PART B QUESTIONS

UNIT-I

VECTOR SPACES

1.In any Vector Space V,the following Statements are true.

(a). $0.x = 0$ *for all* $x \epsilon V$ (b).$(-\alpha)x = -(\alpha x) = \alpha(-x)$ *for each* $\alpha \epsilon F$ *and each* $x \epsilon V$.

(c).$\alpha.0 = 0$ *for each* $\alpha \epsilon F$

2. Prove that a subspace W of a vector space V is a subspace of V if and only if $0 \in W$ and $ax + y \in W$ *whenever* $a \in F$ *and* $x, y \in W$.

3. Prove that the set of all polynomials over afield F is a vector space V

4. Prove that the union of two subspace of a vector space is a subspace if and only if one is contained in the other.

5. Let V be a vector space over a field F and S be a non empty subset of V .The

(i) $L(S)$ *is a subspace of* V.

(ii) $S \subseteq L(S)$ (iii) $L(S)$ *is the smallest subspace of* V *containing* S.

6 .Show that the set $S = \{1, x, x^2, \ldots, x^n\}$ *of* $(n+1)$ polynomial in xis a basis of the Vector spac$P_n(R)$ of all polynomial in x over a field of real numbers.

7. Let V be a vector space over a field F and $S = \{u_1, u_2, u_3, \ldots\ldots u_n\}$be a subset of V. Then S is a basis for V iff every element of V can be uniquely expressed as a linear combination of elements of S.

8. Show that $\{x, 3x^2, 5 + x\}$is a basis of p_2(R)

9. Define linearly independent and dependent, and Let $\{u, v, w, z\}$ be a set of linearly independent vectors of a space. Check, whether the set $S = \{u - 3z, v + 2u, 2v - w, w + z\}$is linearly independent

10.Show that the following vectors form a basis for $V_3(R)\{(1,1,0), (0,1,1), (1,0,1)\}$

11.Evaluate the set of all real valued continuous functions of x defined in some interval [0,1] is a vector space

12.(i) Prove that any intersection of sub spaces of a vector space v is a subspace of V

(ii)Prove that the union of two subspaces is not necessarily a subspace.

13.Point out that the set of all m xn matrices with entries from a field F is a vector space denoted as

$M_{m\times n}(F)$ with the operations of matrix addition and scalar multiplication is a vector space.

14. Let S be a linearly dependent sub set of a vector space V and let v be a vector in V that is not in S.

Then prove that $S \cup \{v\}$ is linearly dependent iff $v \in span(S)$.

15.Identity whether the set { $x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\} in P_2(R)$ is linearly independent or not

16.Illustrate the set{ $1, x, x^2, x^3, ..., x^n$ } is a basis for $P_n(R)$

17.If a vector space V is generated by a finite set S then prove that some subset of S is a basis for V. Hence V has a finite basis

18. Prove that the span of any subset S of a vector space is the smallest subspace of Containing S.

19.Prove that the span of any subset S of a vector space V is a subspace of V.Moreover, any subspace of V that contains S must also contain the span of V

20.Lret V be a vector space and $S_1 \subseteq S_2 \subseteq V$, then prove that (i) If S_1 is linearly dependent then S_2 is also linearly dependent .If S_2 is linearly independent then S_1 is also linearly independent .

UNIT-II

LINEAR TRANSFORMATIONS AND DIAGONALIZATION

1.Show that $T: V_2(R) \rightarrow V_2(R)$ defined by $T(a,b) = (2a - 3b, a + 4b)$ is a linear

Transformation

2..Let V and W be vector spaces and $T; V \to W$ $be\ l; inear\ transformation\ .Then$

(a).N(T) is a subspace of V. (b).R(T) is a subspace of w.

3. State and prove Dimension Theorem (10)

4. Let V and W be vector spaces and let $T: V \to w$ be linear transformation. Then T is 1-1 iffN(T)=0

5.) Let $T: P_2(R) \to P_3(R)$ be the linear Transformation defined by

$T(f(x)) = 2f'(x) + \int_0^3 3f(x)dx$ check(i) T is 1-1 or not (ii) T is onto or not

6.Let $T: V_3(R) \to V_3(R)$ be the linear transformation defined by

$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$ .find a basis and dimension of R(T) and N(T)

7. Let V and W be finite dimensional vector space with ordered bases $\beta\ and\ \gamma$ respectively and $T_{1,}T_2: V \to w$ be linear transformation. Then (a)

