<u>To find y(0.4)</u> :

$$k_1 = h\ f\ (x_1, y_1)$$

$$= (0.2)\left[\frac{(1.196)^2-(0.2)^2}{(1.196)^2+(0.2)^2}\right] = 0.1891$$

$$k_2 = h\ f\left[x_1+\frac{h}{2},\ y_1+\frac{k_1}{2}\right] = (0.2) f\left[0.2+\frac{0.2}{2},\ 1.196+\frac{0.1891}{2}\right]$$

$$= (0.2)\ f\ [0.3, 1.2906]$$

$$= (0.2)\left[\frac{(1.2906)^2-(0.3)^2}{(1.2906)^2+(0.3)^2}\right] = 0.1795$$

$$k_3 = h\ f\left[x_1+\frac{h}{2},\ y_1+\frac{k_2}{2}\right]$$

$$= (0.2)\ f\left[0.2+\frac{0.2}{2},\ 1.196+\frac{0.1795}{2}\right]$$

$$= (0.2)\ f\ [0.3, 1.2858]$$

$$= (0.2)\left[\frac{(1.2858)^2-(0.3)^2}{(1.2858)^2+(0.3)^2}\right] = 0.1793$$

$$k_4 = h\ f\ (x_1+h,\ y_1+k_3)$$

$$= (0.2)\ f\ [0.4,\ 1.3753]$$

$$= (0.2)\left[\frac{(1.3753)^2 - (0.4)^2}{(1.3753)^2 + (0.4)^2}\right] = 0.1688$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1891 + 2(0.1795) + 2(0.1793) + 0.1688] = 0.1793$$

$$y(0.4) = y_2 = y_1 + \Delta y = 1.196 + 0.1793 = 1.3753$$

<u>To find y(0.6)</u> :

$$k_1 = h\, f(x_2, y_2)$$

$$= (0.2)\left[\frac{(1.3753)^2 - (0.4)^2}{(1.3753)^2 + (0.4)^2}\right] = 0.1360$$

$$k_2 = h\, f\left[x_2 + \frac{h}{2},\ y_2 + \frac{k_1}{2}\right] = (0.2)\, f\,[0.5, 1.4597]$$

$$= (0.2)\left[\frac{(1.4597)^2 - (0.5)^2}{(1.4597)^2 + (0.5)^2}\right] = 0.1580$$

$$k_3 = h\, f\left[x_1 + \frac{h}{2},\ y_1 + \frac{k_2}{2}\right] = (0.2)\, f\,[0.5, 1.4543]$$

$$= (0.2)\left[\frac{(1.4543)^2 - (0.5)^2}{(1.4543)^2 + (0.5)^2}\right] = 0.1577$$

$$k_4 = h\, f\,(x_1 + h,\ y_1 + k_3) = (0.2)\, f\,[0.6, 1.533]$$

$$= (0.2)\left[\frac{(1.533)^2 - (0.6)^2}{(1.533)^2 + (0.6)^2}\right] = 0.1469$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1688 + 2(0.1580) + 2(0.1577) + 0.1469] = 0.1579$$

$$y(0.6) = y_3 = y_2 + \Delta y = 1.3753 + 0.1579 = 1.5332$$

To Find y (0.8) by Milne's method :

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}[2y_{n-2}' - y_{n-1}' + 2y_n']$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}[y_{n-1}' + 4y_n' + y_{n+1}']$$

$$y_1' = \left[\frac{y_1^2 - x_1^2}{y_1^2 + x_1^2}\right] = 0.9456 \qquad y_2' = \left[\frac{y_2^2 - x_2^2}{y_2^2 + x_2^2}\right] = 0.8440$$

$$y_3' = \left[\frac{y_3^2 - x_3^2}{y_3^2 + x_3^2}\right] = 0.7349$$

$$y_{4,p} = y_0 + \frac{4(0.2)}{3}[2y_1' - y_2' + 2y_3']$$

$$= 1 + \frac{4(0.2)}{3}[2(1.19598) - 1.3753 + 2(1.5332)] = 2.0888$$

$$y_{4,c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$y_4' = \left[\frac{y_4^2 - x_4^2}{y_4^2 + x_4^2}\right] = 0.7442$$

$$y_{4,c} = 1.3753 + \frac{0.2}{3}[0.8440 + 4(0.7349) + 0.7442] = 1.6772$$

Agni College of Technology

Chennai - 130

B.E./ B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA2266 –STATISTICS AND NUMERICAL METHODS
(Regulation 2008)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. Define Type – I error and Type – II error.
Solution:
Type I error : If H_0 is rejected while it should have been accepted.
Type II error: If H_0 is accepted while it should have been rejected.

2. State the application of Chi- test
Solution:
1. To test the goodness of fit.
2. To test the independent of attributes.
3. To test the homogeneous of independent estimations.

3. State the assumptions involved in ANOVA.
Solution:
1.Normality 2. Homogeneity 3. Square Independence of error.

4. What are the advantages of a Latin square design?
Solution:
1.With a two way stratification or grouping, the Latin Square controls more of the variations than the completely randomized design or the randomized completely block design. The two way elimination of variations often results in small error mean square.
2. The analysis is simple , it is only slightly more complicated than that for the randomized complete block design.

5. Arrive a formula to find the value of $\sqrt[3]{N}$, where N≠0 , using Newton-Raphson method.
Solution:
Let $x = N^{1/3}$ $(ie)\ x^3 = N$ Let $f(x) = x^3 - N$, $f'(x) = 3x^2$

N-R formula : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \left[\frac{x_n^3 - N}{3x_n^2}\right]$

$$= x_n - \frac{1}{3}x_n + \frac{N}{3x_n^2} = \frac{2}{3}x_n + \frac{N}{3x_n^2}$$

$$= \frac{1}{3}\left[2x_n + \frac{N}{x_n^2}\right] \quad , \ n = 0, 1, 2, \ldots\ldots.$$

6. Solve the following system of equations using Gauss-Jordan elimination method
$2x + y = 3$, $x - 2y = -1$
Solution:

The given system is equivalent to

$$|A, B| = \left(\begin{array}{cc|c} 2 & 1 & 3 \\ 1 & -2 & -1 \end{array}\right)$$

$$= \left(\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & -5 & -5 \end{array}\right) \quad R_2 \Leftrightarrow 2R_2 - R_1$$

$$= \left(\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & 1 & 1 \end{array}\right) \quad R_2 \Leftrightarrow \frac{R_2}{-5}$$

$$= \left(\begin{array}{cc|c} 2 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right) \quad R_2 \Leftrightarrow R_1 - R_2$$

$$= \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right) R_1 \Leftrightarrow \frac{R_1}{2}$$

There fore x = 1 ,y = 1.

7. Form the divided difference table for the following data:

X	5	15	22
Y	7	36	160

Solution:

The divided difference table is as follows:

x	f(x)	Δf(x)	Δ^2f(x)
5	7		
		$\frac{36-7}{15-5} = 2.9$	
15	36		$\frac{17.7-2.97}{22-5} = \frac{14.8}{17}$
		$\frac{160-36}{22-15} = 17.7$	
22	160		

8. Evaluate $\int_{0.5}^{1} \frac{dx}{x}$ by Trapezoidal rule, dividing the range into 4 equal parts

Solution:

Here $h = \frac{1-\frac{1}{2}}{4} = \frac{1}{8}$; $y = \frac{1}{x}$

x	$\frac{1}{2}=\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
f(x) : $\frac{1}{x}$	$\frac{8}{4}$	$\frac{8}{5}$	$\frac{8}{6}$	$\frac{8}{7}$	$\frac{8}{8}$

A = Sum of the first and the last ordinates = $\frac{8}{4}+\frac{8}{8}$ = 3

B = Sum of the remaining ordinates = $\frac{8}{5}+\frac{8}{6}+\frac{8}{7}=\frac{856}{210}$

$\int_{\frac{1}{2}}^{1}\frac{dx}{x} = \frac{h}{2}$(A+2B) = $\frac{1}{16}(3+\frac{856\ X2)}{1680}) = \frac{1171}{1680}$ = 0.6971.

9. State the merits of RK – method over Taylor series method.

Solution:

Runge Kutta methods do not require prior calculation of higher derivatives of y(x), as the Taylor method does. Since the differential equations using in applications are often complicated the calculations of derivatives may be difficult.

Also, the Runga Kutta formulae involve the computations of f(x, y) at various positions instead of derivatives and this function occurs in the given equation.

10. Write the central difference approximations for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$.

Solution:

$$y' = \frac{1}{2h}(y_{i+1} - y_{i-1})$$

$$y'' = \frac{1}{h^2}(y_{i-1} - 2y_i + y_{i+1})$$

PART - B (5 x 16 = 80 Marks)

11.a(i) A dice is thrown 400 times and a throw of 3 or 4 is observed 150 times. Test the hypothesis that the dice is fair.

Solution:

Given n = 400, X = Number of successes = 150

P = probability of getting 3 or 4. = $\frac{1}{6}+\frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

Q = 1 –P = 1 - $\frac{1}{3}$ = $\frac{2}{3}$

$$H_0 = P = \frac{1}{3}(unbaised)$$

$H_1 : P \neq \frac{1}{3}$

$Z = \frac{X - n\,P}{\sqrt{nPQ}} = \frac{150 - 400\,\frac{1}{3}}{\sqrt{(400)(\frac{1}{3})(\frac{2}{3})}} = \frac{16.6667}{9.4281} = 1.7678$

Cal $|z| = 1.7678$

For two tailed test = $|z| = 1.96$ $(5\%\ level)$

Cal $|z| < table\ |z|$

Accept H_0. Thus the dice is fair.

(ii). Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory?

Solution:

Given : A,B,C,D in the ratio 9:3:3:1

					Total
E_i	900	300	300	100	1600
O_i	882	313	287	118	1600

$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{18^2}{900} + \frac{13^2}{300} + \frac{13^2}{300} + \frac{18^2}{100} = 4.73$

$\sum E_i = \sum O_{i,}$ degree of freedom = 4 – 1 = 3

$H_0 = The\ experiment\ supports\ the\ theory$

Cal $\chi^2 = 4.73$

Table value for 3 degree of freedom = 7.82 $\Rightarrow$ Cal$cal\ \chi^2 < tab\ \chi^2$

Accept H_0.

11.b(i) The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0inches respectively. Can the samples be regarded as drawn from the same populations of standard deviation 2.5 inches?

Solution:

Given n_1=1000, $n_2 = 2000$,$\overline{x_2} =$ 67.5, $\overline{x_1} =$ 68 $\sigma_1 = \sigma_2 = 2.5$

The parameter of interests is μ_1 and μ_2

1. $H_0 : \mu_1 = \mu_2$ [No significant difference]
2. $H_1 : \mu_1 \neq \mu_2$

3. α = 0.05, (Two Tailed Test)

the test statistic is $z = \frac{\overline{x_1}-\overline{x_2}}{\sqrt{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)}} = \frac{67.5-68}{\sqrt{\frac{2.5^2}{1000}+\frac{2.5^2}{2000}}} = \frac{-0.5}{\sqrt{0.00625+0.003125}} = -5.16$

Conclusion : -1.96 < - 5.16 < 1.96 So we reject $H_{0.}$

(ii). Two random samples gave the following results:

Sample	Size	Sample Mean	Sum of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples have come from the same normal population.

Solution:

A normal population has two parameters namely the mean μ and the variance $\boldsymbol{\sigma}^2$. If we want to test the samples from the same normal population,we have to test

(i) The equality of population variance (Using F-Test)

(ii) The equality of population means (using t-test).

Since t-test assumes $\boldsymbol{\sigma}_1^2 = \boldsymbol{\sigma}_2^2$ we shall first apply F-test and then t-test.

(i) F-test:

Given: $n_1=10,\ n_2 = 12, \overline{x_2} = 14, \overline{x_1} = 15$

$S_1^2 = \frac{\sum(x_1-\overline{x_1})^2}{n_1-1} = \frac{90}{9} = 10 \qquad S_2^2 = \frac{\sum(x_2-\overline{x_2})^2}{n_2-1} = \frac{108}{11} = 9.8181$

$S_1^2 > S_2^2$

The parameters of interest is $\boldsymbol{\sigma}_1^2$ *and* $\boldsymbol{\sigma}_2^2$

1. H_0 : $\boldsymbol{\sigma}_1^2 = \boldsymbol{\sigma}_2^2$
2. H_1 : $\boldsymbol{\sigma}_1^2 \neq \boldsymbol{\sigma}_2^2$
3. α = 0.05, d.f.$(v_1) = n_1 - 1 = 9$ d.f.$(v_2) = n_2 - 1 = 11$
4. Table value of F :2.90
5. The test statistic is F = $\frac{S_1^2}{S_2^2} = \frac{10}{9.8182} = 1.019$
6. Conclusion:

If Cal F < table F, then we accept H_0; Otherwise we reject H_0.

Here, F = 1.019 < 2.90, we accept H_0 at 5% level of significance.

(Note: If F-test failed, then t-test should not be used)

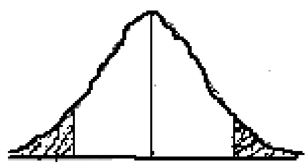
(ii) t-test:

Given $n_1=10, n_2=12, S_1^2=10, S_2^2=9.8181$

$$S^2=\frac{n_1s_1^2+n_2s_2^2}{n_1+n_2-2}=\frac{100+117.82}{10+12-2}=10.9$$

The parameter of interests is μ_1 *and* μ_2

1. $H_0: \mu_1=\mu_2$
2. $H_1: \mu_1 \neq \mu_2$
3. $\alpha = 0.05$, d.f $= n_1+n_2-2=20$ (Two Tailed Test)
4. Critical region



$-t_{\alpha/2} = -2.086$ $\qquad$ $-t_{\alpha/2} = 2.086$

5. the test statistic is $t=\frac{\overline{x_1}-\overline{x_2}}{\sqrt{S^2(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{15-14}{\sqrt{(10.9)(\frac{1}{10}+\frac{1}{12})}} = 0.707$

6. Conclusions:

Calculated $|t| = 0.707 < 2.086$, we accept H_0 at 5% level of significance.

12.(a) Four varieties A, B, C, D of a fertilizer are tested in a RBD with 4 replications. The plot yields in pounds are as follows:

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

Analyse the experimental yield.

Solution:

Let us take 12 as origin for simplifying the calculations

Row	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
(y_1) (1)	A 0	D 8	C 4	B -2	10	0	64	16	4
(y_2) (2)	D 6	A 2	B -1	C 2	9	36	4	1	4
(y_3) (3)	B 0	C 3	D 7	A 1	11	0	9	49	1
(y_4) (4)	C 4	B 1	A 3	D 8	14	16	1	9	64
Total	10	12	13	9	44	52	78	75	73

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = 44

Step 3 : C.F $= \frac{T^2}{N} = \frac{(44)^2}{16} = 121$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

= 52 + 78 + 75 + 73 - 121 = 157

Step 5 : SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(10)^2}{4} + \frac{(12)^2}{4} + \frac{(13)^2}{4} + \frac{(9)^2}{4} - 121 = 2.5$$

Step 6. SSR $= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(10)^2}{4} + \frac{(9)^2}{4} + \frac{(11)^2}{4} + \frac{(14)^2}{4} - 121 = 3.5$$

To Find SSK

Treatment	1	2	3	4	Total
A	0	2	3	1	6
B	0	-1	-1	-2	- 4
C	4	3	4	2	13
D	6	8	7	8	29
					44

SSK $= \frac{(6)^2}{4} + \frac{(-4)^2}{4} + \frac{(13)^2}{4} + \frac{(29)^2}{4} - 121 = 144.5$

SSE = TSS – SSC – SSR = 157 – 2.5 – 3.5 – 144.5 = 6.5

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	F test 1%
Between Rows	SSR = 3.5	3	1.17	1.08	9.78
Between columns	SSC = 2.5	3	0.83	0.77	27.91
Variety	SSK = 144.5	3	48.17	44.60	9.78
Error	SSE = 6.5	6	1.08		

Total	TSS = 157	11			

Step 8 . Conclusion :

The F ratios for rows and columns are not significant at 1 % level while that for varieties is very highly significant. The fact that there are no significant differences between rows and columns. Show that the Latin square arrangement has not been advantageous.

(b). A variable trial was conducted on wheat with 4 varieties in a Latin Square design. The plan of the experiment and per plot yield are given below:

D25	B23	A20	D20
A19	D19	C21	B18
B19	A14	D17	C20
D17	C20	B21	A15

Analyse the data.

Solution:

Subtract 20 from all the items

Y_n	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
y_1	5	3	0	0	8	25	9	0	0
y_2	-1	-1	1	-2	-3	1	1	1	4
y_3	-1	-6	-3	0	-10	1	36	9	0
y_4	-3	0	1	-5	-7	9	0	1	25
Total	0	-4	-1	-7	-12	36	46	11	29

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = -12

Step 3 : $C.F = \frac{T^2}{N} = \frac{144}{16} = 9$

Step 4 : $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 36 + 46 + 11 + 29 - 9 = 113$

Step 5 : $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(0)^2}{4} + \frac{(4)^2}{4} + \frac{(1)^2}{4} + \frac{(7)^2}{4} - 9 = 7.5$

Step 6. $SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(8)^2}{4} + \frac{(-3)^2}{4} + \frac{(-10)^2}{4} + \frac{(-7)^2}{4} - 9 = 46.5$$

To Find SSK

Treatment	1	2	3	4	Total
A	0	-1	-6	-5	-12
B	3	-2	-1	1	1
C	5	1	0	0	6
D	0	-1	-3	-3	-7

$$\text{SSK} = \frac{(-12)^2}{4} + \frac{(1)^2}{4} + \frac{(6)^2}{4} + \frac{(-7)^2}{4} - 9 = 48.5$$

SSE = TSS – SSC – SSR = 113 – 7.5 – 46.5 – 48.5 = 10.5

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	Table value at 5% level
Between Rows	SSR = 46.5	k –1= 3	MSR = $\frac{SSR}{k-1}$ = 15.5	$F_R = \frac{MSR}{MSE}$ = 8.86	$F_R(3, 6)$ = 4.76
Between columns	SSC = 7.5	k – 2 = 3	MSC = $\frac{SSC}{k-1}$ = 2.5	$F_C = \frac{MSC}{MSE}$ = 1.43	$F_C(3, 6)$ = 4.76
Treatment	SSK = 48.5	k – 1 = 3	MSK = $\frac{SSK}{k-1}$ = 16.17	$F_T = \frac{MSK}{MSE}$ = 9.24	$F_T(3, 6)$ = 4.76
Error	SSE = 10.5	(k -1)(k – 2) = 6	MSE = $\frac{SSE}{(k-1)(k-2)}$ = 1.75		
Total	TSS = 113				

Step 8 . Conclusion :

Cal F_R > Table F_R
Cal F_C < Table F_C
Cal F_T > Table F_T

There is significant difference between treatments and rows. But there is no significant difference between columns.

13.a(i) Using Newton-Raphson method, solve $x \log 10\, x = 12.34$ taking the initial value x_0 as 10.
Solution:

Given : $x\log_{10}x = 12.34$

Let $f(x) = x\log_{10}x - 12.34$

Given x_0=10

Newton Raphson formula: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_0)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 10 - \frac{f(10)}{f'(10)}$

$= 10 - \left[\frac{10\log_{10}10 - 12.34}{\log_{10}e + \log_{10}10}\right] = 10 - \left[\frac{-2.34}{1.4343}\right]$

$= 10 + \frac{2.34}{1.4343} = 11.6315$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 11.6315 - \frac{f(11.6315)}{f'(11.6315)}$

$= 11.6315 - \frac{11.6315\log_{10}11.6315 - 12.315}{\log_{10}e + \log_{10}11.6315}$

$= 11.6315 - \frac{0.0549}{1.5} = 11.5949$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 11.5949 - \frac{f(11.5949)}{f'(11.5949)}$

$= 11.5949 - \frac{11.5949\log_{10}11.5949 - 12.34}{\log_{10}e + \log_{10}11.5949}$

$= 11.5949 - \frac{0.00006}{1.4986} = 11.5949$

Here $x_2 = x_3 = 11.5949$.

Hence the better approximate root is 11.5949.

(ii). Find the numerically largest eigen value of $A=\begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}$ by power method.

Solution: Let $X_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$ be an arbitrary initial eigen vector.

$$AX_1 = \begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\4\\6\end{bmatrix} = 6\begin{bmatrix}0.167\\0.667\\1\end{bmatrix} = 6X_2$$

$$AX_2 = \begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0.167\\0.667\\1\end{bmatrix} = \begin{bmatrix}0.166\\2.336\\8.003\end{bmatrix} = 8.003\begin{vmatrix}0.021\\0.292\\1\end{vmatrix} = 8.003x_2$$

$$AX_3 = \begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.021\\0.292\\1\end{bmatrix} = \begin{bmatrix}1.145\\0.252\\6.002\end{bmatrix} = 6.002\begin{bmatrix}0.191\\0.042\\1\end{bmatrix} = 6.002X_4$$

$$AX_4 = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.191\\0.042\\1\end{bmatrix} = \begin{bmatrix}2.065\\-0.068\\6.272\end{bmatrix} = 6.272\begin{bmatrix}0.329\\-0.011\\1\end{bmatrix} = 6.272X_5$$

$$AX_5 = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.329\\-0.011\\1\end{bmatrix} = \begin{bmatrix}2.362\\0.272\\6.941\end{bmatrix} = 6.941\begin{bmatrix}0.34\\0.039\\1\end{bmatrix} = 6.941X_6$$

$$AX_6 = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.34\\0.039\\1\end{bmatrix} = \begin{bmatrix}2.223\\0.516\\7.157\end{bmatrix} = 7.157\begin{bmatrix}0.311\\0.072\\1\end{bmatrix} = 7.157X_7$$

$$AX_7 = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.311\\0.072\\1\end{bmatrix} = \begin{bmatrix}2.095\\0.532\\7.082\end{bmatrix} = 7.082\begin{bmatrix}0.296\\0.075\\1\end{bmatrix} = 7.082X_8$$

$$AX_8 = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.296\\0.075\\1\end{bmatrix} = \begin{bmatrix}2.071\\0.484\\7.001\end{bmatrix} = 7.001\begin{bmatrix}0.296\\0.069\\1\end{bmatrix} = 7.001X_9$$

$$AX_9 = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.296\\0.069\\1\end{bmatrix} = \begin{bmatrix}2.089\\0.46\\6.983\end{bmatrix} = 6.983\begin{bmatrix}0.296\\0.066\\1\end{bmatrix} = 6.983X_{10}$$

$$AX_{10} = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.296\\0.066\\1\end{bmatrix} = \begin{bmatrix}2.101\\0.46\\6.992\end{bmatrix} = 6.992\begin{bmatrix}0.3\\0.066\\1\end{bmatrix} = 6.992X_{11}$$

$$AX_{11} = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.3\\0.066\\1\end{bmatrix} = \begin{bmatrix}2.102\\0.464\\6.998\end{bmatrix} = 6.998\begin{bmatrix}0.3\\0.066\\1\end{bmatrix} = 6.998X_{12}$$

$$AX_{12} = \begin{bmatrix}1 & -3 & 2\\4 & 4 & -1\\6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.3\\0.066\\1\end{bmatrix} = \begin{bmatrix}0.3\\0.066\\1\end{bmatrix} = 6.998\begin{bmatrix}0.3\\0.066\\1\end{bmatrix} = 6.998X_{13}$$

The dominant eigen value is 7and the corresponding eigen vector is $\begin{bmatrix}0.3\\0.066\\1\end{bmatrix}$

b.(i) Using Gauss Jordon method, find the inverse of A = $\begin{bmatrix}1 & 1 & 3\\1 & 3 & -3\\-2 & -4 & -4\end{bmatrix}$

Solution:

Let $[A, I] = \left(\begin{array}{ccc|ccc}1 & 1 & 3 & 1 & 0 & 0\\1 & 3 & -3 & 0 & 1 & 0\\-2 & -4 & -4 & 0 & 0 & 1\end{array}\right)$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right) \quad \begin{array}{l} R_2 \Leftrightarrow R_2 - R_1 \\ R_3 \Leftrightarrow R_3 + 2R_1 \end{array}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -2 & 2 & 1 & 1 & 1 \end{array}\right) \quad R_2 \Leftrightarrow \frac{R_2}{2}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{2} & \frac{-1}{2} & 0 \\ 0 & 1 & -3 & \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right) \quad \begin{array}{l} R_1 \Leftrightarrow R_1 - R_2 \\ R_3 \Leftrightarrow R_3 + 2R_2 \end{array}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{2} & \frac{-1}{2} & 0 \\ 0 & 1 & -3 & \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \end{array}\right) \quad R_3 \Leftrightarrow \frac{-R_3}{4}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & \frac{3}{2} \\ 0 & 1 & 0 & -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right) \quad \begin{array}{l} R_1 \Leftrightarrow R_1 - 6R_3 \\ R_2 \Leftrightarrow R_2 + 3R_3 \end{array}$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & \frac{3}{2} \\ -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 12 & 4 & 6 \\ -5 & -1 & -3 \\ -1 & -1 & -1 \end{bmatrix}$$

(ii). Solve the following system of equations using Gauss-Seidal iterative method
$27x + 6y - z = 85$, $6x + 15y + 2z = 72$, $x + y + 54z = 110$.
Solution:

Write x, y, z as follows:

$$x = \frac{1}{27}[85 - 6y + z] \qquad y = \frac{1}{15}[72 - 6x - 2z] \qquad z = \frac{1}{54}[110 - x - y]$$

Let the initial values be y = 0, z = 0

First Iteration

$$x^1 = \frac{1}{27}[85 - 6y^0 + z^0] = \frac{1}{27}[85 - 6(0) + 0] = 3.148$$

$y^1 = \frac{1}{15}[72 - 6x^1 - 2z^0] = \frac{1}{15}[72 - 6(3.148) - 0] = 3.541$

$z^1 = \frac{1}{54}[110 - x^1 - y^1] = \frac{1}{54}[110 - 3.148 - 3.541] = 1.913$

Second iteration

$x^2 = \frac{1}{27}[85 - 6y^1 + z^1] = \frac{1}{27}[85 - 6(3.541) + 1.913] = 2.432$

$y^2 = \frac{1}{15}[72 - 6x^2 - 2z^1] = \frac{1}{15}[72 - 6(2.432) - 2(1.913)] = 3.572$

$z^2 = \frac{1}{54}[110 - x^2 - y^2] = \frac{1}{54}[110 - 2.432 - 3.572] = 1.926$

Third iteration

$x^3 = \frac{1}{27}[85 - 6y^2 + z^2] = \frac{1}{27}[85 - 6(3.572)+1.926 =2.426$

$y^3 = \frac{1}{15}[72 - 6x^3 - 2z^2] = \frac{1}{15}[72 - 6(2.426) - 2(1.926) =3.573$

$z^3 = \frac{1}{54}[110 - x^3 - y^3] = \frac{1}{54}[110 - 2.426 - 3.573] = 2.426$

Fourth iteration

$x^4 = \frac{1}{27}[85 - 6y^3 + z^3] = \frac{1}{27}[85 - 6(3.573) - 1.926] = 2.426$

$y^4 = \frac{1}{15}[72 - 6x^4 - 2z^3] = \frac{1}{15}[72 - 6(2.426) - 2(10926)= 3.573$

$z^4 = \frac{1}{54}[110 - x^4 - y^4] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$

Hence $x = 2.426$, $y = 3.573$, $z = 1.926$

14.a(i) Using Lagrange's interpolation, find the value of $f(3)$, from the following table:

x	0	1	2	5
f(x)	2	3	12	147

Solution:

By Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$f(3) = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3)$$

$$+ \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$f(3) = \frac{(x-1)(x-2)(x-5)}{-10}(2) + \frac{(x-0)(x-2)(x-5)}{4}(3)$$

$$+\frac{(x-0)(x-1)(x-5)}{-6}(12)+\frac{(x-0)(x-1)(x-2)}{60}$$

$$f(3)=\frac{(3-1)(3-2)(3-5)}{-10}(2)+\frac{(3)(3-2)(3-5)}{4}(3)$$

$$+\frac{3(3-1)(3-5)}{-6}(12)+\frac{(3)(3-1)(3-2)}{60}(147)$$

f(3) = $\frac{4}{10}(2)+\frac{6}{4}(3)+2(12)+\frac{1}{10}(147)$

f(3) = $\frac{8}{10}-\frac{18}{4}+24+\frac{147}{10} = 35$

(ii). Evaluate $\int_0^2 \frac{dx}{x^2+x+1}$ to three decimals, dividing the range of integration into 8 equal parts using Simpson's rule.

Solution:

Let $y=f(x)=\frac{1}{1+x\ +x^2}$, here in the interval (0,2)

Now the values of y are given below:

X	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Y	1	0.7619	0.5714	0.4324	0.333	0.2622	0.2105	0.1720	0.1428

I = $\frac{h}{3}[(y_0+4y_1+y_2)+(y_2+4y_3+y_4)+(y_4+4y_5+y_6)+(y_6+4y_7+y_8)]$

= $\frac{0.25}{3}[4.619+2.6343+1.5926+1.0413] = 0.8239$

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b. (i) Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence evaluate $f(x)$ at $x = 5$.

X	4	6	8	10
f(x)	1	3	8	16

Solution:

We form the difference table

X	y	Δy	$\Delta^2 y$	$\Delta^3 y$	
$(x_o)4$	$(y_0)1$				
		3-1 = 2 Δy_0			
$(x_1)6$	$(y_1)3$		5-2 = 3 $\Delta^2 y_0$		
		8-3 = 5 Δy_1		-3-3 = - 6 $\Delta^3 y_0$	
$(x_2)8$	$(y_2)8$				
		10-8 = 2 Δy_2			
$(x_3)10$	$(y_3)10$				

There are only 4 data given. Hence the polynomial will be degree 3.

$$y(x) = P_3(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

Where u $= \frac{x - x_0}{h}$

Here, $x_0 = 4, \quad h = 6 - 4 = 2 \quad (difference)$

$$y(x) = P_3(x) = 1 + \frac{\frac{(x-4)}{2}}{1!}(2) + \frac{\frac{(x-4)}{2}\frac{(x-6)}{2}}{2!}(3) + \frac{\frac{(x-4)}{2}\frac{(x-6)}{2}\frac{(x-8)}{2}}{3!}(-6)$$

$$= 1 + x - 4 + \frac{(x-4)(x-6)}{8}(3) + \frac{(x-4)(x-6)(x-8)}{(8)(6)}(-6)$$

$$= x - 3 + \frac{3}{8}(x-4)(x-6) - \frac{1}{8}(x-4)(x-6)(x-8)$$

$$= x - 3 + \frac{3}{8}(x^2 - 10x + 24) - \frac{1}{8}(x^2 - 10x + 24)(x-8)$$

$$= x - 3 + \frac{3}{8}(x^2 - 10x + 24) - \frac{1}{8}(x^3 - 10x^2 + 24x - 8x^2 + 80x - 192)$$

$$= \frac{1}{8}[(8x - 24 + 3x^2 - 30x + 72) - x^3 + 10x^2 - 24x + 8x^2 - 80x + 192$$

$$= \frac{1}{8}[- x^3 + 21x^2 - 126x + 240]$$

$$y(5) = \frac{1}{8}[-5^3 + 21(5)^2 - 126(5) + 240]$$

$$= \frac{1}{8}[-125 + 21(25) - 630 + 240] = \frac{1}{8}[10] = 1.25$$

(ii). Compute $f'(0)$ and $f''(4)$ from the following data:

X	0	1	2	3	4

f(x)	1	2.718	7.381	20.086	54.598

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		1.718			
1	2.718		2.945		
		4.663		5.097	
2	7.381		8.042		8.668
		12.705		13.765	
3	20.086		21.807		
		34.512			
4	54.598				

By Newton's forward formula we have

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - ......\right]$$

$$= \frac{1}{1}\left[1.718 - \frac{2.945}{2} + \frac{5.097}{3} - \frac{8.668}{4} ......\right]$$

$$= 1.718 - 1.4725 + 1.699 - 2.167$$

$$= -0.2225$$

By Newton's backward formula we have

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + ...\right]$$

$$= \ = \frac{1}{1}\left[334.512 + \frac{21.807}{2} + \frac{13.765}{3} + \frac{8.668}{4} ......\right]$$

$$= 34.512 + 10.9035 + 4.588 + 2.167$$

$$= 52.1705$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{2}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + ...\right]$$

$$= \frac{1}{1}\left[21.807 + 13.765 + \frac{11}{12}(8.668) ......\right]$$

$$= 21.807 + 13.765 + 7.9457$$

$$= 43.5177$$

15.a(i) Consider the initial value problem $\frac{dy}{dx} = y - x^2 + 1 \quad y(0) = 0.5$.. Using the modified Euler method find $y(0.2)$.

Solution:

f(x, y) = $y - x^2+1$, $x_0 = 0, y_0 = 0.5, h = 0.2, x_1 = 0.2$

By modified Euler Method

$$y_{n+1} = y_n + hf[x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)]$$

$$y_1 = y_0 + hf[x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)]$$

$$= 0.5 + (0.2)\, f\left[0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2} f(0,0.5)\right]$$

$$= 0.5 + (0.2)\, f[0.1, 0.5+(0.1)[0.5-0+1]$$

$$= 0.5 + (0.2) f[0.1, 0.5 + (0.1)[1.5]]$$

$$= 0.5 + (0.2)\, f[0.1, 0.5 + 0.15] \quad = 0.5 + (0.2) f[0.1, 0.65]$$

$$= 0.5 + (0.2)[0.65 - 0.1^2 + 1] \quad = 0.5 + (0.2)[0.65 - 0.01 + 1]$$

$$= 0.5 + 0.2(1.64) \quad = 0.5 + 0.328$$

$$= 0.828$$

$$\boldsymbol{y(0.2) = 0.828}$$

(ii). Using Milne's method find $y(4.4)$ given
$5xy' + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$.

Solution:

$$y' = \frac{2-y^2}{5x}$$

$$x_0 = 4, x_1 = 4.1, x_2 = 4.2, x_3 = 4.3, \ x_4 = 4.4,$$

$$y_0 = 1, \ y_1 = 1.0049, \ y_2 = 1.0097, \ y_3 = 1.0143$$

$$y_1' = \frac{2-y_1^2}{5x_1} = \frac{2-1.0049^2}{5(4.1)} = 0.0493$$

$$y_2' = \frac{2-y_2^2}{5(4.2)} = \frac{2-1.0097^2}{5(4.2)} = 0.0467$$

$$y_3' = \frac{2-y_3^2}{5x_3} = \frac{2-1.0143^2}{5(4.3)} = 0.0452$$

By Milne's Predictor formula

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}[2y_{n-2}' - y_{n-1}' + 2y_n']$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}[y_{n-1}' + 4y_n' + y_{n+1}']$$

$$y_{4,p} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$$

$$= 1 + \frac{4(0.1)}{3}[2(0.0493) - 0.0467 + 2(0.0452)] = 1.01897$$

$$y_{4,c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + 2y_4']$$

$$y_4' = \frac{2 - y_4^2}{5(x_4)} = \frac{2 - (1.01897)^2}{5(4.4)} = 0.0437$$

$$y_{4,c} = 1.0097 + \frac{0.1}{3}[0.0467 + 4(0.0452) + 0.0437] = 1.01874$$

b.(i) Find $y(0.8)$ given that $y' = y - x^2$, $y(0.6) = 1.7379$ by using R-K method of order 4, taking $h = 0.1$.

Solution:

Given $y' = f(x,y) = y - x^2$, $x_0 = 0.6$, $y_0 = 1.7379$, $x_1 = 0.7$, $x_2 = 0.8$, h = 0.1

<u>To find y(0.7)</u> :

$$k_1 = h\,f(x_0, y_0) = (0.1)\left(y_0 - x_0^2\right) = (0.1)(1.7379 - (0.6)^2)) = 0.13779$$

$$k_2 = h\,f\left[x_0 + \frac{h}{2}, \; y_0 + \frac{k_1}{2}\right] = (0.1)\; f\left[0.6 + \frac{0.1}{2}, \; 1.7379 + \frac{0.13779}{2}\right]$$

$$= (0.1)\; f\,[0.65, 1.806795]$$

$$= (0.1)\left[1.806795 - (0.65)^2\right] = 0.13843$$

$$k_3 = h\; f\left[x_0 + \frac{h}{2}, \; y_0 + \frac{k_2}{2}\right]$$

$$= (0.1)\; f\left[0.65, \; 1.7379 + \frac{0.13843}{2}\right]$$

$$= (0.1)\; f\,[0.65, \; 1.807115]$$

$$= (0.1)\left[1.807115 - (0.65)^2\right] = 0.13846$$

$$k_4 = h f(x_0 + h \;,\; y_0 + k_3)$$
$$= (0.1) f [0.6 + 0.1 , 1.7379 + 0.13846]$$
$$= (0.1) f [0.7 , 1.87636]$$
$$= (0.1)\left[1.87636 - (0.7)^2\right] = 0.13864$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$
$$= \frac{1}{6}[0.13779 + 2(0.13843) + 2(0.13846) + 0.13864]$$
$$= 0.13837$$
$$y_1 = y_0 + \Delta y = 1.7379 + 0.13837 = 1.87627$$

<u>To find y(0.8)</u> :

$$k_1 = h f(x_1 , y_1) = (0.1) f [0.7 , 1.876]$$
$$= (0.1)\left[1.876 - (0.7)^2\right] = 0.1386$$
$$k_2 = h f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right] = (0.1) f\left[0.7 + \frac{0.1}{2} , 1.876 + \frac{0.1386}{2}\right]$$
$$= (0.1) f [0.75 , 1.9453]$$
$$= (0.1)\left[1.9543 - (0.75)^2\right] = 0.13828$$
$$k_3 = h f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right]$$
$$= (0.1) f\left[0.7 + \frac{0.1}{2} , 1.876 + \frac{0.13828}{2}\right]$$
$$= (0.1) f [0.75 , 1.94514]$$
$$= (0.1)\left[1.94514 - (0.75)^2\right] = 0.138264$$
$$k_4 = h f(x_1 + h , y_1 + k_3)$$
$$= (0.1) f [0.7 + 0.1 , 1.876 + 0.138264]$$
$$(0.1) f [0.8 , 2.014264]$$
$$= (0.1)\left[2.014264 - (0.8)^2\right]$$
$$= 0.1374264$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$
$$= \frac{1}{6}[0.1386 + 2(0.13828) + 2(0.138264) + 0.1374264]$$
$$= 0.138186$$
$$y_2 = y_1 + \Delta y = 1.876 + 0.138186 = 2.014186$$

(ii) Solve the BVP $\frac{d^2y}{dx^2} - y = 0, y(0) = 0, y(1) = 1$ using finite difference method with $h = 0.2$.

