AGNI COLLEGE OF TECHNOLOGY

Chennai – 600 130

Department of Mathematics

Third Semester – Common to all Branches

MA6351-TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

ANNA UNIVERSITY IMPORTANT QUESTIONS

UNIT –I: PARTIAL DIFFERENTIAL EQUATIONS

PART- A

1. Form the pde by eliminating a and b from $(x+a)^2 + (y+b)^2$.
2. Solve $(D^2 - 2DD' + D'^2)z = 0$
3. Solve $\partial^2 z / \partial x^2 = \sin y$
4. Find the complete integral of $z/pq = x/q + y/p + \sqrt{(pq)}$
5. Obtain the p.d.e. by eliminating f and g from $z = f(x+it) + g(x-it)$
6. Solve $(D^2 - 12DD' + 9D'^2)z = 0$
7. Find the p.d.e. from $\phi[z^2 - xy,\ x/z]$
8. Find the singular sol. of $z = px + qy + p^2 - q^2$
9. Form the p.d.e. from $z = f(x^2 + y^2)$
10. Find the complete integral of $p - y^2 = q + x^2$
11. Find the p.d.e. by eliminating the arbitrary function from $z = f(xy)$.
12. Write down the complete solution of $z = px + qy + c\sqrt{(1 + p^2 + q^2)}$
13. Form the p.d.e. of all spheres whose centres lie on the z-axis.
14. Find the complete integral of $(1-x)p + (2-y)q = 3-z$.
15. Find the p.d.e. of all planes having equal intercepts on the x and y axis.
16. Find the solution of $px^2 + qy^2 = z^2$
17. Find the p.d.e. of all planes passing through the origin.
18. Find the P.I. of $(D^3 - 3D^2D' - 4DD'^2 + 12D'^3)z = \sin(x+2y)$
19. Form the PDE by eliminating a and b from $z = ax^n + bx^n$.
20. Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = 0$
21. Find the complete integral of $p + q = pq$
22. Form the PDE by eliminating a and b from $z = (x^2 + a^2)(y^2 + b^2)$
23. Solve $(D^2 - DD' + D' - 1)z = 0$
24. Obtain the PDE by eliminating a & b: $(x-a)^2 + (y-b)^2 + z^2 = 1$
25. Find the complete integral of $p + q = pq$.
26. Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$
27. Find the PDE of the spheres having their centers on the line $x = y = z$
28. $(D^3 - 2D^2D' - 4DD'^2 + 8D'^3)z = 0$
29. Find the PDE by eliminating f from $z = (x+y)f(x^2 - y^2)$
30. Find the complete integral of $q = 2px$

