VALLIAMMAI ENGINEERING COLLEGE
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DEPARTMENT OF MATHEMATICS

QUESTION BANK

SUBJECT : MA8451 – PROBABILITY AND RANDOM PROCESSES
SEM / YEAR : IV SEM / II YEAR

UNIT- I - PROBABILITY AND RANDOM VARIABLES

Probability – Axioms of probability – Conditional probability – Baye's theorem – Discrete and continuous random variables – Moments – Moment generating functions – Binomial, Poisson, Geometric, Uniform, Exponential and Normal distributions.

Q.No	Question	Bloom's Taxonomy Level	Domain
	PART – A		
1.	List the limitations of Poisson distribution.	BTL -1	Remembering
2.	Write the MGF of Geometric distribution	BTL -1	Remembering
3.	If $f(x) = \begin{cases} ke^{-x}, x > 0 \\ 0, otherwise \end{cases}$ is the pdf of a random variable X, then find the value of k.	BTL -1	Remembering
4.	The mean and variance of binomial distribution are 5 and 4 .Identify the distribution.	BTL -1	Remembering
5.	State Baye's Theorem.	BTL -1	Remembering
6.	Test whether $f(x) = \begin{cases} \lvert x \rvert, -1 \le x \le 1 \\ 0, otherwise \end{cases}$ can be a probability density function of a continuous random variable.	BTL -1	Remembering
7.	A random variable X takes values 1,2,3,4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$. Give the probability distribution of X.	BTL -2	Understanding
8.	Show that f(x) is a pdf, if f(x) $= \begin{cases} xe^{\frac{-x^2}{2}} & x > 0 \\ 0, & x < 0 \end{cases}$	BTL -3	Applying
9.	If 3% of the electric bulb s manufactured by a company are defective, calculate the probability that in a sample of 100 bulbs exactly 5 bulbs are defective	BTL -2	Understanding
10.	A random variable X has the MGF $M_X(t) = \frac{2}{2-t}$ .Examine the standard deviation of X	BTL -4	Analyze
11.	Show that the function $f(x) = \begin{cases} e^{-x} & , x \ge 0 \\ 0, & x < 0 \end{cases}$ is a probability density function of a continuous random variable X.	BTL -3	Applying
12.	Show that the moment generating function of the uniform distribution $f(x) = \frac{1}{2a}, -a < x < a$, about origin is $\frac{\sinh(at)}{at}$.	BTL -3	Applying
13.	The MGF of a uniform distribution for a RV X is $\frac{1}{t}(e^{5t} - e^{4t})$. Calculate E(X).	BTL -3	Applying
14.	Explain two characteristics of the Normal Distribution	BTL -4	Analyzing
15.	Explain the moment generating function of binomial distribution	BTL -4	Analyzing

16.	The CDF of a continuous random variable is given by $F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\frac{x}{5}}, & x > 0 \end{cases}$ Analyze the PDF and E[X] .	BTL -4	Analyzing
17.	The number of hardware failures of a computer system in a week of operations has the following Pdf, Evaluate the mean of the number of failures in a week. No.of failures: 0, 1, 2, 3, 4, 5, 6 Probability: .18, .28, .25, .18, .06, .04, .01	BTL -5	Evaluating
18.	A random variables X has the following probability distribution. Find the value of K and P(X≥ 3) X: 0, 1, 2, 3, 4 P(x): K, 3K, 5K, 7K, 9K	BTL -3	Applying
19.	The number of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function of a month with only one breakdown.	BTL -6	Creating
20.	The probability of a candidate can pass in an examination is 0.6. a) What is the probability that he will pass in third trial? b) What is the probability that if he pass before third trail?	BTL -6	Creating
	PART-B		
1(a).	A random variable X has the following probability distribution: X: 0, 1, 2, 3, 4, 5, 6, 7 P(X): 0, k, $2k$, $2k$, $3k$, k^2, $2k^2$, $7k^2$+k Identify (i) the value of k (ii) $P(1.5 < X < 4.5 / X > 2)$	BTL-1	Remembering
1(b).	Give the MGF of Binomial distribution and hence find its mean and variance.	BTL-2	**Understanding**
2(a).	For the triangular distribution $\begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2 \\ 0, & otherwise \end{cases}$.Find mean, variance and MGF of X.	BTL-1	Remembering
2(b).	Give the MGF of Poisson distribution and hence find its mean and variance.	BTL-2	**Understanding**
3(a).	The probability distribution of an infinite discrete distribution is given by P [X = j] = $\frac{1}{2^j}$ (j = 1,2,3…) Identify(i)Mean of X (ii)P [X is even] (iii) $P[X \ge 5]$(iv)P(X is divisible by 3)	BTL-1	Remembering
3(b).	Calculate the MGF of Geometric distribution and hence find its mean and variance.	BTL -3	Applying
4(a).	A continuous random variable X that can assume any value between X = 2 and X =5 has a probability density function given by $f(x) = k(1+x)$. Estimate P(X<4).	BTL-2	**Understanding**
4(b).	Analyze the MGF of Uniform distribution and hence find its mean and variance.	BTL -4	Analyzing
5.	If f(x) = $\begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0, & elsewhere \end{cases}$ is the p.d.f of X. Calculate (i) The value of a , (ii) The cumulative distribution function of X	BTL -3	**Applying**