Solution:

$y'' - y = 0 \quad , \quad h = 0.2$

$x_0 = 0 \quad , \quad x_n = 1$

wkt $y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \quad , \quad i = 1, 2, 3, ...$

$\therefore$ the equation becomes $\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - y_i = 0$

$\Rightarrow \quad y_{i+1} - 2y_i + y_{i-1} + h^2 y_i = 0$

$\Rightarrow \quad y_{i+1} - 2y_i + y_{i-1} - \frac{1}{25} y_i = 0$

$\Rightarrow \quad 25y_{i+1} - 50y_i + 25y_{i-1} - y_i = 0 \quad , \quad i = 1, 2, 3, ...$

$\Rightarrow \quad 25y_{i+1} - 51y_i + 25y_{i-1} = 0$

put i=1 then $25y_2 - 51y_1 + 25y_0 = 0 \quad ------(1)$

put i=2 then $25y_3 - 51y_2 + 25y_1 = 0 \quad ------(2)$

put i=3 then $25y_4 - 51y_3 + 25y_2 = 0 \quad ------(3)$

put i=4 then $25y_3 - 51y_4 + 25y_3 = 0 \quad ------(4)$

Given

$x_0 = 0 \; , \; x_1 = 0.2 \; , \; x_2 = 0.4 \; , \; x_3 = 0.6 \; , \; x_4 = 0.8 \; , \; x_5 = 1$

$y_0 = 1 \; , \; y_5 = 1$

(1) $\Rightarrow \quad 25y_2 - 51y_1 = 0$

(2) $\Rightarrow \quad 25y_1 - 51y_2 + 25y_3 = 0$

(3) $\Rightarrow \quad 25y_2 - 51y_3 + 25y_4 = 0$

(4) $\Rightarrow \quad 25y_3 - 51y_4 = 0$

Solve by using Gauss elimination method

$$[A, B] = \left[\begin{array}{cccc|c} -51 & 25 & 0 & 0 & 0 \\ 25 & -51 & 25 & 0 & 0 \\ 0 & 25 & -51 & 25 & 0 \\ 0 & 0 & 25 & -51 & -25 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} -51 & 25 & 0 & 0 & 0 \\ 0 & -1976 & 1275 & 0 & 0 \\ 0 & 25 & -51 & 25 & 0 \\ 0 & 0 & 25 & -51 & -25 \end{array}\right] \quad R_2 \Leftrightarrow 51R_2 + 25R_1$$

$$= \left[\begin{array}{cccc|c} -51 & 25 & 0 & 0 & 0 \\ 0 & -1976 & 1275 & 0 & 0 \\ 0 & 0 & -68901 & 49400 & 0 \\ 0 & 0 & 25 & -51 & -25 \end{array}\right] \quad R_3 \Leftrightarrow 1976R_3 + 25R_2$$

$$= \left[\begin{array}{cccc|c} -51 & 25 & 0 & 0 & 0 \\ 0 & -1976 & 1275 & 0 & 0 \\ 0 & 0 & -68901 & 49400 & 0 \\ 0 & 0 & 0 & -2278951 & -1722525 \end{array}\right] \quad R_4 \Leftrightarrow 68901R_4 + 25R_3$$

$-2278951\, y_4 = -1722525 \quad \Rightarrow \quad y_4 = 0.7558$

$-68901\, y_3 + 49400\, y_4 = 0 \quad \Rightarrow \quad -68901\, y_3 = -(49400)0.7558)$

$\Rightarrow \quad y_3 = 0.5419$

$-1976\, y_2 + 1275\, y_3 = 0 \quad \Rightarrow \quad -1976\, y_2 = -1275(0.5419)$

$\Rightarrow \quad y_2 = 0.3497$

$-51\, y_1 + 25\, y_2 = 0 \quad \Rightarrow \quad -51\, y_1 = -25(0.3497)$

$\Rightarrow \quad y_1 = 0.1714$

Agni College of Technology

Chennai - 130

B.E./ B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA6452 –STATISTICS AND NUMERICAL METHODS
(Regulation 2013)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. State level of significance.

Solution:

The probability that the value of the statistic lies in the critical region is called the level of significance.

In general, these levels are chosen as 0.01 or 0.05, called 1% level and 5% level of significance respectively.

2. Define Type I and Type II error.

Solution:

Type-I Error: The rejection of the null hypothesis H_o when it is true is called Type-I Error.

Type-II Error: The acceptance of the null hypothesis H_o when it is called Type II error.

3. What is the aim of design of experiments?

Solution:

The design of experiment may be defined as the logical construction of the experiment in which the degree of uncertainty with which the inference is drawn may be well defined.

4. Define 2^2 factorial design.

Solution:

When there are two factors A,B and two levels 'high' & 'low' for each factor we have a 2^2 factorial design. In spite of its simplicity, the 2^2 design is a powerful tool to improve products and process.

5. State the principle used in Gauss-Jordan method.

Solution:

Coefficient matrix is transformed into diagonal matrix.

6. Solve $x+y+z=6$, $3x+3y+4x=20$, $2x+y+3z=13$ using Gauss elimination method.

Solution:

Given

$$x+y+z=6$$
$$2x+y+3z=13$$
$$3x+3y+4z=20$$

The given system is equivalent to

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \\ 20 \end{bmatrix}$$

$i.e., \quad AX = B$

$$Here, [A, B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & 3 & 13 \\ 3 & 3 & 4 & 20 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \qquad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

This is an upper triangular matrix

Here, $z = 2$ ....(1)

$-y + z = 1$ ....(2)

$x + y + z = 6$ ....(3)

$(2) \Rightarrow \; -y = 1 - z \quad \Rightarrow \quad -y = 1 - 2$

$-y = -1 \quad \Rightarrow \quad y = 1$

$(3) \Rightarrow x + 1 + 2 = 6$

$x = 6 - 3 \quad \Rightarrow \quad x = 3$

7. State the use of Lagrange's interpolation form.

Solution:

The forward and backward interpolation formulae of Newton can be used be only when the values of the independent variable x are equally spaced, can also be used when the differences of the dependent variable y become smaller ultimately. But Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced or not and whether the difference of y become smaller or not.

8. Evaluate $\int_1^2 \frac{dx}{1+x^2}$, using Trapezoidal rule taking h = 0.5

Solution: Here, $y(x) = \frac{1}{1+x^2}$, $h = \frac{1}{2} = 0.5$

x	1	1.5	2
y	0.5	0.3077	0.2

By Trapezoidal rule

$$\int_1^2 \frac{dx}{1+x^2} = \frac{h}{2}[\text{sum of four corners} + 2(\text{sum of remaining vales})]$$

$$= \frac{0.5}{2}[0.5 + .02 + 2(0.3077)]$$

$$= 0.25[0.7 + 0.6154] = 0.25[1.3154] = 0.3289$$

9. Using Taylor's method find y at x = 1.3 given $\frac{dy}{dx} = x^3 + y, y(1) = 1.$

Solution:

$Given: y' = x^3 + y,\ x_0 = 1,\ y_0 = 1$

$y' = x^3 + y \qquad y_0' = (x_0')^3 + y_0 = 1 + 1 = 2$

$y'' = 3x^2 + y' \qquad y_0'' = 3x_0^2 + y_0' = 3(1) + 2 = 5$

$y''' = 6x + y'' \qquad y_0''' = 6x_0 + y'' = 6(1) + 5 = 11$

Taylor's series formula

$$y = y_0 + \frac{x - x_0}{1!} y_0' + \frac{(x - x_0)^2}{2!} y_0'' + \frac{(x - x_0)^3}{3!} y_0''' + ...$$

$$= 1 + 2(x-1) + \frac{5}{2}(x-1)^2 + \frac{11}{6}(x-1)^3$$

$$y(1.1) = 1 + 2(1.1-1) + \frac{5}{2}(1.1-1)^2 + \frac{11}{6}(1.1-1)^3$$

$$= 1 + 2(0.1) + \frac{5}{2}(0.1)^2 + \frac{11}{6}(0.1)^3 \quad = 1 + 0.2 + 0.025 + 0.0018 = 1.2268$$

10. Obtain the finite difference scheme for differential equation $2\frac{d^2y}{dx^2} + y = 5.$

Solution:

$$2y'' + y = 5$$

$$2\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + y_i = 5$$

$$2y_{i+1} - 4y_i + 2y_{i-1} + h^2 y_i = 5h^2$$

$$2y_{i+1} + (h^2 - 4)y_i + 2y_{i-1} = 5h^2$$

PART – B (5 x 15 = 60)

11 a(i) Time taken by workers in performing a job are given below:

Type I :	21	17	27	28	24	23	--
Type II :	28	34	43	36	33	35	39

Test whether there is any significant difference between the variance of time distribution.

Solution:

$Here, n_1 = 6, \& n_2 = 7$

								Total
X_1 :	21	17	27	28	24	23	--	140
X_1^2 :	441	289	729	784	576	529	--	3348
X_2 :	28	34	43	36	33	35	39	248

X_2^2:	784	1156	1849	1296	1089	1225	1521	8920

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{140}{6} = 23.3$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{248}{7} = 35.43$$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - (\bar{x}_1)^2 = \frac{3348}{6} - (23.3)^2 = 15.11$$

$$s_2^2 = \frac{\sum x_2^2}{n_2} - (\bar{x}_2)^2 = \frac{8920}{7} - (35.43)^2 = 19$$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(6)(15.11)}{6-1} = \frac{90.66}{5} = 18.132$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{(7)(19)}{7-1} = \frac{133}{6} = 22.17$$

Here, $S_2^2 > S_1^2$

1. The parameter of interest is σ_1^2 *and* σ_2^2
2. $H_0 : \sigma_1^2 = \sigma_2^2$ [The difference of variance is not significant]
3. $H_1 : \sigma_1^2 \neq \sigma_2^2$
4. $\alpha = 0.05, d.f(v_1) = 5, d.f(v_2) = 6$
5. $F = \frac{S_2^2}{S_1^2}$
6. Reject H_0 if $F > 3.29$ [From F table]
7. Computation:

$$F = \frac{S_2^2}{S_1^2} = \frac{22.17}{18.132}$$

8. Conclusion: $F = 1.22 < 3.29$, we accept H_0 at 5% level of significance.

(ii) 20 people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease is 85 % is favour of the hypothesis that is more at 5% level ?

Solution:

Out of Syllabus.

b(i) Before an increase in excise duty on tea, 900 persons out of a sample 0f 1100 persons were found to be tea drinkers. After an increase in excise duty, 900 person were tea drinkers in a sample of 1300. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty?.

Solution:

Out of Syllabus.

(ii) The sales manager of a large company conducted a sample survey in two places A and B taking 200 samples in each case. The results were the following table. Test whether the average sales in the same in the two areas at 5% level.

Solution:

	Place A	Place B
Average sales	Rs.2,000	Rs.1,700
S.D	Rs.200	Rs.450

$$Given: \quad n_1 = 200, \quad \bar{x}_1 = 2000, \quad s_1 = 200$$

$$n_2 = 200, \quad \bar{x}_2 = 1700, \quad s_2 = 450$$

1. $H_0 : \mu_1 = \mu_2$

2. $H_1 : \mu_1 \neq \mu_2$ [Use two – tailed test]

3. $\alpha = 5\%$

5. The test statistic

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{2000 - 1700}{\sqrt{\frac{(200)^2}{200} + \frac{(450)^2}{200}}} = \frac{300}{34.8} = 8.62$$

6. Conclusion :

If $-Z_{\alpha/2} < Z < Z_{\alpha/2}$, then accept H_0; otherwise, we reject H_0.

Here, $-.196 < 8.62 < 1.96$

So, we reject H_0

12 . a.(i) Analyse the following RBD and find your conclusion.

		Treatments			
		T_1	T_2	T_3	T_4
	B_1	12	14	20	22
	B_2	17	27	19	15
Blocks	B_3	15	14	17	12
	B_4	18	16	22	12
	B_5	19	15	20	14

Solution:

1. H_0 : There is no significant difference between blocks and treatments.

2. H_1 : There is significant difference between blocks and treatments.

Subtract 15 from each number.

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	-3	-1	5	7	8	9	1	25	49
Y_2	2	12	4	0	18	4	144	16	0
Y_3	0	-1	2	-3	-2	0	1	4	9
Y_4	3	1	7	-3	8	9	1	49	9
Y_5	4	0	5	-1	8	16	0	25	1

Total	6	11	23	0	40	38	147	119	68

$step\,1 : N = 20$

$step\,2 : T = 40$

$$step\,3 : \frac{T^2}{N} = \frac{(40)^2}{20} = 80$$

$$step\,4 : TSS = \sum X_1^{\,2} + \sum X_2^{\,2} + \sum X_3^{\,2} + \sum X_4^{\,2} - \frac{T^2}{N}$$

$$= 38 + 147 + 119 + 68 - 80 = 292$$

$$step\,5 : SSC = \frac{\left(\sum X_1\right)^2}{N_1} + \frac{\left(\sum X_2\right)^2}{N_1} + \frac{\left(\sum X_3\right)^2}{N_1} + \frac{\left(\sum X_4\right)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{(6)^2}{5} + \frac{(11)^2}{5} + \frac{(23)^2}{5} - 0 - 80 = 57.2$$

$$step\,6 : SSR = \frac{\left(\sum Y_1\right)^2}{N_2} + \frac{\left(\sum Y_2\right)^2}{N_2} + \frac{\left(\sum Y_3\right)^2}{N_2} + \frac{\left(\sum Y_4\right)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{8^2}{4} + \frac{18^2}{4} + \frac{(-2)^2}{4} + \frac{8^2}{4} + \frac{8^2}{4} - 80 = 50$$

$$SSE = TSS - SSC - SSR$$

$$= 292 - 57.2 - 50 = 184.8$$

Step 7: ANOVA Table

Source of variance	Sum of squares	d.f	Mean square	Variance ratio	Table value at 5% level
Between rows	SSR = 50	r –1 =5 – 1 =4	$MSR = \frac{SSR}{r-1} = \frac{57.2}{3} = 19.1$	$F_R = \frac{MSR}{MSE} = \frac{19.1}{15.4} = 1.24$	$F_R(3,12) = 8.74$
Between column	SSC = 57.2	C-1 = 4 – 1 = 3	$MSC = \frac{SSC}{C-1} = \frac{50}{4} = 12.5$	$F_C = \frac{MSC}{MSE} = 1\frac{12.5}{15.4} = 1.23$	$F_C(12,4) = 3.26$
Residual	SSE = 184.8	N – C – r +1 = 20 - 4 -+1 =12	$MSE = \frac{SSE}{N-C-r+1} = \frac{184.8}{12} = 15.4$		
Total	292				

Step 8: Conclusion: Cal F_C < Table F_C , so accept H_0
Cal F_R < Table F_R , so accept H_0

b. The following is a Latin square of a design when 4 varieties of seed being tested. Set up the analysis of variance table and state your conclusion. You can carry out the suitable charge of origin and scale.

A 110	B 100	C 130	D 120
C 120	D 130	A 110	B 110
D 120	C 100	B 110	A 120
B 100	A 140	D 100	C 120

Solution:

Subtract by 5 we get

A 105	B 95	C 125	D 115
C 115	D 125	A 105	B 105
D 115	C 95	B 105	A 115
B 95	A 135	D 95	C 115

Subtract 100 and then divided by 5 we get

A	1	B	-1	C	5	D	3
C	3	D	5	A	1	B	1
D	3	C	-1	B	1	A	3
B	-1	A	7	D	-1	C	3

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	1	-1	5	3	8	1	1	25	9
Y_2	3	5	1	1	10	9	25	1	1
Y_3	3	-1	1	3	6	9	1	1	9
Y_4	-1	7	-1	3	8	1	49	1	9
Total	6	10	6	10	32	20	76	28	28

H_0 : There is no significant difference between rows, columns and treatments.
H_1: There is significant difference between rows, columns and treatments.

$$\text{Step 1: N} = 16$$

$$\text{Step 2: T} = 32$$

$$\text{Sept 3: } \frac{T^2}{N} = \frac{(32)^2}{16} = 64$$

$$\text{step 4: TSS} = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$$

$$= 20 + 76 + 28 + 28 - 64 = 88$$

$$step\, 5: SSC = \frac{\left(\sum X_1\right)^2}{N_1} + \frac{\left(\sum X_2\right)^2}{N_1} + \frac{\left(\sum X_3\right)^2}{N_1} + \frac{\left(\sum X_4\right)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{(6)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(10)^2}{4} - 64$$

$$= 9 + 25 + 9 + 25 - 64 = 4$$

$$step\,6: SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{8^2}{4} + \frac{10^2}{4} + \frac{(6)^2}{4} + \frac{8^2}{4} - 64 \quad = 16 + 25 + 9 + 16 - 64 = 2$$

To find SSK :

Arrange the elements in the order of treatment.

A	1	1	3	7	12
B	-1	1	1	-1	0
C	5	3	-1	3	10
D	3	5	3	-1	10

$$SSK = \frac{(12)^2}{4} + \frac{0^2}{4} + \frac{(10)^2}{4} + \frac{(10)^2}{4} - \frac{T^2}{N}$$

$$= 36 + 0 + 25 + 25 - 64 = 22$$

$$SSE = TSS - SSC - SSR - SSK$$

$$= 88 - 4 - 2 - 22 = 60$$

Step 7: ANOVA Table

Source of variance	Sum of square	d.f	Mean square	Variance ratio	Table value at 5% level
Between rows	SSR = 2	k – 1 = 3	$MSR = \frac{SSR}{k-1} = 0.67$	$F_R = \frac{MSE}{MSR} = \frac{10}{0.67} = 14.9$	$F_R(6,3) = 8.94$
Between columns	SSC = 4	k–1 = 3	$MSC = \frac{SSC}{k-1} = 1.33$	$F_C = \frac{MSE}{MSC} = \frac{10}{1.33} = 7.52$	$F_C(6,3) = 8.94$
Between Treatments	SSK = 22	k–1 = 3	$MSK = \frac{MSK}{K-1} = 7.33$	$F_T = \frac{MSE}{MSK} = \frac{10}{7.33} = 1.36$	$F_T(6,3) = 8.94$
Residual	SSE = 60	(k-1)(k–2) = 6	$MSE = \frac{SSE}{(K-1)(K-2)} = 10$		
Total	88	15			

Step 8: Conclusion: Cal F_C < Table F_C
Cal F_R > Table F_R
Cal F_T < Table F_T

There is a significant difference between rows as well as between columns. But, there is no significant difference between treatments.

13 a(i) Solve the system of equations by Gauss-Elimination method.

$$x_1 + x_2 + x_3 + x_4 = 1$$
$$2x_1 - x_2 + 2x_3 - x_4 = -5$$
$$3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$$
$$x_1 - 2x_2 - 3x_3 + 2x_4 = 5$$

Solution:

$$[A,B] = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & -1 & 2 & -1 & -5 \\ 3 & 2 & 3 & 4 & 7 \\ 1 & -2 & -3 & 2 & 5 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -3 & 0 & -3 & -9 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & -3 & -4 & 1 & 3 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -3 & 0 & -3 & -9 \\ 0 & 0 & 0 & 6 & 12 \\ 0 & 0 & -4 & 4 & 12 \end{array}\right] \begin{array}{l} R_3 \to 3R_3 - R_2 \\ R_4 \to R_4 - R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] R_3 \to \frac{R_3}{6}, R_4 \to \frac{R_4}{4} \text{ and interchanging } R_3 \,\&\, R_4] \quad R_2 \to \frac{R_2}{-3}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} R_3 \to R_3 - R_4 \\ R_2 \to R_2 - R_4 \\ R_1 \to R_1 - R_4 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} R_2 \to R_2 \\ R_1 \to R_1 + R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] R_1 \to R_1 - R_2$$

Hence, $x = 0, y = 1, z = -1, w = 2$

(ii) Find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ by Gauss-Jordan method.

Solution: $\quad Let\ A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$$[A, I] = \left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 3 & 2 & -1 & 0 & 2 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to 2R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 5 & 2 & -3 & 2 \end{array}\right] R_3 \to R_3 - 3R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 10 & 5 & 0 & 1 & 6 & -4 \\ 0 & 5 & 0 & -3 & 2 & 0 \\ 0 & 0 & 5 & 2 & -3 & 2 \end{array}\right] \begin{array}{l} R_2 \to 5R_2 + R_3 \\ R_1 \to 5R_1 - 2R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 10 & 0 & 0 & 4 & 4 & -4 \\ 0 & 5 & 0 & -3 & 2 & 0 \\ 0 & 0 & 5 & 2 & -3 & 2 \end{array}\right] R_1 \to R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 5 & 0 & 0 & 2 & 2 & -2 \\ 0 & 5 & 0 & -3 & 2 & 0 \\ 0 & 0 & 5 & 2 & -3 & 2 \end{array}\right] R_1 \to \frac{R_1}{2}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{5} & \frac{2}{5} & -\frac{2}{5} \\ 0 & 1 & 0 & \frac{-3}{5} & \frac{2}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} \end{array}\right]$$

$$Hence,\ A^{-1} = \frac{1}{5}\begin{bmatrix} 2 & 2 & -2 \\ -3 & 2 & 0 \\ 2 & -3 & 2 \end{bmatrix}$$

b(i) Find the largest eigen value of the matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{pmatrix}$ by power method. Also find its corresponding eigenvector.

Solution:

$$\text{Let } A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\text{Let } X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{be an arbitrary initial eigenvector.}$$

$$AX_1 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ -2 \\ 0.5 \end{bmatrix} = 2.5\begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 2.8 \\ -2.6 \\ 0.8 \end{bmatrix} = 2.8\begin{bmatrix} 1 \\ -0.93 \\ 0.29 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -0.93 \\ 0.29 \end{bmatrix} = \begin{bmatrix} 2.93 \\ -2.86 \\ 0.93 \end{bmatrix} = 2.93\begin{bmatrix} 1 \\ -0.98 \\ 0.32 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -0.98 \\ 0.32 \end{bmatrix} = \begin{bmatrix} 2.98 \\ -2.96 \\ 0.98 \end{bmatrix} = 2.98\begin{bmatrix} 1 \\ -0.99 \\ 0.33 \end{bmatrix}$$

$$AX_6 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -0.99 \\ 0.33 \end{bmatrix} = \begin{bmatrix} 2.99 \\ 2.98 \\ 0.99 \end{bmatrix} = 2.99\begin{bmatrix} 1 \\ -1 \\ 0.33 \end{bmatrix}$$

$$AX_7 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0.33 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ -1 \\ 0.33 \end{bmatrix}$$

$$AX_8 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0.33 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ -1 \\ 0.33 \end{bmatrix}$$

Hence, the corresponding eigenvalue $= 3$ and eigenvector is $\begin{bmatrix} 1 \\ -1 \\ 0.33 \end{bmatrix}$

(ii) Find a positive root for the equation $3x - \cos x = 1$ by Newton-Raphson method.

Solution:

Let $f(x) = 3x - \cos x - 1, \quad f'(x) = 3 + \sin x$

$f(0) = 0 - 1 - 1 = -2 = -ve$

$f(1) = 3 - \cos 1 - 1 = 2 - \cos 1 = 1.45968 = +ve$

$\therefore$ a root lies between 0 and 1.

$|f(0)| > |f(1)|$

Hence, the root is nearer to 1.

Let $x_0 = 0.6$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow x_1 = 0.6 - \frac{f(0.6)}{f'(0.6)}$$

$$= 0.6 - \left[\frac{3(0.6) - \cos(0.6) - 1}{3 + \sin(0.6)}\right]$$

$$= 0.6 - (-0.007101) = 0.607108$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.607108 - \left[\frac{3(0.607108) - \cos(0.607108) - 1}{3 + \sin(0.607108)}\right]$$

$$= 0.607108 - (0.000006) = 0.607102$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.607102 - \left[\frac{3(0.607102) - \cos(0.607102) - 1}{3 + \sin(0.607102)}\right]$$

$$= 0.607102 - 0.0000004 = 0.607102$$

Here, $x_2 = x_3 = 0.607102$

$\therefore$ The root is 0.607102 correct to six decimals.

14 a(i) Using Lagrange's method, find the value of f(3) from the following table.

X:	0	1	2	5
Y :	2	3	12	147

Solution:

By Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$f(3) = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) + \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$f(3) = \frac{(x-1)(x-2)(x-5)}{-10}(2) + \frac{(x-0)(x-2)(x-5)}{4}(3) + \frac{(x-0)(x-1)(x-5)}{-6}(12) + \frac{(x-0)(x-1)(x-2)}{60}(147)$$

$$f(3) = \frac{(3-1)(3-2)(3-5)}{-10}(2) + \frac{(3)(3-2)(3-5)}{4}(3) + \frac{3(3-1)(3-5)}{-6}(12) + \frac{(3)(3-1)(3-2)}{60}(147)$$

$$f(3) = \frac{4}{10}(2) + \frac{6}{4}(3) + 2(12) + \frac{1}{10}(147)$$

$$f(3) = \frac{8}{10} - \frac{18}{4} + 24 + \frac{147}{10} = 35$$

(ii) Using Newton's divided difference formula, find the value of f(2) and f(14) from the following table.

X :	4	5	7	10	11	13
Y :	48	100	294	90	1210	2028

Solution:

We form the divided difference table, since the intervals are unequal.

X	↑f(x)	↑ f(x)	↑ f(x)	↑ f(x)	↑ f(x)
4	48				
		$\frac{100-48}{5-4}=52$			
5	100		$\frac{97-52}{7-4}=15$		
		$\frac{294-100}{5-7}=97$		$\frac{21-15}{10-4}=1$	
7	294		$\frac{202-97}{10-5}=21$		0
		$\frac{900-294}{10-7}=202$		$\frac{27-21}{11-5}=1$	
10	900		$\frac{310-202}{11-7}=27$		0
		$\frac{1210-900}{11-10}=310$		$\frac{33-27}{13-7}=1$	
11	1210		$\frac{409-310}{13-10}=33$		
		$\frac{2028-1210}{13-11}=409$			
13	2028				

By Newton's divided difference interpolation formula,

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0,x_1,x_2,x_3) + \ldots$$

$$f(2) = 48 + (-2)(52) + (-2)(-3)(15) + (-2)(-3)(-5)(1)$$

$$= 48 - 104 + 90 - 30 \quad = 4$$

$$f(4) = 48 + (10)(52) + (10)(9)(15) + (10)(9)(8)(1) \quad = 2638$$

b(i) Evaluate $\int_0^1 \int_0^1 \frac{dx\,dy}{1+x+y}$ using Trapezoidal rule, taking h =k = 0.5.

Solution:

x/y	0	0.5	1
0	1	0.6667	0.5
0.5	0.6667	0.5	0.4
1	0.5	0.4	0.3333

$$I = \left(\frac{0.5}{3}\right)\left(\frac{0.5}{3}\right)\left[(1+0.5+0.333+0.5)+4(0.6667+0.6667+0.4+0.4)+16(0.5)\right]$$

$$= (0.0278)(18.8669) = 0.5245$$

(ii) Taking $h = k = \frac{\pi}{4}$, evaluate $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sqrt{\sin(x+y)}\, dx\, dy$ by Simpson's $\frac{1}{3}$ rule

Solution:

Let $I = \int_0^{\pi/2}\int_0^{\pi/2} \sqrt{\sin(x+y)}\, dx\, dy$

$x \backslash y$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
0	0	0.8409	1
$\frac{\pi}{4}$	0.8409	1	0.8409
$\frac{\pi}{2}$	1	0.8409	0

Simpson's rule,

$$I = \left[\frac{\left(\frac{\pi}{4}\right)}{3}\right]\left[\frac{\left(\frac{\pi}{4}\right)}{3}\right]\{(0+1+0+1)+4(0.8409+0.8409+0.8409+0.8409)+16(1)\}$$

$$= (0.0685)(31.4544)$$

$$= 2.1546$$

15 a(i) Using Milne's predictor-corrector method, find y(0.4), given that

$$y' = \frac{(1+x^2)y^2}{2},\ y(0) = 1,\ y(0.1) = 1.06,\ y(0.2) = 1.12,\ y(0.3) = 1.21$$

Solution:

$$x_0 = 0,\ x_1 = 0.1,\quad x_2 = 0.2,\ x_3 = 0.3,\quad x_4 = 0.4$$

$$y_0 = 1,\quad y_1 = 1.06,\ y_2 = 1.12,\ y_3 = 1.21,\ y_4 = ?\ h = 0.1$$

$$y' = f(x,y) = \frac{1}{2}(1+x^2)y^2$$

$$y_0' = \frac{1}{2}(1+x_0^2)y_0^2 = \frac{1}{2}(1+0)1 = \frac{1}{2}$$

$$y_1' = \frac{1}{2}(1+x_1^2)y_1^2 = \frac{1}{2}[1+(0.1)^2][1.06]^2 = 0.5674$$

$$y_2' = \frac{1}{2}(1+x_2^2)y_2^2 = \frac{1}{2}[1+(0.2)^2][1.12]^2 = 0.6522$$

$$y_3' = \frac{1}{2}(1+x_3^2)y_3^2 = \frac{1}{2}[1+(0.3)^2][1.21]^2 = 0.7979$$

Milne's method

$$y_4, p = y_0 + \frac{4h}{3}\left[2y_1^{'} - y_2^{'} + 2y_3^{'}\right]$$

$$= 1 + \frac{4(0.1)}{3}\left[2(0.5674) - 0.6522 + 2(0.7979)\right]$$

$$= 1.2771$$

$$y_4^{'} = \frac{1}{2}\left(1 + x_4^{2}\right)y_4^{2} = \left[1 + 0.16\right]\left[1.2771\right]^2 = 0.9460$$

By corrector method,

$$y_4, c1 = y_2 + \frac{h}{3}\left[y_2^{'} - 4y_3^{'} + y_4^{'}\right]........................(2)$$

$$= 1.12 + \frac{0.1}{3}[0.6522 + 4(0.7979) + 0.9460]$$

Now, use this on the R.H.S of (2) *and we get* $y_4 c1$

$$y'_4 c1 = \frac{1}{2}[1 + 0.16][1.2797]^2 = 0.9498$$

Agani using (2),

$$y_4, c2 = 1.12 + \frac{0.1}{3}\left[0.6522 + 4(0.7979) + 0.9498\right] = 1.2798$$

$$y(0.4) = 1.2798$$

(ii) Solve by Euler's method, the equation $\frac{dy}{dx} = x + y, y(0) = 1,$ choose $h = 0.2$ and compute y(0.4) and y(0.6).

Solution:

$$y^{'} = f(x, y) = x + y, x_0 = 0,\ y_0 = 1\ \ h = 0.2$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y(0.4) = 1 + 0.2 f(0,0) \qquad = 1 + 0.2[0 + 1]$$

$$y(0.4) = 1.2$$

$$y(0.6) = 1.2 + f(x_1, y_1) \qquad = 1.2 + 0.2[0.4 + 1.2]$$

$$= 1.2 + [0.096] = 1.296$$

b(i) Given $y^{'} = x^2 - y,\ y(0) = 1, y(0.1) = 0.9052, y(0.2) = 0.8213,$ find $y(0.3)$ using Taylor's series method.

Solution:

Given $y^{'} = x^2 - y,\ x_0 = 0,\ y_0 = 1,\ x_1 = 0.1, h = 0.1$

$y^{'} = x^2 - y$	$y_0^{'} = x_0^{2} - y_0 = (0) - 1 = -1$
$y^{''} = 2x - y^{'}$	$y_0^{''} = 2x_0 - y_0^{'} = (0) - (-1) = 1$
$y^{'''} = 2 - y^{''}$	$y_0^{'''} = 2 - y_0^{''} = 2 - (1) = 1$
$y^{iv} = -y^{'''}$	$y_0^{iv} = -y_0^{'''} = -1$

By Taylor's series formula

$$y_1 = y_0 + \frac{h}{1!}y_0^{'} + \frac{h^2}{2!}y_0^{''} + \frac{h^3}{3!}y_0^{'''} + \frac{h^4}{4!}y_0^{iv} + ...$$

$$y(0.1) = 1 - 0.1 + \frac{(0.1)^2}{2} + \frac{(0.2)^3}{6} - \frac{(0.1)^4}{24}$$

$$= 1 - 0.1 + 0.005 + 0.000167 + 0.000004$$

$$= 0.905163$$

$y_1^{'} = x_1^{\,2} - y_1 = -0.895$	$y_2^{'} = x_2^{\,2} - y_2 = -0.7813$
$y_1^{''} = 2x_1 - y_1^{'} = 1.095$	$y_2^{''} = 2x_2 - y_2^{'} = 1.1813$
$y_1^{'''} = 2 - y_1^{''} = 0.905$	$y_2^{'''} = 2 - y_2^{''} = 0.8187$
$y_1^{iv} = -y_1^{'''} = -0.905$	$y_2^{iv} = -y_2^{'''} = -0.8187$

$$y_2 = y_0 + \frac{h}{1!}y_1^{'} + \frac{h^2}{2!}y_1^{''} + \frac{h^3}{3!}y_1^{'''} + \frac{h^4}{4!}y_1^{iv} + ...$$

$$y(0.2) = 0.9052 + (0.1)(-0.895) + \frac{(0.1)^2}{2}(1.095) + \frac{(0.2)^3}{6}(0.905) - \frac{(0.1)^4}{24}(-0.905)$$

$$= 0.9052 - 0.0895 + 0.005475 + 0.00015$$

$$= 0.8213$$

$$y(0.3) = 0.8213 + (0.1)(-0.7813) + \frac{(0.1)^2}{2}(1.1813) + \frac{(0.2)^3}{6}(0.8187) - \frac{(0.1)^4}{24}(-0.81$$

$$= 0.8213 - 0.07813 + 0.0059 + 0.00014$$

$$= 0.7492$$

(ii) Using Runge-Kutta method of fourth order, given $y^{''} + xy^{'} + y = 0, y(0) = 1, y^{'}(0) = 0,$ find the value of y at x = 0.1.

Solution:

Given

$$y^{''} = -xy^{'} - y \quad , \quad y(0) = 1$$

$$y^{'} = 0 \quad , y_0 = 1$$

Setting

$y^{'} = z$ and $y^{''} = z^{'}$

The equations becomes

$$y^{''} = -xz - y$$

$$\frac{dy}{dx} = z = f_1(x, y, z) \qquad \frac{dz}{dx} = -xz - y = f_2(x, y, z)$$

(ie) $y_0 = 1$, $z_0 = y_0^{'} = 0$

By algorithm,

$$k_1 = h\, f_1(x_0, y_0, z_0) = h\ f_1(0,1,0) = h(0) = 0$$

$$l_1 = h\, f_2(x_0, y_0, z_0) = h\ f_2(0,1,0) = h(-0,-1) = -h$$

$$k_2 = h\ f_1\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\ ,\ z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\ f_1\left[0 + \frac{0.1}{2}, 1+0\ ,\ 0 + \frac{(-0.1)}{2}\right] = 0.1\ f_1[0.05\ , 1, -0.05]$$

$$= (0.1)(-0.05) = -0.005$$

$$l_2 = h\ f_2\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\ ,\ z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\ f_2\left[0 + \frac{0.1}{2}, 1+0\ ,\ 0 + \frac{(-0.1)}{2}\right] = 0.1\ f_2[0.05\ , 1, -0.05]$$

$$= (-0.1)[(0.05)(-0.05) + 1] = -0.0998$$

$$k_3 = h\ f_1\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\ ,\ z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)\ f_1\left[0 + \frac{0.1}{2}, 1 + \frac{(-0.005)}{2}\ ,\ 0 + \frac{(-0.0998)}{2}\right]$$

$$= 0.1\ f_1[0.05\ , 0.9975, -0.0499]$$

$$= (0.1)(-0.0499) = -0.00499$$

$$l_3 = h\ f_2\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\ ,\ z_0 + \frac{l_2}{2}\right]$$

$$= 0.1\ f_2[0.05\ , 0.9975, -0.0499]$$

$$= (-0.1)[(0.05)(-0.0499) + 0.9975] = -0.0995$$

$$k_4 = h\ f_1[x_0 + h\ , y_0 + k_3\ ,\ z_0 + l_3]$$

$$= (0.1)\ f_1[0.1\ , 1 + (-0.00499)\ ,\ 0 + (-0.0995)]$$

$$= 0.1\ f_1[0.05\ , 0.995, -0.0995]$$

$$= (0.1)(-0.0995) = -0.00995$$

$$l_4 = h\ f_2[x_0 + h\ , y_0 + k_3\ ,\ z_0 + l_3]$$

$$= 0.1\ f_2[0.1\ , 0.995, -0.0995]$$

$$= (0.1)[(0.1)(-0.0995) + 0.995] = -0.0985$$

Now

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad = \quad \frac{1}{6}[0 + 2(-0.005) + 2(-0.00499) + (-0.00995)]$$

$$= -0.00498$$

$$\therefore \; y_1 \;=\; y_0 + \Delta y \;=\; 1 - 0.00498 \;=\; 0.9950$$

Agni College of Technology
Chennai - 130

B.E./ B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA6452 –STATISTICS AND NUMERICAL METHODS
(Regulation 2013)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. It has been found that 2% of the tools produced by a certain machine defective. What is the probability that in a shipment of 400 such tools
   (a) 3% or more
   (b) 2% or less will prove defective

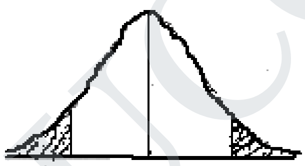
Ans : Out of Syllabus

2. A random sample of 200 tins of coconut oil gave an average weight of 4.95. With a standard deviation of 0.21kg. Do we accept that the net weight is 5 kgs per tin at 5% level?