PART B

1. Form the pde by eliminating the arbitrary functions from $z = x f(2x+y) + g(2x+y)$

2. Solve $p^2 y(1+x^2) = q x^2$.
3. Find the singular integral of $z = px + qy + p^2 - q^2$.
4. Solve $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$
5. Solve $(D^3 - 2D^2D')z = 4\sin(x+y) + e^{x+2y}$
6. Form the p.d.e. by eliminating a & b from $(x-a)^2 + (y-b)^2 + z^2 = c^2$
7. Solve $(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x+y)$
8. Solve $p(1+q) = qz$
9. Solve $x(y^2+z) + y(x^2+z)q = z(x^2-y^2)$
10. Solve $(x+px)^2 + (y+qz)^2 = 1$
11. Solve $(D^2 - 5DD' + 6D'^2)z = y\sin x$
12. Solve $x(y^2+z^2)p + y(x^2+z^2)q = z(y^2-x^2)$
13. Solve $(D^2 - 4DD' + D'^2)z = e^{3x-2y} + \sin x$
14. Solve $(y^2+z^2)p - xyq + xz = 0$,
15. Slove $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y + xy$
16. Solve $p(1-q^2) = q(1-z)$
17. Solve $(D^2 - 4DD' + 4D'^2)z = e^{(2x+y)}$
18. Find the general solution of $z(x-y) = px^2 - qy^2$
19. Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = \cos(2x+y)$
20. Solve $9(p^2z + q^2) = 4$
21. Solve $(D^2 + DD' - 2D'^2)z = \sinh(x+y) + xy$
22. Solve $(x-2z)p + (2z-y)q = y - x$
23. Solve $(D^2 + 4DD' - 5D'^2)z = \sin(2x+3y)$
24. Find the singular integral of $z = px + qy + p^2 + q^2 + pq$
25. Solve $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$
26. Solve $(D^2 - 5DD' + 6D'^2)z = y\sin x$
27. Solve $(3z-4y)p + (4x-2z)q = (2y-3x)$
28. Solve $(x^2+y^2+yz) + (x^2+y^2-xz)q = z(x+y)$
29. Solve $(D^2 + DD' - 20D'^2)z = \sin(4x-y) + e^{5x+y}$
30. Solve $(D^2 + DD' - 6D'^2)z = x^2y + e^{3x+y}$
31. Find the singular integral of $z = px + qy + p^2 - q^2$
32. Form the PDE by eliminating f and g in $z = x^2 f(y) + y^2 g(x)$
33. Form the PDE by eliminating f, g from $z = f(x^3 + 2y) + g(x^3 - 2y)$
34. Find the complete and singular solution of $z = px + qy + p^2 - q^2$
35. Find the general solution of $x(z^2-y^2)p + y(x^2-z^2)q = z(y^2-x^2)$
36. Solve $(D^2 - 4D'^2)z = \cos 2x \cos 3y$
37. Find the singular solution of $z = px + qy + \sqrt{(p^2 + q^2 + 16)}$
38. Find the general solution of $(3z-4y)p\ (4x-2z)q = 2y - 3x$
39. Form the PDE by eliminating f and g from $z = f(y) + g(x+y+z)$
40. Solve $[(D+D'-1)(D+2D'-3)]z = 4 + 3x + 6y + e^{x+y}$

UNIT – 2: FOURIER SERIES

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PART – A

1. State the sufficient condition for a function $f(x)$ to be expressed as a Fourier series.
2. Obtain the first term of the Fourier series for the function $f(x)=x^2, -\pi<x<\pi$.
3. Find the constant term in the expansion of $\cos^2 x$ as a Fourier series in the interval $(-\pi,\pi)$.
4. Define Root mean square value of a function $f(x)$ over the interval (a,b).
5. State the Dirichlet's conditions for Fourier series.
6. What is meant by Harmonic Analysis?
7. Write the condition for a function $f(x)$ to satisfy for the existence of a Fourier series.
8. If $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$, deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^3}+....=\frac{\pi^2}{6}$.
9. Find the root mean square value of $f(x)=x^2$ in $(0, l)$.
10. Find the value of a_0 in the Fourier series expansion of $f(x)=e^x$ in $(0,2\pi)$
11. Find the value of b_n in the Fourier series expansion of $f(x)=x\sin x$ in $(-2,2)$
12. State the Dirichlet's conditions for the existence of the Fourier expansion of $f(x)$ in the interval $(0,2\pi)$.
13. Find the root mean square value of $f(x)=x$ in $(0, l)$.
14. Find the half range sine series expansion of $f(x)=1$ in $(0, 2)$.
15. Give the expression for the Fourier series co efficient b_n for the function $f(x)$ define in the interval $(-2, 2)$.
16. To which value, the half range sine series corresponding to $f(x)=x^2$ expressed in the interval (0,5) converges at x = 5.
17. Let $f(x)$ be defined in $(0,2\pi)$ by $f(x)=\begin{cases}\frac{1+\cos x}{\pi-x}, & 0<x<\pi\\ \cos x, & \pi<x<2\pi\end{cases}$ and $f(x+2\pi)=f(x)$. Find the value of $f(\pi)$.
18. Find the sum of the Fourier series for $f(x)=\begin{cases}x, & 0<x<1\\ 2, & 1<x<2\end{cases}$ at x = 1.
19. The cosine series for $f(x)=x\sin x$ for $0<x<\pi$ is given as $x\sin x=1-\frac{1}{2}\cos x-2\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2-1}\cos nx$. Deduce that $1+2\left(\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-...\right)=\frac{\pi}{2}$.