	(iii) If X_1, X_2 and X_3 are 3 independent observations of X. Find the probability that exactly one of these 3 is greater than 1.5?		
6.	A random variable X has the p.d.f. $f(x) = \begin{cases} 2x, 0 < x < 1 \\ 0, \quad elsewhere \end{cases}$ Find (i) $P\left(X < \frac{1}{2}\right)$ (ii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$ (iii) $P(X > \frac{3}{4} / X > \frac{1}{2})$ (iv) $P(X < \frac{3}{4} / X > \frac{1}{2})$	**BTL -3**	**Applying**
7(a).	Explain the MGF of Normal distribution and hence find its mean and variance.	**BTL -4**	Analyzing
7(b).	If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8 Evaluate the probability that he will finally pass the test (i) on the fourth trial and (ii) in less than 4 trials.	**BTL -6**	Creating
8(a).	The marks obtained by a number of students for a certain subject is assumed to be normally distributed with mean 65 and standard deviation 5.If 3 students are taken at random from this set Identify the probability that exactly 2 of them will have marks over 70?	**BTL -1**	Remembering
8(b).	The contents of urns I, II, III are as follows: 1 white, 2 black and 3 red balls; 2 white, 1 black and 1 red balls; 4 white, 5 black and 3 red balls; One urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urns I, II, III?	**BTL -3**	Applying
9(a).	Out of 800 families with 4 children each, how many families would be expected to have (1) 2 boys and 2 girls (2) at least one boy (3) at most 2 girls (4) children of both sexes. Assume equal probability for boys and girls.	**BTL-2**	Understanding
9(b).	In 1989, there were three candidates for the position of principal Mr. Chatterji, Mr. Ayangar and Dr. Singh. Whose chances of getting the appointment are in the proportion 4:2:3 respectively. The probability that Mr. Chatterji is selected, would introduce co-education in the college is 0.3. The probabilities of Mr. Ayangar and Dr. Singh doing the same are respectively 0.5 and .08. What is the probability that there was co-education in the college in 1990?	**BTL -3**	Applying
10(a).	In a certain city, the daily consumption of electric power in millions of kilowatt hours can be treated as a RV having Gamma distribution with parameters $\lambda = ½$ and k =3. If the power plant of this city has a daily capacity of 12 million kilowatts – hours, Find the probability that this power supply will be inadequate on any given day?	**BTL -3**	**Applying**
10(b).	Suppose that the life of an industrial lamp in 1,000 of hours is exponentially distributed with mean life of 3,000 hours. Find the probability that (i) The lamp last more than the mean life (ii) The lamp last between 2,000 and 3,000 hours (iii) The lamp last another 1,000 hours given that it has already lasted for 250 hours.	**BTL -3**	**Applying**
11.	A random variable X has the following probability distribution X: -2, -1, 0, 1, 2, 3 P(x): 0.1, K, 0.2, 2K, 0.3, 3K (1) Find the value of K. (2) Evaluate P(X< 2) and P(-2 < X < 2). (3) Find the cumulative distribution of X. (4) Find the mean of X.	**BTL -3**	**Applying**