Ans :

Given $n = 200$, $\mu = 5$, $s = 0.21$, $\overline{x} = 4.95$, $\alpha = 5\,\%$

1. $H_0 : \mu = 5$
2. $H_1 : \mu \neq 5$ [use two tailed test]
3. $\alpha = 5\%$
4. critical region



$-Z_{\alpha/2} = -1.96$ $\qquad$ $Z_{\alpha/2} = 1.96$

5. The test statistic $Z = \dfrac{\overline{x} - \mu}{\left(\dfrac{s}{\sqrt{n}}\right)} = \dfrac{4.95 - 5}{\left(\dfrac{0.21}{\sqrt{200}}\right)} = -3.36$

$|\, Z \,| = 3.36$

6. Conclusion : If $|\, Z \,| = 3.36 \nless 1.96$ we accept H_0

3. What do you understand by "Design of an experiments" ?

Ans :

The design of experiment may be defined as " the logical construction of the experiment in which the degree of uncertainly with which the inference is drawn may be well defined.

4. Write down the ANOVA table for one way classification.

Ans :

Analaysis of variance (ANOVA) table : one-way classification model

Sources of variation	Sum of squares	v d.f	Mean square	Variance ratio
Between sample	SSC	$v_1 = c-1$	MSC = $\frac{SSC}{c-1}$	$F_C = \frac{MSC}{MSE}$
Within samples	SSE	$v_2 = n-c$	MSE = $\frac{SSE}{n-c}$	
Total	TSS	$n-1$		

TSS = Total sum of squares of variations
SSC = Sum of squares between samples (columns)
SSE = Sum of squares within samples (rows)
MSC = Mean sum of squares between samples
MSE = Mean sum of squares within samples

5. Find the real positive root of 3x- cos x – 1 = 0 by Newton's method correct decimal places.

Ans :

Let $f(x) = 3x - \cos x - 1$

$f(0) = -2 \ (-ve)$

$f(1) = 1.4597 \ (+ve)$

A root is nearer to 1

$\Rightarrow$ Let $x_0 = 0.6$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad f(x) = 3x - \cos x - 1 \quad \text{and} \quad f'(x) = 3 + \sin x$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.6 - \frac{f(0.6)}{f'(0.6)} = 0.6 - \left[\frac{3(0.6) - \cos(0.6) - 1}{3 + \sin(0.6)}\right] = 0.607108$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.607108 - \frac{f(0.607108)}{f'(0.607108)}$$

$$= 0.607108 - \left[\frac{3(0.607108) - \cos(0.607108) - 1}{3 + \sin(0.607108)}\right] = 0.607102$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.607102 - \frac{f(0.607102)}{f'(0.607102)}$$

$$= 0.607102 - \left[\frac{3(0.607102) - \cos(0.607102) - 1}{3 + \sin(0.607102)}\right] = 0.607102$$

6. Solve the equations A + B + C = 6, 3A + 3B + 4C = 20, 2A + B + 3C = 13 using Gauss elimination method.

Ans :

Given A + B + C = 6, 3A + 3B + 4C = 20, 2A + B + 3C = 13

The given system is equivalent to

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 6 \\ 20 \\ 13 \end{bmatrix}$$

(ie) A X = B

Here ,

$$[A , B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{matrix} R_2 \Leftrightarrow R_2 - 2R_1 \\ R_3 \Leftrightarrow R_3 - 3R_1 \end{matrix}$$

By upper triangular matrix

Here C = 2 -----------(1)

-B + C = 1 -----------(2)

A + B + C = 1 -----------(3)

(2) ⇒ -B = 1 - C = 1 – 2 ⇒ B = 1

(3) ⇒ A + 1 + 2 = 6 ⇒ A = 3

7. Use Lagrange's formula to fit a polynomial to the data and find y at x = 1

X	-1	0	2	3
Y	-8	3	1	12

Ans :

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	-8			
		11		
0	3		-4	
		-1		2
2	1		4	
		11		
3	12			

8. Show that the divided difference of second order can be expressed as the quotient of two determinants of third order.

Ans :

$$f(x_0, x_1, x_2) = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)} + \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)} + \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)}$$

$$= \frac{(x_2 - x_1) f(x_0) + (x_0 - x_2) f(x_1) + (x_1 - x_0) f(x_2)}{(x_0 - x_1)(x_1 - x_2)(x_2 - x_0)}$$

$$= -\left[\frac{(x_1 - x_2) f(x_0) + (x_2 - x_0) f(x_1) + (x_0 - x_1) f(x_2)}{(x_0 - x_1)(x_1 - x_2)(x_2 - x_0)} \right]$$

$$= \frac{(x_1 - x_2) f(x_0) + (x_2 - x_0) f(x_1) + (x_0 - x_1) f(x_2)}{(x_0 - x_1)(x_1 - x_2)(x_0 - x_2)}$$

$$= \frac{\begin{vmatrix} f(x_0) & f(x_1) & f(x_2) \\ x_0 & x_1 & x_2 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} x_0^2 & x_1^2 & x_2^2 \\ x_0 & x_1 & x_2 \\ 1 & 1 & 1 \end{vmatrix}}$$

9. Using Taylor series method, find y at x = 0.1, 0.2 given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ (correct to 4 decimal places).

Ans : Given $\frac{dy}{dx} = x^2 - y$

$x_0 = 0$; $y_0 = 1$

$x_1 = 0.1$; $y_1 = ?$ $h = 0.1$

$$y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \ldots\ldots$$

$y' = x^2 - y$ $\quad y_0' = x_0^2 - y_0 = -1$

$y'' = 2x - y'$ $\quad y_0'' = 2x_0 - y_0' = 1$

$y''' = 2 - y''$ $\quad y_0''' = 2 - y_0'' = 1$

$y^{iv} = -y'''$ $\quad y^{iv} = -y_0''' = -1$

$$y_1 = 1 + \frac{0.1}{1!}(-1) + \frac{(0.1)^2}{2!}(1) + \frac{(0.1)^3}{3!}(1) + \frac{(0.1)^4}{4!}(-1) + \ldots\ldots$$

$= 0.905163$

To find y_2 :

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1''' + \ldots\ldots$$

$y' = x^2 - y$ $\quad y_1' = x_1^2 - y_1 = -0.895$

$y'' = 2x - y'$ $\quad y_1'' = 2x_1 - y_1' = 1.095$

$y''' = 2 - y''$ $\quad y_1''' = 2 - y_1'' = 0.905$

$$y_1 = 0.9052 + \frac{0.1}{1!}(-0.895) + \frac{(0.1)^2}{2!}(1.095) + \frac{(0.1)^3}{3!}(0.905) + ...$$

$$= 0.8213$$

10. Compute y at x = 0.25 by modified Euler Method given $y' = 2xy$, $y(0) = 1$

Ans :

$y' = 2xy$, $x_0 = 0$, $y_0 = 1$, $h = 0.25$, $x_1 = 0.25$

By Modified Euler method ,

$$y_{n+1} = y_n + h\, f\left[x_n + \frac{h}{2} , y_n + \frac{h}{2} f(x_n , y_n)\right]$$

$$y_1 = y_0 + h\, f\left[x_0 + \frac{h}{2} , y_0 + \frac{h}{2} f(x_0 , y_0)\right]$$

$$= 1 + (0.25)\, f\left[0 + \frac{0.25}{2} , 1 + \frac{0.25}{2} f(0, 0)\right]$$

$$= 1 + (0.25)\, f\,[0.125 , 1 + (0.25)(0)(1)]$$

$$= 1 + (0.25)[(2)(0.125)(1)]$$

$$= 1 + 0.00625$$

$$= 1.0625$$

PART – B (5 x 15 = 60)

11. a(i) Random samples drawn from two countries gave the following data relating to the heights of adult males. Is the difference between standard deviation significant?

	Country A	Country B
Mean height (in inches)	67.42	67.25
S.D (in inches)	2.58	2.50
Number in samples	1000	1200

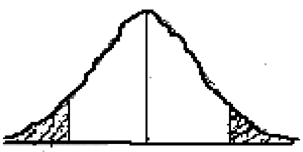
Ans :

Given

$n_1 = 1000$, $\overline{x_1} = 67.42$, $s_1 = 2.58$

$n_2 = 1200$, $\overline{x_2} = 67.25$, $s_2 = 2.50$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 \neq \mu_2$ [use two tailed test]
3. $\alpha = 5\%$
4. critical region



$-Z_{\alpha/2} = -1.96$ $\qquad$ $Z_{\alpha/2} = 1.96$

5. The test statistic $Z = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{67.42 - 67.25}{\sqrt{\frac{(2.58)^2}{1000} + \frac{(2.50)^2}{1200}}} = 1.03$

6. Conclusion : If $-Z_{\alpha/2} < Z < Z_{\alpha/2}$ then accept H_0 , otherwise we reject H_0 . Here - 1.96 $\not<$ - 5.16 < 1.96 So we reject H_0.

(ii) 1000 students at college level were graded according to their I.Q. and their economic conditions. What conclusion can you draw from the following data:

Economic conditions	I.Q. level	
	High	Low
Rich	460	140
Poor	240	160

Ans :

The parameter of interest is χ^2

1. H_0 : The given attributes are independent
2. H_1 : The given attributes are not independent le.
3. α = 0.05, d. f = (r-1)(s-1) = (2-1)(2-1) = 1
4. Table value of χ^2 = 3.841
5. The test statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$

The expected frequencies are calculated using the following formulae

Expected frequency for 460 $= \frac{600 \times 700}{1000} = 420$

Expected frequency for 140 $= \frac{600 \times 300}{1000} = 80$

Expected frequency for 240 $= \frac{700 \times 400}{1000} = 280$

Expected frequency for 160 $= \frac{300 \times 400}{1000} = 120$

O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
460	400	40	1600	3.81
140	180	- 40	1600	8.88
240	100	-40	1600	5.714
160	100	-40	1600	13.33
				31.7373

$\therefore \chi^2 = \sum \frac{(O-E)^2}{E} = 31.7373$

8. Conclusion:

If Cal χ^2 < table χ^2, then we accept H_0 . Otherwise , we reject H_0

Here, $\chi^2 = 31.7373 > 3.841 .16$. so reject H_0 at 5 % level of significance.
We conclude that the attributes I.Q as economic conditions are not independent.

b(i) The sales manager of a large company conducted a sample survey in states A and B taking 400 samples in each case. The results were in the following table. Test whether the average sales in the same in the 2 states at 1 % level.

	State A	State B
Average sales	Rs. 2,500	Rs. 2,200
S.D.	Rs. 400	Rs. 550

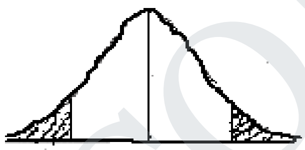
Ans :

Given

$n_1 = 400$, $\overline{x_1} = 2500$, $s_1 = 400$

$n_2 = 400$, $\overline{x_2} = 2200$, $s_2 = 550$ $\alpha = 1\%$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 \neq \mu_2$ [use two tailed test]
3. $\alpha = 1\%$
4. critical region



$-Z_{\alpha/2} = -2.58$ $\qquad$ $Z_{\alpha/2} = 2.58$

5. The test statistic $Z = \dfrac{(\overline{x_1} - \overline{x_2})}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{2500 - 2000}{\sqrt{\dfrac{(400)^2}{400} + \dfrac{(550)^2}{400}}} = 8.82$

6. Conclusion :

If $-Z_{\alpha/2} < Z < Z_{\alpha/2}$ then accept H_0 , otherwise we reject H_0 .

Here $-2.58 < 8.82 \nless 2.58$ So we reject H_0.

(ii) Find if there is any association between extravagance in fathers and extravagance in sons from the following data. Determine the coefficient of association also

	Extravagant father	Misserly father
Extrav. sons	Under 327	741
Misser. Sons	545	234

Ans :

1. The parameter interest is χ^2
2. H_0 : Namely that the extravagance in sons and fathers are not significant.
3. H_1 : Significant

4. $\alpha = 0.05$, $d.f = (r-1)(s-1) = (2-1)(2-1) = 1$

5. The test statistic is $\chi^2 = \dfrac{(ad-bc)^2(a+b+c+d)}{(a+b)(c+d)(a+c)(b+d)}$

6. Reject H_0 if $\chi^2 > 3.84$

7. Computations :

$$\chi^2 = \frac{[(327)(234)-(545)(741)]^2(327+545+741+234)}{(872)(975)(1068)(779)} = 279.77$$

8. Conclusion :

Since $\chi^2 = 230.24 > 3.841$ solve reject H_0 at 5 % level of significance

∴ There is dependence between the attributes

Coefficient of attributes $= \dfrac{ad-bc}{ad+bc} = \dfrac{-327330}{480363} = -0.6814$

12. a(i) The following data represent the number of units of production per day turned out by 5 different workers using 4 different types of machine

		Machine Type			
		A	B	C	D
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(1) Test whether the mean production is the same for the different machine types

(2) Test whether the 5 men differ with mean productivity.

Ans :

The code data is

Row	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
y_1	4	-2	7	-4	5	16	4	49	16
y_2	6	0	12	3	21	36	0	144	9
y_3	-6	-4	4	-8	-14	36	16	16	64
y_4	3	-2	6	-7	0	9	4	36	49
Y_5	-2	2	9	-1	8	4	4	81	1
Total	5	-6	38	-17	20	101	28	326	139

H_0 : (i) the mean productivity is the same for four different machines and
H_1 : (ii) the 5 men do not differ with respect to mean productivity code the data by subtracting 40 from each value.

Step 1 : N = 20

Step 2 : T = 20

Step 3 : C.F $= \frac{T^2}{N} = \frac{(20)^2}{20} = 20$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 101 + 28 + 326 + 139 - 20 = 574$

Step 5 : SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(5)^2}{5} + \frac{(-6)^2}{5} + \frac{(38)^2}{5} + \frac{(-17)^2}{5} - 20 = 338.8$

Step 6. SSR $= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} + \frac{(\sum Y_5)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{(5)^2}{4} + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{(0)^2}{4} + \frac{(8)^2}{4} - 20 = 161.5$

SSE = TSS – SSC – SSR = 574 – 338.8 – 161.5 = 73.7

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	Table value at 5% level
Between Rows	SSR = 161.5	r – 1 = 5-1 = 4	MSR = $\frac{SSR}{k-1}$ = 40.375	$F_R = \frac{MSR}{MSE}$ = 6.574	$F_R(4, 6)$ = 3.26
Between columns	SSC = 338.8	c – 1 = 4-1 = 3	MSC = $\frac{SSC}{k-1}$ = 112.933	$F_C = \frac{MSC}{MSE}$ = 18.38	$F_C(3, 12)$ = 3.49
Residual	SSE = 73.7	N - c - r + 1 = 12	MSE = $\frac{SSE}{N-c-r+1}$ = 6.142		

Step 8 . Conclusion :

(i) Table F_C (3, 12) at 5 % level = 3.49

Cal F_C > Table F_C . Reject H_0 .

(ii) Table F_R (4, 12) at 5 % level = 3.26

Cal F_R > Table F_R . Reject H_0

∴ The worker differ with respect to mean productivity.

(ii) The following is a Latin square of design when 4varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You can carry out suitable change of origin and scale.

A 105	B 95	C 125	D 115
C 115	D 125	A 105	B 105
D 115	C 95	B 105	A 115
B 95	A 135	D 95	C 115

Ans :

Subtract 100 and then divided by 5 we get

A 1	B -1	C 5	D 3
C 3	D 5	A 1	B 1
D 3	C -1	B 1	A 3
B -1	A 7	D -1	C 3

Y_n	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
y_1	1	-1	5	3	8	1	1	25	9
y_2	3	5	1	1	10	9	25	1	1
y_3	3	-1	1	3	6	9	1	1	9
y_4	-1	7	-1	3	8	1	49	1	9
Total	6	10	6	10	32	20	76	28	28

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = 32

Step 3 : $C.F = \frac{T^2}{N} = \frac{(32)^2}{16} = 64$

Step 4 : $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 20 + 76 + 28 + 28 - 64 = 88$

Step 5 : $SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(6)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(10)^2}{4} - 64 = 4$

Step 6. SSR $= \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(8)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(8)^2}{4} - 64 = 2$$

To Find SSK

Treatment	1	2	3	4	Total
A	1	1	3	7	12
B	-1	1	1	-1	0
C	5	3	-1	3	10
D	3	5	3	-1	10

SSK $= \frac{(12)^2}{4} + \frac{(0)^2}{4} + \frac{(10)^2}{4} + \frac{(10)^2}{4} - 64 = 22$

SSE = TSS – SSC – SSR = 88 – 4 – 2 –22 = 60

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	Table value at 5% level
Between Rows	SSR = 2	k –1 = 3	MSR = $\frac{SSR}{k-1}$ = 0.67	$F_R = \frac{MSR}{MSE}$ = 14.9	$F_R(6, 3)$ = 8.94
Between columns	SSC = 4	k – 2 = 3	MSC = $\frac{SSC}{k-1}$ = 1.33	$F_C = \frac{MSC}{MSE}$ = 7.52	F_C6, 3) = 8.94
Treatment	SSK = 22	k – 1 = 3	MSK = $\frac{SSK}{k-1}$ = 7.33	$F_T = \frac{MSK}{MSE} = \frac{10}{7.33}$ = 1.36	$F_T(6, 3)$ = 8.94
Error	SSE = 60	(k -1)(k – 2) = 6	MSE = $\frac{SSE}{(k-1)(k-2)}$ = 10		
Total	TSS = 88	15			

Step 8 . Conclusion :

Cal F_R > Table F_R
Cal F_C < Table F_C
Cal F_T > Table F_T

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There is significant difference between treatments and rows. But there is no significant difference between columns.

b(i) Compare and contrast the Latin square design with the randomized block design.

Ans :

S.No	LSD	RBD
1.	It is suitable for small number of treatments, between 5 and 12.	No such restrictions suitable for upto 24 treatments.
2.	The number of rows and columns are equal and hence the number of replication is equal to the number of treatments.	There is no such restriction. It can have any number replications and treatments.
3.	Experimental error is reduced to a large extent, because variation is controlled in two directions.	Variations is controlled in one directions only.
4.	LSD is preferred over RBD because of (3)	RBD is the most popular one for its simplicity , flexibility and validity.
5.	Experimental area must be a square.	Suitable if it is a rectangle or square.

(ii) Analyse the following of Latin square experiment

Column row	1	2	3	4
1	A(12)	D(20)	C(16)	B(10)
2	D(18)	A(14)	B(11)	C(14)
3	B(12)	C(15)	D(19)	A(13)
4	C(16)	B(11)	A(15)	D(20)

Ans :

Let us take 12 as origin for simplifying the calculations

Row	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
(y_1) (1)	A 0	D 8	C 4	B -2	10	0	64	16	4
(y_2) (2)	D 6	A 2	B -1	C 2	9	36	4	1	4
(y_3) (3)	B 0	C 3	D 7	A 1	11	0	9	49	1
(y_4) (4)	C 4	B 1	A 3	D 8	14	16	1	9	64
Total	10	12	13	9	44	52	78	75	73

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = 44

Step 3 : C.F $= \frac{T^2}{N} = \frac{(44)^2}{16} = 121$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 52 + 78 + 75 + 73 - 121 = 157$

Step 5 : SSC $= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(10)^2}{4} + \frac{(12)^2}{4} + \frac{(13)^2}{4} + \frac{(9)^2}{4} - 121 = 2.5$

Step 6. SSR $= \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{(10)^2}{4} + \frac{(9)^2}{4} + \frac{(11)^2}{4} + \frac{(14)^2}{4} - 121 = 3.5$

To Find SSK

Treatment	1	2	3	4	Total
A	0	2	3	1	6
B	0	-1	-1	-2	- 4
C	4	3	4	2	13
D	6	8	7	8	29
					44

SSK $= \frac{(6)^2}{4} + \frac{(-4)^2}{4} + \frac{(13)^2}{4} + \frac{(29)^2}{4} - 121 = 144.5$

SSE = TSS – SSC – SSR = 157 – 2.5 – 3.5 – 144.5 = 6.5

Step 7 : ANOVA Table

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	F test 1%
Between Rows	SSR = 3.5	3	1.17	1.08	9.78
Between columns	SSC = 2.5	3	0.83	0.77	27.91
Variety	SSK = 144.5	3	48.17	44.60	9.78
Error	SSE = 6.5	6	1.08		
Total	TSS = 157	11			

Step 8 . Conclusion :

The F ratios for rows and columns are not significant at 1 % level while that for varieties is very highly significant. The fact that there are no significant differences between rows and columns. Show that the Latin square arrangement has not been advantageous.

13. a(i) Solve the system of equations by Gauss Jordan Method

$x + y + z + w = 1$; $2x - y + 2z - w = -5$,

$3x + 2y + 3z + 4w = 7$; $x - 2y - 3z + 2w = 5$

Ans :

$$[A, B] = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & -1 & 2 & -1 & -5 \\ 3 & 2 & 3 & 4 & 7 \\ 1 & -2 & -3 & 2 & 5 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -3 & 0 & -3 & -9 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & -3 & -4 & 1 & 3 \end{array}\right] \quad \begin{array}{l} R_2 \Leftrightarrow R_2 - 2R_1 \\ R_3 \Leftrightarrow R_3 - 3R_1 \\ R_4 \Leftrightarrow R_4 - R_1 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -3 & 0 & -3 & -9 \\ 0 & 0 & 0 & 6 & 12 \\ 0 & 0 & -4 & 4 & 12 \end{array}\right] \quad \begin{array}{l} R_3 \Leftrightarrow 3R_3 - 2R_2 \\ R_4 \Leftrightarrow R_4 - R_2 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R_2 \Leftrightarrow \frac{R_2}{3}, \quad R_3 \Leftrightarrow \frac{R_3}{6} \\ R_4 \Leftrightarrow \frac{R_4}{4} \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R_3 \Leftrightarrow R_3 - 2R_4 \\ R_2 \Leftrightarrow R_2 - 3R_4 \\ R_1 \Leftrightarrow R_1 - R_4 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R_2 \Leftrightarrow R_2 \\ R_1 \Leftrightarrow R_1 + R_2 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 \Leftrightarrow R_1 - R_2$$

Hence $x = 0$, $y = 1$, $z = -1$, $w = 2$

(ii) Solve by Gauss- Seidal method the following system:
28x + 4y – z = 32; x + 3y + 10z = 24; 2x + 17y + 4z =35

Ans :

Write x, y, z as follows:

$$x = \frac{1}{28}[32 - 4y + z] \qquad y = \frac{1}{17}[35 - 2x - 4z] \qquad z = \frac{1}{10}[24 - x - 3y]$$

Let the initial values be y = 0, z = 0

First Iteration

$$x^1 = \frac{1}{28}[32 - 4y^0 + z^0] = \frac{1}{27}[32 - 4(0) + 0] = 1.1429$$

$$y^1 = \frac{1}{17}[35 - 2x^1 - 4z^0] = \frac{1}{17}[35 - 2(1.1429) - 0] = 1.9244$$

$$z^1 = \frac{1}{10}[24 - x^1 - 3y^1] = \frac{1}{10}[24 - 1.1429 - 3(1.9244)] = 1.7084$$

Second iteration

$$x^2 = \frac{1}{28}[32 - 4y^1 + z^1] = \frac{1}{28}[32 - 4(1.9244) + 1.7084] = 0.9230$$

$$y^2 = \frac{1}{17}[35 - 2x^1 - 4z^1] = \frac{1}{17}[35 - 2(0.9230) - 4(1.7084)] = 1.5483$$

$$z^2 = \frac{1}{10}[24 - x^2 - 3y^2] = \frac{1}{10}[24 - 0.9230 - 3(1.5483)] = 1.8432$$

Third iteration

$$x^3 = \frac{1}{28}[32 - 4y^2 + z^2] = \frac{1}{28}[32 - 4(1.5483) + 1.8432] = 0.9875$$

$$y^3 = \frac{1}{17}[35 - 2x^3 - 4z^2] = \frac{1}{17}[35 - 2(0.9875) - 4(1.8432)] = 1.509$$

$$z^3 = \frac{1}{10}[24 - x^3 - 3y^3] = \frac{1}{10}[24 - 0.9875 - 3(1.509)] = 1.8486$$

Fourth iteration

$$x^4 = \frac{1}{28}[32 - 4y^3 + z^3] = \frac{1}{28}[32 - 4(1.509) + 1.8486] = 0.9933$$

$$y^4 = \frac{1}{17}[35 - 2x^4 - 4z^3] = \frac{1}{17}[35 - 2(0.9933) - 4(1.8486)] = 1.507$$

$$z^4 = \frac{1}{10}[24 - x^4 - 3y^4] = \frac{1}{10}[24 - 0.9933 - 3(1.507)] = 1.8486$$

Fifth iteration

$$x^5 = \frac{1}{28}[32 - 4y^4 + z^4] = \frac{1}{28}[32 - 4(1.507) + 1.8486] = 0.9936$$

$$y^5 = \frac{1}{17}[35 - 2x^5 - 4z^4] = \frac{1}{17}[35 - 2(0.9936) - 4(1.8486)] = 1.507$$

$$z^5 = \frac{1}{10}[24 - x^5 - 3y^5] = \frac{1}{10}[24 - 0.9936 - 3(1.507)] = 1.8486$$

sixth iteration

$$x^6 = \frac{1}{28}[32 - 4y^5 + z^5] = \frac{1}{28}[32 - 4(1.507) + 1.8486] = 0.9936$$

$$y^6 = \frac{1}{17}[35 - 2x^6 - 4z^5] = \frac{1}{17}[35 - 2(0.9936) - 4(1.8486)] = 1.507$$

$$z^6 = \frac{1}{10}[24 - x^6 - 3y^6] = \frac{1}{10}[24 - 0.9936 - 3(1.507)] = 1.8486$$

Hence $x = 0.9936$, $y = 1.507$, $z = 1.8486$

b(i) Solve by Gauss Elimination method
$3x + 4y + 5z = 18$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$

Ans : The augmented matrix is

$$[A, B] = \left(\begin{array}{ccc|c} 3 & 4 & 5 & 18 \\ 2 & -1 & 8 & 13 \\ 5 & -2 & 7 & 20 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 3 & 4 & 5 & 18 \\ 0 & -11 & 14 & 3 \\ 0 & -26 & -4 & -30 \end{array}\right) \quad \begin{array}{l} R_2 \Leftrightarrow 3R_2 - 2R_1 \\ R_3 \Leftrightarrow 3R_3 - 5R_1 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 3 & 4 & 5 & 18 \\ 0 & -11 & 14 & 3 \\ 0 & 0 & -408 & -408 \end{array}\right) \quad R_3 \Leftrightarrow 11R_3 - 26R_2$$

By back substitution method

$$-408z = -408 \quad \Rightarrow \quad z = 1$$

$$-11y + 14z = 3$$

$$-11y = 3 - 14(1) \quad \Rightarrow \quad y = 1$$

$$3x + 4y + 5z = 18$$

$$3x = 18 - 4(1) - 5(1) \quad \Rightarrow \quad x = 3$$

(ii) Using Power method, find all the Eigen values of A= $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$

Ans :

Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an approximate eigen vector

$$AX_1 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 0 \\ 0.2 \end{bmatrix} = 5X_2$$

$$AX_2 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 5.2 \\ 0 \\ 2 \end{bmatrix} = 5.2 \begin{bmatrix} 1 \\ 0 \\ 0.39 \end{bmatrix} = 5.2X_3$$

$$AX_3 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.39 \end{bmatrix} = \begin{bmatrix} 5.39 \\ 0 \\ 2.95 \end{bmatrix} = 5.39 \begin{bmatrix} 1 \\ 0 \\ 0.55 \end{bmatrix} = 5.39X_4$$

$$AX_4 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 5.55 \\ 0 \\ 3.75 \end{bmatrix} = 5.55 \begin{bmatrix} 1 \\ 0 \\ 0.68 \end{bmatrix} = 5.55X_5$$

$$AX_5 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.68 \end{bmatrix} = \begin{bmatrix} 5.68 \\ 0 \\ 4.4 \end{bmatrix} = 5.68 \begin{bmatrix} 1 \\ 0 \\ 0.78 \end{bmatrix} = 5.68X_6$$

$$AX_6 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.78 \end{bmatrix} = \begin{bmatrix} 5.78 \\ 0 \\ 4.9 \end{bmatrix} = 5.78 \begin{bmatrix} 1 \\ 0 \\ 0.85 \end{bmatrix} = 5.78X_7$$

$$AX_7 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.85 \end{bmatrix} = \begin{bmatrix} 5.85 \\ 0 \\ 5.25 \end{bmatrix} = 5.85 \begin{bmatrix} 1 \\ 0 \\ 0.9 \end{bmatrix} = 5.85X_8$$

$$AX_8 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.9 \end{bmatrix} = \begin{bmatrix} 5.9 \\ 0 \\ 5.5 \end{bmatrix} = 5.9 \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = 5.9X_9$$

$$AX_9 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \begin{bmatrix} 5.93 \\ 0 \\ 5.65 \end{bmatrix} = 5.93 \begin{bmatrix} 1 \\ 0 \\ 0.95 \end{bmatrix} = 5.93X_{10}$$

$$AX_{10} = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.95 \end{bmatrix} = \begin{bmatrix} 5.95 \\ 0 \\ 5.75 \end{bmatrix} = 5.95 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 5.95X_{11}$$

$$AX_{11} = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 6X_{12}$$

$$AX_{12} = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 6X_{13}$$

$$\therefore \lambda_1 = 6 \quad ; \quad \text{Eigenvector} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$B = A - 6I = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & -1 \end{bmatrix} \text{ Take } Y_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$BY_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = -1Y_2$$

$$BY_2 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = -2Y_3$$

$$BY_3 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Greater Eigen value of B = - 2
$\therefore$ Smallest Eigen value of A = - 2 + 6 = 4

$$\lambda_1 + \lambda_2 + \lambda_3 = -2 + 5 = 8$$
$$6 + 4 + \lambda_3 = 8 \Rightarrow \lambda_3 = -2$$

Eigen values are 6 ,4, -2

14. a(i)

By dividing the range into ten equal parts, evaluate $\int_0^\pi \sin x \, dx$ by Trapezoidal and Simpson's rule. Verify your answer with integration.

Ans :

Range $\pi - 0 = \pi$ hence h = $\frac{\pi}{10}$

x	0	$\frac{\pi}{10}$	$\frac{2\pi}{10}$	$\frac{3\pi}{10}$	$\frac{4\pi}{10}$	$\frac{5\pi}{10}$
$y = \sin x$	0	0.3090	0.5878	0.8090	0.9511	1
x	$\frac{6\pi}{10}$	$\frac{7\pi}{10}$	$\frac{8\pi}{10}$	$\frac{9\pi}{10}$	π	

$y=\sin x$	0.9511	0.8090	0.5878	0.3090	0	

(i) Trapezoidal rule

$$I = \frac{\pi}{20}[(0+0) + 2(0.3090 + 0.5878 + 0.8090 + 0.9511 + 1 + 0.9511 + 0.8090 + 0.5878 + 0.3090)]$$

$= 1.9843$

(ii) Simpson's rule

	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
	1	4	1								
			1	4	1						
					1	4	1				
							1	4	1		
									1	4	1
Sum	1	4	2	4	2	4	2	4	2	4	1

$$I = \frac{h}{3}[(y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)]$$

$$= \frac{\pi/10}{3}[(0+0) + 4(0.3090 + 0.8090 + 1 + 0.8090 + 0.3090) + 2(0.5878 + 0.9511 + 0.9511 + 0.5878)]$$

$= 2.0001$

(iii) By actual integration, $I = \int_0^{\infty} \sin x \, dx = (-\cos x)_0^{\pi} = 2$

(ii) If $f(x) = \frac{1}{x}$, show that $f(x_0, x_1, \ldots\ldots\ldots, x_n) = \frac{(-1^r)}{x_0 x_1, \ldots x_2}$. Where r is any positive integer.

Ans : Given $f(x) = \frac{1}{x}$

wkt the result is true for r = 1

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{\frac{1}{x_0} - \frac{1}{x_1}}{x_1 - x_0} = \frac{-1}{x_0 x_1} = \frac{(-1)^n}{x_0 x_1} \text{ ------(1)}$$

To prove the result by induction

Assume that the result is true for r = n

$$f(x_0, x_1 \ldots\ldots, x_n) = \frac{(-1)^n}{x_0 x_1 \ldots\ldots x_n} \text{ ------(2)}$$

Let r = n + 1

$$f(x_0, x_1 \ldots\ldots, x_{n+1}) = \frac{f(x_1 \ldots\ldots, x_{n+1}) - f(x_0, x_1 \ldots\ldots, x_n)}{x_{n+1} \ldots\ldots x_0}$$

$$= \frac{(-1)^n}{x_{n+1}\ x_n \ldots\ldots x_1} - \frac{(-1)^n}{x_n \ldots\ldots x_0}$$

$$= \frac{\left[\dfrac{(-1)^n(.x_0 - x_{n+1})}{x_0, x_1 \ldots\ldots, x_{n+1}}\right]}{x_{n+1} - .x_0} = \frac{(-1)^{n+1}}{x_0, x_1 \ldots\ldots, x_{n+1}}$$

The result is true for r = n+1

$$f(x_0\ , x_1\ \ldots\ldots, x_n) = \frac{(-1)^r}{x_0, x_1 \ldots\ldots, x_r}$$ where r is any positive integer.

b(i) The population of a certain town is given below. Find the rate of growth of the population in 1931, 1941,1961 and 1971

Year (x)	1931	1941	1951	1961	1971
Population in thousand (y)	40.62	60.80	79.95	103.56	132.65

Ans :

we form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^3 y$
1931	40.62				
		20.18			
1941	60.80		-1.03		
		19.15		5.49	
1951	79.95		4.46		-4.47
		23.61		1.02	
19561	103.56		5.48		
		29.09			
1971	132.65				

We use the same table for backward and forward differences.

(i) To get $f'(1931)$ and $f'(1941)$ we use forward formula

$x_0 = 1931\ ,\ x_1 = 1941$

Where u $= \frac{x-x_0}{h}$ ⇒ u = 0

$$\left(\frac{dy}{dx}\right)_{x=1931} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}[\ \Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \ldots\ldots\ldots]$$

$$= \frac{1}{10}[\ 20.18 - \frac{1}{2}(-1.03) + \frac{1}{3}(5.49) - .\frac{1}{4}(-4.47) + \ldots.]$$

$$= \frac{1}{10}[\ 20.18 + 0.515 + 1.83 + 1.1175]$$

$$= 2.36425$$

If $x = 1941$

Where u $= \frac{x-x_0}{h}$ ⇒ u = 1

$$\frac{dy}{dx} = \frac{1}{h}\left[\ \Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0 + \Delta^3 y_0 + ..\right.$$

$$\left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{10}[\ 20.18 - \frac{1}{2}(-1.03) - \frac{1}{6}(5.49) - .\frac{1}{12}(-4.47)\ ]$$

$$= \frac{1}{10}[\ 20.18 - 0.515 - 0.915 - 0.3725] \qquad = 1.83775$$

(ii) To get $f^{'}(1971)$ we use forward formula

$$\left(\frac{dy}{dx}\right)_{x=1961} = \left(\frac{dy}{dx}\right)_{u=-1} = \frac{1}{h}[\ \nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{4}\nabla^3 y_n - ....]$$

$$= \frac{1}{10}[\ 20.09 - \frac{1}{2}(5.48) + \frac{1}{3}(1.02) + .\frac{1}{4}(-4.47) + ...] \quad = 3.10525$$

(iii) To get $f^{'}(1961)$ we use forward formula

Where u $= \frac{x-x_n}{h}$ $\Rightarrow$ u = -1

$$\left(\frac{dy}{dx}\right)_{x=1961} = \left(\frac{dy}{dx}\right)_{v=-1} = \frac{1}{h}[\ \nabla y_n + \frac{2v+1}{2}\nabla^2 y_n + \frac{3v^2+6u+2}{6}\nabla^3 y_n - ....]$$

$$= \frac{1}{10}[\ 20.09 - \frac{1}{2}(5.48) - \frac{1}{6}(1.02) - .\frac{1}{12}(-4.47) + ...]$$

$$= 2.65525$$

(ii) Using Newton's divided difference formula, find the values of f(2), f(8) and f(15) given the following table.