PART – B

1. Find the Fourier series of $f(x)=x^2$ in $[0,2\pi]$ and periodic with period 2π. Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.

2. Find the Fourier series of $f(x)=(\pi-x)^2$ in $(0,\ 2\pi)$ of periodicity 2π.

3. Obtain the Fourier series of the periodic function defined by $f(x)=\begin{cases}1 & ,\ 0<x<\pi \\ 2 & ,\ \pi<x<2\pi\end{cases}$.

4. Find the Fourier series expansion of $f(x)=\begin{cases}x, & 0<x<\pi \\ 2\pi-x, & \pi<x<2\pi\end{cases}$. Also deduce that $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+....=\frac{\pi^4}{8}$.

5. Expand $f(x)=x(2\pi-x)$ as Fourier series in $0<x<2\pi$ and hence deduce that the sum of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+.....$

6. Obtain the Fourier series for the function $f(x)$ given by $f(x)=\begin{cases}1-x, & -\pi<x<0 \\ 1+x, & 0<x<\pi\end{cases}$

   Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$.

7. Obtain the Fourier series of $f(x)=x\sin x$ in $-\pi<x<\pi$.

8. Obtain the Fourier series to represent the function $f(x)=|x|,\ -\pi<x<\pi$ and deduce $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$.

9. Obtain the Fourier series of the periodic function defined by $f(x)=\begin{cases}-\pi, & -\pi<x<0 \\ x, & 0<x<\pi\end{cases}$.

   Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....=\frac{\pi^2}{8}$.

10. Find the Fourier series of the function $f(x)=\begin{cases}0, & -\pi<x<0 \\ \sin x, & 0<x<\pi\end{cases}$ and hence evaluate $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+....$

11. Expand $f(x)=x-x^2$ as a Fourier series in $-1<x<1$ and using this series find the R.M.S value of $f(x)$ in the interval (-1, 1).

12. Expand $f(x)=x-x^2$ as a Fourier series in $-L<x<L$ and using this series find the root mean square value of $f(x)$ in the interval.

13. Find the Fourier series for $f(x)=2x-x^2$ in interval $0<x<2$.

14. Obtain the Fourier series of the periodic function defined by $f(x)=\begin{cases}x & ,\ 0\le x\le 1 \\ 2-x & ,\ 1\le x\le 2\end{cases}$.

    Also deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....=\frac{\pi^2}{8}$.

15. Find the Fourier series expansion of $f(x)=1-x^2$ in the interval (-1, 1).

16. Find the Fourier series expansion of $f(x) = 2x - x^2$ in the interval (0, 3).

17. Find the half- range cosine series of $f(x) = \pi x - x^2$ in $(0, \pi)$. Deduce that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

18. Find the half range Fourier cosine series of $f(x) = (\pi - x)^2$ in $(0, \pi)$. Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\infty$.

19. Find the half range cosine series of the function $f(x) = x(\pi - x)$ in the interval $0 < x < \pi$. Hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots = \frac{\pi^4}{90}$.