$[T_1 + T_2]_\beta^\gamma = [T_1]_\beta^\gamma + [T_2]_\beta^\gamma$ (ii) $[cT_1]_\beta^\gamma = c[T_1]_\beta^\gamma$ for all scalars c

8.Find the Eigen value s and Eigen vectors of the matrix$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

9. Find the Eigen value s and Eigen vectors of the matrix$A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$

10.Let $T_1 : V_2(R) \to V_3(R)$ and $T_2 : V_2(R) \to V_3(R)$ be the linear transformation respectively defined by $T_1(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ and $T_2(a_1, a_2) = (a_1 - a_2 2a_1, 3a_1 + 2a_2)$ .Let $\beta\ and\ \gamma$ be the Standard bases of $V_2(R)$ and $V_3(R)$ respectively. Verify

$[T_1 + T_2]_\beta^\gamma = [T_1]_\beta^\gamma + [T_2]_\beta^\gamma$

11.Let $T : P_2(R) \to P_2(R)$ *be defined by* $T[f(x)] = 2f'(x) + \int_0^x 3f(t)dt$.

Prove that T is linear, find the bases for N(T) and R(T).Compute the nullity and rank of T .Determine whether T is 1-1 or onto

12. .Let $T : P_2(R) \to P_2(R)$ *be defined by* $T[f(x)] = xf(x) + f'(x)$.

Prove that T is linear, find the bases for N(T) and R(T).Compute the nullity and rank of T .Determine whether T is 1-1 or onto

13.Let $T : R^3 \to R^3$ be a linear Transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$

Evaluate a basis and dimension of null space N(T) and range space R(T).Also verify dimension theorem.

14.find a linear map $T : R^2 \to R^4$ whose image is generated by (1,2,0,-4) and (2,0,-1,-3)

15.Let T be the linear operator on R^3 $defin T(x, y, z) = (2x - 7y - 4z, 3x + y + 4z, 6x - 8y + z)$ (i)

Find the matrix of T in the basis $f_1 = (1,1,1), f_2 = (1,1,0), f_3 = (1,0,0)$

16.Let V and W be vector spaces over F ,and suppose that $\{v_1, v_2, \ldots\ldots\ldots, v_n\}$ is a basis for V.For $\{w_1, w_2, \ldots\ldots\ldots, w_n\}$ in W.Prove that there exists exactly one linear transformation $T : V \to W$ *such tht* $T(v_i) = w_i$ *for* $i = 1,2,\ldots n$

17.Find the eigen values and a basis of each eigen space $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$

18.Let D be the differential operator $D(f) = \frac{df}{dt}$ find the matrix of D for the basis $(i)\{e^{5t}, te^{5t}, t^2e^{5t}\}$ $(ii)\{\sin t, \cos t\}$

19.Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ point out all eigen values of A and corresponding eigen vectors find an invertible matrix P such that $P^{-1}AP$ is diagonal

20. Let $A=\begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$ point out all eigen values of A and corresponding eigen vectors find an invertible matrix P such that $P^{-1}AP$ is diagonal

UNIT-III

INNER PRODUCT SPACES

1.State and prove Cauchy-schwartz inequality and Triangle inequality in an inner product space

2.Let V be an inner product space .Prove that

$(a)\|x\pm y\|^2=\|x\|^2\pm 2R<x,y>+\|y\|^2\ for\,all\, x,y\in V, where\ R=<x,y>$ denotes the real part of the complex number $<x,y>$

(b) $\|\|x\|-\|y\|\|^2\leq\|x-y\| all\, x,y\in V$

3.Let x=(2,1+I,i) and y = (2-I,2,1+2i)be vectors in C^3.$compute<x,y>.\|x\|,\|y\|,\|x+y\|$ .Then verify

Schwartz inequality and the triangle inequality.

4. In C[0,1].let $f(t)=t\, and\, g(t)=e^t.compute<f,g>,\|f\|,\|g\|,\|f+g\|$. hen verify Schwartz

inequality and the triangle inequality.

5.Let V be an inner product space , $x,y,z\in V and\ C\in F$.check whether the following are true

(i) (i)$<x,y+z>=<x,y>+<x,z>$

(ii) $<x,cy>=\bar{c}<x,y>$

(iii) $<x,0>=<0,x>=0$

(iv) $<x,x>=0\ iff\ x=0$

(v) $<x,y>=<y,z>$

6. Let V be an inner product space, and suppose that x and y are orthogonal vectors in V. Prove that

$\|\|x\|^2+\|y\|\|^2=\|x+y\|^2 all\, x,y\in V$ .Deduce the Pythagorean theorem in R^2. (a)V=R^3.