X	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

Ans :

Since the argument are not equally spaced, we will use Newton's divided difference

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
4	48			
		52		
5	100		15	
		97		1
7	294		21	
		202		1
10	900		27	
		310		1
11	1210		23	

13	2028	409			

By Newton's divided difference formula we have

$$y = f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + \dots\dots$$

$$y(8) = 48 + (8-4)(52) + (8-4)(8-5)(15) + (8-4)(8-5)(8-7)(1) + \dots\dots$$
$$= 448$$

$$y = f(x) = 48 + (x-4)(52) + (x-4)(x-5)(15) + (x-4)(x-5)(x-7)(1) + \dots$$

$$y(2) = 48 + (2-4)(52) + (2-4)(2-5)(15) + (2-4)(2-5)(2-7)(1) + \dots\dots$$
$$= 4$$
$$y(15) = 48 + (15-4)(52) + (15-4)(15-5)(15) + (15-4)(15-5)(115-7)(1$$
$$= 3150$$

15. a(i) Solve $y_{x+2} - 7y_{x+1} - 8y_x = x(x-1)2^x$

Ans :

The auxiliary equation is

$$a^2 - 7a - 8 = 0$$

$$(a-8)(a+1) = 0 \quad \therefore\ a = 8, -1$$

C.F. = $A8^x + B(-1)^x$

P.I. $= \dfrac{1}{E^2 - 7E - 8} x(x-1).2^x$

$$= 2^x \frac{1}{(2E)^2 - 7(2E) - 8} x(x-1). \quad = 2^x \frac{1}{4E^2 - 14(E) - 8} x(x-1).$$

$$= 2^x \frac{1}{4(1+\Delta)^2 - 14(1+\Delta) - 8} x(x-1).$$

$$= 2^x \frac{1}{4\Delta^2 - 6\Delta - 18} x(x-1). \quad = 2^{x-1} \frac{1}{2\Delta^2 - 3\Delta - 9} x(x-1).$$

$$= 2^{x-1} \frac{1}{4\Delta^2 - 6\Delta - 18} x(x-1).$$

$$= \frac{2^{x-1}}{-9} \left(1 + \frac{\left(2\Delta^2 - 3\Delta\right)}{-9} \right)^{-1} x(x-1)$$

$$= \frac{2^{x-1}}{-9} \left(1 + \frac{2\Delta^2 - 3\Delta}{-9} + \left(\frac{2\Delta^2 - 3\Delta}{-9} \right)^2 + \dots\dots\dots \right) x^2$$

$$= \frac{2^{x-1}}{-9} \left(1 - \frac{-\Delta}{-3} + \frac{\Delta^2}{3} + \dots\dots\dots \right) x^2 \Rightarrow \frac{2^{x-1}}{-9} \left(x^{(2)} - \frac{2}{3} x^1 + \frac{1}{3}(2) \right)$$

$$= \frac{2^{x-1}}{-9}\left(x(x-1)-\frac{2}{3}x^1+\frac{1}{3}(2)\right)$$

$$= \frac{2^{x-1}}{-9}\left(x^2-\frac{5}{3}x^1+\frac{1}{3}(2)\right)$$

Hence , the solution is

$$A8^x + B(-1)^x - \frac{2^{x-1}}{9}\left(x^2-\frac{5}{3}x^1+\frac{1}{3}(2)\right).$$

(ii) Using Runge- Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ given y(0) = 1 at x = 0.2, 0.4

Ans :

Given $y' = f(x,y) = \frac{y^2-x^2}{y^2+x^2}$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.2$, $x_2 = 0.4$, $x_3 = 0.6$

h = 0.2

To find y(0.2) :

$$k_1 = h f(x_0, y_0) = (0.2)\left[\frac{y_0^2 - x_0^2}{y_0^2 + x_0^2}\right] = (0.2)\left[\frac{1-0}{1+0}\right] = 0.2$$

$$k_2 = h f\left[x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right] = (0.2) f\left[0+\frac{0.2}{2}, 1+\frac{0.2}{2}\right]$$

$$= (0.2) f[0.1, 1.1] = (0.2)\left[\frac{(1.1)^2-(1.1)^2}{(1.1)^2+(0.1)^2}\right]$$

$$= (0.2)(0.9836) = 0.19672$$

$$k_3 = h f\left[x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right] = (0.2) f\left[0+\frac{0.2}{2}, 1+\frac{0.19672}{2}\right]$$

$$= (0.2) f[0.1, 1.0983606]$$

$$= (0.2)\left[\frac{(1.0983606)^2-(0.1)^2}{(1.0983606)^2+(0.1)^2}\right] = 0.1967$$

$$k_4 = h f(x_0+h, y_0+k_3) = (0.2) f[0.2, 1.1967]$$

$$= (0.2)\left[\frac{(1.1967)^2-(0.2)^2}{(1.1967)^2+(0.2)^2}\right] = 0.1891$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.2+2(0.19672)+2(1.1967)+0.1891] = 0.19598$$

$$y(0.2) = y_1 = y_0 + \Delta y = 1 + 0.19598 = 1.19598$$

To find y(0.4) :

$k_1 = h\,f(x_1, y_1)$

$= (0.2)\left[\frac{(1.196)^2 - (0.2)^2}{(1.196)^2 + (0.2)^2}\right] = 0.1891$

$k_2 = h\,f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right] = (0.2)\,f\left[0.2 + \frac{0.2}{2}, 1.196 + \frac{0.1891}{2}\right]$

$= (0.2)\,f\,[0.3, 1.2906]$

$= (0.2)\left[\frac{(1.2906)^2 - (0.3)^2}{(1.2906)^2 + (0.3)^2}\right] = 0.1795$

$k_3 = h\,f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right]$

$= (0.2)\,f\left[0.2 + \frac{0.2}{2}, 1.196 + \frac{0.1795}{2}\right]$

$= (0.2)\,f\,[0.3, 1.2858]$

$= (0.2)\left[\frac{(1.2858)^2 - (0.3)^2}{(1.2858)^2 + (0.3)^2}\right] = 0.1793$

$k_4 = h\,f(x_1 + h, y_1 + k_3)$

$= (0.2)\,f\,[0.4, 1.3753]$

$= (0.2)\left[\frac{(1.3753)^2 - (0.4)^2}{(1.3753)^2 + (0.4)^2}\right] = 0.1688$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.1891 + 2(0.1795) + 2(0.1793) + 0.1688] = 0.1793$

y(0.4) = $y_2 = y_1 + \Delta y$ = 1.196 + 0.1793 = 1.3753

b. Given y''+ xy' + y =0, y(0)= 1, y'(0)= 0, find the value of y(0.1) by using Runge-Kutta method for fourth order .

Ans : Given

$y'' = -x\,y' - y$, $y(0) = 1$

$y' = 0$, $y_0 = 1$

Setting

$y' = z$ and $y'' = z'$

The equations becomes

$y'' = -x\,z - y$

$$\frac{dy}{dx} = z = f_1(x,y,z) \qquad \frac{dz}{dx} = -xz - y = f_2(x,y,z)$$

$(ie)\ \ y_0 = 1,\ \ z_0 = y_0' = 0$

By algorithm,

$$k_1 = h\, f_1(x_0, y_0, z_0) = h\ f_1(0,1,0) = h(0) = 0$$

$$l_1 = h\, f_2(x_0, y_0, z_0) = h\ f_2(0,1,0) = h(-0,-1) = -h$$

$$k_2 = h\ f_1\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\ f_1\left[0 + \frac{0.1}{2}, 1+0, 0 + \frac{(-0.1)}{2}\right] = 0.1\ f_1[0.05, 1, -0.05]$$

$$= (0.1)(-0.05) = -0.005$$

$$l_2 = h\ f_2\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\ f_2\left[0 + \frac{0.1}{2}, 1+0, 0 + \frac{(-0.1)}{2}\right] = 0.1\ f_2[0.05, 1, -0.05]$$

$$= (-0.1)[(0.05)(-0.05) + 1] = -0.0998$$

$$k_3 = h\ f_1\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)\ f_1\left[0 + \frac{0.1}{2}, 1 + \frac{(-0.005)}{2}, 0 + \frac{(-0.0998)}{2}\right]$$

$$= 0.1\ f_1[0.05, 0.9975, -0.0499]$$

$$= (0.1)(-0.0499) = -0.00499$$

$$l_3 = h\ f_2\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= 0.1\ f_2[0.05, 0.9975, -0.0499]$$

$$= (-0.1)[(0.05)(-0.0499) + 0.9975] = -0.0995$$

$$k_4 = h\ f_1[x_0 + h, y_0 + k_3, z_0 + l_3]$$

$$= (0.1)\ f_1[0.1, 1 + (-0.00499), 0 + (-0.0995)]$$

$$= 0.1\ f_1[0.05, 0.995, -0.0995]$$

$$= (0.1)(-0.0995) = -0.00995$$

$$l_4 = h\ f_2[x_0 + h, y_0 + k_3, z_0 + l_3]$$

$$= 0.1\ f_2[0.1, 0.995, -0.0995]$$

$$= (0.1)[(0.1)(-0.0995) + 0.995] = -0.0985$$

Now

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad = \quad \frac{1}{6}[0 + 2(-0.005) + 2(-0.00499) + (-0.00995)]$$

$$= -0.00498$$

$$\therefore \; y_1 \;=\; y_0 + \Delta y \;=\; 1 - 0.00498 \;=\; 0.9950$$

Agni College of Technology
Chennai - 130

B.E./ B.Tech. DEGREE EXAMINATION, NOV/DEC 2014
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA6452 –STATISTICS AND NUMERICAL METHODS
(Regulation 2013)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. Twenty people were attacked by a disease and only 18 survived. The hypothesis is set in such a way that the survival rate is 85% if attacked by this disease. Will you reject the hypothesis that it is more at 5% level $(Z_{0.05} = 1.645)$?

   Ans : Out of syllabus

2. Write the formula for the Chi-square test of goodness of fit of a random sample to a hypothetical distribution.
   Ans :
   Chi square test of goodness of fit is a test to find if the derivation of the experiment from theory is just by chance or it is due to the inadequacy of the theory to fit the observed data. By this test, we test whether differences between observed and expected frequencies are significant or not.

   Chi square test statistic of goodness of fit is defined by $\chi^2 = \sum \frac{(O-E)^2}{E}$, where O is the observed frequency and E the expected frequency.

3. Explain the situations in which randomized block design is considered an improvement over a completely randomized design.
   Ans :
   1. RBD is more efficient (or) accurate than CRD for most types of experiment.
   2. In RBD, no restrictions are placed on number of treatments on the number of replicates.

4. State the advantage of a factorial experiment over a simple experiment.
   Ans :
   Factorial experiment is the procedure of varying all factors simultaneously. A major conceptual advancement in experimental design is exemplified by factorial design.
   In factorial designs, an assessment of each individual factor effect is based on the whole set of measurements so that a more efficient utilization of experimental resources is achieved in these designs.

5. Find a real root of the equation $x = e^x$, using Newton-Raphson method.
   Ans :
   Let $f(0) = x - e^{-x}$ then $f(0) = -1$, $f(1) = 0.6321$

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Choose $x_0 = 1$ then $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$x_1 = 1 - \frac{0.6321}{1.3679} = 0.5379$$

$$f(x_1) = -0.0461$$

$$x_2 = 0.5379 - \frac{(-0.0461)}{1.584} = 0.5670$$

$$f(x_2) = -0.0002$$

$\therefore$ The root is $x = 0.567$

6. Write down the iterative formula of Gauss-Seidal method.

Ans :

Consider the system of equations

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

Let us assume

$$|a_1| > |b_1| + |c_1|$$
$$|b_2| > |a_2| + |c_2|$$
$$|c_3| > |a_2| + |b_2|$$

$x^{(r)}$, $y^{(r)}$, $z^{(r)}$ are the rth iterates , then

If $$x^{(r+1)} = \frac{1}{a_1}\left(d_1 - b_1 y^{(r)} - c_1 z^{(r)}\right)$$

$$y^{(r+1)} = \frac{1}{b_2}\left(d_2 - a_2 x^{(r+1)} - c_2 z^{(r)}\right)$$

$$z^{(r+1)} = \frac{1}{c_3}\left(d_3 - a_3 x^{(r+1)} - b_3 y^{(r+1)}\right)$$

7. Write the Lagrange's formula for interpolation and state its uses.

Ans :

Let $y = f(x)$ be a function which takes the values y_0 , y_1 , ..., y_n corresponding to x_0 , x_1 , ..., x_n

Then Lagrange's interpolation function,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \ldots\ldots + \frac{(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_2)\ldots\ldots\ldots(x_n-x_{n-1})}y_n$$

The forward and backward interpolation formulae of Newton can be used only when the values of the independent variable x are equally spaced, can also be used when the differences of the dependent variable y become smaller ultimately. But Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced or not and whether the difference of y become smaller or not.

8. Evaluate $I = \int_0^1 \frac{1}{1+x}dx$, correct to three decimal places using trapezoidal rule with $h = 0.25$

Ans :

x	0	0.25	0.5	0.75	1
$y = \frac{1}{1+x}$	1 y_0	0.8 y_1	0.6667 y_2	0.5714 y_3	0.5 y_4

$$I = \int_0^1 \frac{dx}{1+x^2} = \frac{0.25}{3}[1(1+0.5)+4(0.9412+0.64)+2(0.8)] = 0.785$$

9. Use the Runge-Kutta forth order method to find the value of y when $x=1$ given that $y=1$ when $x=0$ and that $\frac{dy}{dx} = \frac{y-x}{y+x}$

Ans :

$$y' = f(x,y) = \frac{y-x}{y+x},\ x_0 = 0,\ y_0 = 1$$

h = 1

$$k_1 = h\,f(x_0, y_0) = 1$$

$$k_2 = h\ f\left[x_0+\frac{h}{2},\ y_0+\frac{k_1}{2}\right] = f\left[\frac{1}{2},\frac{3}{2}\right] = 0.25$$

$$k_3 = h\,f\left[x_0+\frac{h}{2},\ y_0+\frac{k_2}{2}\right] = f\left[\frac{1}{2},\frac{5}{4}\right] = \frac{3/4}{7/4} = \frac{3}{7}$$

$$\Delta y = \frac{1}{6}[k_1+2k_2+2k_3+k_4] = 0.4342$$

$$y_1 = y_0 + \Delta y = 1 + 0.4342 = 1.4342.$$

10. Given $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$. Determine $y(0.02)$ using Euler's method.

Ans : $f(x, y) = x^2 + y$, $x_0 = 0$, $y_0 = 1$, $h = 0.02$

$$y_{n+1} = y_n + h\, f\left[x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right]$$

$$y_1 = y_0 + h\, f\left[x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right]$$

$$y(0.02) = 1 + 0.02\, f\left[0 + \frac{0.02}{2}, 1 + \frac{0.02}{2} f(0, 1)\right] = 1 + 0.02\, f(0.01, 1.01) = 1.0202$$

PART-B (5 x 15 = 60)

11a(i) Explain clearly the procedure generally followed in testing of a hypothesis.

Ans :

1. Set up null hypothesis H_0: It is a statement of no difference about the population parameters.
2. Set up alternative hypothesis H_1. It is a statement, complement to H_0
3. Compute the test statistic.
4. Obtain the tabulated value of the statistic at given level of significance (α)
5. Write the conclusion. If tabulated value of the statistic is less than the calculated value, the reject H_0.

(ii) The demand for a particular spare part in a factory was found to vary for day-to-day. In a sample study the following information was obtained.

Days	Mon	Tues	Wed	Thu	Fri	Sat
No. of spare parts demanded	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week $(\chi^2_{0.05}(5) = 11.07)$

Ans :

Null hypothesis H_0: The number of parts demanded does not depend on the day of the week.

Attractive hypothesis H_1: The number of parts demanded depend on the day of the week.

Statistic : $\chi^2 = \sum \frac{(O-E)^2}{E}$

$$E = \frac{1124 + 1125 + 1110 + 1120 + 1126 + 1115}{6} \Rightarrow E = 1120$$

O	1124	1125	1110	1120	1126	1115	
E	1120	1120	1120	1120	1120	1120	
$\frac{(O-E)^2}{E}$	0.0143	0.0223	0.0893	0	0.0321	0.0223	0.1803

Calculated $\chi^2 = 0.1803$

Table $\chi^2(5) = 11.07$

Conclusion : Table $\chi^2 >$ Cal χ^2 . Accept H_0.

b(i) Explain briefly the procedure involved in testing the significance for difference of proportions in the case of large samples.

Ans :

1. Null hypothesis H_0: There is no significant difference.
2. Alternative hypothesis $H_1 : p_1 \neq p_2$
3. Statistic : $Z = \left| \dfrac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \right|$ where $H_1 : p_1 \neq p_2$

   Where $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ and Q = 1 - P

   n_1 , n_2 are sample sizes.
4. Obtain the tabulated Z for given LOS (α)
5. If tab Z < Cal Z , Reject H_0

(ii) The height of six randomly chosen sailors are (in inches) : 63, 65, 68, 69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Discuss, the height that these data thrown on the suggestion that sailors are on the average taller than soldiers $(t_{0.01}(14) = 1.76)$

Ans:

$\sum x_1 = 63 + 65 + 68 + 69 + 71 + 72 = 408$

$\sum x_1^2 = 63^2 + 65^2 + 68^2 + 69^2 + 71^2 + 72^2 = 27804$

$\sum x_2 = 61 + 62 + 65 + 66 + 69 + 69 + 70 + 71 + 72 + 73 = 678$

$\sum x_2^2 = 61^2 + 62^2 + 65^2 + 66^2 + 69^2 + 69^2 + 70^2 + 71^2 + 72^2 + 73^2 = 46122$

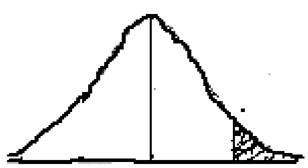
$$\overline{x_1} = \frac{\sum x_1}{n_1} = \frac{408}{6} = 68 \qquad \& \qquad \overline{x_2} = \frac{\sum x_2}{n_2} = \frac{678}{10} = 67.8$$

$$s_1^{\,2} = \frac{\sum x_1^{\,2}}{n_1} - (\overline{x_1})^2 = \frac{27804}{6} - (68)^2 = 10$$

$$s_2^{\,2} = \frac{\sum x_2^{\,2}}{n_2} - (\overline{x_2})^2 = \frac{46122}{10} - (67.8)^2 = 15.26$$

$$s^2 = \frac{n_1 s_1^{\,2} + n_2 s_2^{\,2}}{n_1 + n_2 - 2} = \frac{(6)(10) + (10)(15.36)}{6 + 10 - 2} = 15.26$$

1. H_0 ; $\mu_1 = \mu_2$
2. H_1 ; $\mu_1 > \mu_2$ $= 50$
3. $\alpha = 5\,\%$, $d.f = n - 1 = 5 - 1 = 4$
4. critical region



$-t_\alpha = 7.76$

$= 2.26$

5. The test statistic $t = \dfrac{\overline{x_1} - \overline{x_2}}{\sqrt{S^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{68 - 67.8}{\sqrt{15.26\left(\dfrac{1}{6} + \dfrac{1}{10}\right)}} = 0.099$

6. Conclusion : If $t < t_\alpha$ we accept H_0 , otherwise we reject H_0

Here $0.099 < 1.76$ $\therefore$ we accept H_0

The sailors are not an the average taller than the soldiers

12.(a) A company wants to procedure cars for its own use. It has to select the make of the car out of the four makes A, B, C, D available in the market. For this he tries four cars of each make by assigning the cars to four drivers to run on four different routes. The efficiency of cars is measured in terms of time in hours. The layout and time consumed is as given below.

	Drivers			
Routes	1	2	3	4
1	18(C)	12(D)	16(A)	20(B)
2	26(D)	34(A)	25(B)	31(C)
3	15(B)	22(C)	10(D)	28(A)
4	30(A)	20(B)	15(C)	9(D)

Analyse the experimental data and draw conclusions. $(F_{0.05}(3,5) = 5.41)$

Ans:

H_0 : There is no significant difference between routes and drivers.

H_1 : There is significant difference between routes and drivers

Subtract 25 from all numbers

	X_1 A	X_2 B	X_3 C	X_4 D	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	-9	-5	-7	-13	-34	81	25	49	169
Y_2	9	0	6	6	21	81	0	36	36
Y_3	3	-10	-3	-15	-25	9	100	9	225
Y_4	5	-5	-10	-16	-26	25	25	100	256
	8	-20	-14	-38	-64	296	150	194	686

Step 1 : N = 16

Step 2. T = -64

Step 3. C.F. $= \frac{T^2}{N} = \frac{(-64)^2}{16} = 256$

Step 4. TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 196 + 150 + 194 + 686 - 256 = 970$

Step 5. SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(8)^2}{4} + \frac{(-20)^2}{4} + \frac{(-14)^2}{4} + \frac{(-38)^2}{4} - 256 = 270$

Step 6. SSR $= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{(-34)^2}{4} + \frac{(21)^2}{4} + \frac{(-25)^2}{4} + \frac{(-26)^2}{4} - 256 = 468.5$

SSE = TSS – SSC – SSR = 970 - 270 – 468.5 = 231.5

Step 7. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 270	c –1 = 4 – 1 = 3	MSC $= \frac{SSC}{c-1}$ $= \frac{270}{3}$ = 90	$F_C = \frac{MSC}{MSE}$ $= \frac{90}{25.72} = 3.50$	F_C (3, 9) = 3.86
Between Rows	SSR = 468.5	r–1 = 4 – 1 = 3	MSR $= \frac{SSR}{r-1}$ $= \frac{468.5}{3}$ = 156.2	$F_R = \frac{MSR}{MSE}$ $= \frac{156.2}{25.72}$ = 6.07	F_R (5,15) = 4.5
Residual	SSE = 231.5	N – c –r + 1 = 16 – 4 – 4 + 1 = 9	MSE = $\frac{SSE}{N-c-r+1}$ $= \frac{231.5}{9} = 25.72$		

Step 7 : Conclusion :

Cal F_C < Table F_C . So we accept H_0 .

Cal F_R > Table F_R . So we reject H_0

(b). Consider the results given in the following table for an experiment involving six treatments in four randomized blocks. The treatments are indicated by numbers within parenthesis.

Blocks	Yield for a randomized block experiment treatment and yield					
1	(1) 24.7	(3) 27.7	(2) 20.6	(4) 16.2	(5) 16.2	(6) 24.9
2	(3) 22.7	(2) 28.8	(1) 27.3	(4) 15.0	(6) 22.5	(5) 17.0
3	(6) 26.3	(4) 19.6	(1) 38.5	(3) 36.8	(2) 39.5	(5) 15.4
4	(5) 17.7	(2) 31.0	(1) 28.5	(4) 14.1	(3) 34.9	(6) 22.6

Test whether the treatments differ significantly. $(F_{0.05}(3,15) = 5.42\ ;\ F_{0.05}(5,15) = 4.5)$

Ans:

H_0 : There is no significant difference between blocks and treatments.

H_1: There is significant difference between blocks and treatments

Subtract 20 from all the numbers

	X_1 A	X_2 B	X_3 C	X_4 D	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	4.7	-7.3	18.5	8.5	39	22.09	53.29	342.25	72.25
Y_2	0.6	8.8	19.5	11.0	39.9	0.36	77.44	380.25	121
Y_3	7.7	2.7	16.8	14.9	42.1	59.29	7.29	282.24	222.01
Y_4	-3.8	-5	-0.4	-5.9	-15.1	14.44	25	0.16	34.81
Y_5	-3.8	-3	-4.6	-2.3	-13.7	14.44	9	21.16	5.29
Y_6	4.9	2.5	6.3	2.6	16.3	24.01	6.25	36.69	6.76
	10.3	13.3	56.1	28.8	108.5	134.63	178.27	1065.75	462.37

Step 1 : N = 24

Step 2. T = 108.5

Step 3. C.F. = $\frac{T^2}{N} = \frac{(108.5)^2}{24} = 490.5$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 134.63 + 178.27 + 1065.75 + 462.37 - 490.5 = 1350.52$

Step 5. SSC = $\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(10.3)^2}{6} + \frac{(13.3)^2}{6} + \frac{(56.1)^2}{6} + \frac{(28.8)^2}{6} - 490.5 = 219.44$

Step 6. SSR = $\frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} + \frac{(\sum Y_5)^2}{N_2} + \frac{(\sum Y_6)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{(39)^2}{6} + \frac{(39.9)^2}{6} + \frac{(42.1)^2}{6} + \frac{(-15.1)^2}{6} + \frac{(-13.7)^2}{6} + \frac{(16.3)^2}{6} - 490.5$

$= 901.2$

SSE = TSS – SSC – SSR = 1350.52 - 219.44 - 901.2 = 229.9

Step 6. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 219.44	C –1 = 4 – 1 = 3	MSC $= \frac{SSC}{C-1}$ $= \frac{219.44}{3}$ = 73.15	$F_C = \frac{MSC}{MSE}$ $= \frac{73.05}{15.33} = 4.8$	F_C (3, 15) = 5.42
Between rows	SSR =901.2	R–1 = 6 – 1 = 5	MSC $= \frac{SSC}{R-1}$ =180.24	$F_C = \frac{MSR}{MSE}$ $= \frac{180.24}{15.33}$ = 11.76	F_R (5,15) = 4.5
Residual	SSE =229.9	N – C-R+1 = 24 – 4 -6 +1 = 15	MSE = $\frac{SSE}{N-C-R+1}$ $= \frac{229.9}{15} = 15.3$		

Step 7 : Conclusion :

Cal F_C < Table F_C . So we accept H_0 .

Cal F_R < Table F_R . So we reject H_0

13.a(i) Find the dominant eigen value and its eigenvector of the matrix by power method

$$A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$$

Ans: Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an approximate eigen vector

$$AX_1 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 0 \\ 0.2 \end{bmatrix} = 5X_2$$

$$AX_2 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 5.2 \\ 0 \\ 2 \end{bmatrix} = 5.2 \begin{bmatrix} 1 \\ 0 \\ 0.39 \end{bmatrix} = 5.2X_3$$

$$AX_3 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.39 \end{bmatrix} = \begin{bmatrix} 5.39 \\ 0 \\ 2.95 \end{bmatrix} = 5.39 \begin{bmatrix} 1 \\ 0 \\ 0.55 \end{bmatrix} = 5.39 X_4$$

$$AX_4 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 5.55 \\ 0 \\ 3.75 \end{bmatrix} = 5.55 \begin{bmatrix} 1 \\ 0 \\ 0.68 \end{bmatrix} = 5.55 X_5$$

$$AX_5 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.68 \end{bmatrix} = \begin{bmatrix} 5.68 \\ 0 \\ 4.4 \end{bmatrix} = 5.68 \begin{bmatrix} 1 \\ 0 \\ 0.78 \end{bmatrix} = 5.68 X_6$$

$$AX_6 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.78 \end{bmatrix} = \begin{bmatrix} 5.78 \\ 0 \\ 4.9 \end{bmatrix} = 5.78 \begin{bmatrix} 1 \\ 0 \\ 0.85 \end{bmatrix} = 5.78 X_7$$

$$AX_7 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.85 \end{bmatrix} = \begin{bmatrix} 5.85 \\ 0 \\ 5.25 \end{bmatrix} = 5.85 \begin{bmatrix} 1 \\ 0 \\ 0.9 \end{bmatrix} = 5.85 X_8$$

$$AX_8 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.9 \end{bmatrix} = \begin{bmatrix} 5.9 \\ 0 \\ 5.5 \end{bmatrix} = 5.9 \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = 5.9 X_9$$

$$AX_9 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \begin{bmatrix} 5.93 \\ 0 \\ 5.65 \end{bmatrix} = 5.93 \begin{bmatrix} 1 \\ 0 \\ 0.95 \end{bmatrix} = 5.93 X_{10}$$

$$AX_{10} = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.95 \end{bmatrix} = \begin{bmatrix} 5.95 \\ 0 \\ 5.75 \end{bmatrix} = 5.95 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 5.95 X_{11}$$

$$AX_{11} = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 6 X_{12}$$

$$AX_{12} = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 6 X_{13}$$

$$\therefore \lambda_1 = 6 \quad ; \quad \text{Eigenvector} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

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$$B = A - 6I = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & -1 \end{bmatrix} \text{ Take } Y_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$BY_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = -1Y_2$$

$$BY_2 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = -2Y_3$$

$$BY_3 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Greater Eigen value of B = - 2

$\therefore$ Smallest Eigen value of A = - 2 + 6 = 4

$$\lambda_1 + \lambda_2 + \lambda_3 = -2 + 5 = 8$$

$$6 + 4 + \lambda_3 = 8 \Rightarrow \lambda_3 = -2$$

Eigen values are 6 ,4, -2

(ii) Find the solution, to three decimals, of the system using Gauss-Seidal method $8x + 11y - 4z = 95$; $7x + 52y + 13z = 104$ *and* $3x + 8y + 29z = 71$.

Ans :

The diagonal elements of the given system or not dominant. Hence the system of equations cannot be solved by Gauss seidal method.

b(i). Find the inverse of the matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{pmatrix}$ using Gauss-Jordan method.

Ans : $[A, I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & -2 & 4 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 1 & 0 \\ 0 & 3 & 1 & -1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \Leftrightarrow R_2 - R_1 \\ R_3 \Leftrightarrow R3 - R_1 \end{array}$$

$$\sim\left[\begin{array}{ccc|ccc}1&-1&1&1&0&0\\0&1&-3&1&-1&0\\0&3&1&-1&0&1\end{array}\right]\quad R_2\Leftrightarrow -R_2$$

$$\sim\left[\begin{array}{ccc|ccc}1&-1&1&1&0&0\\0&1&-3&1&-1&0\\0&0&10&-4&3&1\end{array}\right]\quad R_3\Leftrightarrow R_3-3R_2$$

$$\sim\left[\begin{array}{ccc|ccc}1&-1&1&1&0&0\\0&1&-3&1&-1&0\\0&0&1&-2/5&3/10&1/10\end{array}\right]\quad R_3\Leftrightarrow R_3/10$$

$$\sim\left[\begin{array}{ccc|ccc}1&-1&0&7/5&-3/10&-1/10\\0&1&0&-1/5&-1/10&3/10\\0&0&1&-2/5&3/10&1/10\end{array}\right]\quad \begin{array}{l}R_1\Leftrightarrow R_1-R_3\\R_2\Leftrightarrow R_2+3R_3\end{array}$$

$$\sim\left[\begin{array}{ccc|ccc}1&0&0&6/5&-2/5&1/5\\0&1&0&-1/5&-1/10&3/10\\0&0&1&-2/5&3/10&1/10\end{array}\right]$$

$$A^{-1}=\frac{1}{10}\begin{bmatrix}12&-4&2\\-2&-1&3\\-4&3&1\end{bmatrix}$$

(ii) Solve the system of equations using Gauss-elimination method.
$5x-2y+z=4$; $7x+y-5z=8$ *and* $3x+7y+4z=10$.

Ans : The augmented matrix is

$$[A\,,B]=\left(\begin{array}{ccc|c}5&-2&1&4\\7&1&-5&8\\3&7&4&10\end{array}\right)$$

$$\sim\left(\begin{array}{ccc|c}5&-2&1&4\\0&19&-32&12\\0&41&17&38\end{array}\right)\quad \begin{array}{l}R_2\Leftrightarrow 5R_2-7R_1\\R_3\Leftrightarrow 5R_3-3R_1\end{array}$$

$$\sim\left(\begin{array}{ccc|c}5&-2&1&4\\0&19&-32&12\\0&0&1635&230\end{array}\right)\quad R_3\Leftrightarrow 19R_3-41R_2$$

By back substitution method

$1635z = 230$

$$z=\frac{230}{1635}\quad\Rightarrow\quad z=0.1407$$

$$19y - 32z = 12$$
$$19y = 12 + 32(0.1407) = 16.5024$$
$$y = 0.8685$$
$$5x - 2y + z = 4$$
$$5x = 4 + 2(0.8685) - 0.1407$$
$$x = 1.1193$$

14.a(i) Given the table of values

x	50	52	54	56
$\sqrt[3]{x}$	3.684	3.732	3.779	3.825

Using Lagrange's formula to find $\sqrt[3]{53}$

Ans :

Let $y = \sqrt[3]{x}$

Given

x	(x_0) 50	(x_1) 52	(x_2) 54	(x_3) 56
$f(x)$	3.684 (y_0)	3.732 (y_1)	3.779 (y_2)	3.825 (y_3)

By Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-52)(x-54)(x-56)}{(50-52)(50-54)(50-56)}(3.684) + \frac{(x-50)(x-54)(x-56)}{(52-50)(52-54)(52-56)}(3.732) + \frac{(x-50)(x-52)(x-56)}{(54-50)(54-52)(54-56)}(3.779) + \frac{(x-50)(x-52)(x-54)}{(56-50)(56-52)(56-54)}(3.825)$$

When $x = 53$

$$= \frac{(53-52)(53-54)(53-56)}{(50-52)(50-54)(50-56)}(3.684) + \frac{(53-50)(53-54)(53-56)}{(52-50)(52-54)(52-56)}(3.732) + \frac{(53-50)(53-52)(53-56)}{(54-50)(54-52)(54-56)}(3.779) + \frac{(53-50)(53-52)(53-54)}{(56-50)(56-52)(56-54)}(3.825)$$

$$= -0.2305 + 2.09925 + 2.1256875 - 0.23906$$
$$= 3.7556$$

(ii). Given the set of tabulated points (1, -3), (3, 9), (4, 30) and (6, 132) obtain the value of y when $x = 2$ using Newton's divided difference formula.

Ans :

Since the argument are not equally spaced, we will use Newton's divided difference

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
1	-3			
		6		
3	9		5	
		21		1
4	30		10	
		51		
6	132			

By Newton's divided difference formula we have

$$y = f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + \dots\dots$$
$$y = f(x) = -3 + (x-1)(6) + (x-1)(x-3)(5) + (x-1)(x-3)(x-4)(1) + \dots\dots$$
$$y(2) = -3 + (2-1)(6) + (2-1)(2-3)(5) + (2-1)(2-3)(2-4)(1) + \dots\dots$$
$$= 0$$

b.(i) The velocities of a car (running on a straight road) at intervals of 2 minutes are given below.

Time in minutes	0	2	4	6	8	10	12
Velocities in km/hr	0	22	30	27	18	7	0

Apply Simpson's rule to find the distance covered by the car.

Ans :

WKT $v = \dfrac{ds}{dt}$, $t = \int_0^{12} \dfrac{1}{v}\, ds$, take $y = \dfrac{1}{v}$

v	0	2	4	6	8	10	12
$y = \dfrac{1}{v}$	0 (y_0)	22 (y_1)	30 (y_2)	27 (y_3)	18 (y_4)	7 (y_5)	0 (y_6)

By Simpson's 1/3 rule

$$\therefore t = \int_0^{12} \frac{1}{v}\, ds = \frac{h}{3}[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$
$$= \frac{2}{3}[(0 + 0) + 2(30 + 18) + 4(22 + 27 + 7)]$$
$$= 213.33 \text{ km}$$

(ii). Find the first and second derivatives of $f(x)$ at $x=1.5$ if

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.000	13.625	24.000x	38.875	59.000

Ans :

x	y	Δ	Δ^2	Δ^3	Δ^4
1.5	3.375				
		3.625			
2.0	7.000		3.000		
		6.625		0.750	
2.5	13.625		3.750		0
		10.375		0.750	
3.0	24.000		4.500		0
		14.875		0.750	
3.5	38.875		5.250		
		20.125			
4.0	59.000				

$x_0 = 1.5$, $y_0 = 3.375$, $\Delta y_0 = -2$, $\Delta^2 y_0 = 3$, $\Delta^3 y_0 = 0.75$, $h = 0.5$

$$\left[\frac{dy}{dx}\right]_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 - \frac{1}{3}\Delta^3 y_0 + ...\right]$$

$$\left[\frac{dy}{dx}\right]_{x=x_0} = f'(1.5) = \frac{1}{0.5}\left[3.625 - \frac{1}{2}(3) + \frac{1}{3}(4.75)\right] = 4.75$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_0} = f''(1.5) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + ...\right]$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_0} = f''(1.5) = \frac{1}{(0.5)^2}\left[3 - 0.75\right] = 9$$

15.a(i) Compute y(0.4) and y(0.5) give that $y' = y - \frac{2x}{y}$, $y(0)=1$, $y(0.1)=1.0954$, $y(0.2)=1.1832$, $y(0.3)=1.2649$ using Milne's predictor-corrector method.