20. Obtain the half range cosine series for $f(x) = x$ in $0 < x < \pi$.

21. Obtain the Fourier cosine series expansion of $f(x) = x \sin x$ in $0 < x < \pi$. Hence find the interval of $1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} - \ldots$

22. Find the hale range Sine series of $f(x) = lx - x^2$ in the interval $(0, l)$.

23. Obtain the Fourier sine series for $f(x) = \begin{cases} x, & 0 < x < l/2 \\ l - x, & l/2 < x < l \end{cases}$.

24. Find the Fourier series as far as the second harmonic to represent the function $f(x)$ with period 6, given in the following table

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

25. Calculate the first harmonics of the Fourier of $f(x)$ from the following data:

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

26. Calculate the first harmonics of the Fourier of $f(x)$ from the following data:

x	0	T/6	T/3	T/2	2T/3	5T/6	T
f(x)	1.98	1.3	1.05	1.3	-0.88	-0.25	1.98

27. Calculate the first 2 harmonics of the Fourier of $f(x)$ from the following data:

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1

28. Calculate the first 3 harmonics of the Fourier of $f(x)$ from the following data:

x	0	30	60	90	120	150	180	210	240	270	300	330
f(x)	298	356	376	337	254	155	80	51	60	93	147	221

29. Calculate the first 3 harmonics of the Fourier of $f(x)$ from the following data:

x	0	30	60	90	120	150	180	210	240	270	300	330
f(x)	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2.0

30. If a is not integer, find the complex Fourier series of $f(x) = \cos nx \ in \ (-\pi, \pi)$.

31. Find the complex form of the Fourier series of $f(x) = e^{-x}$ in $-1 < x < 1$.

32. Find the complex form of the Fourier series of $f(x) = e^{ax}$ in $-\pi < x < \pi$.

UNIT – 3: APPLICATION OF PARTIAL DIFFERENTIAL EQUATIONS

PART – A

1. Write down the possible solution of the one dimensional heat equation.
2. State the governing equation for one dimensional heat equation and necessary to solve the problem.
3. Write the boundary conditions for the following problem. A rectangular plate is bounded by the line $x = 0$, $y = 0$, $x = a$ and $y = b$. Its surfaces are insulated. The temperature along $x = 0$ and $y = 0$ are kept at 0^o C and the others at 100^oC.
4. What is meant by steady state condition in one dimensional heat flow?
5. Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.
6. Write down the possible solution of the one dimensional wave equation.
7. Classify the partial differential equation $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$.
8. Write down the partial differential equation that represents steady state heat flow in two dimensions and name the variables involved.
9. In the one dimensional heat equation $c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, what is c^2 ?
10. Write all three possible solutions of steady state two dimensional heat equation.
11. A tightly stretched string with fixed end point x = 0 and x = l is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi n}{l}\right)$. If it is released from rest in this position, write the boundary condition.
12. Write down the three possible solutions of Laplace equation in two dimensions.
13. A rod 40 cm long with insulated sides has its ends A and N kept at 20^oC and 60^oC respectively. Find the steady state temperature at a location 15 cm from A.

PART – B

1. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibium position. If it is set vibrating giving each point a initial velocity $\lambda(l-x)$ find the displacement.
2. A tightly stretched string of length 'l' has its ends fastened at $x=0$ and $x=l$. The midpoint of the string is then taken to height b and released from rest in that position. Find the lateral displacement of a point of the string at t from the instant of release.
3. A tightly stretched string of length 2l is fastened at both ends. The midpoint of the string is displaced by a distance b transversely and the string is released from rest in this position.

Find an expression for the transverse displacement of the string at any time during the subsequent motion.