,S={(1,0,1),(0,1,1),(1,3,3)} and x=(1,1,2)

7. Let V be an inner product space, and suppose that x and y are orthogonal vectors in V.Prove that

$\left\| \|x\|^2 + \|y\| \right\|^2 = \|x+y\|^2 \, all\, x, y \in V$ .Deduce the Pythagorean theorem in R^2. (a)V= R^3.

,S={(1,0,1),(0,1,1),(0,0,1)} and x=(1,0,1)

8. Let V be an inner product space, and suppose that x and y are orthogonal vectors in V. Prove that

$\left\| \|x\|^2 + \|y\| \right\|^2 = \|x+y\|^2 \, all\, x, y \in V$ .Deduce the Pythagorean theorem in R^3. (a)V= $P_2(R)$ with inner

product space $< f(x), g(x) >= \int_0^1 f(t)g(t)dt, S = \{1, x, x^2\} \, and\, h(c) = 1 + x$

9.Stete and prove Gram Schmidt Orthogonal process

10.For each of the sets of data tha2t follows,use the least squares approximation to find the best fits with both(i) a linear function and (ii)a quadratic function.Compute the ErrorE in bote cases.{(-3,9),(-2,6),(0,2),(1,1)}

11. Evaluate by the gram Schmidt process to the given subset S={(1,-2,-1,3),(3,6,3,-1),(1,4,2,8)} and x=(-1,2,1,1) of the inner product space V=R4 to obtain an orthogonal basis for span(S).Then normalize the vectors in this basis to obtain an orthogonal basis β for span(S) and compute the fourier coefficients of the given vector relative to β

12.Let $V = P_2(R) \, and < f(x), g(x) >= \int_{-1}^{1} f(t)g(t)dt, \beta$ be the standard ordered basis using Gram Schmidt process .obtain orthonormal basis for $P_2(R)$

13.Let $A \in M_{m\times n}(F) \, and\, b \in F^m .Suppose\, that\, Ax = b$ is consistent. Then prove the following statements are true(a).There exists exactly one minimal solution s of Ax=b that lies in $R(L_{A^*}) \; that is\, , if\, u\, satisfies (AA^*)u = b\, then s = A^* u$

14.Let V be a finite dimensional inner product space and T be a linear operator on V. then there exists a unique function $T^{*} : V \to V\, such that < T(x), y >=< x, T^*(y) > for\, all\, x, y \in V$

15.Find the minimal solution to the system $x + 2y - z = 12$

UNIT-IV

PARTIAL DIFFERENTIAL EQUATIONS

1. . Solve (mz –ny)p + (nx –lz)q = ly – mn

2. Solve $(D^2-4DD'+4D'^2)z=xy+e^{2x+y}$.

3.Form the PDE by eliminating the arbitrary functions from the f (x + y + z, $x^2+y^2+z^2$) =0

4.Solve $z = px + qy + p^2 + q^2$

5. Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ (8)

6. Solve ($D^2 - DD' - 30D'^2)Z = e^{6x+y}$)

7. Form the PDE by eliminating the arbitrary function φ

from $\phi(x^2 + y^2 + z^2, x + y + z) = 0$.

8.Find the singular integral of $Z = px + qy + \sqrt{1 + p^2 + q^2}$

9.Solve (i) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

10. Solve: $[D^3 + D^2D' - 4DD'^2 - 4D'^3]z = \cos(2x + y)$

11. Form the PDE by eliminating the arbitrary function

$\Phi(x^2+y^2+z^2, ax+by+cz)=0$

12.Solve $(D^2-3DD'+2D'^2+2D-2D')z=x+y+\sin(2x+3y)$

13.Solve($x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

14) Solve $z = 1 + p^2 + q^2$

15) Solve $z = p^2 + q^2$.

16) Solve $p(1-q^2) = q(1-z)$.

17) Solve $9(p^2z + q^2) = 4$.

18) Solve $p(1+q) = qz$.

19) Solve : $x^2p^2 + y^2q^2 = z^2$

20) Solve $p^2y(1+x^2) = qx^2$.

UNIT-V

FOURIER SERIES AND SOLUTIONS PARTIAL DIFFERENTIAL EQUATIONS

1. Find the Fourier series expansion for f $f(x)=\begin{cases}-\pi & in\ -\pi<x<0\\ x & in\ \ 0<x<\pi\end{cases}$

   Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ ....=\frac{\pi^2}{8}$.