Ans :

Given $y' = f(x,y) = y - \frac{2x}{y}$

$x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$

$y_0 = 1$, $y_1 = 1.0954$, $y_2 = 1.1832$, $y_3 = 1.2649$

To find y (0.4)

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}[2y_{n-2}' - y_{n-1}' + 2y_n']$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}[y_{n-1}' + 4y_n' + y_{n+1}']$$

$$y_{4,p} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$$

$$y_1' = y_1 - \frac{2x_1}{y_1} = 1.0954 - \frac{2(0.1)}{1.0954} = 0.91128$$

$$y_2' = y_2 - \frac{2x_2}{y_2} = 1.1832 - \frac{2(0.2)}{1.1832} = 0.8451$$

$$y_3' = y_3 - \frac{2x_3}{y_3} = 1.2649 - \frac{2(0.3)}{1.2649} = 0.7906$$

$$y_{4,p} = 1 + \frac{4(0.1)}{3}[2(0.9128) - 0.8451 + 2(0.7906)] = 1.3415$$

$$y_{4,c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$y_4' = y_4 - \frac{2x_4}{y_4} = 1.3415 - \frac{2(0.4)}{1.3415} = 0.7452$$

$$y_{4,c} = 1.1832 + \frac{0.1}{3}[0.8451 + 4(0.7906) + 0.7452] = 1.3415$$

To find y (0.5)

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}[2y_{n-2}' - y_{n-1}' + 2y_n']$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}[y_{n-1}' + 4y_n' + y_{n+1}']$$

$$y_{5,p} = y_1 + \frac{4h}{3}[2y_2' - y_3' + 2y_4']$$

$$y_{5,p} = 1.0954 + \frac{4(0.1)}{3}[2(0.8451) - 0.7906 + 2(0.7452)] = 1.414$$

$$y_{5,c} = y_3 + \frac{h}{3}[y_3' + 4y_4' + y_5']$$

$$y_5' = y_5 - \frac{2x_5}{y_5} = 1.414 - \frac{2(0.5)}{1.414} = 0.7068$$

$$y_{5,c} = 1.2649 + \frac{0.1}{3}[0.7906 + 4(0.7452) + 0.7068] = 1.414$$

(ii). Solve by Taylor's method to find an approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$. Compare the numerical solution with exact solution. Use first three non-zero terms in the series.

Ans :

Here $h = 0.1$, $x_0 = 0$, $y_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$

$y' = 2y + 3e^x$ $\quad\quad$ $y_0' = 2y_0 + 3e^{x_0} = 3$

$y'' = 2y' + 3e^x$ $\quad\quad$ $y_0'' = 2y_0' + 3e^{x_0} = 9$

$y''' = 2y'' + 3e^x$ $\quad\quad$ $y_0''' = 2y_0'' + 3e^{x_0} = 21$

$y^{iv} = 2y''' + 3e^x$ $\quad\quad$ $y_0^{iv} = 2y_0''' + 3e^{x_0} = 45$

By Taylor's series formula

$$y_1 = y_0 + \frac{h}{1}y_0' + \frac{h^2}{2}y_0'' + \frac{h^3}{6}y_0''' + \frac{h^4}{24}y_0^{iv} + ...$$

$$y(0.1) = y_1 = 0 + (0.1)(3) + \frac{(0.01)}{2}(9) + \frac{(0.001)}{6}(21) + \frac{(0.0001)}{24}(45) + ...$$

$= 0.3 + 0.045 + 0.0035 + 0.0001875 + \ldots$

$= 0.3486875 \quad = \quad 0.349$

$y_1' = 2y_1 + 3e^{x_1} = 0.3486875 \text{ x } 2 + 3\,e^{0.1} = 4.012887$

$y_1'' = 2y_1' + 3e^{x_1} = 11.34$

$y_1''' = 2y_1'' + 3e^{x_1} = 25.996$

$$y_2 = y\,(0.2) = y_1 + \frac{h}{1}y_1' + \frac{h^2}{2}y_1'' + ...$$

$= 0.3486875 + (0.1)(4.012887) + \frac{0.01}{2}(11.34) + \frac{0.001}{6}(25.996) + ...$

$= 0.8110156 \quad = \quad 0.8110$

b.(i) Consider the initial value problem $\frac{dy}{dx} = y - x^2 + 1$; $y(0) = 0.5$.

Compare y (0.2) by Euler's method and modified Euler's method.

Ans :

$f(x, y) = y - x^2 + 1$, $x_0 = 0$, $y_0 = 0.5$, $h = 0.2$, $x_1 = 0.2$

Modified Euler method is

$$y_{n+1} = y_n + h\, f\left[x_n + \frac{h}{2}\ ,\ y_n + \frac{h}{2}\, f\,(x_n\,, y_n\,)\right]$$

$$y_1 = y_0 + h\, f\left[x_0 + \frac{h}{2}\ ,\ y_0 + \frac{h}{2} f(x_0, y_0)\right]$$

$$y(0.02) = 0.5 + (0.2)\, f\left[0 + \frac{0.2}{2}\ ,\ 0.5 + \frac{0.2}{2} f(0, 0.5)\right]$$

$$= 0.5 + (0.2)\, f\,[0.1\ ,\ 0.5 + 0.1(0.5 - 0 + 1)\ ]$$

$$= 0.5 + (0.2)\, f\,[0.1\ ,\ 0.5 + 0.15] = 0.5 + (0.2)\, f\,[0.1\ ,\ 0.65]$$

$$= 0.5 + (0.2)[0.65 - (0.1)^2 + 1] = 0.5 + 0.328$$

$$= 0.828$$

(ii). Solve $\frac{dy}{dx} = xy + y^2$; $y(0) = 1$ find $y(0.1)$, $y(0.2)$ using fourth order Runge – Kutta method

Ans :

Given $y' = f(x,y) = xy + y^2$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.1$, $x_2 = 0.2$, $h = 0.1$

To find y(0.1) :

$$k_1 = h\, f(x_0\ ,\ y_0) = (0.1)[\,x_0\, y_0 + y_0^2\,] = (0.1)(0 + 1) = 0.1$$

$$k_2 = h\, f\left[x_0 + \frac{h}{2}\ ,\ y_0 + \frac{k_1}{2}\right] = (0.1) f\left[0 + \frac{0.1}{2}\ ,\ 1 + \frac{0.1}{2}\right]$$

$$= (0.1)\ f\ [0.05\ ,1.05\ ]$$

$$= (0.1)\left[(0.05)(1.05) + (1.05)^2\right] = 0.1155$$

$$k_3 = h\, f\left[x_0 + \frac{h}{2}\ ,\ y_0 + \frac{k_2}{2}\right] = (0.1) f\left[0 + \frac{0.1}{2}\ ,\ 1 + \frac{0.1155}{2}\right]$$

$$= (0.1)\ f\ [0.05\ ,1.05775\ ]$$

$$= (0.1)\left[(0.05)(1.05775) + (1.05775)^2\right] = 0.1172$$

$$k_4 = h\ f(x_0 + h, y_0 + k_3) = (0.1)\ f\ [\,0.1\ ,1.1172\ ]$$

$$= (0.1)\left[(0.1)(1.1172) + (1.1172)^2\right]$$

$$= 0.1136$$

$$\Delta y = \frac{1}{6}[\,k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[\,0.1 + 2(0.1155) + 2(0.1172) + 0.136\ ]$$

$$= 0.1169$$

$$y(0.1) = y_1 = y_0 + \Delta y = 1 + 0.1169 = 1.1169$$

To find y(0.2) :

$k_1 = h f(x_1, y_1) = (0.1)[x_1 y_1 + y_1^2]$

$= (0.1)\left[(0.1)(1.1169) + (1.1169)^2\right]$

$= 0.1359$

$k_2 = h f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right] = (0.1) f\left[0.1 + \frac{0.1}{2}, 1.1169 + \frac{0.1359}{2}\right]$

$= (0.1) f [0.15, 1.1849]$

$= (0.1)\left[(0.15)(1.1849) + (1.1849)^2\right]$

$= 0.1582$

$k_3 = h f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_3}{2}\right] = (0.1) f\left[0.1 + \frac{0.1}{2}, 1.1169 + \frac{0.1582}{2}\right]$

$= (0.1) f [0.15, 1.196]$

$= (0.1)\left[(0.15)(1.196) + (1.196)^2\right] = 0.16098$

$k_4 = h f(x_1 + h, y_1 + k_3) = (0.1) f [0.2, 1.1169 + 0.16098]$

$(0.1) f [0.2, 1.2779]$

$= (0.1)\left[(0.2)(1.2779) + (1.2779)^2\right]$

$= 0.1825$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.1359 + 2(0.1582) + 2(0.16098) + 0.1825]$

$= 0.1595$

$y(0.2) = y_2 = y_1 + \Delta y = 1.1169 + 0.1595 = 1.2764$

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B.E./ B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA6452 –STATISTICS AND NUMERICAL METHODS
(Regulation 2013)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. Define Type – I and Type – II errors.

Ans:

In sampling theory to draw valid inferences about the population parameter on the basis of the sample results. We decide to accept or to reject the H_o after examining a sample from it.

Type I Error : If H_o is rejected while it should have been accepted.
Type II Error: If H_o is accepted while it should have been rejected.

	Decision	
	Accept H_o	**Reject H_o**
H_0 True	Correct decision	Type I error
H_0 False	Type II error	Correct decision

2. State the conditions for applying χ^2 test.

Ans: (i) The Sample observations should be independent.
(ii) Constraints on the cell frequencies, if any, must be linear.
(iii) N, the total frequency, should be atleast 50.
(iv) No theoretical cell frequency should be less than 5.

3. What are the basic principles of experimental design ?

Ans:

There are three basic principles of experimental design.
They are (i) Randomization
(ii) Replication (iii) Local control (error control)

4. State any two advantages of a completely Randomized Experimental Design.

Ans: The following are the main advantages of this type of design:
1. It is easy, to lay out the design.
2. It allows for complete flexibility. Any number of factor classics and replications may be used.
3. The Statistical analysis is relatively simple, even if we do not have the same number of replicates for each factor class or if the experimental errors are not the same from class to class of this factor.
4. The method of analysis remains simple when data re missing or rejected and the loss of information due to missing data is smaller than with any other design.

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5. State the order of convergence and condition for convergence of Newton-Raphson method.

Ans:

The order of convergence is 2.

Condition for convergence is $|f(x) f''(x)| < |f'(x)|^2$

6. Write the procedure involved in Gauss elimination method.

Ans:

In this method, starting with the augmented matrix of the system, using elementary row operations, we transform the augmented matrix into an upper triangular matrix.

7. State any two properties of divided differences.

Ans:

(i) The divided differences are symmetrical in all their arguments. i.e., the value of any difference is independent of the order of the arguments.

(ii) The divided differences of the sum or difference of two functions is equal to the sum or difference of the corresponding separate divided differences.

8. What is inverse interpolation ?

Ans:

Suppose we are given a table of values of x and y. Direct interpolation is the process of finding the vales of y corresponding to a value of x, not present in the table. Inverse interpolation is the process of finding the values of x corresponding to a Value of y, not present in the table.

9. State the advantages of Runge-Kutta method over Taylor series method.

Ans:

R-K methods do not require prior calculation of higher derivatives of y(x) as the Taylor method does.

Since the differential equations are using in applications often complicated, the calculation of derivatives may be difficult.

Also the R-K formulas involve the computation of f(x,y) at various positions, instead of derivatives and this function occurs in the given equation.

10. Convert the differential equation $y''(x) + y'(x) + y = 0$ into finite difference equivalent form.

Ans:

$$\frac{1}{h^2}(y_{i-1} - 2y_i + y_{i+1}) + \frac{1}{2h}(y_{i+1} - y_{i-1}) + y_i = 0$$

$$\left(1 - \frac{h}{2}\right) y_{i-1} + (-2 + h^2) y_i + \left(1 + \frac{h}{2}\right) y_{i+1} = 0$$

Agni College of Technology

Chennai - 130

PART – B (5 x 15 = 60)

11.a(i) A manufacture of light bulbs claims that an average of 2% of the bulbs manufactured by him are defective. A random sample of 400 bulbs contained 13 defective bulbs. On the basis of the sample, can you support the manufacturer's claim at 5% level of significance ?

Ans: Out of syllabus

(ii) A survey of 320 families with 5 children each revealed the following distribution :

No. of boys	5	4	3	2	1	0
No. of girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

Is this result consistent with the hypothesis that male and female births are equally probable ?

Ans:

1. H_0: Male and Female births are equally probable.
2. H_1: Male and Female births are not equally probable.
3. α = 0.05, d.f = n-1 = 6-1 = 5
4. Table value of χ^2 =11.07
5. The test statistic is $\chi^2 = \sum\frac{(O-E)^2}{E}$

On the assumption H_0,

the expected frequencies frequencies are given by the terms of $N(q+p)^n$

$$= 320\left(\frac{1}{2}+\frac{1}{2}\right)^5 = 320\left[5C_0 + 5C_1 + 5C_2 + 5C_3 + 5C_4 + 5C_5\right]$$

$$= 10\,[1+5+10+10+5+1]$$

The expected frequencies are 10, 50, 100, 100, 50, 10

No. of boys	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
5	14	10	4	16	1.6
4	56	50	6	36	0.72
3	110	100	10	100	1
2	88	100	-12	144	1.44
1	40	50	-10	100	2
0	12	10	2	4	0.4
Total	320				7.16

$$\therefore\ \chi^2 = \sum\frac{(O-E)^2}{E} = 7.16$$

8. Conclusion:

If Cal χ^2 < table χ^2, then we accept H_0 . Otherwise , we reject H_0

Here, 7.16 < 11.06 , so we accept H_0

b(i) In a random sample of 100 men taken from village A, 60 were found to be consuming alcohol. In another sample of 200 men taken from village B, 100 were found to be consuming alcohol. Do the two villages differ significantly in respect of the proportion of men who consume alcohol ?

Ans: Out of syllabus

(ii) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables.

Sample I	18	13	12	15	12	14	16	14	15
Sample II	16	19	13	16	18	13	15		

Do the estimates of population variance differ significantly at 5% level of significance ?

Ans:

Given $n_1 = 9$, $n_2 = 7$

											Total
Sample I	x_1	18	13	12	15	12	14	16	14	15	129
	x_1^2	18^2	13^2	12^2	15^2	12^2	14^2	16^2	14^2	15^2	1871
Sample II	x_2	16	19	13	16	18	13	15			110
	x_2^2	16^2	19^2	13^2	16^2	18^2	13^2	15^2			1760

$$\overline{x_1} = \frac{\sum x_1}{n_1} = \frac{129}{9} = 14.333 \quad \& \quad \overline{x_2} = \frac{\sum x_2}{n_2} = \frac{110}{7} = 15.7143$$

$$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x_1})^2 = \frac{1879}{9} - (14.3333)^2 = 3.3342$$

$$s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x_2})^2 = \frac{1760}{7} - (15.7143)^2 = 4.4894$$

$$s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(9)(3.3342)}{8} = 3.751$$

$$s_2^2 = \frac{n_{21} s_2^2}{n_2 - 1} = \frac{(7)(4.4894)}{6} = 5.2376$$

$$s_2^2 > s_1^2$$

The parameter of interest is σ_1^2 and σ_1^2

1. $H_0 : \sigma_1^2 = \sigma_2^2$
2. $H_1 : \sigma_1^2 \neq \sigma_2^2$
3. $\alpha = 0.05$, $d.f\,(v_1) = n_1 - 1 = 9 - 1 = 8$, $d.f\,(v_2) = n_2 - 1 = 7 - 1 = 6$
4. Table value of $F = 3.58$
5. The test statistic is $F = \dfrac{s_2^2}{s_1^2} = \dfrac{5.2376}{3.7510} = 1.3963$

6.Conclusion :

If Cal F < table F , then we accept H_0 ; otherwise we reject H_0

Here , F = 1.3963 < 3.58 , we accept H_0 at 5 % level of significance.

We conclude that the difference is not significant.

12.a. Four varieties A, B, C and D of a fertilizer are tested in a Randomized Block Design with four replications. The plot yields in pounds are as follows :

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

Analyses the experimental yield.

Ans :

H_0 : Four varieties are similar.

H_1 : Four varieties are not similar

Variety	(X_1)	(X_2)	(X_3)	(X_4)	Total	X_1^2	X_2^2	X_3^2	X_4^2
A	12	14	15	13	54	144	196	225	169
B	12	11	11	10	44	144	121	121	100
C	16	15	16	14	61	256	225	256	196
D	18	20	19	20	77	324	400	361	400
Total	58	60	61	57	236	868	942	363	865

Step 1 : N = 16

Step 2. T = 236

Step 3. C.F. = $\dfrac{T^2}{N} = \dfrac{(236)^2}{16} = 3481$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \dfrac{T^2}{N}$

$= 868 + 942 + 963 + 865 - 3481 = 157$

Step 5. SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(58)^2}{4} + \frac{(60)^2}{4} + \frac{(61)^2}{4} + \frac{(57)^2}{4} - 3481 = 2$$

Step 6. SSC $= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2= number of elements in each rows]

$$= \frac{(54)^2}{4} + \frac{(44)^2}{4} + \frac{(61)^2}{4} + \frac{(77)^2}{4} - 3481$$

$= 144.5$

Step 7: The analysis of variance table

SSE = TSS – SSC – SSR = 157 – 2 - 144.5 = 10.5

ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 2	c –1 = 4 – 1 = 3	MSC $= \frac{SSC}{C-1}$ $= \frac{2}{3} = 0.67$	$F_C = \frac{MSE}{MSC}$ $= \frac{1.17}{0.67}$ $= 1.75$	$F_C(9,3) = 8.81$
Between blocks	SSR = 144.5	r –1 = 4 – 1 = 3	MSR $= \frac{SSR}{r-1}$ $= \frac{144.5}{3} = 48.17$	$F_R = \frac{MSR}{MSE}$ $= \frac{48.17}{1.17}$ $= 41.17$	$F_R(3,9) = 3.86$
Residual	SSE = 10.5	N – c - r + 1 = 9	MSE = $\frac{SSE}{N - c - r + 1} =$ $\frac{10.5}{9} = 1.17$		

Step 7 : Conclusion :

Cal F_C < Table F_C ; Cal F_R > Tab F_R

b. Analyse the variance in the Latin square of yields (in kgs) of paddy where P, Q , R , S denote the different method of cultivation :

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Estimate whether different method of cultivation have significantly different yields.

Ans :

Subtract 120 we get

S	2	P	1	R	3	Q	2
Q	4	R	3	P	2	S	5
P	0	Q	-1	S	0	R	1
R	2	S	3	Q	1	P	2

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	2	1	3	2	8	4	1	9	4
Y_2	4	3	2	5	14	16	9	4	25
Y_3	0	-1	0	1	0	0	1	0	1
Y_4	2	3	1	2	8	4	9	1	4
Total	8	6	6	10	30	24	20	14	34

Step 1. N = 16 [Total number of entries]

Step 2. T = 30

Step 3. $\frac{T^2}{N} = \frac{(30)^2}{16} = 56.25$

Step 4. TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 24 + 20 + 14 + 34 - 56.25 = 35.75$

Step 5. SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{64}{4} + \frac{36}{4} + \frac{36}{4} + \frac{100}{4} - 56.25 = 2.75$

Step 6. SSR $= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{64}{4} + \frac{196}{4} + \frac{0}{4} + \frac{64}{4} - 56.25 = 24.75$

To find SSK :

Arrange the elements in the order of treatment

					Total
P	0	1	2	2	5
Q	4	-1	2	2	6
R	2	3	1	1	9
S	2	3	5	5	10

$$SSK = \frac{(5)^2}{4} + \frac{(6)^2}{4} + \frac{(9)^2}{4} + \frac{(10)^2}{4} - \frac{T^2}{N}$$

$$= \frac{25}{4} + \frac{36}{4} + \frac{81}{4} + \frac{100}{4} - 56.25 = 4.25$$

SSE = TSS – SSC – SSR – SSK = 35.75 – 2.75 – 24.75 – 4.25 = 4

Step 7. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 1% level
Between Rows	SSR = 24.75	k –1 = 3	$MSR = \frac{SSR}{k-1}$ = 8.25	$F_R = \frac{MSR}{MSE}$ = 12.31	$F_R(3, 6)$ = 4.76
Between columns	SSC = 2.75	k –1 = 3	$MSC = \frac{SSC}{k-1}$ = 0.92	$F_C = \frac{MSC}{MSE}$ = 1.37	$F_C(3, 6)$ = 4.76
Between treatments	SSK = 4.25	k –1 = 3	$MSK = \frac{SSK}{k-1}$ = 1.42	$F_T = \frac{MSK}{MSE}$ = 2.12	$F_R(3, 6)$ = 4.76
Residual	SSE = 4	(k –1)(k - 2) = 6	$MSE = \frac{SSE}{(k-1)(k-2)}$ = 0.67		
Total	TSS = 35.75	$n^2 - 1$ = 15			

Step 8 : Conclusion :

(i) Cal F (F_T) < Table F . There is no significant difference between treatments

(ii) Cal F (F_R) > Table F . There is significant difference rows

(iii) Cal F (F_C) < Table F . There is no significant difference columns.

13.a(i) Solve the equation $x\log_{10} x = 1.2$ using Newton – Raphson method.

Ans : Let $f(x) = x\log_{10} x - 1.2$

$f(1) = \log_{10} 1 - 1.2 = -1.2 \quad (-ve)$

$f(2) = \log_{10} 2 - 1.2 = -0.598 \quad (-ve)$

$f(3) = \log_{10} 3 - 1.2 = 0.231 \quad (+ve)$

A root lies between 2 and 3

Let $x_0 = 2.7$

$f(x) = x\log_{10} x - 1.2$

$$f'(x) = \left[x\frac{1}{x}\log_{10} e\right] + \log_{10} x \quad = \log_{10} e + \log_{10} x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.7 - \frac{f(2.7)}{f'(2.7)} = 2.7 - \left[\frac{(2.7)\log_{10}(2.7) - 1.2}{\log_{10} e + \log_{10} 2.7}\right] = 2.740$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.740 - \frac{f(2.74)}{f'(2.74)} = 2.74 - \left[\frac{(2.74)\log_{10}(2.74) - 1.2}{\log_{10} e + \log_{10} 2.74}\right] = 2.741$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.741 - \frac{f(2.741)}{f'(2.741)} = 2.741 - \left[\frac{(2.741)\log_{10}(2.741) - 1.2}{\log_{10} e + \log_{10} 2.741}\right] = 2.741$$

Hence the root is 2.741

(ii) By Gauss Jordan elimination method. Find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 2 & 1 & 2 \end{pmatrix}$

Ans :

$$[A, I] = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 2 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \quad R_1 \Leftrightarrow \frac{1}{2}R_1$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & -1/2 & -3/2 & -1/2 & 1 & 0 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \Leftrightarrow R_2 - R_1 \\ R_3 \Leftrightarrow R_3 - 2R_1 \end{array}$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 3 & 1 & -2 & 0 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right] \quad R_2 \Leftrightarrow -2R_2$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 & -2 & 0 \\ 0 & 0 & 7 & 1 & -4 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \Leftrightarrow R_1 - \frac{1}{2}R_2 \\ R_3 \Leftrightarrow R_3 + 2R_2 \end{array}$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 3 & -1 & -2 & 0 \\ 0 & 0 & 1 & 1/7 & -4/7 & 1/7 \end{array}\right] \quad R_3 \Leftrightarrow \frac{R_3}{7}$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/7 & 3/7 & 1/7 \\ 0 & 1 & 0 & 4/7 & -2/7 & -3/7 \\ 0 & 0 & 1 & 1/7 & -4/7 & 1/7 \end{array}\right] \quad \begin{array}{l} R_1 \Leftrightarrow R_1 + R_3 \\ R_2 \Leftrightarrow R_2 - 3R_3 \end{array}$$

$$A^{-1} = \begin{bmatrix} 1/7 & 3/7 & 1/7 \\ 4/7 & -2/7 & -3/7 \\ 1/7 & -4/7 & 1/7 \end{bmatrix}$$

(b)(i) Solve the following set of equation using Gauss- Seidal iterative procedure
$-10x + 2y + 2z = 4 \;;\; x - 10y + 2z = 18 \;;\; x + y - 10z = 45$

Ans :

$$x = \frac{1}{10}(2y + 2z - 4) \quad ; \quad y = \frac{1}{10}(x + 2z - 18) \quad ; \quad z = \frac{1}{10}(x + y - 45)$$

Let $y_0 = z_0 = 0$

Gauss Seidel is

x	y	z
2.5	2.8235	1.6667
2.5216	2.8725	1.0752
2.5840	2.8017	1.0673
2.5801	2.7971	1.06825
2.5797	2.7975	1.0692

$\therefore \quad x = 2.580 \;;\; y = 2.797 \;;\; z = 1.069$

(ii) Find the numerically largest eigen value of $\begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$ by using power method.

Ans : Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an arbitrary initial eigenvector

$$AX_1 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} = 6 \begin{bmatrix} 0.167 \\ 0.667 \\ 1 \end{bmatrix} = 6X_2$$

$$AX_2 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.167 \\ 0.667 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.166 \\ 2.336 \\ 8.003 \end{bmatrix} = 8.003 \begin{bmatrix} 0.021 \\ 0.292 \\ 1 \end{bmatrix} = 8.003X_3$$

$$AX_3 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.021 \\ 0.292 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.145 \\ 0.252 \\ 6.002 \end{bmatrix} = 6.002 \begin{bmatrix} 0.191 \\ 0.042 \\ 1 \end{bmatrix} = 6.002X_4$$

$$AX_4 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.191 \\ 0.042 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.0685 \\ -0.068 \\ 6.272 \end{bmatrix} = 6.272 \begin{bmatrix} 0.329 \\ -0.011 \\ 1 \end{bmatrix} = 6.272X_5$$

$$AX_5 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.329 \\ -0.011 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.362 \\ 0.272 \\ 6.941 \end{bmatrix} = 6.941 \begin{bmatrix} 0.34 \\ 0.039 \\ 1 \end{bmatrix} = 6.941X_6$$

$$AX_6 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.34 \\ 0.039 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.223 \\ 0.516 \\ 7.157 \end{bmatrix} = 7.157 \begin{bmatrix} 0.311 \\ 0.072 \\ 1 \end{bmatrix} = 7.157X_7$$

$$AX_7 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.311 \\ 0.072 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.095 \\ 0.532 \\ 7.082 \end{bmatrix} = 7.082 \begin{bmatrix} 0.296 \\ 0.075 \\ 1 \end{bmatrix} = 7.082X_8$$

$$AX_8 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.296 \\ 0.075 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.071 \\ 0.484 \\ 7.001 \end{bmatrix} = 7.001 \begin{bmatrix} 0.296 \\ 0.069 \\ 1 \end{bmatrix} = 7.001X_9$$

$$AX_9 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.296 \\ 0.069 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.089 \\ 0.46 \\ 6.983 \end{bmatrix} = 6.983 \begin{bmatrix} 0.296 \\ 0.066 \\ 1 \end{bmatrix} = 6.983\, X_{10}$$

$$AX_{10} = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.296 \\ 0.066 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.101 \\ 0.46 \\ 6.992 \end{bmatrix} = 6.992 \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = 6.992\, X_{11}$$

$$AX_{11} = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.102 \\ 0.464 \\ 6.998 \end{bmatrix} = 6.998 \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = 6.998\, X_{12}$$

$$AX_{12} = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.102 \\ 0.464 \\ 6.998 \end{bmatrix} = 6.998 \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix}$$

The largest eigen value $= 7$

14.(a)(i) Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(4)$ for

x	1	3	5	7
$f(x)$	24	120	336	720

Ans :

Given

x	(x_0) 1	(x_1) 3	(x_2) 5	(x_3) 7
$f(x)$	24 (y_0)	120 (y_1)	336 (y_2)	720 (y_3)

By Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-3)(x-5)(x-7)}{(1-3)(1-5)(1-7)}(24) + \frac{(x-1)(x-5)(x-7)}{(3-1)(3-5)(3-7)}(120)$$

$$+ \frac{(x-1)(x-3)(x-7)}{(5-1)(5-3)(5-7)}(336) + \frac{(x-1)(x-3)(x-5)}{(7-1)(7-3)(7-5)}(720)$$

$$= \frac{-1}{2}(x-3)(x-5)(x-7) + \frac{15}{2}(x-1)(x-5)(x-7)$$

$$-21(x-1)(x-3)(x-7) + 15(x-1)(x-3)(x-5)$$

$$= \frac{-1}{2}[x^3 - 15x^2 + 71x - 105] + \frac{15}{2}[x^3 - 13x^2 + 47x - 35]$$
$$- 21[x^3 - 11x^2 + 31x - 21] + 15[x^3 - 9x^2 + 23x - 15]$$

$$= \left[\frac{-1}{2} + \frac{15}{2} - 21 + 15\right]x^3 + \left[\frac{15}{2} - \frac{195}{2} + 231 - 135\right]x^2$$
$$+ \left[\frac{-71}{2} + \frac{705}{2} - 605 + 345\right]x + \left[\frac{105}{2} - \frac{525}{2} + 441 - 225\right]$$
$$= x^3 + 6x^2 + 11x + 6$$
$$f(4) = (4)^3 + 6(4)^2 + 11(4) + 6 = 210$$

(ii) Evaluate $\int_0^1 \frac{dx}{1+x}$ by Simpson's 1/3 rule and hence deduce the value of $\log_2^e$.

Ans :

x	0	0.5	1
$y = \frac{1}{1+x}$	1	0.6667	0.5

$$I = \int_0^1 \frac{dx}{1+x} = \frac{h}{3}[(y_0 + y_2) + 4(y_1)]$$
$$= \frac{0.5}{3}[(1+0.5) + 4(0.6667)] = 0.6945$$
$$\int_0^1 \frac{dx}{1+x} = \left[\log_e(1+x)_0^1\right] = \log 2 - \log 1 = 0.693$$

b(i) Construct Newton's forward interpolation polynomial for the following data :

x	1	2	3	4	5
$f(x)$	1	-1	1	-1	1

And hence find f(3.5) , f '(3.5)

Ans :

x	y	Δ	Δ^2	Δ^3	Δ^4
1	1				
		-2			
2	-1		4		
		2		- 8	
3	1		- 4		16
		-2		8	
4	-1		4		
		2			
5	1				

$$x_0 = 1 \;,\; y_0 = 1 \;,\; \Delta y_0 = -2 \;,\; \Delta^2 y_0 = 4 \;,\; \Delta^3 y_0 = -8$$

$$h = 1 \quad \therefore \; p = \frac{x - x_0}{h} = \frac{x-1}{1} = x - 1$$

$$y(x) = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + ...$$

$$y(x) = 1 + \frac{(x-1)}{1\,!}(-2) + \frac{(x-1)(x-2)}{2\,!}(4) + \frac{(x-1)(x-2)(x-3)}{3\,!}(-8) + \frac{(x-1)(x-2)(x-3)(x-4)}{4\,!}(16)..$$

$$y(3.5) = 1 + \frac{(3.5-1)}{1\,!}(-2) + \frac{(3.5-1)(3.5-2)}{2\,!}(4) + \frac{(3.5-1)(3.5-2)(3.5-3)}{3\,!}(-8) + \frac{(3.5-1)(3.5-2)(3.5-3)(3.5-4)}{4\,!}(16)$$

$$y\,(3.5) = 0.375$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\,\Delta y_0 + \left(\frac{2p-1}{2}\right)\Delta^2 y_0 + \left(\frac{3p^2 - 6p + 2}{6}\right)\Delta^3 y_0 + \left(\frac{4p^3 - 18p^2 + 22p - 6}{24}\right)\Delta^4 y_0 + ...\right.$$

$when \;\; x = 3.5 \quad p = 2.5$

$$\frac{dy}{dx} = -2 + \frac{2(2.5)-1}{2}(4) + \frac{3(2.5)^2 - 6(2.5) + 2}{6}(-8) + \frac{4(2.5)^3 - 18(2.5)^2 + 22(2.5) - 6}{24}(16)$$

$$= -2.3334$$

(ii) The velocity v of a particle at a distance s from a point on its path is given as follows :

s in meter	0	10	20	30	40	50	60
v m/sec	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by using Trapezoidal rule and Simpson's 1/3 rule.

Ans :

WKT $\quad v = \dfrac{ds}{dt} \;,\; t = \displaystyle\int_0^{60} \frac{1}{v}\,ds \quad ,$ take $\; y = \dfrac{1}{v}$

v	47	58	64	65
$y = \frac{1}{v}$	0.02127 (y_0)	0.01724 (y_1)	0.015625 (y_2)	0.01538 (y_3)
v	61	52	38	
$y = \frac{1}{v}$	0.01639 (y_4)	0.01923 (y_5)	0.026316 (y_6)	

By Simpson's 1/3 rule

	y_0	y_1	y_2	y_3	y_4	y_5	y_6
	1	4	1				
			1	4	1		
					1	4	1
Sum	1	4	2	4	2	4	1

$$\therefore t = \int_0^{60} \frac{1}{v}\, ds = \frac{h}{3}[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

$$= \frac{10}{3}[\,(0.02127 + 0.026316) + 2(0.015625 + 0.01639)$$

$$+ 4(0.01724 + 0.1538 + 0.01923)\,] = 1.06338$$

15a(i) Apply Taylor's series method to find and approximation value of y when $x = 0.1, 0.2$ given that $\frac{dy}{dx} = x + y$, $y(0) = 1$

Ans :

Here $h = 0.1$, $x_0 = 0$, $y_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$

$y' = x + y$ $\qquad y_0' = x_0 + y_0 = 1$

$y'' = 1 + y'$ $\qquad y_0'' = 1 + y_0' = 2$

$y''' = y''$ $\qquad y_0''' = y_0'' = 2$

$y^{iv} = y'''$ $\qquad y^{iv} = y_0''' = 2$

By Taylor's series formula

$$y_1 = y_0 + \frac{h}{1} y_0' + \frac{h^2}{2} y_0'' + \frac{h^3}{6} y_0''' + \frac{h^4}{24} y_0^{iv} + \ldots..$$

$$y(0.1) = y_1 = 1 + (0.1)(1) + \frac{(0.01)}{2}(2) + \frac{(0.001)}{6}(2) + \frac{(0.0001)}{24}(2) + \ldots\ldots..$$

$= 1 + 0.1 + 0.01 + 0.00033$

$= 1.11033$

$$y_2 = y\,(0.2) = y_1 + \frac{h}{1} y_1' + \frac{h^2}{2} y_1'' + \ldots\ldots\ldots.$$

$y_1' = x_1 + y_1 = 0.1 + 1.110$

$= 1.2103$

$y_0'' = 1 + y_0' = 1 + 1.2103$

$= 2.2103$

$y_0''' = y_0'' = 2.2103$

$y^{iv} = y_0''' = 2.2103$

$$y(0.1) = y_1 = 1.1103 + (0.1)(1.2103) + \frac{(0.01)}{2}(2.2103) + \frac{(0.001)}{6}(2.2103) + ...$$

$$= 1.24275$$

(ii) Solve the BVP $y'' + y = 0$, $y(0) = 1$, $y(1) = 0$ using finite difference method, taking h = 0.25

Ans :

Given $y'' + y = 0$, $y(0) = 1$, $y(1) = 0$ $h = 0.25$

$x_0 = 0$, $x_n = 1$

since $h = 0.25$, $n = 4$

$x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 0.75$, $y_0 = 1$, $y_4 = 0$

wkt $y_i'' = \dfrac{y_{i+1} - 2y_i + y_{i-1}}{\text{h}^2}$, $i = 1, 2, 3, ...$

$\therefore$ the equation becomes $\dfrac{y_{i+1} - 2y_i + y_{i-1}}{\text{h}^2} + y_i = 0$

$$\Rightarrow \frac{y_{i+1} - 2y_i + y_{i-1}}{(0.25)^2} + y_i = 0$$

$\Rightarrow (16)\,(y_{i+1} - 2y_i + y_{i-1}) + y_i = 0$

$\Rightarrow 16y_{i+1} - 31y_i + 16y_{i-1} = 0$, $i = 1, 2, 3, ...$

put i=1 then $16y_2 - 31y_1 + 16y_0 = 0$

$\Rightarrow 16y_2 - 31y_1 + 16(1) = 0 \;\Rightarrow 16y_2 - 31y_1 = -16$

$\Rightarrow y_1 = \dfrac{16}{31} + \dfrac{16}{31}y_2$ ------(1)

put i=2 then $16y_3 - 31y_2 + 16y_1 = 0$ ------(2)

put i=3 then $16y_4 - 31y_3 + 16y_2 = 0$

$-31y_3 - 16y_2 = 0 \quad \Rightarrow y_3 = \dfrac{16}{31}y_2$ ------(3)

substituting the values of y_1 , y_3 in (2) we get

$$16\,\frac{16}{31}y_2 - 31y_2 + 16\left(\frac{16}{31} + \frac{16}{31}y_2\right) = 0$$

$$\Rightarrow \left(\frac{256}{31}+\frac{256}{31}-31\right)y_2+\frac{256}{31}=0$$

$$\Rightarrow -14.4839\,y_2 = -8.2581 \quad \Rightarrow \quad y_2 = 0.5702$$

$$\therefore (3) \Rightarrow \quad y_3 = \frac{16}{31}(0.5702) = 0.2943$$

$$(1) \Rightarrow y_1 = \frac{16}{31}+\frac{16}{31}(0.5702) = 0.8104$$

$$\therefore \quad y_1 = 0.8104 \;,\; y_2 = 0.5702 \;,\; y_3 = 0.2943$$

b(i) Using Milne's predictor and corrector method find y (4.4) given $5xy'+y^2-2=0$ given y (4) = 1 , y (4.1) = 1.0049 , y (4.2) = 1.0097 and y (4.3) = 1.0143

Ans :

$$y' = \frac{2-y^2}{5x} \;,\quad x_0 = 4 ,\; x_1 = 4.1 ,\; x_2 = 4.2 ,\; x_3 = 4.3$$

$$x_4 = 4.4 \;,\; y_0 = 1 \;,\; y_1 = 1.0049 \;,\; y_2 = 1.0097 \;,\; y_3 = 1.0143$$

$$y_{n+1,p} = y_{n-3}+\frac{4h}{3}[2y_{n-2}'-y_{n-1}'+2y_n']$$

$$y_{n+1,c} = y_{n-1}+\frac{h}{3}[y_{n-1}'+4y_n'+y_{n+1}']$$

$$y_{4,p} = y_0+\frac{4h}{3}[2y_1'-y_2'+2y_3']$$

$$y_1' = \frac{2-y_1^2}{5x_1} = \frac{2-(1.0049)^2}{5(4.1)} = 0.0493$$

$$y_2' = \frac{2-y_2^2}{5x_2} = \frac{2-(1.0097)^2}{5(4.2)} = 0.0467$$

$$y_3' = \frac{2-y_3^2}{5x_3} = \frac{2-(1.0413)^2}{5(4.3)} = 0.0452$$

$$y_{4,p} = 1+\frac{4(0.1)}{3}[2(0.0493)-0.0467+2(0.0452)]$$

$$= 1.01897$$

$$y_{4,c} = y_2+\frac{h}{3}[y_2'+4y_3'+y_4']$$

$$y_4' = \frac{2-y_4^2}{5x_4} = \frac{2-(1.01897)^2}{5(4.4)} = 0.0437$$

$$y_{4,c} = 1.0097+\frac{0.1}{3}[0.0467+4(0.0452)+0.0437] = 1.01874$$

(ii) Evaluate y (1.2) and y (1.4) correct to three decimal places by the modified Euler method, given that $\frac{dy}{dx} = (y - x^2)^3$; $y(1) = 0$ taking h=0.2 .