4. A tightly stretched string with fixed end points x = 0 and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a initial velocity $\lambda x(l-x)$ find the displacement.
5. A tightly stretched string with fixed end points x = 0 and x = 10 is initially at rest in its equilibrium position. If it is set vibrating giving each point a initial velocity *3x(l-x)* find the displacement y at any time and at any distance from end x = 0.
6. A tightly stretched flexible string has its ends fixed at x = 0 and x = L. At time t = 0, the string is given a shape defined by $f(x) = kx^2(L-x)$ and then released from rest. Find the displacement of any point x of the string at any time t > 0.
7. A tightly stretched string of length $2l$ has its ends at x = 0 and x = 2*l*. The mid point of the string is then taken to a height 'b' and then released from rest in that position. Obtain an expression for the displacement.
8. A tightly stretched string of length l has its ends fixed at x = 0 and x = *l*. The point x = *l* / 3 of the string is then taken to a height h and then released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time.
9. A string is stretched between two fixed points at a distance 2*l* apart and the points of the string are given initial velocities v where
$v = \begin{cases} cx/l & 0 < x < l \\ (c/l)(2l-x) & l < x < 2l \end{cases}$. Find the displacement of the string at any time.
10. A tightly stretched string with fixed end point x = 0 and x = l is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi n}{l}\right)$. If it is released from rest in this position, write the boundary condition and also find the displacement.
11. A string is stretched between two fixed points at a distance l apart and the points of the string are given initial velocities v where
$v = \begin{cases} cx & 0 < x < l/2 \\ (c)(l-x) & l/2 < x < l \end{cases}$. Find the displacement of the string at any time.
12. A rod, 30 cm long has it ends A and B kept at 20^oC and 80oC respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^oC and kept so. Find the resulting temperature function u(x, t) taking x =0 at A.
13. A bar of 10 cm long, with insulated sides has its ends A and B maintained at temperature 50^oC and 100^oC respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 90^oC and B is lowered to 60^oC. Find the displacement.
14. A rod, 40 cm long has it ends A and B kept at 0^oC and 80^oC respectively, until steady state conditions prevail. The temperature at each end B is then suddenly reduced to 40^oC and kept so. While that of the end A is kept at0^oC. Find the resulting temperature function u(x, t).

15. A rod of length l cm long has its ends A and B at 40° C and 90 C respectively until steady state condition prevail. The temperature at the end B is then suddenly reduced to 40° C and at the end A is raised to 90° C and maintained so. Find the resulting temperature u(x , t). Also show that the temperature at the mid point of the rod remains unaltered for all time, regardless of the material of the rod.
16. A rectangular plat with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge y = 0 is $u(x,0) = 100\sin\left(\frac{\pi x}{8}\right)$, 0 < x < 8, while two long edges x = 0 and x = 8 as well as the other short edge are kept at 0°C. Find the steady state temperature at any point of the plate.
17. A rod 10 cm long has its ends A and B at 20° C and 40 C respectively until steady state condition prevail. The temperature at the end B is then suddenly reduced to 10° C and at the end A is raised to 50° C and maintained so. Find the resulting temperature u(x , t).

UNIT –IV: FOURIER TRANSFORMS

PART – A

1. Find the Fourier cosine transform of $e^{-x}, x \geq 0$.
2. State parseval's identity in Fourier transforms.
3. Define the Fourier transforms pair.
4. Find the Fourier sine transform of $1/x$.
5. State Fourier integral theorem.
6. State convolution theorem for Fourier transforms.
7. Find the Fourier transform of $f(x) = \begin{cases} e^{ikx}, & a < x < b \\ l - x, & x < a \text{ and } x > b \end{cases}$.
8. What is the Fourier transform of f(x-a), if the Fourier transform of f(x) is F(s))?
9. Find the Fourier sine transform of $f(x) = e^{-ax}, a \geq 0$.
10. Find the Fourier transform of $f(x) = e^{-a|x|}, a \geq 0$.
11. Find the Fourier sine transform of $f(x) = e^{-3x}$.
12. If $F\{f(x)\} = F(s)$, prove that $F\{f(ax)\} = \frac{1}{a}F(s/a)$.
13. If $F(s)$ is the Fourier transform of $f(x)$ show that $F\{f(x-a)\} = e^{ias}F(s)$.
14. State and prove the change of scale property of Fourier transform.
15. If $F_C(s)$ is the Fourier cosine transform of $f(x)$ prove that Fourier cosine transform of $f(ax)$ is $\frac{1}{a}F_C(s/a)$.