12(a).	The density function of a random variable 'X' is given by $f(x) = Kx(2-x), 0 \le x \le 2$. Find K, mean, variance and rth moment.	**BTL-2**	Understanding
12(b).	If X and Y are independent Poisson variate such that P(X = 1) = P(X = 2) and P(Y = 2) = P(Y = 3).Find the variance of X− 2Y.	**BTL-3**	Applying
13(a).	State and prove memoryless property for geometric distribution.	**BTL-1**	Remembering
13(b).	The mileage which car owners get with certain kind of radial tyre is a RV having an exponential distribution with mean 4,000 km. Find the probabilities that one of these tyres will last (i) at least 2,000 km (ii) atmost 3,000 km.	**BTL-3**	Applying
14(a).	A coin is biased so that a head is twice as likely to appear as a tail. If the coin is tossed 6 times, find the probabilities of getting (1) Exactly 2 heads, (2) at least 3 heads, (3) at most 4 heads. .	**BTL-3**	Applying
14(b).	The length of time a person speaks over phone follows exponential distribution with mean 6 mins. What is the probability that the person will talk for (1) more than 8 mins, (2) between 4 and 8 mins.	**BTL-2**	Understanding

UNIT II - TWO – DIMENSIONAL RANDOM VARIABLES

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and linear regression – Transformation of random variables – Central limit theorem (for independent and identically distributed random variables).

Q. No.	Question	BT Level	Competence
	PART – A		
1.	Define the distribution function of two dimensional random variables (X,Y) . State any two properties.	**BTL -1**	Remembering
2.	The joint pdf of (X, Y) is $f(x,y) = \begin{cases} \frac{1}{4}, 0 < x, y < 2 \\ 0, \ otherwise \end{cases}$. Find $P(X + Y \le 1)$.	**BTL -1**	Remembering
3.	Examine the value of k if the joint probability density function of a bivariate random variable (X,Y) is given by $f(x,y) = \begin{cases} k(1-x)(1-y) \ if \ 0 < x < 4, 1 < y < 5 \\ 0, \quad otherwise \end{cases}$	**BTL -1**	Remembering
4.	The joint pdf of the random variable (X,Y) is given by f(x,y) = $Kxye^{-(x^2+y^2)}$, $x > 0, \ y > 0$.Calculate the value of K.	**BTL -3**	Applying
5.	Let X and Y be independent random variables with variance 2 and 3. Find the variance of 3X+4Y.	**BTL -1**	Remembering
6.	Can you tell that the joint distributions of two random variables X and Y be got if their Marginal distributions are known?	**BTL -1**	Remembering
7.	The two random variables X and Y have probability density function $f(x,y) = k(2x + y)$ for $0 \le x \le 2$ and $0 \le y \le 2$, Calculate k.	**BTL -3**	Applying
8.	The joint probability density function of a random variable X and Y is given by $f(x,y) = \begin{cases} \frac{x^3y^3}{16}, 0 < x < 2, 0 < y < 2 \\ 0, \quad otherwise \end{cases}$. Find the marginal functions of X and Y.	**BTL -3**	Applying
9.	If X and Y have joint pdf f(x,y) = $\begin{cases} x + y, 0 < x < 1, \ 0 < y < 1 \\ 0, \ otherwise \end{cases}$ Discuss whether X and Y are independent.	**BTL -2**	Understanding
10.	The joint probability mass function of a two dimensional random variable (X,Y) is given by $p(x,y) = k(2x+3y), x = 0,1,2; y = 1,2,3$. Evaluate k.	**BTL -5**	Evaluating
11.	If X and Y are RVs such that Y = aX + b where a and b are real constants, show that the correlation coefficient between them has magnitude 1.	**BTL -3**	Applying