2. Find the half range Fourier Cosine Series to represent function

   $f(x)=2x-x^2\ \ in\ \ (0,2)$. Hence deduce the value of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+....$

3. Find the Fourier series expansion for f(x) = $\begin{cases} x\ ; & 0\le x\le \pi\\ 2\pi-x; & \pi\le x\le 2\pi\end{cases}$

   and hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.........=\frac{\pi^2}{8}$

4. Find the Fourier series expansion of f(x) = $x+x^2$ in the

   interval (-π, π). Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.....=\frac{\pi^2}{6}$

5. Find the Fourier series expansion of the periodic function f(x) of the period defined by

   $f(x)=\begin{cases}1+x & in\ -2<x<0\\ 1-x & in\ \ 0<x<2\end{cases}$

6.Obtain the Fourier series for f(x) of periods (-l,l) and defined as follows $f(x)=\begin{cases}L+x & in\ (-L,0)\\ L-x & in\ (0,L)\end{cases}$

(6)

7..Obtain the Fourier series expansion of f(x) given by

$$f(x)=\begin{cases}1+\frac{2x}{\pi}\ ; -\pi<x<0\\ 1-\frac{2x}{\pi}\ ; 0<x<\pi\end{cases}\quad \text{in } (-\pi,\pi).$$

8.A tightly stretched string with fixed end points x=0 and x=*l* is initially at rest in its equilibrium position. Motion is started by displacing the string into the form y= kx(*l*-x) from which it is released at time t=0. Find the displacement of any point on the string at a distance of x from one end at time t.

9. A rod of 30cm long has its ends A and B kept at 20^0 and 80^0 respectively until steady state conditions prevail the temperature at each end is then suddenly reduced to 0^0c and kept so. Find the resulting temperature function u (x, t) taking x = 0 at A.

10. A tightly stretched string with fixed end $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set to vibrating giving each point a velocity $\lambda x\ (l\text{-}x)$, find the displacement y (x, t).

11..An infinitely long plane uniform plate with insulated surfaces is 10 cm wide. The two long edges and one short edge are kept at zero temperature, while the other short edge x=0 is kept at temperature given by $u = \begin{cases} 20y & 0 \le y \le 5 \\ 20(10-y) & 5 \le y \le 10 \end{cases}$. Find the steady state temperature distribution in the plate.

12. Find the steady state temperature at any point of a square plate whose two adjacent edges are kept at 0^0C and the other two edges are kept at the constant temperature 100^0C.

13. The temperature at short edge y=0 is given by u = 20x, 0≤x≤5 = 20(10-x), 5≤x≤10 and the two long edges x=0 and x=10 as well as the other short edge are at 0^0C. Find the temperature u(x,y) at any point (x,y).

14.A tightly stretched string with fixed end points x=0 and x=l is initially at rest in its Equilibrium position. Motion is started by displacing the string into the form y= 3x(l-x) from which it is released at time t=0. Find the displacement of any point on the string at a distance

15.A tightly stretched string with fixed end $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set to vibrating giving each point a velocity $V = \begin{cases} cx & 0 \le x \le l/2 \\ c(l-x) & l/2 \le x \le l \end{cases}$.

S.T the displacement y(x,t) at any time t

16. A rectangular plate is bounded by the lines x=0, y=0, x=a, y=b. Its surfaces are insulated. The temperature along x=0 and y=0 are kept at 0^0C and the others at 100^0C. Find the steady state temperature at any point of the plate.

17. A rod 20cm long has its ends A and B kept at 30^0C and 90^0C respectively until steady state conditions prevail. If the temperature at each end is then suddenly reduced to 0^0C and maintained so, find the temperature distribution at a distance x from A at time t.

18. A bar 10cm long with insulated sides, has its ends A and B kept at $20^{\circ}C$ and $40^{\circ}C$ respectively until steady state conditions prevail. The temperature at A is then suddenly raised to $50^{\circ}C$ and at the same instant that at B is lowered to $10^{\circ}C$ .find the subsequent temperature at any point of the bar at any time.

19. Obtain the half range cosine series for f(x) = x in 0<x< π and deduce that

$$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+\dots\dots\dots=\frac{\pi^4}{96}$$

20. A tightly stretched string between the fixed end points x=0 and x=l is initially in a position given by $y(x,0)= y_0 \sin^3\left(\frac{n\pi}{l}\right)$. It is released from rest this position. Find the expression for the displacement at any time t.

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