PART – B

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1. Find the Fourier transform of $f(x)=\begin{cases}1-|x|, & |x|<1\\ 0, & |x|\geq 1\end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin^4 t}{t^4}\,dt.$

2. Evaluate $\int_0^\infty \frac{dx}{(4+x^2)(25+x^2)}$ using transform method.

3. Find the Fourier cosine transform of e^{-x^2}.

4. Prove that $1/\sqrt{x}$ is self reciprocal under Fourier sine and cosine transforms.

5. Derive the parseval's indentify for Fourier transform.

6. Find the Fourier integral representation of $f(x)$ defined as $f(x)=\begin{cases}0, & x<0\\ 1/2, & x=0\\ e^{-x}, & x>0\end{cases}$.

7. State and prove convolution theorem on Fourier transform.

8. Find the Fourier sine and cosine transform of x^{n-1} and hence prove $1/\sqrt{x}$ is self reciprocal under Fourier sine and cosine transformed.

9. Find the Fourier transform of $f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|\geq a\end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin x}{x}\,dx.$

10. Verify the convolution theorem under Fourier transform, for $f(x)=g(x)=e^{-x^2}$.

11. Obtain the Fourier transform of $e^{-x^2/2}$.

12. Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$ using Parseval's identity.

13. Show that the Fourier transform of $e^{-x^2/2}$ is $e^{-s^2/2}$.

14. Show that the Fourier cosine transform of $e^{-a^2x^2}$ $a>0$. Hence show that the function $e^{-x^2/2}$ is self reciprocal.

15. Find the Fourier sine and cosine transforms of $f(x)=\begin{cases}\sin x, & 0<x<a\\ 0, & x\geq a\end{cases}$.

16. Using Fourier cosine transform method, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$.

17. Find the Fourier sine transform of $f(x)=\begin{cases}x, & 0<x<1\\ 2-x, & 1<x<2\\ 0, & x>2\end{cases}$.

18. Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier cosine transforms of e^{-ax} and e^{-bx}.

19. Find the Fourier sine transform of e^{-ax} and hence evaluate Fourier cosine transforms of xe^{-ax} and $e^{-ax}\sin ax$.

20. Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|<1\\ 0, & |x|\ge 1\end{cases}$ and hence evaluate

(i) $\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)\cos(x/2)dx$. (ii) $\int_0^\infty \left(\frac{\cos s-\sin s}{s^3}\right)\cos\left(\frac{s}{2}\right)ds$. (ii) $\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)^2 dx$.

21. Find the Fourier transform of $f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|\ge a\end{cases}$ and deduce the value of

(i) $\int_0^\infty \frac{\sin x}{x}dx$. and $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$.

22. Find Fourier cosine transform of e^{-x^2}.

23. Find the Fourier sine transformation of $\frac{e^{-ax}}{x}$ where $a > 0$.

24. Show that the Fourier transform of $f(x)=\begin{cases}a^2-x^2, & |x|<a\\ 0, & |x|\ge a\end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as-as\cos as}{s^3}\right)$. Hence deduce that $\int_0^\infty \frac{\sin t-t\cos t}{t^3}dt=\frac{\pi}{4}$. Using Parserval's identity show that $\int_0^\infty \frac{\sin t-t\cos t}{t^3}dt=\frac{\pi}{15}$.

25. Find $F_C(e^{-ax})$, $F_C\left(\frac{1}{1+x^2}\right)$ and $F_C\left(\frac{x}{1+x^2}\right)$.

UNIT –V: Z –TRANSFORMS

PART - A

1. Form the difference equation from the relation $y_n = a + b\,3^n$
2. Find the **Z** transform of $\frac{1}{(n+1)(n+2)}$
3. Find the **Z** transform of $\frac{1}{n(n-1)}$
4. Find the inverse **Z** transform of $\frac{10z}{z^2-3z+2}$
5. Form the difference equation whose solution is $y_n = (A + B_n)\,2^n$.
6. Find the **Z** transform of $\sin^2(\frac{n\pi}{4})$
7. Find the **Z** transform of $\cos n\theta$ and hence find **Z** [$n\cos n\theta$]
8. Find the **Z** transform of (i) $\sin n\theta$ (ii) $n^2 e^{an}$
9. Determine the **Z** transforms of $\cos\frac{n\pi}{2}$ *and* $a^n\cos\frac{n\pi}{2}$