Ans :

$$\frac{dy}{dx} = (y - x^2)^3 \; ; \; x_0 = 1 \; , \; y_0 = 0 \; , \; h = 0.2 \; , \; x_1 = 1.2$$

Modified Euler' s method is

$$y_{n+1} = y_n + h\, f\left[x_n + \frac{h}{2} \, , \, y_n + \frac{h}{2} f(x_n, y_n)\right]$$

$$y_1 = y_0 + h\, f\left[x_0 + \frac{h}{2} \, , \, y_0 + \frac{h}{2} f(x_0, y_0)\right]$$

$$y(1.2) = 0 + (0.2)\, f\left[1 + \frac{0.2}{2} \, , \, 0 + \frac{0.2}{2} f(1, 0)\right]$$

$$= (0.2)\, f\left[1.1 \, , (0.1)(0-1)^3\right]$$

$$= (0.2)\, f\left[1.1 \, , -0.1\right] = (0.2)[-0.1 - (1.1)^2]^3$$

$$= (0.2)\,[-1.3676]$$

$$= -0.274$$

Agni College of Technology
Chennai - 130

B.E./ B.Tech. DEGREE EXAMINATION, NOV/DEC 2015
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA6452 –STATISTICS AND NUMERICAL METHODS
(Regulation 2013)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. What is random sampling ?
Ans :
A random sampling is one in which each number of population has an equal chance of being included in it. There are NC_n different samples of size n that can be picked up from a population size N.

2. Write about F-test.
Ans :
A random variable F is said to follow F-distribution with (v_1, v_2) degrees of freedom if its density function given by $f(F) = K F^{\frac{v_1}{2}-1}\left(1+\frac{v_1 F}{v_2}\right)_{\infty}^{-\frac{(v_1+v_2)}{2}}$ where F > 0 and k is constant to be determined such that $\int_0^{\infty} f(F)\, dF = 1$

3. Write two probability advantages of completely randomized experimental design.
Ans :
1. It is easy to layout the design.
2. It allows for complete flexibility. Any number of factor classes and replications may be used.

4. Is a 2 x 2 Latin square design possible ? Why ?
Ans :

Consider, a n x n Latin square design, then the degrees of freedom for SSE is

$$= (n^2 - 1) - (n-1) - (n-1) - (n-1)$$
$$= n^2 - 1 - 3n + 3 \quad = n^2 - 3n + 2$$
$$= (n-1)(n-2)$$

For n=2, d.f. of SSE = 0 and hence, MSE is not defined.

$\therefore$ Comparisons are not possible. Hence, 2 x 2 Latin square design is not possible.

5. Compare Gauss elimination with Gauss seidel.

Ans :

	Gauss Elimination Method	Gauss-Sediel method
1.	It works on the basis of elimination of variables.	It employs initial values and iterates to obtain refined estimates.
2.	These can be used for small number of co-efficients.	These can be used for large number of co-efficient.
3.	Its performance is affected due to round-off errors.	There is no round-off error problem. Error is controlled by number of iterations.
4.	Direct method for solving linear simultaneous equations.	Iterative method for solving linear simultaneous equations.
5.	It gives the exact solution in finite number of steps.	Successive approximations get the solutions.

6. Obtain the iterative formula to find $\frac{1}{N}$ using Newton-Raphson's method.

Ans : Let $x = \frac{1}{N}$ $(ie)\ N = \frac{1}{x}$ Let $f(x) = \frac{1}{x} - N$, $f'(x) = \frac{-1}{x^2}$

N-R formula : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \left[\frac{\frac{1}{x_n} - N}{-\frac{1}{x_n^2}}\right] = x_n + x_n^2\left[\frac{1}{x_n} - N\right]$

$= x_n + x_n - Nx_n^2 = 2x_n - Nx_n^2$ $x_{n+1} = x_n[2 - Nx_n]$.

7. Give the Newton's backward difference table for

X	0	1	2	3
Y	-1	-2	-1	2

Ans :

The difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-1			
		-1		
1	-2		2	
		1		0
2	-1		2	
		3		
3	2			

$v = \frac{x - x_3}{h} = \frac{x-3}{1} = x - 3$

8. Compare Trapezoidal rule with Simpson's 1/3 rule.

Ans :

	Trapezoidal rule	Simpson's 1/3 rule
1.	y is a liner function of x	y is a polynomial of degree two
2.	It is least accurate result.	It is more accurate result.

9. If $y' = -y, y(0) = 1$ then find $y(0.1)$ by Euler method.

Ans : Given $f(x,y) = -y$, $x_0 = 0$, $y_0 = 1$

By Euler

$$y_1 = y_0 + h\,f(x_0, y_0)$$

$$= 1 + h[-y_0] = 1 + (x - x_0)[-y_0] = 1 + (x-0)[-1] = 1 - x$$

10. What are single step and multistep methods ? Give an example.

Ans :

Single step : It is not possible to get any information about truncation error.

Example : 1. Taylor's series method 2. Modified Euler's method

Multi-step method : It is possible to get easily a good estimate of the truncation error.

Example : 1. Milne's predictor-corrector method 2. Adam's-Bashforth predictor-corrector method.

PART – B (5 x 15 = 60)

11.a(i) Test if the variances are significantly different for

X_1	24	27	26	21	25	
X_2	27	30	32	36	28	23

Ans :

Given $n_1 = 5$, $n_2 = 6$

x_1	x_1^2	x_2	x_2^2
24	576	27	729
27	729	30	900
26	676	32	1024
21	441	36	1296
25	625	28	784
		23	529
123	3047	176	5262

$\overline{x}_1 = \frac{123}{5} = 24.6$ & $\overline{x}_2 = \frac{176}{6} = 29.33$

$s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x}_1)^2 = \frac{3047}{5} - (24.6)^2 = 4.24$ & $s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x}_2)^2 = \frac{5262}{6} - (29.33)^2 = 16.75$

$s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(5)(4.24)}{4} = 5.3$ & $s_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{(6)(16.7511)}{5} = 20.10$

$s_2^2 > s_1^2$

$H_0 : \sigma_1^2 = \sigma_2^2$ & $H_1 : \sigma_1^2 \neq \sigma_2^2$

$\alpha = 5\% = 0.05$ $d.f.\ v_1 = n_1 - 1 = 4$ & $v_2 = n_2 - 1 = 5$

$F = \frac{s_2^2}{s_1^2} = \frac{20.10}{5.3} = 3.79$ $F(5,4) = 6.26$

$\therefore$ Cal F < Table F , $\therefore H_0$ is accepted.

(ii). The number of automobile accidents in a certain locality was 12, 8, 20, 2, 14, 10, 15, 6, 9, 4. Are these frequencies in agreement with the belief that accident conditions were the same during this 10 week period.

Ans : Given $n = 10$

x	12	8	20	2	14	10	15	6	9	4	100
x^2	12^2	8^2	20^2	2^2	14^2	10^2	15^2	6^2	9^2	4^2	1266

$\overline{x} = \frac{\sum x}{n} = \frac{100}{10} = 10$

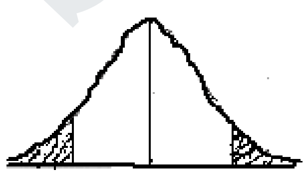
$s^2 = \frac{\sum x^2}{n} - (\overline{x})^2 = \frac{1266}{10} - 100 = 26 \Rightarrow s = 5.16$

1. $H_0 : \mu = 0$ [*No change in during this 10 week period*]

2. $H_1 : \mu \neq 0$ [*There is a change in this 10 week period*]

3. $\alpha = 5\%$, $d.f = n - 1 = 10 - 1 = 9$

4. critical region



$-t_{\alpha/2} = -2.26 \sim 5\%$ $-t_{\alpha/2} = 2.26 \sim 5\%$

5. The test statistic $t = \dfrac{\bar{x}-\mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)} = \dfrac{10-0}{\left(\dfrac{5.16}{\sqrt{10-1}}\right)} = 5.81$

6. Conclusion : If $-t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}}$ we accept H_0 , otherwise we reject H_0

Here -2.26 < 5.81 $\not<$ 2.26

$\therefore\ H_0$ at 5% level of significance.

b.(i) A certain pesticide is packed into bags by a machine. A random sample of 10 bags is chosen and the contents of the bags is found to have the following weights (in kgs) 50, 49, 52, 44, 45, 48, 46, 45, 49 and 45. Test if the average quantity packed be taken as 50 kg.

Ans :

Given $n = 10$, μ =50 kg

x	50	49	52	44	45	48	46	45	49	45	473
x^2	50^2	49^2	52^2	44^2	45^2	48^2	46^2	45^2	49^2	45^2	22437

$\bar{x} = \dfrac{\sum x}{n} = \dfrac{473}{10} = 47.3$

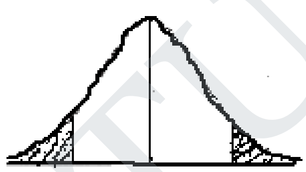
$s^2 = \dfrac{\sum x^2}{n} - (\bar{x})^2 = \dfrac{22437}{10} - (47.3)^2 = 6.41 \quad \Rightarrow \quad s = 2.53$

1. $H_0 : \mu = 50$

2. $H_1 : \mu \neq 50$

3. $\alpha = 5\%$, $d.f = n-1 = 10-1 = 9$

4. critical region



$-t_{\alpha/2} = -2.26$ $\qquad$ $-t_{\alpha/2} = 2.26$

5. The test statistic $t = \dfrac{\bar{x}-\mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)} = \dfrac{47.3-50}{\left(\dfrac{2.53}{\sqrt{10-1}}\right)} = -3.2$

6. Conclusion : If $-t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}}$ we accept H_0 , otherwise we reject H_0

Here -2.26 $\not<$-3.2< 2.26

$\therefore\ H_0$ at 5% level of significance.

b.(ii) Given $\overline{X_1}=72, \overline{X_2}=74, s_1=8, s_2=6, n_1=32, n_2=36$. Test if the means are significant.

Ans :

$$s_1^2=\frac{n_1 s_1^2}{n_1-1}=\frac{(32)(8^2)}{32-1}=66.07 \quad \& \quad s_2^2=\frac{n_2 s_2^2}{n_2-1}=\frac{(36)(6^2)}{36-1}=37.03 \quad \Rightarrow \quad s_1^2>s_2^2$$

1. $H_0: \sigma_1^2=\sigma_2^2$
2. $H_1: \sigma_1^2 \neq \sigma_2^2$
3. $\alpha=5\%$ $\quad d.f.\ v_1=31 \ \& \ v_2=35$
4. Table value of F is <1.80
5. The test statistic is $F=\frac{s_1^2}{s_2^2}=\frac{66.07}{37.03}=1.80$
6. Conclusion :

If Cal F<table F , then we accept H_0; otherwise , we reject H_0

$\therefore$ Cal F $\not<$ table F , we reject H_0.

12.a(i) Given

Detergent	Engine		
	1	2	3
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Perform ANOVA and test at 0.05 level of significance whether these are differences in the detergents or in the engines.

Ans :

Detergent	Engine			Total
	1	2	3	
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

The above data are classified according to criteria (i) Detergent (ii) Engine.

In order to simplify calculations, we code the data by subtracting 50 from each figure.

Detergent	Engine			Total			
	(X_1)	(X_2)	(X_3)		X_1^2	X_2^2	X_3^2
$A(Y_1)$	-5	-7	1	-11	25	49	1
$B(Y_2)$	-3	-4	2	-5	9	16	4
$C(Y_3)$	-2	0	5	3	4	0	25
$D(Y_4)$	-8	-13	-1	-22	64	1691	1
Total	-18	-24	7	-35	102	234	31

1. H_0 : There is no significant diference between columns means as well as row means.
2. H_1 : There is significant diference between columns means or the row means.

Step 1. N = 12 [Total number of entries]

Step 2. T = -35

Step 3. $\frac{T^2}{N} = \frac{(-35)^2}{12} = 102.08$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$

= (102) + (234) + (31) –(102.08)

= 264.92

Step 5. SSC = $\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{(7)^2}{4} - 102.08$

= 135.17

Step 6. SSR = $\frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{(-11)^2}{3} + \frac{(-5)^2}{3} + \frac{(3)^2}{3} + \frac{(-22)^2}{3} - 102.08$

= 110.91

SSE = TSS – SSC – SSR

= 264.92 – 135.17 - 110.91

= 18.84

Step 7. Table of analysis of variance

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 1% level
Between columns	SSC = 135.17	c –1 = 3-1 = 2	MSC = $\frac{SSC}{c-1}$ $=\frac{135.17}{2}=67.585$	F_C = $\frac{MSC}{MSE}$ = $\frac{67.585}{3.14}=21.52$	$F_C(2, 6)$ =10.92
Between Rows	SSR = 110.91	r – 1 = 4-1 = 3	MSR = $\frac{SSR}{r-1}$ $\frac{110.91}{3}=36.97$	F_R = $\frac{MSR}{MSE}$ = $\frac{36.9}{3.14}=11.77$	$F_R(3, 6)$ = 9.78
Residual	SSE = 18.84	N – c- r +1 = 12-3-4+1 = 6	MSE = $\frac{SSE}{N-c-r+1}$ = $\frac{18.84}{6}=3.14$		
Total	TSS = 264.92	11			

Step 8 . Conclusion :

Cal F_C > Table F_C . So we reject H_0

Cal F_R> Table F_R . So we reject H_0

(ii). Find out the main effects and interactions in the following 2^2factorial experiment and write down the ANOVA table.

	I	a	b	ab
Block	00	10	01	11
I	64	25	30	6
II	75	14	50	33
III	76	12	41	17
IV	75	33	25	10

Ans :

Taking deviation from y = 37 we get

Treatment combination	Blocks				Total				
	I X_1	II X_2	III X_3	IV X_4		X_1^2	X_2^2	X_3^2	X_4^2
(y_1) (1)	27	38	39	38	142	729	1444	1521	1444
(y_2) a	-12	-23	-25	-4	-64	144	529	625	15
(y_3) b	-7	13	4	-12	-2	49	159	16	144
(y_4) ab	-31	-4	-20	-27	-82	961	16	400	729
Total	-23	24	-2	-5	-6	1883	2158	2562	2333

Step 1 : N = 16

Step 2 : T = - 6

Step 3 : C.F $= \frac{T^2}{N} = \frac{36}{16} = 2.25$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 1883 + 2158 + 2562 + 2333 + 2.25 = 8933.75$

Step 5 : SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(-23)^2}{4} + \frac{(24)^2}{4} + \frac{(-2)^2}{4} + \frac{(-5)^2}{4} - 2.25 = 281.25$

Step 6. SSR $= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{(142)^2}{4} + \frac{(-64)^2}{4} + \frac{(-2)^2}{4} + \frac{(-82)^2}{4} - 2.25 = 7744.75$

Step 7 : SSE = TSS – SSC – SSR = 8933.75-281.25-7744.75 = 907.75

For 2^2 experiment
Contrast A = [a + ab – b-(1)] = - 64 – 82 + 2 -142 = -286
Contrast B = [b + ab – a-(1)] = - 2 – 82 + 64 -142 = -162
Contrast AB = [ab +(1) – a - b] = - 82+142 + 64 + 2 = 126

Main effect A $= \frac{1}{2}$ [a + ab – b – (1)] = -143

B $= \frac{1}{2}$ [b + ab – a – (1)] = -81

AB $= \frac{1}{2}$ [ab + (1) - a – b] = 63

$SS_A = \frac{[a+ab-b-(1)]^2}{16} = \frac{(-286)^2}{16} = 5112.25$

$SS_B = \frac{[b+ab-a-(1)]^2}{16} = \frac{(-162)^2}{16} = 1640.25$

$SS_{AB} = \frac{[ab+(1)-a-b]^2}{16} = \frac{(-126)^2}{16} = 992.25$

Analysis of variance table for the 2^2 experiment

s.v	d.f	s.s	MSS	Variance ratio	Table value of F	
					5 %	1 %
Blocks	3	281.5	93.83	$\frac{100.86}{93.83} = 1.075$	F(9, 3) = 8.81	27.35
Treatments	3	7744.75	2581.58	$\frac{2581.88}{100.86} = 25.60$	F(3, 9) = 3.86	6.99
A	1	5112.25	5112.25	$\frac{5112.25}{100.86} = 50.69$	$F_A(1, 9)$ = 5.12	6.99
B	1	1640.25	1640.25	$\frac{1640.25}{100.86} = 16.26$	$F_B(1, 9)$ = 5.12	6.99
AB	1	992.25	992.25	$\frac{992.25}{100.86} = 9.84$	$F_{AB}(1, 9)$ = 5.12	6.99
Error	9	907.75	100.86			

Error (d.f) = N – c – r +1 = 16 – 4 - 4 + 1 = 9

Cal F_A > Table F_A

Cal F_B > Table F_B

Cal F_{AB} > Table F_{AB} .

13.a(i) Find the +ve root of $x^4 - x - 9 = 0$ using Newton method.

Ans : Let $f(x) = x^4 - x - 9 = 0$

$f(0) = -9 \;\; (-ve)$

$f(1) = 1 - 1 - 9 = -9 \;\; (-ve)$

$f(2) = 16 - 2 - 9 = 5 \;\; (+ve)$

A root is nearer to 2 $\Rightarrow$ Let $x_0 = 2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad f(x) = x^4 - x - 9 = 0 \text{ and } f'(x) = 4x^3 - 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 2 - \left[\frac{2^4 - 2 - 9}{4(2^3) - 1}\right] = 1.839$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.839 - \frac{f(1.839)}{f'(1.839)} = 1.839 - \left[\frac{1.839^4 - 1.839 - 9}{4(1.839^3) - 1}\right] = 1.814$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.841 - \frac{f(1.814)}{f'(1.814)} = 1.814 - \left[\frac{1.814^4 - 1.814 - 9}{4(1.814^3) - 1}\right] = 1.813$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.813 - \frac{f(1.813)}{f'(1.813)} = 1.813 - \left[\frac{1.813^4 - 1.813 - 9}{4(1.813^3) - 1}\right] = 1.813$$

Hence the approximate root is 1.813

(ii). Find the largest eigen value and its corresponding eigen vector using Power method, for $A = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$.

Ans : Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an arbitrary initial eigenvector

$$AX_1 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 0.167 \\ 0.667 \\ 1 \end{bmatrix} = 6X_2$$

$$AX_2 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.167 \\ 0.667 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.166 \\ 2.336 \\ 8.003 \end{bmatrix} = 8.003\begin{bmatrix} 0.021 \\ 0.292 \\ 1 \end{bmatrix} = 8.003X_3$$

$$AX_3 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.021 \\ 0.292 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.145 \\ 0.252 \\ 6.002 \end{bmatrix} = 6.002\begin{bmatrix} 0.191 \\ 0.042 \\ 1 \end{bmatrix} = 6.002X_4$$

$$AX_4 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.191 \\ 0.042 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.065 \\ -0.068 \\ 6.272 \end{bmatrix} = 6.272\begin{bmatrix} 0.329 \\ -0.011 \\ 1 \end{bmatrix} = 6.272X_5$$

$$AX_5 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.329 \\ -0.011 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.362 \\ 0.272 \\ 6.941 \end{bmatrix} = 6.941\begin{bmatrix} 0.34 \\ 0.039 \\ 1 \end{bmatrix} = 6.941X_6$$

$$AX_6 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.34 \\ 0.039 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.223 \\ 0.516 \\ 7.157 \end{bmatrix} = 7.157\begin{bmatrix} 0.311 \\ 0.072 \\ 1 \end{bmatrix} = 7.157X_7$$

$$AX_7 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.311 \\ 0.072 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.095 \\ 0.532 \\ 7.082 \end{bmatrix} = 7.082\begin{bmatrix} 0.296 \\ 0.075 \\ 1 \end{bmatrix} = 7.082X_8$$

$$AX_8 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.296 \\ 0.075 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.071 \\ 0.484 \\ 7.001 \end{bmatrix} = 7.001\begin{bmatrix} 0.296 \\ 0.069 \\ 1 \end{bmatrix} = 7.001X_9$$

$$AX_9 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.296 \\ 0.069 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.089 \\ 0.46 \\ 6.983 \end{bmatrix} = 6.983 \begin{bmatrix} 0.296 \\ 0.066 \\ 1 \end{bmatrix} = 6X_{10}$$

$$AX_{10} = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.296 \\ 0.066 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.101 \\ 0.46 \\ 6.992 \end{bmatrix} = 6.992 \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = 6.992X_{11}$$

$$AX_{11} = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.102 \\ 0.464 \\ 6.998 \end{bmatrix} = 6.998 \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = 6.998X_{12}$$

$$AX_{12} = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.102 \\ 0.464 \\ 6.998 \end{bmatrix} = 6.998 \begin{bmatrix} 0.3 \\ 0.066 \\ 1 \end{bmatrix} = 6.998X_{12}$$

The largest eigen value = 7

b.(i) Solve by Gauss Seidel : $5x - 2y + z = -4$; $x + 6y - 2z = -1$; $3x + y + 5z = 13$.

Ans :

$$x = \frac{1}{5}(2y - z - 4) \quad ; \quad y = \frac{1}{6}(x + 2z - 1) \quad ; \quad z = \frac{1}{5}(-3x - y + 13)$$

Let $y_0 = z_0 = 0$

Gauss Seidel is

x	y	z
-0.8	-0.03	3.09
-1.43	1.102	3.24
-1.01	1.08	2.99
-0.97	0.99	2.98
-1	0.99	3.00
-1	1	3
-1	1	3

$\therefore \quad x = -1 \; ; \; y = 1 \; ; \; z = 3$

(ii). Find the inverse of $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ by Gauss Jordan method.

Ans :

$$[A, I] = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 7 & 17 & -1 & 0 & 2 \end{array}\right] \quad \begin{array}{l} R_2 \Leftrightarrow 2R_2 - 3R_1 \\ R_3 \Leftrightarrow 2R3 - R_1 \end{array}$$

$$\sim\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & -4 & 20 & -14 & 2 \end{array}\right] \quad R_3 \Leftrightarrow R_3 - 7R_2$$

$$\sim\left[\begin{array}{ccc|ccc} 8 & 4 & 0 & 24 & -14 & 2 \\ 0 & 4 & 0 & 48 & -34 & 6 \\ 0 & 0 & -4 & 20 & -14 & 2 \end{array}\right] \quad \begin{array}{l} R_2 \Leftrightarrow 4R_2 + 3R_3 \\ R_1 \Leftrightarrow 4R_1 + R_3 \end{array}$$

$$\sim\left[\begin{array}{ccc|ccc} 8 & 0 & 0 & -24 & 20 & -4 \\ 0 & 4 & 0 & 48 & -34 & 6 \\ 0 & 0 & -4 & 20 & -14 & 2 \end{array}\right] \quad R_1 \Leftrightarrow R_1 - R_2$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 5/2 & -1/2 \\ 0 & 1 & 0 & 12 & -17/2 & 3/2 \\ 0 & 0 & 1 & -5 & 7/2 & -1/2 \end{array}\right]$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -6 & 5 & -1 \\ 24 & -17 & 3 \\ -10 & 7 & -1 \end{bmatrix}$$

14.a(i) Given :

X	0	2	3	4	7	9
Y	4	26	58	112	466	922

Find $y(10)$, $y'(6)$ using Newton's divided difference formula.

Ans :

Since the argument are not equally spaced, we will use Newton's divided difference

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	4				
		11			
2	26		7		
		32		1	
3	58		11		0
		54		1	
4	112		16		0
		118		1	
7	466		22		
		228			
9	922				

By Newton's divided difference formula we have

$$y = f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + \ldots\ldots$$

$$= 4 + (x-0)11 + (x-0)((x-2)7 + (x-0)(x-2)(x-3)1 = x^3 + 2x^2 + 3x + 4$$

Therefore

$y'(x) = 3x^2 + 4x + 3$

$y'(6) = 3(6)^2 + 4(6) + 3 = 135$

$y'(5) = 3(5)^2 + 4(5) + 3 = 98$

$y(x)$ is maximum if $y'(x) = 0$ $3x^2 + 4x + 3 = 0$. But the roots are imaginary. Therefore there is no extremum value in the range. In fact, it is an increasing curve.

(ii). Evaluate the integral I = $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's 1/3 rule by taking h = 1/4.

Ans :

x	0	0.25	0.5	0.75	1
$y = \frac{1}{1+x^2}$	1	0.9412	0.8	0.64	0.5
	1	4	1 1	4	1
sum	1	4	2	4	1

$$I = \int_0^1 \frac{dx}{1+x^2} = \frac{0.25}{3}[1(1 + 0.5) + 4(0.9412 + 0.64) + 2(0.8)] = 0.7854.$$

b.(i) Evaluate the integral I = $\int_1^2 \frac{dx}{1+x^2}$ by taking h = 0.2 using trapezoidal rule.

Ans :

x	1	1.2	1.4	1.6	1.8	2
$y = \frac{1}{1+x^2}$	0.5	0.4098	0.3378	0.2809	0.2358	0.2

$$I = \int_1^2 \frac{dx}{1+x^2} = \frac{h}{2}[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)]$$

$$= \frac{0.2}{2}[(0.5+0.2) + 2(0.4098+0.3378+0.2809+0.2358)]$$

$$= 0.3229$$

(ii). Given :

x 140 150 160 170 180

y 3.685 4.854 6.302 8.076 10.225

Find y (175).

Ans :

x	y	Δ	Δ^2	Δ^3	Δ^4
140	3.685				
		1.169			
150	4.854		0.279		
		1.448		0.047	
160	6.302		0.326		0.002
		1.774		0.049	
170	8.076		0.375		
		2.149			
180	10.225				

(1) $x_n = 180$, $y_n = 10.225$, $\nabla y_n = 2.149$, $\nabla^2 y_n = 0.375$, $\nabla^3 y_n = 0.049$

$x = 175$, $h = 10$ $\therefore$ $p = \dfrac{x - x_n}{h} = -0.5$

$$y(x) = y_n + \frac{p}{1!}\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + ...$$

$$y(175) = 10.225 + \frac{(-0.5)}{1!}(2.149) + \frac{(-0.5)(-0.5+1)}{2!}(0.375)$$

$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(0.049) + ...$$

$y(175) = 9.1003$

15.a(i) Using Runge-Kutta method of fourth order solve $y' = \dfrac{y^2 - x^2}{x^2 + y^2}$ given y(0) = 1. Find y at $x = 0.2 , 0.4 , 0.6$.

Ans : Given $y' = f(x, y) = \dfrac{y^2 - x^2}{y^2 + x^2}$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.2$, $x_2 = 0.4$, $x_3 = 0.6$

h = 0.2

<u>To find y(0.2)</u> :

$$k_1 = h\, f(x_0, y_0) = (0.2)\left[\frac{y_0^{\,2} - x_0^{\,2}}{y_0^{\,2} + x_0^{\,2}}\right] = (0.2)\left[\frac{1-0}{1+0}\right] = 0.2$$

$$k_2 = h\,f\left[x_0+\frac{h}{2},\ y_0+\frac{k_1}{2}\right] = (0.2)\,f\left[0+\frac{0.2}{2},\ 1+\frac{0.2}{2}\right]$$

$$= (0.2)\ f\ [0.1\ ,1.1] = (0.2)\left[\frac{(1.1)^2-(1.1)^2}{(1.1)^2+(0.1)^2}\right]$$

$$= (0.2)(0.9836) = 0.19672$$

$$k_3 = h\,f\left[x_0+\frac{h}{2},\ y_0+\frac{k_2}{2}\right] = (0.2)\,f\left[0+\frac{0.2}{2},\ 1+\frac{0.19672}{2}\right]$$

$$= (0.2)\ f\ [0.1\ ,\ 1.0983606]$$

$$= (0.2)\left[\frac{(1.0983606)^2-(0.1)^2}{(1.0983606)^2+(0.1)^2}\right] = 0.1967$$

$$k_4 = h\,f(x_0+h\ ,y_0+k_3) = (0.2)\ f[\ 0.2\ ,\ 1.1967\ ]$$

$$= (0.2)\left[\frac{(1.1967)^2-(0.2)^2}{(1.1967)^2+(0.2)^2}\right] = 0.1891$$

$$\Delta y = \frac{1}{6}[k_1+2k_2+2k_3+k_4]$$

$$= \frac{1}{6}[0.2+2(0.19672)+2(1.1967)+0.1891] = 0.19598$$

$$y(0.2) = y_1 = y_0+\Delta y = 1+0.19598 = 1.19598$$

<u>To find y(0.4)</u> :

$$k_1 = h\,f(x_1,y_1) = (0.2)\left[\frac{(1.196)^2-(0.2)^2}{(1.196)^2+(0.2)^2}\right] = 0.1891$$

$$k_2 = h\,f\left[x_1+\frac{h}{2},y_1+\frac{k_1}{2}\right] = (0.2)\,f\left[0.2+\frac{0.2}{2},1.196+\frac{0.1891}{2}\right]$$

$$= (0.2)\ f\ [0.3\ ,1.2906] = (0.2)\left[\frac{(1.2906)^2-(0.3)^2}{(1.2906)^2+(0.3)^2}\right]$$

$$= 0.1795$$

$$k_3 = h\,f\left[x_1+\frac{h}{2},y_1+\frac{k_2}{2}\right] = (0.2)\,f\left[0.2+\frac{0.2}{2},1.196+\frac{0.1795}{2}\right]$$

$$= (0.2)\,f\,[0.3\ ,\ 1.2858] = (0.2)\left[\frac{(1.2858)^2-(0.3)^2}{(1.2858)^2+(0.3)^2}\right]$$

$$= 0.1793$$

$$k_4 = h\, f(x_1 + h\, , y_1 + k_3) = (0.2)\ f[0.4\ , 1.3753\ ]$$

$$= (0.2)\left[\frac{(\ 1.3753\ )^2 - (\ 0.4\ )^2}{(\ 1.3753\ )^2 + (\ 0.4\ )^2}\right] = 0.1688$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1891 + 2(0.1795) + 2(0.1793) + 0.1688] = 0.1793$$

$$y(0.4) = 1.196 + 0.1793 = 1.3753$$

<u>To find y(0.6)</u> :

$$k_1 = h\, f(x_2\ , y_2) = (\ 0.2\ )\left[\frac{(\ 1.3753\ \ )^2 - (\ 0.4)^2}{(\ 1.3753\ \ )^2 + (\ 0.4\ )^2}\right] = 0.1360$$

$$k_2 = h\, f\left[x_2 + \frac{h}{2}\ \ ,\ y_2 + \frac{k_1}{2}\right] = (0.2)\ f[0.5\ , 1.4597\ ]$$

$$= (0.2)\left[\frac{(\ 1.4597\ \ )^2 - (0.5)^2}{(\ 1.4597\ \ )^2 + (0.5)^2}\right] = 0.1580$$

$$k_3 = h\, f\left[x_1 + \frac{h}{2}\, , y_1 + \frac{k_2}{2}\right] = (0.2)\ f\,[0.5\ , 1.4543\ \ ]$$

$$= (0.2)\left[\frac{(\ 1.4543\ \ )^2 - (0.5)^2}{(\ 1.4543\ \ )^2 + (0.5)^2}\right] = 0.1577$$

$$k_4 = h\, f(x_1 + h\, , y_1 + k_3) = (0.2)\ f[0.6\, , 1.533\ ]$$

$$= (0.2)\left[\frac{(1.533\ )^2 - (0.6)^2}{(\ 1.533\ )^2 + (0.6)^2}\right] = 0.1469$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1688 + 2(0.1580) + 2(0.1577) + 0.1469]$$

$$= 0.1579$$

$$y(0.6) = y_2 = y_2 + \Delta y = 1.3753 + 0.1579$$

$$= 1.5332$$

(ii). Compute y(0.5) , y(1) and y(1.5) using Taylor's series for $y' = \frac{x+y}{2}$ with y(0) = 2 and hence find y(2) using Milne's method.

Ans :

Given

$x_0 = 0 \;;\; y_0 = 2 \quad x_1 = 0.5 \;;\; y_1 = ?$

$x_2 = 1 \;;\; y_2 = ? \quad x_3 = 1.5 \;;\; y_3 = ?$ by using Taylor's series

To find y(0.5) :

$$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \ldots\ldots$$

$$y' = \frac{x+y}{2} \qquad y_0' = \frac{x_0+y_0}{2} = 1$$

$$y'' = \frac{1+y'}{2} \qquad y_0'' = \frac{1+y_0'}{2} = 1$$

$$y''' = \frac{y''}{2} \qquad y_0''' = \frac{y_0''}{2} = \frac{1}{2}$$

$$y^{iv} = \frac{y'''}{2} \qquad y^{iv} = \frac{y_0'''}{2} = \frac{1}{4}$$

$$y_1 = 2 + \frac{0.5}{1!}(1) + \frac{(0.5)^2}{2!}(1) + \frac{(0.5)^3}{3!}\left(\frac{1}{2}\right) + \frac{(0.5)^4}{4!}\left(\frac{1}{4}\right) + \ldots\ldots$$

$$= 2.6361$$

To find y(1) :

$$y_2 = y_1 + \frac{h}{1!}y_1' + \frac{h^2}{2!}y_1'' + \frac{h^3}{3!}y_1''' + \ldots\ldots$$

$$y' = \frac{x+y}{2} \qquad y_1' = \frac{x_1+y_1}{2} = 1.5681$$

$$y'' = \frac{1+y'}{2} \qquad y_1'' = \frac{1+y_1'}{2} = 1.2841$$

$$y''' = \frac{y''}{2} \qquad y_1''' = \frac{y_1''}{2} = 0.6421$$

$$y^{iv} = \frac{y'''}{2} \qquad y^{iv} = \frac{y_1'''}{2} = 0.3211$$

$$y_2 = 2.6361 + \frac{0.5}{1!}(1.5681) + \frac{(0.5)^2}{2!}(1.2841) + \frac{(0.5)^3}{3!}(0.6421) + \frac{(0.5)^4}{4!}(0.3211) + ...$$
$$= 3.5949$$

To find y(1.5) :

$$y_3 = y_2 + \frac{h}{1!}y_2' + \frac{h^2}{2!}y_2'' + \frac{h^3}{3!}y_2''' + ...$$

$$y' = \frac{x+y}{2} \qquad y_2' = \frac{x_2 + y_2}{2} = 2.2975$$

$$y'' = \frac{1+y'}{2} \qquad y_2'' = \frac{1+y_2'}{2} = 1.6488$$

$$y''' = \frac{y''}{2} \qquad y_2''' = \frac{y_2''}{2} = 0.8422$$

$$y^{iv} = \frac{y'''}{2} \qquad y^{iv} = \frac{y_2'''}{2} = 0.4122$$

$$y_3 = 3.5949 + \frac{0.5}{1!}(2.2975) + \frac{(0.5)^2}{2!}(1.6488) + \frac{(0.5)^3}{3!}(0.8244) + \frac{(0.5)^4}{4!}(0.4122) + ...$$
$$= 4.9681$$

To find y(2) by Milne's method :

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}[2y_{n-2}' - y_{n-1}' + 2y_n']$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}[y_{n-1}' + 4y_n' + y_{n+1}']$$

$$y_1' = \frac{x_1 + y_1}{2} = 1.5681 \qquad y_2' = \frac{x_2 + y_2}{2} = 2.2975$$

$$y_3' = \frac{x_3 + y_3}{2} = 3.2341$$

$$y_{4,p} = y_0 + \frac{4(0.5)}{3}[2y_1' - y_2' + 2y_3']$$

$$= 2 + \frac{4(0.5)}{3}[2(1.5681) - 2.2975 + 2(3.2341)] = 6.8713$$

$$y_{4,c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$y_4' = \frac{x_4 + y_4}{2} = 4.4357$$

$$y_{4,c} = 3.5949 + \frac{0.5}{3}[2.2975 + 4(3.2341) + 4.4357] = 6.8732$$

Tables Values for relevant problem

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Table values : (at 5% los)

$|Z| = 1.96$, $t_8 = 2.31$, $t_9 = 2.26$, $F_{2,6} = 5.14$, $F_{3,6} = 4.76$, $F_{9,3} = 8.81$, $F_{3,9} = 3.86$,

$F_{1,9} = 5.12$, $F_{4,5} = 5.19$, $F_{5,4} = 6.26$, $\psi_9^{\,2} = 16.9$, $\psi_{10}^{\,2} = 18.3$.