12.	What do you mean by correlation between two random variables	**BTL -1**	Remembering
13.	Distinguish between correlation and regression.	**BTL -2**	Understanding
14.	Let (X, Y) be a two-dimensional random variable. Define covariance of (X, Y). If X and Y are independent, what will be the covariance of (X, Y)?	**BTL -4**	Analyzing
15.	The regression equations are x + 6y = 14 and 2x + 3y = 1. Find the mean values of X & Y **.**	**BTL -4**	Analyzing
16.	If $\bar{X} = 970, \bar{Y} = 18, \sigma_x = 38, \sigma_y = 2$ and r =0.6 , Find the line of regression and obtain the value of X and Y = 20.	**BTL -4**	Analyzing
17.	Give the acute angle between the two lines of regression.	**BTL -2**	Understanding
18.	The joint probability density of a two dimensional random variable (X,Y) is given by $f(x,y) = \begin{cases} kxe^{-y}; 0 \le x2, y > 0 \\ 0, \quad otherwise \end{cases}$. Evaluate k.	**BTL -5**	Evaluating
19.	Let X and Y be random variables with joint density function $f_{XY}(x,y) = \begin{cases} 4xy, & 0 < x < 1, \ 0 < y < 1 \\ 0, & otherwise \end{cases}$ formulate the value of E(XY)	**BTL -6**	Creating
20.	The random variables X and Y are having the joint density function f(x,y) = 1/8 (6 – x – y), 0 < x < 2, 2 < y < 4, formulate the value of P(X+Y < 3).	**BTL -6**	Creating
	PART -B		
1.	If X, Y are RV's having the joint density function $f(x,y) = k(6-x-y), 0 < x < 2, 2 < y < 4$, Point out $(i)\ P(x < 1, y < 3)$ $ii)\ P\left(x < 1 \frac{}{y} < 3\right)$ $iii) P(y < 3/x < 1)$ $iv)\ P(X + Y < 3)$	**BTL -4**	Analyzing
2(a).	The joint distribution of X and Y is given by $f(x,y) = \frac{x+y}{21}, x = 1,2,3; y = 1,2.$ Find the marginal distributions.	**BTL-3**	Applying
2(b).	If the joint probability distribution function of a two dimensional random variable (X,Y) is given by $F(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}); x > 0, y > 0 \\ 0, \quad otherwise \end{cases}$. Calculate the marginal densities of X and Y. Are X and Y independent? $Find\ P[1 < X < 3, 1 < Y < 2]$	**BTL -3**	Applying
3(a).	If the joint pdf of (X, Y) is given by p(x, y)= K(2x+3y), x=0, 1, 2, 3 and y=1,2. Find all the marginal probability distribution. Also find the probability distribution of (X+Y).	**BTL-3**	Applying
3(b).	The joint pdf of X and Y is given by f(x,y)=$\begin{cases} kx(x-y), 0 < x < 2, -x < y < x \\ 0, \quad otherwise \end{cases}$ (i)Find K (ii) Find $f_x(x)$ and $f_y(y)$ (iii) $f_{\frac{y}{x}}\left(\frac{y}{x}\right)$	**BTL -1**	Remembering
4(a).	The joint pdf in given as $f(x,y) = \begin{cases} K(4-x-y); 0 \le x \le 2, 0 \le y \le 2 \\ 0, \quad otherwise \end{cases}$ (i)Find K (ii) Find conditional density function (iii) Check whether X & Y are independent.	**BTL-3**	Applying
4(b).	The joint pdf a bivariate R.V(X, Y) is given by $f(x,y) = \begin{cases} Kxy; 0 < x < 1, 0 < y < 1 \\ 0, \quad otherwise \end{cases}$ (1) Find K. (2) Find P(X+Y<1). (3) Are X and Y independent R.V's.	**BTL-3**	Applying