10. Find the **Z** transform of $\frac{1}{n}$ and $\cos\frac{n\pi}{2}$
11. Find the **Z** transform of $\{a^n\}$ *and* $\{na^n\}$
12. Find the **Z** transform of $\frac{1}{n(n+1)}$
13. Find the **Z** transform of f(n) = $\frac{2n+3}{(n+1)(n+2)}$
14. Find the **Z** transform of a^n and $a^n \cos n\theta$
15. State and Prove second shifting theorem in **Z** -transform.
16. Prove that $Z\left[\frac{1}{n+1}\right] = z\log\left(\frac{z}{z-1}\right)$

PART - B

1. Find the **Z** transform of sin nθ and cosnθ. Hence deduce the **Z** transforms of cos(n+1)θ and sin(n+1)θ
2. Find the inverse **Z** transform of $\frac{z(z+1)}{(z-1)^3}$ by residue method.
3. 4. Solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 2^n$, $y_0 = 0$, $y_1 = 1$
5. Determine the **Z** transforms of $\sin\frac{n\pi}{2}$ *and* $\cos\frac{n\pi}{2}$
6. Find $Z^{-1}\left[\frac{z^3}{(z-2)^2(z-3)}\right]$
7. Solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, $u_0 = 0$, $u_1 = 1$

9. Find the Z transform of i. $r^n \cos n\theta$
10. Find the Z transform of f(n) * g(n) where f(n) = $(\frac{1}{2})^n$ & g(n) = cos nπ
11. Solve f(n+2) – 7 f(n+1) + 12 f(n) = 2^n , f(0) = f(1) = 0
12. Solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 2^n$, $u_0 = 2$, $u_1 = 1$
13. Using convolution theorem find inverse **Z** transform of $\left(\frac{z}{z-4}\right)^3$
14. Find **Z** [n a^n sinnθ]
15. Find (i) $Z^{-1}\left[\frac{z}{(z-1)\,(z-2)}\right]$ (ii) $Z^{-1}\left[\frac{z(z^2-z+2)}{(z-1)^2(z+1)}\right]$
16. Find **Z** [f(n)], if $f(n) = \frac{1}{\sqrt{5}}\left\{\left(\frac{\sqrt{5}+1}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}$
18. Solve y(k+2) + 4 y(k+1) + 4 y(k) = k , y(0) = 0, y(1) = 1