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B.E./ B.Tech. DEGREE EXAMINATION, MAY/JUNE 2015
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA6452 –STATISTICS AND NUMERICAL METHODS
(Regulation 2013)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. What are the expected frequencies of 2 x 2 contingency table

a	b
c	d

Ans :

The complete table is

Attributes	B	β	Total
A	a	B	a + b
α	c	D	c + d
Total	(B) = a + c	(β) = b + d	a +b + c + d

The expected frequencies are

$$e(a) = e(A, B) = \frac{(a+b)(a+c)}{a+b+c+d} \quad ; \quad e(b) = e(A, \beta) = \frac{(a+b)(b+d)}{a+b+c+d}$$

$$e(c) = e(\alpha, A) = \frac{(c+d)(a+c)}{a+b+c+d} \quad ; \quad e(d) = e(\alpha, \beta) = \frac{(c+d)(b+d)}{a+b+c+d}$$

2. Write down the formula of test statistic t to test the significance of difference between the means of large samples.

Ans :

$$Z = \frac{\left|\overline{x}_1 - \overline{x}_2\right|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where $\overline{x}_1$, $\overline{x}_2$ sample means ; s_1^2 & s_2^2 sample variances ; n_1 & n_1 sample sizes.

3. What do you understand by design of an experiment?

Ans :

The design of an experiment may be defined as “the logical construction of the experiment in which the degree of uncertainty with which the inference is drawn may be well defined.

4. What are the basis principles of the design of experiments ?

Ans :

There are tree basic principles of experimental design.

They are : (1) Randomization (2) Replication (3) Local control (error control).

5. Perform four iterations of the Newton-Raphson method to find the smallest positive root of the equation $f(x)=x^3-5x+1=0$.

Ans :

Let $f(x)=x^3-5x+1=0$

$f(0)=1\ (+ve)$

$f(1)=1-5-1=-3\ (-ve)$

A root is nearer to 0 and 1

Let $x_0=0$

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \quad f(x)=x^3-5x+1=0 \ \text{ and } \ f'(x)=3x^2-5$$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)} = 0-\frac{f(0)}{f'(0)}=0-\left[\frac{1}{-5}\right]=0.2$$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)} = 0.2-\frac{f(0.2)}{f'(0.2)}=0.2-\left[\frac{0.2^3-5(0.2)+1}{3(0.2^2)-5}\right]=0.2016393$$

$$x_3=x_2-\frac{f(x_2)}{f'(x_2)}$$

$$= 0.2016393-\frac{f(0.2016393)}{f'(0.2016393)}=0.2016393-\left[\frac{0.2016393^3-5(0.2016393)+1}{3(0.2016393^2)-5}\right]$$

$=0.2016397$

$$x_4=x_3-\frac{f(x_3)}{f'(x_3)}$$

$$= 0.2016397-\frac{f(0.2016397)}{f'(0.2016397)}=0.2016397-\left[\frac{0.2016397^3-5(0.2016397)+1}{3(0.2016397^2)-5}\right]$$

$=0.2016397$

Hence the approximate root is 0.2016397

6. Solve the equation $10x-y+2z=4$; $x+10y-z=3$; $2x+3y+20z=7$ using the Gauss elimination method.

Ans :

Given

$10x-y+2z=4$; $x+10y-z=3$; $2x+3y+20z=7$

$$[A, B] = \left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 1 & 10 & -1 & 3 \\ 2 & 3 & 20 & 7 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & 101 & -12 & 26 \\ 0 & 16 & 98 & 31 \end{array}\right] \quad \begin{array}{l} R_2 \Leftrightarrow 10R_2 - R_1 \\ R_3 \Leftrightarrow 5R3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & 101 & -12 & 26 \\ 0 & 0 & 10090 & 2715 \end{array}\right] \quad R_3 \Leftrightarrow 101R_3 - 16R_2$$

This is an upper triangular matrix

$\therefore \Rightarrow 10090\,z = 2715 \Rightarrow z = \dfrac{543}{2018} \Rightarrow z = 0.269$

$\Rightarrow 101\,y - 12\,z = 26 \Rightarrow 101y = 12(0.269) + 26 \Rightarrow 101\,y = 29.228 \Rightarrow y = 0.289$

$\Rightarrow 10\,x - y + 2z = 4 \Rightarrow 10x = 4 + 0.289 - 2(0.269) = 3.751$

$\Rightarrow x = 0.375$

Hence $x = 0.375$, $y = 0.289$, $z = 0.269$

7. Given $f(2) = 5, f(2.5) = 5.5$ find the linear interpolating polynomial using Lagrange's interpolation.

Ans :

x	x_0 2	x_1 2.5
y	5 y_0	5.5 y_1

By Lagrange's interpolation formula we have

$$y = f(x) = \frac{(x - x_1)}{x_0 - x_1} y_0 + \frac{(x - x_0)}{x_1 - x_0} y_1$$

$$= \frac{x - 2.5}{2 - 2.5}(5) + \frac{x - 2}{2.5 - 2}(5.5) = -10(x - 2.5) + 11(x - 2)$$

$$= -10x + 25 + 11x - 22 = x + 3$$

8. Construct the divided difference table for the data.

X	0.5	1.5	3.0	5.0	6.5	8.0
F(x)	1.625	5.875	31	131	282.125	521

Ans :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0.5	1.625				
		$\frac{5.875-1.625}{1.5-0.5} = 11$			
1.5	5.875		5		
		$\frac{31-5.875}{3.0-1.5} = 1.675$		1	
3.0	31		9.5		0
		$\frac{131-131}{5.0-3.0} = 50$		1	
5.0	131		14.5		0
		$\frac{282.125-131}{6.5-5.0} = 100.75$		1	
6.5	282.125		19.5		
		$\frac{521-282.125}{8.0-6.5} = 159.25$			
8.0	521				

9. Given $y' = \frac{y-x}{x+y}$ with initial condition $y=1$ at $x=0$ find y for $x=0.1$ by Euler's method.

Ans :

Given $y' = \frac{y-x}{x+y} = f(x,y)$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.1$, $y_1 = ?$, $h = 0.1$

By Euler's algorithm,

$$y_1 = y_0 + h\, f(x_0, y_0)$$

$$= 1 + (0.1)\, f(0, 1) = 1 + (0.1)\left[\frac{1-0}{0+1}\right] = 1.1$$

10. Given the initial value problem $u' = -2tu^2$ estimate using modified Euler- Cauchy method.

Ans :

$u' = -2tu^2$, $u(0) = 1$, $u(0.4) = ?$, $h = 0.4$ $\quad t_0 = 0, u_0 = 1, t_1 = 0.4$, $u_1 = ?$

$f(u,t) = -2tu^2$

By Modified Euler's method

$$u_1 = y_0 + h\left[f\left[x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right]\right]$$

$$u_1 = u_0 + h\left[f\left[t_0 + \frac{h}{2}, u_0 + \frac{h}{2} f(t_0, u_0)\right]\right] = 1 + (0.4)\,[\,f\,(0.2, 1 + 0.2(0)\,] = 1 + 0.4[-2(0.2)]$$

$$= 1 - 0.16 \quad = 0.84$$

PART –B (5 x 15 = 60)

11.(i) Fit a binomial distribution for the following data and also test the goodness of fit.

X	0	1	2	3	4	5	6	Total
F(X)	5	18	28	12	7	6	4	80

Ans :

1. H_0 : Binomial is good fit
2. H_1 : Binomial is not good fit
3. $\alpha = 0.05$ $d.f. = n-1 = 7-1 = 6$
4. Table value of $\psi^2 = 12.592$
5. The test statistic is $\psi^2 = \sum \frac{(O-E)^2}{E}$

On the assumption H_0, the expected frequencies frequencies are given by the terms of $N(q+p)^n$

$$= 80\left(\frac{1}{2}+\frac{1}{2}\right)^6 = 80\left[6C_0 + 6C_1 + 6C_2 + 6C_3 + 6C_4 + 6C_5 + 6C_6\right]$$

$$= \frac{80}{64}[1+6+15+20+15+6+1] = 1.25\,[1+6+15+20+15+6+1]$$

x	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
0	5	1.25	3.75	14.1	11.28
1	18	7.5	10.5	110.3	14.71
2	28	18.75	9.25	85.6	4.57
3	12	25	-13	169	6.76
4	7	18.75	-11.75	138.1	7.37
5	6	7.5	-1.5	2.3	0.31
6	4	1.25	2.75	7.6	6.08
	N = 80				51.08

$\therefore \psi^2 = 51.08$

6. Conclusion :

If cal ψ^2 < table ψ^2

Then we accept H_0 , here 12.592 < 51.08

$\therefore$ we accept H_0

(ii) The mean value of a random sample of 60 items was found to be 145, with a standard deviation of 40. Find the 95% confidence limits for the population mean. What size of the sample is required to estimate the population mean within 5 of its actual value 95% or more confidence, using the sample mean ?

Ans :

S.D. is not given

$\therefore$ limits of μ is $\left|\frac{\mu - \bar{x}}{s/\sqrt{n}}\right| \leq 1.96$

$$\bar{x} - 1.96\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + 1.96\frac{s}{\sqrt{n}}$$

$$134.9 \leq \mu \leq 155.1$$

$$p\left(\bar{x} - 5 \leq \mu \leq \bar{x} + 5\right) \geq 0.95 \quad \Rightarrow \quad p\left(\left|\bar{x} - \mu\right| \leq 5\right) \geq 0.95$$

$$(ie)\ p\left(|z| \leq \frac{5\sqrt{n}}{\sigma}\right) \geq 0.95$$

$$wkt \quad p\left(|z| \leq 1.96\right) = 0.95$$

$$\therefore \quad n = n\frac{5\sqrt{n_1}}{\sigma} = 1.96 \quad \Rightarrow \quad \sqrt{n_1} = \frac{1.95}{5} \quad \Rightarrow \quad n_1 = \left(\frac{1.96 \times 40}{5}\right)^2 = 245.86$$

$\therefore$ Least sample since is 246

(iii) Test made on the breaking strength of 10 pieces of a metal gave the following results 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg.

Ans :

Find $\bar{x}$ and sample S. D. s and then test if $\bar{x}$ differs from $\mu = 577$

Assume A = 582

$$d_i = x_i - A \quad , \quad x_i = d_i + A$$

$$\bar{x} = \frac{\sum d_i}{n} + A = 575.2 \quad , \quad s^2 = \frac{1}{n}\sum d_i^2 - \left(\frac{\sum d_i}{n}\right)^2 = 8.26$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = -0.65 \quad , \quad v = n - 1 = 9$$

n_i :	578	572	570	568	572	570	570	572	596	584
d_i :	-4	-10	-12	-14	-10	-12	-12	-10	14	2
d_i^2 :	16	100	144	196	100	144	144	100	196	4

$H_0 : \bar{x} = \mu \qquad H_1 : \bar{x} \neq \mu$

$t_{0.05} = 2.26 \qquad |t| < t_{0.05} \qquad \therefore\ H_0$ is accepted.

$\therefore$ we can assumed 577 kg at 5% loss.

(iv) A group of 10 rats fed on diet A and another group of 8 rats fed on diet B recorded the following increase in weight

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	3		

Show that the estimates of the population variance from the samples are not significantly different.

Ans :

Given $n_1 = 10 \quad , \quad n_2 = 8$

												Total
Sample I	x_1	5	6	8	1	12	4	3	9	6	10	64
	x_1^2	5^2	6^2	8^2	1^2	12^2	4^2	3^2	9^2	6^2	10^2	512
Sample II	x_2	2	3	6	8	10	1	2	8			400
	x_2^2	2^2	3^2	6^2	8^2	10^2	1^2	2^2	8^2			282

$$\overline{x_1} = \frac{\Sigma x_1}{n_1} = \frac{64}{10} = 6.4 \qquad \& \qquad \overline{x_2} = \frac{\Sigma x_2}{n_2} = \frac{40}{8} = 5$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - (\overline{x_1})^2 = \frac{512}{10} - (6.4)^2 = 10.24 \qquad \& \qquad s_2^2 = \frac{\Sigma x_2^2}{n_2} - (\overline{x_2})^2 = \frac{282}{8} - (5)^2 = 10.25$$

$$s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{(10)(10.24)}{9} = 11.3777 \qquad \& \qquad s_2^2 = \frac{n_{21} s_2^2}{n_2 - 1} = \frac{(8)(10.25)}{7} = 11.7143$$

$$s_2^2 > s_1^2$$

The parameter of interest is σ_1^2 and σ_1^2

1. $H_0 : \sigma_1^2 = \sigma_2^2$ [The difference of a varaince is not significant]
2. $H_1 : \sigma_1^2 \neq \sigma_2^2$
3. $\alpha = 0.05 \quad , \ d.f\,(v_1) = 9 \quad , \ d.f\,(v_2) = 7$
4. Table value of $F = 3.29$
5. The test statistic is $F = \dfrac{s_2^2}{s_1^2} = \dfrac{11.7143}{11.3777} = 1.02958$

6.Conclusion :

If Cal F < table F , then we accept H_0 ; otherwise we reject H_0

Here , F = 1.02958 < 3.29 , we accept H_0 at 5 % level of significance.

We conclude that the two samples have come from populations with equal variances.

12.(i) The following table shows the live in hours of four brands of electric lamps brand.

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps.

Ans :

H_0 : There is no significant difference between the four brands.

H_1 : There is significant difference between the four brands.

Subtract 1600 and then divided by 10 we get

X_1 A	X_2 B	X_3 C	X_4 D	Total	X_1^2	X_2^2	X_3^2	X_4^2
1	-2	-14	-9	-24	1	4	196	81
1	4	-5	-8	-8	1	16	25	64
5	4	0	-7	2	25	16	0	49
8	10	2	-3	17	34	100	4	9
10	15	4	0	29	100	225	16	0
12	-	6	8	26	144	-	36	64
20	-	14	-	34	400	-	196	-
-	-	22	-	22	-	-	484	-
57	31	29	-19	98	735	361	957	267

Step 1 : N = 26

Step 2. T = 98

Step 3. C.F. = $\frac{T^2}{N} = \frac{9604}{26} = 369.39$

Step 4. TSS = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 735 + 361 + 957 + 267 - 369.39 = 1950.61$

Step 5. SSC = $\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(57)^2}{7} + \frac{(31)^2}{5} + \frac{(29)^2}{8} + \frac{(-19)^2}{6} - 369.39 = 452.25$$

SSE = TSS – SSC = 1950.61 – 452.25 = 1498.336

Step 6. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 452.225	C –1 = 4 – 1 = 3	$MSC = \frac{SSC}{C-1}$ $= \frac{452.25}{3}$ $= 150.75$	$F_C = \frac{MSC}{MSE}$ $= \frac{150.75}{68.11}$ = 2.21 > 1 Since $\frac{MSE}{MSC} < 1$	$F_C(3, 22)$ = 3.05
Error	SSE = 1498.36	N – C = 26 – 4 = 22	$MSE = \frac{SSE}{N-C} = \frac{1498.36}{22}$		

Step 7 : Conclusion : Cal F_C < Table F_C . So we accept H_0 .

(ii) Analyze the variance in the following Latin square of yields of paddy where A, B, C, D denote the different methods of cultivation.

D122 A121 C123 B122
B124 C123 A122 D125
A120 B119 D120 C121
C122 D123 B121 A122

Examine whether the different methods of cultivation have given significantly different yields.

Ans :

Subtract 120 we get

S	2	P	1	R	3	Q	2
Q	4	R	3	P	2	S	5
P	0	Q	-1	S	0	R	1
R	2	S	3	Q	1	P	2

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	2	1	3	2	8	4	1	9	4
Y_2	4	3	2	5	14	16	9	4	25
Y_3	0	-1	0	1	0	0	1	0	1
Y_4	2	3	1	2	8	4	9	1	4
Total	8	6	6	10	30	24	20	14	34

Step 1. N = 16 [Total number of entries]

Step 2. T = 30

Step 3. $\frac{T^2}{N} = \frac{(30)^2}{16} = 56.25$

Step 4. TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 24 + 20 + 14 + 34 - 56.25 = 35.75$

Step 5. SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{64}{4} + \frac{36}{4} + \frac{36}{4} + \frac{100}{4} - 56.25 = 2.75$

Step 6. SSR $= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = number of elements in each row]

$= \frac{64}{4} + \frac{196}{4} + \frac{0}{4} + \frac{64}{4} - 56.25 = 24.75$

To find SSK :

Arrange the elements in the order of treatment

					Total
P	0	1	2	2	5
Q	4	-1	2	2	6
R	2	3	1	1	9
S	2	3	5	5	10

SSK $= \frac{(5)^2}{4} + \frac{(6)^2}{4} + \frac{(9)^2}{4} + \frac{(10)^2}{4} - \frac{T^2}{N} = \frac{25}{4} + \frac{36}{4} + \frac{81}{4} + \frac{100}{4} - 56.25 = 4.25$

SSE = TSS – SSC – SSR – SSK = 35.75 – 2.75 – 24.75 – 4.25 = 4

Step 7. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 1% level
Between Rows	SSR = 24.75	k –1 = 3	MSR $= \frac{SSR}{k-1}$ = 8.25	$F_R = \frac{MSR}{MSE}$ = 12.31	$F_R(3, 6)$ = 4.76
Between columns	SSC = 2.75	k –1 = 3	MSC $= \frac{SSC}{k-1}$ = 0.92	$F_C = \frac{MSC}{MSE}$ = 1.37	F_C (3, 6) = 4.76

Between treatments	SSK = 4.25	k –1 = 3	MSK = $\frac{SSK}{k-1}$ = 1.42	F_T = $\frac{MSK}{MSE}$ = 2.12	$F_R(3, 6)$ = 4.76
Residual	SSE = 4	(k –1)(k - 2) = 6	MSE = $\frac{SSE}{(k-1)(k-2)}$ = 0.67		
Total	TSS = 35.75	$n^2 - 1 = 15$			

Step 8 : Conclusion :
(i) Cal F (F_T) < Table F . There is no significant difference between treatments
(ii) Cal F (F_R) > Table F. There is significant difference rows
(iii) Cal F (F_C) < Table F . There is no significant difference columns.

13.(i) Find the inverse of the coefficient matrix of the system $\begin{pmatrix}1&1&1\\4&3&-1\\3&5&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\6\\4\end{pmatrix}$ by the Gauss Jordan method, also solve the system.

Ans : $[A, I] = \left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\4&3&-1&0&1&0\\3&5&3&0&0&1\end{array}\right]$

$$\sim\left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&-1&-5&-4&1&0\\0&2&0&-3&0&1\end{array}\right] \quad \begin{array}{l}R_2 \Leftrightarrow 2R_2 - 4R_1\\R_3 \Leftrightarrow R_3 - 3R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&-1&-5&-4&1&0\\0&0&10&-11&2&1\end{array}\right] \quad R_3 \Leftrightarrow R_3 + 2R_2$$

$$\sim\left[\begin{array}{ccc|ccc}10&10&0&-1&2&1\\0&2&0&-3&0&1\\0&0&-10&-11&2&1\end{array}\right] \quad \begin{array}{l}R_2 \Leftrightarrow -2R_2 + R_3\\R_1 \Leftrightarrow 10R_1 + R_3\end{array}$$

$$\sim\left[\begin{array}{ccc|ccc}10&0&0&14&2&-4\\0&2&0&-3&0&1\\0&0&-10&-11&2&1\end{array}\right] \quad R_1 \Leftrightarrow R_1 - 5R_2$$

$$\sim\left[\begin{array}{ccc|ccc}1&0&0&1.4&0.2&-0.4\\0&1&0&-1.5&0&0.5\\0&0&1&1.1&-0.2&-0.1\end{array}\right] \quad R_1 \Leftrightarrow \frac{R_1}{10} \quad R_2 \Leftrightarrow \frac{R_2}{2} \quad R_3 \Leftrightarrow \frac{R_3}{-10}$$

$$A^{-1} = \begin{bmatrix} 1.4 & 0.2 & -0.4 \\ -1.5 & 0 & 0.5 \\ 1.1 & -0.2 & -0.1 \end{bmatrix}$$

(ii) Find the smallest eigen value in magnitude of the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ using four iterations of the inverse power method.

Ans : Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an arbitrary initial eigenvector

$$AX_1 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix} = 2X_2$$

$$AX_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ -2 \\ 0.5 \end{bmatrix} = 2.5\begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix} = 2.5X_3$$

$$AX_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 2.8 \\ -4.6 \\ 4.8 \end{bmatrix} = 4.8\begin{bmatrix} 0.6 \\ -0.96 \\ 1 \end{bmatrix} = 4.8X_4$$

$$AX_4 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.6 \\ -0.96 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.16 \\ -3.52 \\ 2.996 \end{bmatrix} = -3.52\begin{bmatrix} -0.61 \\ 1 \\ -0.84 \end{bmatrix} = -3.52X_5$$

$$AX_5 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -0.61 \\ 1 \\ -0.84 \end{bmatrix} = \begin{bmatrix} -2.22 \\ 3.45 \\ -2.68 \end{bmatrix} = 3.45\begin{bmatrix} -0.64 \\ 1 \\ -0.78 \end{bmatrix} = 3.45X_6$$

$$AX_6 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -0.64 \\ 1 \\ -0.78 \end{bmatrix} = \begin{bmatrix} -2.28 \\ 3.42 \\ -2.56 \end{bmatrix} = 3.42\begin{bmatrix} -0.67 \\ 1 \\ -0.75 \end{bmatrix} = 3.42X_7$$

$$AX_7 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -0.67 \\ 1 \\ -0.75 \end{bmatrix} = \begin{bmatrix} -2.34 \\ 3.42 \\ -2.5 \end{bmatrix} = 3.42\begin{bmatrix} -0.69 \\ 1 \\ -0.73 \end{bmatrix} = 3.42X_8$$

$$AX_8 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -0.69 \\ 1 \\ -0.73 \end{bmatrix} = \begin{bmatrix} -2.38 \\ 3.42 \\ -2.46 \end{bmatrix} = 3.42 \begin{bmatrix} -0.7 \\ 1 \\ -0.72 \end{bmatrix} = 3.42\, X_9$$

$$AX_9 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -0.7 \\ 1 \\ -0.72 \end{bmatrix} = \begin{bmatrix} -2.4 \\ 3.42 \\ -2.44 \end{bmatrix} = 3.42 \begin{bmatrix} -0.7 \\ 1 \\ -0.7 \end{bmatrix} = 3.42\, X_{10}$$

The largest eigen value = 3.42 eigen vector = $\begin{bmatrix} -0.7 \\ 1 \\ -0.7 \end{bmatrix}$

$$B = A - 3I = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$BY_1 = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = -1Y_2$$

$$BY_2 = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix} = -2Y_3$$

$$BY_3 = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -2 \\ -2.5 \\ -1.5 \end{bmatrix} = -2.5 \begin{bmatrix} 0.8 \\ 1 \\ 0.6 \end{bmatrix} = 4.8Y_4$$

$$BY_4 = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0.8 \\ 1 \\ 0.6 \end{bmatrix} = \begin{bmatrix} -1.8 \\ -2.4 \\ -1.6 \end{bmatrix} = -2.4 \begin{bmatrix} 0.75 \\ 1 \\ 0.7 \end{bmatrix} = -2.4Y_5$$

$$BY_5 = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0.75 \\ 1 \\ 0.7 \end{bmatrix} = \begin{bmatrix} -1.75 \\ -2.45 \\ -1.7 \end{bmatrix} = -2.45 \begin{bmatrix} 0.7 \\ 1 \\ 0.7 \end{bmatrix} = -2.45\, Y_6$$

$$BY_6 = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0.7 \\ 1 \\ 0.7 \end{bmatrix} = \begin{bmatrix} -1.7 \\ -2.4 \\ -1.7 \end{bmatrix} = -2.4 \begin{bmatrix} 0.7 \\ 1 \\ 0.7 \end{bmatrix} = -2.4Y_7$$

$$BY_7 = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0.7 \\ 1 \\ 0.7 \end{bmatrix} = \begin{bmatrix} -1.7 \\ -2.4 \\ -1.7 \end{bmatrix} = -2.4 \begin{bmatrix} 0.7 \\ 1 \\ 0.7 \end{bmatrix} = -2.4 Y_8$$

Greatest eigen value = - 2.4
Smallest eigen value of A = - 2.4 + 3.42 = 1.02
Eigen values are - 2 , 4 , 1

(iii) Solve the equations $5x+2y+z=12$; $x+4y+2z=15$; $x+2y+5z=20$ by (i) Jacobi's method (ii) Gauss Seidal method.

Ans :

$5x+2y+z=12$; $x+4y+2z=15$; $x+2y+5z=20$

$$x = \frac{1}{5}(12-2y-z) \quad ; \quad y = \frac{1}{4}(15-x-2z) \quad ; \quad z = \frac{1}{5}(20-x-2z)$$

Let $x_0 = y_0 = z_0 = 0$

(i) Gauss Jacobi's method

x	y	z
2.4	3.75	4
0.1	1.15	2.02
1.54	2.71	3.52
0.61	1.61	2.61
1.24	2.29	3.23
0.83	1.82	2.83
1.10	2.12	3.10
0.93	1.92	2.93
1.04	2.05	3.04
0.97	1.96	2.97
1.02	2.02	3.02

The solutions are $x = 1.02$; $y = 2.02$; $z = 3.02$

(ii) Gauss Seidal method

x	y	z
2.4	3.15	2.26
0.688	2.448	2.883
0.844	2.097	2.992
0.9628	2.0133	3.0021
0.994	2.0004	2.999
1.00004	2.000049	2.9998

The solutions are $x = 1$; $y = 2$; $z = 3$

14.(i) Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ by Simpson's rule and Trapezoidal rule with $h = 0.5$ and $k = 0.25$.

Ans :

$I = \int_1^2\int_1^2 \frac{dxdy}{x+y}$, h = 0.5 , k = 0.25

Trapezoidal rule :

$$I = \frac{0.5\times 0.25}{4}[f(1,1)+f(2,1)+f(1,2)+f(2,2)+2\{(f(3/2,1)+f(1,5/4)$$
$$+f(3,3/2)+f(1,7/4)+f(2,3/2)+f(2,7/4)+f(3/2,2)\}$$
$$+4f(3/2,5/4)+f(3/2,3/2)+f(3/2,7/4)]$$

$$= \frac{1}{32}[1.4167+2(2.7539)+4(1.09)] = 0.3420$$

Simpson's rule

$$I = \frac{0.5\times 0.25}{9}[f(1,1)+f(2,1)+f(1,2)+f(2,2)+2\{(f(1,3/2)+f(2,3/2)\}$$
$$+4\{f(3/2,1)+f(3/2,2)+f(2,5/4)+f(2,7/4)+f(1,7/4)+f(1,5/4)\}$$
$$+8f(3/2,3/2)+16\{f(3/2,5/4)+f(3/2,7/4)\}]$$

$= 0.3398.$

(ii) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface.

X	100	150	200	250	300	350	400
Y	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values y of when $x = 218$ ft and 410 ft.

Ans :

x	y	Δ	Δ^2	Δ^3	Δ^4
100	10.63				
		2.40			
150	13.03		-0.39		
		2.01		0.15	
200	15.04		-0.24		-0.07
		1.77		0.08	
250	16.81		-0.16		-0.05
		1.61		0.03	
300	18.42		-0.13		-0.01
		1.48		0.02	
350	19.90		-0.11		
		1.37			
400	21.27				

(1) $x_0 = 200$, $y_0 = 15.04$, $\Delta y_0 = 1.77$, $\Delta^2 y_0 = -0.16$, $\Delta^3 y_0 = 0.03$

$x = 218$, $h = 50$ $\therefore$ $p = \frac{x - x_0}{h} = 0.36$

Using formula $f(218) = 16.696 \sim 15.7$

(2) $x = 400$, $p = \frac{x - x_n}{h} = 0.2 \Rightarrow y_{410} = 21.53$

(iii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Trapezoidal rule and Simpson's 1/3 rule and compare with its exact solution.

Ans :

Here $b - a = 6 - 0 = 6$. Divide into 6 equal parts $h = \frac{6}{6} = 1$

x	0	1	2	3	4	5	6
$\frac{1}{1+x^2} = f(x)$	1.000	0.500	0.200	0.100	0.058824	0.038462	0.27027

There are 7 ordinates (n = 6). We can use all the formula

1. Trapezoidal rule

$$I = \int_0^6 \frac{dx}{1+x^2}$$

$$= \frac{1}{2} [(1 + 0.027027) + 2 (0.5 + 0.2 + 0.1 + 0.058824 + 0.038462)]$$

$$= 1.41079950$$

2. Simpson's 1/3 rule

y_0	y_1	y_2	y_3	y_4	y_5	y_6
1	4	1				
		1	4	1		
				1	4	1
1	4	2	4	2	4	1

$$\therefore I = \frac{h}{3} [(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

$$= \frac{1}{3} [(1 + 0.027027) + 2(0.2 + 0.58824) + 4(0.5 + 0.1 + 0.038462)]$$

$$= 1.36617433$$

3. Actual integration

$$I = \int_0^6 \frac{dx}{1+x^2} = (\tan^{-1} x)_0^6 = \tan^{-1} 6 = 1.40564765$$

Here the value by trapezoidal rule is closer to the actual value than the value by Simpson's 1/3 rule.

(iv) Given that :

X	1	1.1	1.2	1.3	1.4	1.5	1.6
Y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and y'' at $x=1.1$ and $x=1.6$.

Ans :

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
1	7.989						
		0.414					
1.1	8.403		-0.036				
		0.378		0.006			
1.2	8.781		-0.030		-0.002		
		0.348		0.004		0.002	
1.3	9.129		-0.026		0		-0.003
		0.322		0.004		-0.001	
1.4	9.451		-0.023		-0.001		
		0.299		0.005			
1.5	9.750		-0.018				
		0.281					
1.6	10.031						

$$y' = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} + \ldots\ldots\right]$$

Where $h = 0.1$, $x_0 = 1.1$, $\Delta y_0 = 0.378$, $\Delta^2 y_0 = -0.030$, $\Delta^4 y_0 = 0$

$y'_{1.1} = 3.941$, $(y'')_{x=1.1} = -3.3167$

Similarly $h = 0.1$, $x_n = 1.6$, $\nabla y_n = 0.281$, *etc*

$(y')_{x=1.6} = 2.732$, $(y'')_{x=1.6} = -1.475$

15.(i) The deflection of a beam is governed by the equations $y'' + 81y = \phi(x)$ where $\phi(x)$ is given by the table :

x	1/3	2/3	1
$\phi(x)$	81	162	243

And the boundary conditions $y(0) = y'(0) = y''(1) = y'''(1) = 0$. Evaluate the deflection at the pivotal points of the beam using three subintervals.

Ans :

Out of syllabus.

(ii) Apply Taylor's method to obtain approximate value of y at $x = 0.2$ for the differential equation $y' = 2y + 3e^{x}$. Compare the numerical solution with its exact solution.

Ans :

Here $h = 0.1$, $x_0 = 0$, $y_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$

$y' = 2y + 3e^{x}$ $\quad$ $y_0' = 2y_0 + 3e^{x_0} = 3$

$y'' = 2y' + 3e^{x}$ $\quad$ $y_0'' = 2y_0' + 3e^{x_0} = 9$

$y''' = 2y'' + 3e^{x}$ $\quad$ $y_0''' = 2y_0'' + 3e^{x_0} = 21$

$y^{iv} = 2y''' + 3e^{x}$ $\quad$ $y_0^{iv} = 2y_0''' + 3e^{x_0} = 45$

By Taylor's series formula

$$y_1 = y_0 + \frac{h}{1}y_0' + \frac{h^2}{2}y_0'' + \frac{h^3}{6}y_0''' + \frac{h^4}{24}y_0^{iv} + ...$$

$$y(0.1) = y_1 = 0 + (0.1)(3) + \frac{(0.01)}{2}(9) + \frac{(0.001)}{6}(21) + \frac{(0.0001)}{24}(45) + ...$$

$= 0.3 + 0.045 + 0.0035 + 0.0001875 + \ldots$

$= 0.3486875 = 0.349$

$y_1' = 2y_1 + 3e^{x_1} = 0.3486875 \times 2 + 3e^{0.1} = 4.012887$

$y_1'' = 2y_1' + 3e^{x_1} = 11.34$

$y_1''' = 2y_1'' + 3e^{x_1} = 25.996$

$$y_2 = y\,(0.2) = y_1 + \frac{h}{1}y_1' + \frac{h^2}{2}y_1'' + ...$$

$= 0.3486875 + (0.1)(4.012887) + \frac{0.01}{2}(11.34) + \frac{0.001}{6}(25.996) + ...$

$= 0.8110156 = 0.811$

The exact value of $y(0.1) = 0.3486955$ and $y(0.2) = 0.8112658$

(iii) Using R.K. fourth order method to find y at $x = 0.1, 0.2, 0.3$ given that $y' = xy + y^2$. Continue the solution at x = 0.4 using Milne's P-C method.

Ans :

Given $y' = f(x,y) = xy + y^2$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$ $\quad$ h = 0.1

<u>To find y(0.1)</u> :

$k_1 = h\,f(x_0, y_0) = (0.1)\left(x_0 y_0 + y_0^{\,2}\right)$

$= (0.1)(0+1) = 0.1$

$$k_2 = h\,f\left[x_0 + \frac{h}{2}\ ,\ y_0 + \frac{k_1}{2}\right] = (0.1)\ f\,[0.05\,, 1.05\,]$$

$= (0.1)\left[(0.05)(1.05) + (1.05)^2\right] = 0.1155$

$$k_3 = h\, f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right]$$

$$= (0.1)\ f\left[0.05\ , 1.0578\right] = (0.1)\left[(0.05)(1.0578) + (1.0578)^2\right] = 0.1172$$

$$k_4 = h\, f(\ x_0 + h\ , y_0 + k_3) = (0.1)\ f\left(0.1\ , 1.1172\right)$$

$$= (0.1)\left[(0.1)(1.1172) + (1.1172)^2\right] = 0.1360$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1 + 2(0.1155) + 2(0.1172) + 0.1360] = 0.1169$$

$$y_1 = y_0 + \Delta y = 1 + 0.1169 = 1.1169$$

To find y(0.2) :

$$k_1 = h\ f(x_1, y_1) = (0.1)\left[(0.1)(1.1169) + (1.1169)^2\right]$$

$$= 0\cdot 1359$$

$$k_2 = h\, f\left[x_1 + \frac{h}{2}\ ,\ y_1 + \frac{k_1}{2}\right] = (0.1)\ f\ [\ 0.15, 1.1849\ ]$$

$$= (0.1)\left[(0.15)(1.1849) + (1.1849)^2\right] = 0.1582$$

$$k_3 = h\, f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right]$$

$$= (0.1)\, f\ [\ 0.15, 1.196\ ] = (0.1)\left[(0.15)(1.196) + (1.196)^2\right] = 0.1610$$

$$k_4 = h\, f(x_1 + h\ ,\ y_1 + k_3) = (0.1)\ f\ [0.2, 1.2779\ ]$$

$$= (0.1)\left[(0.2)(1.2779) + (1.2779)^2\right] = 0.1889$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1359 + 2(0.1582) + 2(0.1610) + 0.1889] = 0..9632$$

$$y_2 = y_1 + \Delta y = 1.1169 + 0.9632 = 2.0801$$

To find y(0.3) :

$$k_1 = h\ f(x_2\ ,\ y_2) = (0.1)\left[(0.2)(2.0801) + (2.0801)^2\right]$$

$$= 0.4743$$

$$k_2 = h f\left[x_1+\frac{h}{2}, y_1+\frac{k_1}{2}\right] = (0.1)\ f\ [\ 0.25, 2.3173\ ]$$

$$= (0.1)\left[(0.25)(2.3173)+(2.3173)^2\right] = 0.5949$$

$$k_3 = h f\left[x_1+\frac{h}{2}, y_1+\frac{k_2}{2}\right] = (0.1)\ f\ \left[0.25,\ 2.3776\right]$$

$$= (0.1)\left[(0.25)(2.3776)+(2.3776)^2\right] = 0.6247$$

$$k_4 = h f(x_1+h,\ y_1+k_3) = (0.1)\ f\ [0.3, 2.7048\ ]$$

$$= (0.1)\left[(0.3)(2.7048)+(2.7048)^2\right] = 0.8127$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.4743 + 2(0.5949) + 2(0.6247) + 0.8127] = 0..6210$$

$$y_3 = y_2 + \Delta y = 2.0801 + 0.6210 = 2.7011$$

By Milne's predictor formula

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}[2y_{n-2}' - y_{n-1}' + 2y_n']$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}[y_{n-1}' + 4y_n' + y_{n+1}']$$

$$y_1' = x_1 y_1 + y_1^2 = (0.1)(1.1169) + (1.1169)^2 = 1.3592$$

$$y_2' = x_2 y_2 + y_2^2 = (0.2)(2.0801) + (2.0801)^2 = 4.7428$$

$$y_3' = x_3 y_3 + y_3^2 = (0.3)(2.7011) + (2.7011)^2 = 8.1063$$

$$y_{4,p} = y_0 + \frac{4(0.1)}{3}[2y_1' - y_2' + 2y_3']$$

$$= 1 + \frac{4(0.1)}{3}[2(1.3592) - 4.7428 + 2(8.1063)] = 2.8918$$

$$y_{4,c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + 2y_4']$$

$$y_4' = x_4 y_4 + y_4^2 = (0.4)(2.8918) + (2.8918)^2 = 9.5192$$

$$y_{4,c} = 2.0801 + \frac{0.1}{3}[4.7428 + 4(8.1063 + 9.5192] = 3.6363$$

Agni College of Technology
Chennai - 130

B.E./ B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016
(Common to Mechanical Engineering and Mechatronics Engineering)
Fourth Semester
MA6452 –STATISTICS AND NUMERICAL METHODS
(Regulation 2013)

Time : Three hours **Maximum : 100 marks**

Answer ALL questions
PART A – (10 x 2 = 20 marks)

1. What are Type-I and Type-II errors?
Solution:
Type-I Error: The rejection of the null hypothesis H_o when it is true is called Type-I Error.
Type-II Error: The acceptance of the null hypothesis H_o when it is called Type II error.

