MATRICES FORMULA SHEET

1) Characteristic Equation:

i) For a 3×3 matrix $\lambda^3-\beta_1\lambda^2+\beta_2\lambda-\beta_3=0$ is the characteristic equation where
β_1 = sum of the leading diagonal elements,
β_2 = sum of the minors of the leading diagonal elements, $\beta_3 = |A|$

ii) For a 2×2 matrix $\lambda^2-\beta_1\lambda+\beta_2=0$ is the characteristic equation where
β_1 = sum of the leading diagonal elements, $\beta_2 = |A|$

Eigen values : roots of the Characteristic equation.Can be obtained from calculator.

Eigen vectors: a nonzero vector X= $\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$ satisfies the equation $(A-\lambda I)X = 0$.

2) Cross Rule (Use only for two distinct equations)

$$a_1x+b_1y+c_1z=0$$
$$a_2x+b_2y+c_2z=0$$

	x	y	z
b_1	c_1	a_1	b_1
b_2	c_2	a_2	b_2

then $\dfrac{x}{b_1c_2-b_2c_1}=\dfrac{y}{a_2c_1-a_1c_2}=\dfrac{z}{a_1b_2-a_2b_1}$

3) If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of A then

(i) $K\lambda_1, K\lambda_2, K\lambda_3$ are the eigen values of KA. (ii) $\dfrac{1}{\lambda_1},\dfrac{1}{\lambda_2},\dfrac{1}{\lambda_3}$ are the eigen values of A^{-1}.

(iii) $\lambda_1^p,\lambda_2^p,\lambda_3^p$ are the eigen values of A^p (iv) $\dfrac{|A|}{\lambda_1},\dfrac{|A|}{\lambda_2},\dfrac{|A|}{\lambda_3}$ are the eigen values of adj A.

(v) $\lambda_1-K,\lambda_2-K,\lambda_3-K$ are the eigen values of $(A-KI)$.

4) (i) Sum of the eigen values = Sum of the leading diagonal elements.

(ii) Product of the eigen values $=|A|$.

5. (i) If A is a triangular matrix then the eigen values are its diagonal elements.

(ii)If A is a singular matrix (i.e. $|A|=0$) then one of the eigen values is 0.

6. (i)If A is an orthogonal matrix then $AA^T = A^TA = I$.(ii) If A is symmetric matrix then $A = A^T$

7) Conditions for orthogonal transformation:

A real symmetric matrix A with

(i) eigen vectors X_1,X_2,X_3 are linearly independent. **i.e.)** $|X_1\ \ X_2\ \ X_3|\neq 0$ and

(ii) eigen vectors X_1,X_2,X_3 are pair wise orthogonal. i.e.) $X_1\cdot X_2^T = 0$, $X_2\cdot X_3^T = 0$ and $X_3\cdot X_1^T = 0$

Note: If the eigen values of real symmetric matrix are distinct then the eigen vectors are linearly independent and pair wise orthogonal

8) Diagonalisation: The process of transforming a square matrix A in to a diagonal matrix D.

A real symmetric matrix is said to be orthogonal diagonalizable if there exist an orthogonal matrix N such that $N^TAN = D$.

9) Quadratic form: A homogeneous polynomial of second degree in any number of variables.

Matrix form of the Q.F**:** Every quadratic form can be expressed as $\mathbf{X^TAX}$ **,** where A is a symmetric matrix with a_{ii} = coefficient of x_i^2 and $a_{ij} = \left(\frac{1}{2} \times \text{coefficient of } x_i x_j\right) = a_{ji}$

10) Canonical form = sum of squares of any number of variable.

Matrix form of the C.F**:** Every canonical form can be expressed as $\mathbf{Y^TDY}$**,** where D is a diagonal matrix.

11) Orthogonal reduction: The orthogonal transformation: **X=NY** reduces the quadratic form to canonical form provided $\mathbf{N^TAN = D}$ where N is normalized modal (orthogonal) matrix.

Proof: Q.F= $X^TAX = (NY)^TA(NY) = (Y^TN^T)A(NY) = Y^T(N^TAN)Y = Y^T(D)Y$ = C.F.

12) Rank (r) of the Quadratic form: The number of nonzero terms in the resulting canonical form is called rank of the quadratic form.

13) Index (p) of the Quadratic form: The number of positive terms in the resulting canonical form is called index of the quadratic form.

14) Signature of the Quadratic form: s = 2p-r

15) Nature of the Quadratic form

S.No.	**Nature**	**If the eigen values are known**	**If the eigen values are not known**
1	Positive definite	All the eigen values are positive	D_1, D_2, D_3 are positive
2	Negative definite	All the eigen values are negative	D_1, D_3 are negative D_2 is positive
3	Positive semi definite	All the eigen values are positive and atleast one is zero	$D_1 \geq 0$, $D_2 \geq 0$ $D_3 \geq 0$ and atleast one is zero
4	Negative semi definite	All the eigen values are negative and atleast one is zero	$D_1 \leq 0$, $D_3 \leq 0$, $D_2 \geq 0$ and atleast one is zero
5	Indefinite	eigen values are positive and negative	All the other cases

where $D_1 = |a_{11}| = a_{11}$ $D_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ $D_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

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