19. Find the inverse **Z** transform of $\frac{z^2}{(z+2(z^2+4)}$ by the method of partial fraction
20. Solve y(n+3) – 3y(n+1) + 2y(n) = 0 , y(0) = 4, y(1) = 0, y(2) = 8
21.
22. Solve the difference equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$, $u_0 = u_1 = 0$
23. Using convolution theorem, find the $\mathbf{Z}^{-1}$ of $\frac{z^2}{(z-4)(z-3)}$
24. Find the inverse **Z** transform of $\frac{z^3-20z}{(z-2)^3(z-4)}$
26. Find the inverse **Z** transform of $\frac{z^2}{(z-5)(z-4)}$ by convolution theorem.
28. Solve the difference equation $y_{(k+2)} + 2y_{(k+1)} + y_{(k)} = k$, $y_{(0)} = y_{(1)} = 0$
29. Find the **Z** transform of $r^n \cos\theta$ and $r^n \sin\theta$
30. Find the inverse **Z** transform of $\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}$ and $\frac{4z^3}{(2z-1)^2(z-1)}$
31. Solve the difference equation $y_{n+2} + 4y_{n+1} + 4y_n = n$, $y_0 = 0$, $y_1 = 1$
32. Using convolution theorem, prove that $\mathbf{Z}^{-1}$ of $\frac{z^2}{(z+a)(z+b)}$ is $\frac{(-1)^n}{b-a}\left[b^{n+1}-a^{n+1}\right]$
33. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = y_1 = 0$
34.
35. Solve the difference equation y(n+2) – 5 y(n+1) + 6 y(n) = 36 , y(0) = y(1) = 0
36. By using convolution theorem, find the inverse **Z** transform of $\frac{z^2}{(z-a)(z-b)}$
38. Solve the difference equation y(n+2) + 4 y(n+1) + 3 y(n) = 3^n , y(0) = 0, y(1) = 1
39. Find the inverse **Z** transform of $\frac{z^3+3z}{(z-1)^2(z^2+1)}$
41. Using convolution theorem, find the inverse **Z** transform of $\frac{12z^2}{(3z-1)(4z+1)}$
42. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = y_1 = 0$
43. Using convolution theorem, find the inverse **Z** transform of $\frac{8z^2}{(2z-1)(4z-1)}$
45. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = y_1 = 0$
47. Solve the difference equation $y_{n+3} - 12y_{n+2} + 48y_{n+1} - 64y_n = 5.\,4^n$, $y_0 = y_1 = y_2 = 0$
48. Find the inverse **Z** transform of $\frac{z}{z^2+7z+10}$ by residue method

49. Using convolution theorem find $Z^{-1}\left[\frac{z^2}{(z+a)^2}\right]$

52. Solve y(n+3) – 3y(n+1) + 2y(n) = 0 , y(0) = 4, y(1) = 0, y(2) = 8

53. Find $Z^{-1}\left[\frac{z^2}{(z+2)(z^2+4)}\right]$ by the method of partial fractions.

54. Solve the difference equation $y_{n+2} + 2y_{n+1} + y_n = n$, $y_0 = y_1 = 0$

55. Find the inverse Z transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$

56. Using convolution theorem find $Z^{-1}\left[\frac{z^2}{(z-a)^2}\right]$

57. Solve the difference equation $y_{n+2} - 6y_{n+1} + 5y_n = 2^n$, $y_0 = y_1 = 0$

58. Solve y(n + 2) + y(n) = 2 , y(0) = y(1) = 0

60. Find the inverse Z transform of $\frac{z(z+1)}{(z-1)^3}$

61. State & prove final value theorem in Z transform

62. Find the Z transform of $\frac{1}{(n+1)(n+2)}$

63. Find the inverse Z transform of $\frac{z(z^2 - z + 2)}{(z+1)(z-1)^2}$ using partial fraction

64. Find $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ by convolution theorem.

65. Solve the difference equation $y_{n+2} - 4y_{n+1} + 4y_n = 0$, $y_0 = 1$, $y_1 = 0$

66. Find the inverse Z transform of $\frac{z^2}{(z-1)(z-3)}$ by convolution theorem.

67. Solve y(n) + 3 y(n –1) – 4 y(n –2) = 0 , n ≥ 2, y(0) = 3, y(1) = –2

68. Solve $y_{x+1} - 2y_x \cos a + y_{x-1} = 0$

69. Show that $Z\left[\frac{1}{n!}\right] = e^{\frac{1}{z}}$ and hence evaluate $Z\left[\frac{1}{(n+1)!}\right]$ *and* $Z\left[\frac{1}{(n+2)!}\right]$

70. If $Z[f_n] = F(z)$, show that $Z\left[a^{-n} f_n\right] = F(az)$ and find the Z transform of $n^2 a^n$

71. Solve $y_{n+2} - 4y_{n+1} + 3y_n = 2^n n^2$, $y_0 = y_1 = 0$

72. Find the inverse Z transform of $\frac{z^3}{(z-1)^2(z-2)}$ using partial fraction

73. Solve y(k+2) – 4 y(k+1) + 4 y(k) = 0 , y(0) = 1, y(1) = 0