2. Give the formula for the χ^2-test of independence for
Solution:
$$\text{The value of } \chi^2 = \frac{N(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

3. State the principles of Design of Experiments.
Solution:
Three important designs are the following:
(i)Completely Randomized Design(CRD)
(ii) Randomized Block Design(RBD)
(iii) Latin Square Design(LSD)

4. Is 2×2 Latin Square Design possible? Why?
Solution:
Consider a $n\times n$ Latin square design, then the degrees of freedom for SSE is
$$=(n^2-1)-(n-1)-(n-1)-(n-1)=n^2-1-3n+3=n^2-3n+2$$
$$=(n-1)(n-2)$$
For n = 2, degree of freedom of SSE = 0 and hence MSE is not defined. Therefore comparisons are not possible. Hence a 2×2 Latin Square Design is not possible.

5. Mention the order and condition for the convergence of Newton-Raphson method.
Solution:
Order is 2
The condition is $\left|f(x).f''(x)\right| < \left(f'(x)\right)^2$ in a neighborhood of the root.

6. What is the procedure of Gauss-Jordan method?
Solution:
Consider the augmented matrix $[A, I]$, where I is the identity matrix of the same Order as A. By row operations reduce A into a unit matrix, then correspondingly I will be changed into a matrix X. This matrix X is the inverse of A.

It is advisable to change the pivot element to 1 before applying row operations at each step.

7. Specify the Newton's backward difference formulae for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \ldots\ldots\ldots\ldots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \ldots\ldots\ldots\ldots\right]$$

8. Write down the errors in Trapezoidal and Simpson's rules of numerical integration.

Solution :

Trapezoidal rule $Error \Rightarrow |E| < \frac{(b-a)h^2}{12}M$

$Order \Rightarrow h^2$

Simpson's rule $Error \Rightarrow |E| < \frac{(b-a)h^4}{180}M$

$order \Rightarrow h^4$

9. Find $y(0.1)$ by Euler's method, if $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 0.1$

Solution :

$$y_{n+1} = y_n + h\,f(x_n, y_n) \quad n = 0,1,2\ldots\ldots$$

$$x_0 = 0 \quad y_0 = 0.1 \quad h = 0.1$$

$$Given\ y' = f(x, y) = x^2 + y^2$$

$$y_1 = y_0 + 0.1 f(x_0, y_0)$$

$$y_1 = 0.1 + 0.1[x_0{}^2 + y_0{}^2]$$

$$= 0.1 + 0.1[0 + (0.1)^2]$$

$$= 0.1 + 0.1[0 + 0.01]$$

$$= 0.1 + 0.0001 = 0.1001$$

$$y(0.1) = 0.1001$$

10. Give the central difference approximations for $y'(x), y''(x)$.

Solution:

$$y_i' = \frac{1}{2h}[y_{i+1} - y_{i-1}]$$

$$y_i'' = \frac{1}{h^2}[y_{i-1} - 2y_i + y_{i+1}]$$

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PART – B (5x 15 = 60)

11.a(i) A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with an SD of 6 and the boys made an average grade of 82 with an SD of 2. Test whether there is any difference between the performance of boys and girls.

Solution:

$$Given \quad n_1 = 50 \quad \overline{x_1} = 76 \quad S_1 = 6$$

$$n_2 = 75 \quad \overline{x_2} = 82 \quad S_2 = 2$$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ [Two tailed test]

$\alpha = 5\%$

The test statistic

$$z = \frac{\left(\overline{x_1} - \overline{x_2}\right)}{\sqrt{\frac{S_1^{\,2}}{n_1} + \frac{S_2^{\,2}}{n_2}}} = \frac{(76-82)}{\sqrt{\frac{(6)^2}{50} + \frac{(2)^2}{75}}} = \frac{-6}{\sqrt{0.72 + 0.053333}} = \frac{-6}{0.8794} = -6.8231$$

If $-Z_{\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}}$ then accept H_0; otherwise we reject H_0

$-1.96 < -6.8231 < 1.96$

$|z| = 6.8231$

$6.8231 > 1.96$

$\therefore$ We reject H_0

(ii) Theory predicts the proportion of beans in the groups A,B,C,D as 9:3:3:1. In an experiment among beans the numbers in the groups were 882, 313, 287 and 118. Does the experiment support the theory?

Solution:

Given n = 4

H_0 : The results in the four categories are in the ratio 9 : 3 : 3 : 1

H_1 : The results in the four categories are not in the ratio 9 : 3 : 3 : 1

$\alpha = 0.05$ degree of freedom $= n - 1 = 4 - 1 = 3$

Table value of $\chi^2 = 7.815$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

On the assumption H_0, the expected frequencies of the 4 classes are

$$\frac{9}{16} \times 1600\ ,\ \frac{3}{16} \times 1600\ ,\ \frac{3}{16} \times 1600\ ,\ \frac{1}{16} \times 1600$$

900, 300, 300, 100

	O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
A	882	900	-18	324	0.36
B	313	300	13	169	0.5633
C	287	300	-13	169	0.5633
D	118	100	18	324	3.24
	1600				4.7266

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 4.7266$$

Conclusion:

If Cal $\chi^2 <$ Tab χ^2, then we accept H_0, otherwise we reject H_0

Here $\chi^2 = 4.7266 < 7.815$

So, we accept H_0 at 5% level of significance

Therefore The Results of the four categories are in the ratio $9:3:3:1$

b(i) 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favor of the proposal. Test whether these two propositions are same.

Solution:

Let p_1, p_2 be the proportions favorable to have over among men and women respectively.

$$\therefore p_1 = \frac{200}{400}; \quad p_2 = \frac{325}{600}$$

p is not known $\quad \therefore$ The best eliminate of $P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

$$= \frac{400 \times \frac{200}{400} + 600 \times \frac{325}{600}}{400 + 600} = \frac{525}{100} = 0.525$$

$$Q = 1 - P = 1 - 0.525 = 0.475$$

$H_0 : p_1 = p_2$

$H_1 : p_1 \neq p_2 \quad [two-tailed\ test]$

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.5 - 0.542}{\sqrt{0.525 \times 0.475\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= \frac{0.5 - 0.542}{\sqrt{0.525 \times 0.475(0.0025 + 0.00167)}}$$

$$= \frac{-0.042}{0.0324} = -1.30$$

$$|Z| = 1.30$$

The table value of Z at 5% level is 1.96

Inference : Since $|Z| < 1.96$, H_0 is accepted at 5% level of significance.

The difference is not significant i.e. Men and women are equally favourable for fly over near their residence.

(ii) The IQ's of 10 girls are respectively 120, 110, 70, 88, 101, 100, 83, 98, 95, 107. Test whether the population mean IQ is 100.

Solution:

Given $n = 10$ $\mu = 100$

X	120	110	70	88	101	100	83	98	95	107	972
X^2	14400	12100	4900	7744	10201	10000	6889	9604	9025	11449	96312

$$\bar{X} = \frac{\sum x}{n} = \frac{972}{10} = 97.2 \quad i.e\ \bar{X} = 97.2$$

$$s^2 = \frac{\sum x^2}{n} - (\bar{x}^2) = \frac{96312}{10} - (97.2)^2 = 9631.2 - 9447.84$$

$$s^2 = 183.96 \quad \Rightarrow \quad s = 13.5$$

$H_0 : \mu = 100$

$H_1 : \mu_1 \neq 100$ [Two – tailed test]

$\alpha = 5\%$ degree of freedom $= n - 1 = 10 - 1 = 9$

The test statistic

$$t = \frac{\bar{X} - \mu}{\left(\frac{s}{\sqrt{n-1}}\right)} = \frac{97.2 - 100}{\left(\frac{13.5}{\sqrt{9}}\right)} = \frac{-2.8}{4.5} = -0.62$$

Conclusion

If $-t_{\alpha/2} < t < t_{\alpha/2}$ then we accept H_0; otherwise we reject H_0

Here $-2.262 < -0.62 < 2.262$ So we accept H_0

We may conclude that the data are consistent with the assumption of mean I.Q. of 100 in the population.

12.a Three varieties of coal were analysed by 4 chemists and the ash content is tabulated here. Perform an analysis of variance.

		Chemists			
Coal		A	B	C	D
	I	8	5	5	7
	II	7	6	4	4
	III	3	6	5	4

Solution:

Coal	Chemists				Total of variance	X_1^2	X_2^2	X_3^2	X_4^2
	A	B	C	D					
I	8	5	5	7	25	64	25	25	49
II	7	6	4	4	21	49	36	16	16
III	3	6	5	4	18	9	36	25	16
Total	18	17	14	15	64	122	97	66	81

H_0 : There is no siginificant difference

H_1 : There is siginificant difference

Step 1: $N = 12$

Step 2: $T = 64$

Step 3: $\frac{T^2}{N} = \frac{(64)^2}{12} = 341.3$

Step 4: $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 64 + 122 + 97 + 66 + 81 - 341.3$

$= 88.7$

Step 5: $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

$= \frac{324}{3} + \frac{289}{3} + \frac{196}{3} + \frac{225}{3} - 341.31$

$= 108 + 96.33 + 65.33 + 75 - 341.3 = 3.36$

Step 6: $SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} - \frac{T^2}{N}$

$= \frac{625}{4} + \frac{441}{4} + \frac{324}{4} - 341.3$

$= 156.25 + 110.25 + 81 - 341.3 = 6.2$

$SSE = TSS - SSC - SSR = 88.7 - 3.36 - 6.2 = 79.14$

$SSE = 79.14$

Source of variance	Sum of squares	d.f.	Mean square	Variance ratio	Table value at 5% level
Between columns	SSC $= 3.36$	C-1 $= 3 - 1 =$ 2	$MSC = \frac{SSC}{C-1}$ $= \frac{3.36}{2}$ $= 1.68$	$F_C = \frac{MSC}{MSE}$ $= \frac{1.68}{13.19}$ $= 0.1274$	$F_C(2,6)$ $= 5.14$

Between rows	SSR = 6.2	r-1 = 4-1 = 3	$MSR = \frac{SSR}{r-1} = \frac{6.2}{3} = 2.0666$	$F_R = \frac{MSR}{MSE} = \frac{2.0666}{13.19} = 0.1566$	$F_R(3,6) = 4.76$
Residual	SSE = 79.14	N – C- r+1 = 12 -4- 3+1 = 6	$MSE = \frac{SSE}{N-C} = \frac{79.14}{6} = 13.19$		
Total	88.7				

Conclusion

Cal F_R < Tab F_R

Cal F_C < Tab F_C

In both the cases the calculated value is less than tabulated value Therefore null hypothesis is accepted.

b. The result of an RBD experiment on 3 blocks with 4 treatments A,B,C,D are tabulated here. Carry out an analysis of variance.

Blocks	Treatment effects			
I	A36	D35	C21	B36
II	D32	B29	A28	C31
III	B28	C29	D29	A26

Solution:

Variety	Block			Total of variance	X_1^2	X_2^2	X_3^2
	I	II	III				
A	36	28	26	90	1296	784	676
B	36	29	28	93	1296	841	784
C	21	31	29	81	441	961	841
D	35	32	29	96	1225	1024	841
Total	128	120	112	360	4258	3610	3142

H_0 : There is no siginificant difference

H_1 : There is siginificant difference

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Step1: $N = 12$

Step 2: $T = 360$

Step 3: $\frac{T^2}{N} = \frac{(360)^2}{12} = 10800$

Step 4: $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$

$= 4258 + 3610 + 3142 - 10800 = 210$

Step 5: $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} - \frac{T^2}{N}$

$= \frac{16384}{4} + \frac{14400}{4} + \frac{12544}{4} - 10800$

$= 4096 + 3600 + 3136 - 10800 = 32$

Step 6: $SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

$= \frac{8100}{3} + \frac{8649}{3} + \frac{6561}{3} + \frac{9216}{3} - 10800$

$= 2700 + 2883 + 2187 + 3072 - 10800 = 42$

$SSE = TSS - SSC - SSR = 210 - 32 - 42 = 136$

Source of variance	Sum of squares	D.f	Mean square	Variance ratio	Table value at 5% level
Between rows	SSR $= 42$	$r-1$ $= 3-1$ $= 2$	$MSR = \frac{SSR}{r-1} = \frac{42}{2} = 21$	$F_R = \frac{MSR}{MSE} = \frac{21}{22.6666} = 0.9265$	$F_R(2,6) = 5.14$
Between column	SSC $= 32$	C-1 $= 4-1 =$ 3	$MSC = \frac{SSC}{C-1} = \frac{32}{3} = 10.6666$	$F_C = \frac{MSC}{MSE} = \frac{10.6666}{22.6666} = 0.4706$	$F_C(3,6) = 4.76$
Residual	SSE $= 136$	$N-C-r+1$ =12 -4 - 3+1 =6	$MSE = \frac{SSE}{N-C-r+1} = \frac{136}{3} = 22.6666$		
Total	210				

Conclusion

Cal F_R < Tab F_R

Cal F_C < Tab F_C

In both the cases the calculated value is less than tabulated value Therefore null hypothesis is accepted.

13.a(i) Solve the following equations by Gauss elimination method:

$$2x + y + 4z = 12,$$
$$8x - 3y + 2z = 20,$$
$$4x + 11y - z = 33$$

Solution: The augmented matrix is

$$[A, B] = \begin{bmatrix} 2 & 1 & 4 : 12 \\ 8 & -3 & 2 : 20 \\ 4 & 11 & -1 : 33 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 : 12 \\ 0 & -7 & -14 : -28 \\ 0 & 9 & -9 : 9 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + (-4)R_1 \\ R_3 \to R_3 + (-2)R_1 \end{matrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 : 12 \\ 0 & 1 & 2 : 4 \\ 0 & 1 & -1 : 1 \end{bmatrix} \quad \begin{matrix} R_2 \to \frac{1}{7} R_2 \\ R_3 \to \frac{1}{9} R_1 \end{matrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 : 12 \\ 0 & 1 & 2 : 4 \\ 0 & 0 & -3 : -3 \end{bmatrix} \quad R_3 \to R_3 + (-1)R_2$$

$\therefore$ *The equivalent reduced equations are*

$$2x + y + 4z = 12$$
$$\Rightarrow y + 2z = 4 \quad \Rightarrow \quad -3z = -3 \quad \Rightarrow \quad z = 1$$
$$\therefore y + 2 = 4 \quad \Rightarrow \quad y = 2$$
$$2x + 2 + 4 = 12 \quad \Rightarrow 2x = 6 \quad \Rightarrow x = 3$$

$\therefore$ *Solution is* $x = 3, \quad y = 2, \quad z = 1$

(ii) Using power method find the dominant eigen value of the matrix $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$

Solution:

Let $X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be an arbitrary initial Eigenvector.

$$AX_1 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 25 \\ 1 \\ 2 \end{pmatrix} = 25 \begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = 25X_2$$

$$AX_2 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = \begin{pmatrix} 25.2 \\ 1.12 \\ 1.68 \end{pmatrix} = 25.2\begin{pmatrix} 1 \\ 0.0444 \\ 0.0667 \end{pmatrix} = 25.2X_3$$

$$AX_3 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0444 \\ 0.0667 \end{pmatrix} = \begin{pmatrix} 25.1778 \\ 1.1332 \\ 1.7337 \end{pmatrix} = 25.1778\begin{pmatrix} 1 \\ 0.0450 \\ 0.06888 \end{pmatrix} = 25.1778X_4$$

$$AX_4 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0450 \\ 0.06888 \end{pmatrix} = \begin{pmatrix} 25.1826 \\ 1.135 \\ 1.7248 \end{pmatrix} = 25.1826\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = 25.1826X_5$$

$$AX_5 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \begin{pmatrix} 25.1821 \\ 1.1353 \\ 1.7260 \end{pmatrix} = 25.1821\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = 25.1821X_6$$

we have reached the limit.

$\therefore \lambda_1 = 25.1821$ and the corresponding Eigenvector is $\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$

b(i) If $A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$, find A^{-1} by Gauss-Jordan method.

Solution:

Consider the augumented matrix

$$[A, I] = \begin{bmatrix} 4 & 1 & 2 & : & 1 & 0 & 0 \\ 2 & 3 & -1 & : & 0 & 1 & 0 \\ 1 & -2 & 2 & : & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{2} & : & \frac{1}{4} & 0 & 0 \\ 2 & 3 & -1 & : & 0 & 1 & 0 \\ 1 & -2 & 2 & : & 0 & 0 & 1 \end{bmatrix} \qquad R_1 \to \frac{1}{4}R_1$$

[use pivot 4 in R_1 is reduced to 1]

$$\sim \begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{2} & : & \frac{1}{4} & 0 & 0 \\ 0 & \frac{5}{2} & -2 & : & -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{9}{4} & \frac{3}{2} & : & -\frac{1}{4} & 0 & 1 \end{bmatrix} \qquad \begin{aligned} R_2 &\to R_2 + (-2)R_1 \\ R_3 &\to R_3 - R_1 \end{aligned}$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -\frac{4}{5} & -\frac{1}{5} & \frac{2}{5} & 0 \\ 0 & -\frac{9}{4} & \frac{3}{2} & -\frac{1}{4} & 0 & 1 \end{array}\right] \quad R_2 \to \frac{2}{5} R_2$$

[The pivot $\frac{5}{2}$ in R_2 is reduced to 1]

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{7}{10} & \frac{3}{10} & \frac{-1}{10} & 0 \\ 0 & 1 & -\frac{4}{5} & -\frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 0 & -\frac{3}{10} & -\frac{7}{10} & \frac{9}{10} & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + \left(-\frac{1}{4}\right) R_2 \\ R_3 \to R_3 + \frac{9}{4} R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{7}{10} & \frac{3}{10} & \frac{-1}{10} & 0 \\ 0 & 1 & -\frac{4}{5} & -\frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 0 & 1 & \frac{7}{3} & -3 & -\frac{10}{3} \end{array}\right] \quad R_3 \to -\frac{10}{3} R_3$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -\frac{4}{3} & 2 & \frac{7}{3} \\ 0 & 1 & 0 & \frac{5}{3} & -2 & -\frac{8}{3} \\ 0 & 0 & 1 & \frac{7}{3} & -3 & -\frac{10}{3} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + \left(-\frac{7}{10} R_3\right) \\ R_2 \to R_2 + \frac{4}{3} R_3 \end{array}$$

$$\therefore \text{The inverse of A is } A^{-1} = \begin{bmatrix} -\frac{4}{3} & 2 & \frac{7}{3} \\ \frac{5}{3} & -2 & -\frac{8}{3} \\ \frac{7}{3} & -3 & -\frac{10}{3} \end{bmatrix}$$

(ii) Solve the following equations by Gauss-seidel method

$x + y + 9z = 15,$

$x + 17y - 2z = 48,$

$30x - 2y + 3z = 75$

Solution:

As the coefficient matrix is not diagonally dominant we rewrite the equations.

$30x - 2y + 3z = 75$

$x + 17y - 2z = 48$

$x + y + 9z = 15$

Since, the diagonal elements are dominant in the coefficient matrix, we write x , y , z as follows:

$$x=\frac{1}{30}[75+2y-3z] \qquad y=\frac{1}{17}[48-x+2z]$$

$$z=\frac{1}{15}[15-x-y]$$

Let the initial values be $y=0 \; z=0$

First Iteration

$$x^{(1)}=\frac{1}{30}\left[75+2y^{(0)}-3z^{(0)}\right]=\frac{1}{30}[75+2(0)-3(0)]=2.5$$

$$y^{(1)}=\frac{1}{17}\left[48-x^{(1)}+2z^{(0)}\right]=\frac{1}{17}[48-2.5+2(0)]=2.6765$$

$$z^{(1)}=\frac{1}{15}\left[15-x^{(1)}-y^{(1)}\right]=\frac{1}{15}[15-2.5-2.6765]=1.0915$$

Second Iteration

$$x^{(2)}=\frac{1}{30}\left[75+2y^{(1)}-3z^{(1)}\right]=\frac{1}{30}[75+2(2.6765)-3(1.0915)]=2.5693$$

$$y^{(2)}=\frac{1}{17}\left[48-x^{(2)}+2z^{(1)}\right]=\frac{1}{17}[48-2.5693+2(1.0915)]=2.8008$$

$$z^{(2)}=\frac{1}{15}\left[15-x^{(2)}-y^{(2)}\right]=\frac{1}{15}[15-2.5693-2.8008]=1.0700$$

Third Iteration

$$x^{(3)}=\frac{1}{30}\left[75+2y^{(2)}-3z^{(2)}\right]=\frac{1}{30}[75+2(2.5693)-3(1.0700)]=2.5800$$

$$y^{(3)}=\frac{1}{17}\left[48-x^{(3)}+2z^{(2)}\right]=\frac{1}{17}[48-2.5800+2(1.0700)]=2.7976$$

$$z^{(3)}=\frac{1}{15}\left[15-x^{(3)}-y^{(3)}\right]=\frac{1}{15}[15-2.5800-2.7976]=1.0692$$

Fourth Iteration

$$x^{(4)}=\frac{1}{30}\left[75+2y^{(3)}-3z^{(3)}\right]=\frac{1}{30}[75+2(2.5800)-3(1.0692)]=2.5796$$

$$y^{(4)}=\frac{1}{17}\left[48-x^{(4)}+2z^{(3)}\right]=\frac{1}{17}[48-2.5796+2(1.0692)]=2.7976$$

$$z^{(4)}=\frac{1}{15}\left[15-x^{(4)}-y^{(4)}\right]=\frac{1}{15}[15-2.5796-2.7976]=1.0692$$

$$x=2.2800 \quad y=2.7976 \quad z=1.0692$$

14.a(i) Interpolate $y(12)$, if

X:	10	15	20	25	30	35
Y(x):	35	33	29	27	22	14

Solution:

Given

$x_0=10, x_1=15, x_2=20, x_3=25, x_4=30, x_5=35$

$y_0=35, y_1=33, y_2=29, y_3=27, y_4=22, \; y_5=14$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)}y_0$$
$$+\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}y_1$$
$$+\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)}y_2$$
$$+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)}y_3$$
$$+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)}y_4$$
$$+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_5-x_0)(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)}y_5$$

$$y(12) = \frac{(12-15)(12-20)(12-25)(12-30)(12-35)}{(10-15)(10-20)(10-25)(10-30)(10-35)}35$$
$$+\frac{(12-10)(12-20)(12-25)(12-30)(12-35)}{(15-10)(15-20)(15-25)(15-30)(15-35)}33$$
$$+\frac{(12-10)(12-15)(12-25)(12-30)(12-35)}{(20-10)(20-15)(20-25)(20-30)(20-35)}29$$
$$+\frac{(12-10)(12-15)(12-20)(12-30)(12-35)}{(25-10)(25-15)(25-20)(25-30)(25-35)}27$$
$$+\frac{(12-10)(12-15)(12-20)(12-25)(12-35)}{(30-10)(30-15)(30-20)(30-25)(30-35)}22$$
$$+\frac{(12-10)(12-15)(12-20)(12-25)(12-35)}{(35-10)(35-15)(35-20)(35-25)(35-30)}14$$

$$y(12) = \frac{(-3)(-8)(-13)(-18)(-23)}{(-5)(-10)(-15)(-20)(-25)}35 + \frac{(2)(-8)(-13)(-18)(-23)}{(5)(-5)(-10)(-15)(-20)}33$$
$$+\frac{(2)(-3)(-13)(-18)(-23)}{10(5)(-5)(-10)(-15)}29 + \frac{2(-3)(-8)(-18)(-23)}{(15)(10)(5)(-5)(-10)}27$$
$$+\frac{2(-3)(-8)(-13)(-23)}{20(15)(10)(5)(-5)}22 + \frac{2(-3)(-8)(-13)(-23)}{25(20)(15)(10)(5)}14$$

$$y(12) = 12.056 + 37.8893 - 24.9725 + 14.3078 - 4.20992 + 0.535808$$
$$= 35.6065$$

(ii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's 1/3 rule, dividing the range into four equal parts.

Solution:

X	0	0.25	0.5	0.75	1
f(x)= $\frac{1}{1+x^2}$	1	0.9412	0.8	0.64	0.5

By Simpson's $\frac{1}{3}$rd rule formula is

$$\int_0^1 \frac{dx}{1+x^2} = \left(\frac{h}{3}\right)[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)]$$

$$= \frac{0.25}{3}[(1+0.5) + 4(0.9412 + 0.64) + 2(0.8)]$$

$$= 0.08333[1.5 + 4(1.5812) + 1.6]$$

$$= 0.08333[1.5 + 6.3248 + 1.6]$$

$$= 0.08333[9.4248] \quad = \quad 0.7854$$

b(i) Find $y'(1)$, if

X:	-1	0	2	3
Y(x):	-8	3	1	12

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	-8			
		11		
0	3		-13	
		-2		26
2	1		13	
		11		
3	12			

By Newton's forward fromula, we have,

$$\left(\frac{dy}{dx}\right)_{x=x_0} = y'(1) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + ...\right]$$

$$= \frac{1}{1}\left[11 - \frac{1}{2}(-13) + \frac{1}{3}(26)\right]$$

$$= \left[11 + \frac{13}{2} + \frac{26}{3}\right] = \frac{157}{6} = 26.1666$$

(ii) Using Trapezoidal rule, evaluate $\int_1^2\int_1^2 \frac{dx+dy}{x+y}$ with $h = k = 0.5$

Solution:

x/y	1	1.5	2
1	0.5	0.4	0.3333
1.5	0.4	0.3333	0.2857
2	0.3333	0.2857	0.25

By using Trapezoidal rule

$$I=\frac{hk}{4}\begin{bmatrix}\text{(sum of values of f at the four corners)}\\ +2\text{(sum of the values of f at the remaining nodes on the boundary)}\\ +4\text{(sum of the values of f at the interior nodes)}\end{bmatrix}$$

$$I=\frac{(0.5)(0.5)}{4}\begin{bmatrix}(0.5+0.3333+0.25+0.3333)\\ +2(0.4+0.4+0.2857+0.2857)+4(0.3333)\end{bmatrix}$$

$$=\frac{0.25}{4}[(1.4166)+2(1.3714)+1.3332]$$

$$=0.0625[1.4166+2.7428+1.3332]$$

$$=0.0625[5.4926]$$

$$I=0.3433$$

15.a If $\frac{dy}{dx}=x^2+y^2, y(0)=1$, find $y(0.1)$, $y(0.2)$ and $y(0.3)$ by Taylor series method. Hence find $y(0.4)$ by Milne's Predictor – Corrector method.

Solution:

Given $y'=y^2+x^2$, $x_0=0$, $y_0=1$

By Taylor's series formula

$$y=y_0+\frac{(x-x_0)}{1!}y_0'+\frac{(x-x_0)^2}{2!}y_0''+\frac{(x-x_0)^3}{3!}y_0'''+......(1)$$

$y'=x^2+y^2$ $\qquad y_0'=x_0^2+y_0^2=1+0=1$

$y''=2yy'+2x$ $\qquad y_0''=2y_0y_0'+2x_0=2(1)(1)+0=2$

$y'''=2[yy''+y'']+2$ $\qquad y_0'''=2y_0y_0''+2(y_0')^2+2$

$=2yy''+2(y')^2+2$ $\qquad =2(1)+2+2(1)^2+2$

$=2+2+2+2=8$

$$(1)\Rightarrow y=1+\frac{x}{1!}(1)+\frac{x^2}{2!}(2)+\frac{x^3}{3!}(8)+.......$$

$$=1+x+x^2+\frac{4x^3}{3}+............$$

$$y(0.1)=1+0.1+(0.1)^2+\frac{4(0.1)^3}{3}=1+0.1+0.01+0.00133$$

$$=1.11133$$

$$y(0.2)=1+0.2+(0.2)^2+\frac{4(0.2)^3}{3}=1+0.2+0.04+0.01067$$

$$=1.25067$$

$$y(0.3)=1+0.3+(0.3)^2+\frac{4(0.3)^3}{3}=1+0.3+0.09+0.12$$

$$=1.51$$

$x_0 = 0 \quad y_0 = 1 \quad y_0' = x_0^2 + y_0^2 = 0 + 1 = 1$

$x_1 = 0.1 \quad y_1 = 1.11133 \quad y_1' = x_1^2 + y_1^2 = 1.2451$

$x_2 = 0.2 \quad y_2 = 1.25067 \quad y_2' = x_2^2 + y_2^2 = 1.6042$

$x_3 = 0.3 \quad y_3 = 1.51 \quad y_3' = x_3^2 + y_3^2 = 2.3701$

$x_4 = 0.4 \quad y_4 = ?$

By Milne's predictor formula

$$y_{4}, p = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 1 + \frac{4(0.1)}{3}[2(1.2451) - 1.6042 + 2(2.3701)]$$

$$= 1 + 0.1333[2.4902 - 1.6042 + 4.7402]$$

$$= 1 + 0.1333[5.6262] \quad = \quad 1 + 0.7499$$

$$y_4 = 1.7499$$

$x_4 = 0.4, \quad y_4 = 1.7499 \quad y_4' = x_4^2 + y_4^2 = 3.2222$

$$y_{4,c} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 1.25067 + \frac{0.1}{3}[1.6042 + 4(2.3701) - 3.2222]$$

$$= 1.25067 + 0.0333[1.6042 + 9.4804 - 3.2222]$$

$$= 1.25067 + 0.0333[7.8624]$$

$$= 1.3752 + 0.2618$$

$$y_{4}, c = 1.6370$$

b. If $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$, find $y(0.2), y(0.4), y(0.6)$ by Runge – Kutta method. Hence f by Milne's method.

Solution:

$$Given \frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, \quad y(0) = 1$$

$$Then\ f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}, x_0 = 0, y_0 = 1, h = 0.2$$

(since the valeus are required at $x = 0.2, 0.4, 0.6$ given $x = 0$)

The fourth – order Runge – Kutta method is

$$k_1 = h\, f(x_n, y_n)$$

$$k_2 = h\, f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h\, f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h\, f(x_n + h, y_n + k_3),\ n = 0,1,2....$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

To find y_1

$$k_1 = h\, f(x_0, y_0) = h\left(\frac{y_0^{\,2} - x_0^{\,2}}{y_0^{\,2} + x_0^{\,2}}\right) = 0.2\left(\frac{1-0}{1+0}\right) = 0.2$$

$$k_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\ f[0.1,\ 1+0.1]$$

$$= 0.2\ f[0.1,\ \ 1.1] = 0.2\left(\frac{1.1^2 - 0.1^2}{1.1^2 - 0.1^2}\right) = 0.2\left(\frac{1.20}{1.22}\right) = 0.1967$$

$$k_3 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.2\ f\left[0.1,\ 1 + \frac{0.1967}{2}\right] = 0.2\ f[0.1,\ 1.09835]$$

$$= 0.2\left(\frac{1.09835^2 - 0.1^2}{1.09835^2 + 0.1^2}\right) = 0.2\left(\frac{1.2064 - 0.01}{1.2064 + 0.01}\right) = 0.2\left(\frac{1.1964}{1.2164}\right) = 0.1967$$

$$k_4 = h\, f(x_0 + h, y_0 + k_3) = 0.2\ f[0.2, 1 + 0.1967] = 0.2\left(\frac{1.1967^2 - 0.2^2}{1.1967^2 + 0.2^2}\right)$$

$$= 0.2\left(\frac{1.4321 - 0.04}{1.4321 + 0.04}\right) = 0.2\left(\frac{1.3921}{1.4721}\right) = 0.1891$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= \frac{1}{6}(0.2 + 2\times 0.1967 + 2\times 0.1967 + 0.1891) = 0.19598$$

$$\therefore\ y_1 = y_0 + \Delta y = 1 + 0.19598 = 1.196$$

Now $x_1 = 0.2, y_1 = 1.196, h = 0.2$

To find y_2

$$k_1 = h\, f(x_1, y_1) = h\left(\frac{y_1^{\,2} - x_1^{\,2}}{y_1^{\,2} + x_1^{\,2}}\right) = 0.2\left(\frac{1.4304 - 0.04}{1.4304 + 0.04}\right) = 0.2\frac{1.3904}{1.4704} = 0.1891$$

$$k_2 = h\, f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = 0.2\ f\left(0.2 + \frac{0.2}{2}, 1.196 + \frac{0.1891}{2}\right)$$

$$=\ 0.2\ f[0.3,\ 1.2906] = 0.1795$$

$$k_3 = h\,f\left(x_1+\frac{h}{2}, y_1+\frac{k_2}{2}\right) = h\,f\left(0.2+\frac{0.2}{2}, 1.196+\frac{0.1795}{2}\right)$$

$$= \;0.2\,f[0.3,\,1.2858] = 0.1793$$

$$k_4 = h\,f(x_1+h, y_1+k_3) = 0.2\,f[0.2+0.2, 0.1891+0.1793]$$

$$= 0.2\,f(0.4, 1.3753) = 0.1688$$

$$\Delta y = \frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

$$= \frac{1}{6}[0.1891+2\times 0.1795+2\times 0.1793+0.1688] = 0.1792$$

$$\therefore\; y_2 = y_1+\Delta y = 1.196+0.1792 = 1.3752$$

$Now \quad x_2 = 0.4,\; y_2 = 1.3752,\; h = 0.2$

To find y_3

$$k_1 = h\,f(x_2, y_2) = h\left(\frac{y_2{}^2 - x_2{}^2}{y_2{}^2 + x_2{}^2}\right) = 0.2\left(\frac{1.8912-0.16}{1.8912+0.16}\right)$$

$$= 0.2\frac{1.7312}{2.0512} = 0.1688$$

$$k_2 = h\,f\left(x_2+\frac{h}{2}, y_2+\frac{k_1}{2}\right) = 0.2\,f\left(0.4+\frac{0.2}{2}, 1.3752+\frac{0.1688}{2}\right)$$

$$= 0.2\,f[0.5,\,1.4596] = 0.1579$$

$$k_3 = h\,f\left(x_2+\frac{h}{2}, y_2+\frac{k_2}{2}\right) = h\,f\left(0.4+\frac{0.2}{2}, 1.13752+\frac{0.1579}{2}\right)$$

$$= 0.2\,f[0.5,\,1.4542] = 0.15771$$

$$k_4 = h\,f(x_2+h, y_2+k_3) = 0.2\,f[0.4+0.2, 1.3752+0.15771]$$

$$= 0.2\,f(0.6, 1.5329) = 0.1469$$

$$\Delta y = \frac{1}{6}(k_1+2k_2+2k_3+k_4) = \frac{1}{6}[0.1688+2\times 0.1579+2\times 15771+0.1469]$$

$$= 0.15782$$

$$y_3 = y_2+\Delta y = 1.3752+0.15782 = 1.53302$$

$x_0 = 0$	$y_0 = 0.1$	$y_0{}' = 1$
$x_1 = 0.2$	$y_1 = 1.196$	$y_1{}' = 0.9456$
$x_2 = 0.4$	$y_2 = 1.3752$	$y_2{}' = 0.84399$
$x_3 = 0.6$	$y_3 = 1.53302$	$y_3{}' = 0.7343$
$x_4 = 0.8$	$y_4 = ?$	

To find $y(0.8)$

By Milne's predictor formula

$$y_4, p = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 0.1 + \frac{4(0.2)}{3}\left[2(0.9456) - 0.84399 + 2(0.7343)\right]$$

$$= 0.1 + 0.2666[1.8912 - 0.84399 + 1.4686]$$

$$= 0.1 + 0.2666[2.51581]$$

$$= 0.1 + 0.67087$$

$$y_4 = 0.7709$$

$$x_4 = 0.8, \quad y_4 = 0.7709 \quad y_4' = -0.03702$$

$$y_4, c = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 1.3752 + \frac{0.2}{3}\left[0.84399 + 4(0.7343) - 0.03702\right]$$

$$= 1.3752 + 0.0666[0.84399 + 2.9372 - 0.03702]$$

$$= 1.3752 + 0.0666[3.74417]$$

$$= 1.3752 + 0.2494$$

$$y_4, c = 1.6246$$