5.	The joint pdf of a two dimensional random variable (X, Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$. Compute (i) $P\left(X > 1 / Y < \frac{1}{2}\right)$ (ii)$P\left(Y < \frac{1}{2} / X > 1\right)$ (iii) $P(X < Y)$ (iv) $P(X + Y) \le 1$.	**BTL-1**	Remembering
6(a).	The two dimensional random variable (X, Y) has the joint probability mass function $f(x,y) = \frac{x+2y}{27}, x = 0,1,2; y = 0,1,2$. Find the conditional distribution of Y for X= x. Also find the conditional distribution of Y given X=1.	**BTL-3**	Applying
6(b).	The joint density function of the RVs X and Y is given by $f(x,y) = \begin{cases} 8xy; 0 < x < 1, 0 < y < x \\ 0, \; elsewhere \end{cases}$. Find $P\left(Y < \frac{1}{8} / X < \frac{1}{2}\right)$	**BTL-3**	Applying
7.	If the joint pdf of a two-dimensional RV(X,Y) is given by, $f(x,y) = \begin{cases} x^2 + \frac{xy}{3}; 0 < x < 1, 0 < y < 2 \\ 0, \; elsewhere \end{cases}$. Find (i) $P\left(X > \frac{1}{2}\right)$ (ii) $P(Y < X)$ and (iii) $P\left(Y < \frac{1}{2} / X < \frac{1}{2}\right)$	**BTL-3**	Applying
8.	Two independent random variables X and Y are defined by $f_X(x) = \begin{Bmatrix} 4ax: \; 0 < x < 1 \\ 0: otherwise \end{Bmatrix}$ And $f_Y(y) = \begin{Bmatrix} 4by: \; 0 < y < 1 \\ 0: otherwise \end{Bmatrix}$ Show that U=X+Y and V=X-Y are uncorrelated	**BTL -3**	Applying
9.	From the following data , Give (i)The two regression equations (ii) The coefficient of correlation between the marks in Mathematics and Statistics (iii) The most likely marks in Statistics when marks in Mathematics are 30 Marks in Maths : 25 28 35 32 31 36 29 38 34 32 Marks in Statistics: 43 46 49 41 36 32 31 30 33 39	**BTL -2**	Understanding
10.	If X and Y are independent normal variates with zero mean and standard deviation σ. Identify the densities of $R = \sqrt{X^2 + Y^2}$ and $\theta = tan^{-1}\left(\frac{y}{x}\right)$.	**BTL -1**	Remembering
11.	Let (x, y) be a two-dimensional non-negative continuous random variable having the joint density. $f(x,y) = \begin{cases} 4xy\, e^{-(x^2+y^2)}; x \ge 0, y \ge 0 \\ 0, \; otherwise \end{cases}$. Find the density function of $U = \sqrt{x^2 + y^2}$.	**BTL-1**	Remembering
12.	The random variable (X,Y) has the joint p.d.f $f(x,y) = \begin{Bmatrix} (x+y): \; 0 \le x \le 1 \; and \; 0 \le y \le 1 \\ 0: elsewhere \end{Bmatrix}$ point out r(X,Y)	**BTL -4**	Analyzing
13.	Two random variables X and Y have the joint density f(x,y) = $\begin{cases} 2 - x - y, 0 < x < 1, 0 < y < 1 \\ 0, \; otherwise \end{cases}$ Create the Correlation coefficient between X and Y is -1 /11.	**BTL -6**	Creating
14(a).	The two lines of regression are 8x-10y+66=0, 40x-18y-214=0. The variance of X is 9. Find (i) The mean values of X and Y (ii) Correlation coefficient between X and Y.	**BTL-4**	**Analyzing**
14(b).	The life time of a certain brand of an electric bulb may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Find the Probability using central limit theorem that the average life time of 60 bulbs exceed 1250 hours.	**BTL -3**	Applying

UNIT -III -RANDOM PROCESSES

Classification – Stationary process – Markov process – Markov chain – Poisson process – Random telegraph process.

PART – A

Q. No.	Question	BT Level	Competence
1.	Define a random process? When do you say a random process is a random variable?	BTL -1	Remembering
2.	State strict sense and wide sense stationary process.	BTL -1	Remembering
3.	Define a semi-random telegraph signal process.	BTL -1	Remembering
4.	State Markov process.	BTL -1	Remembering
5.	Define Binomial process and state its properties. Give an example for its sample function.	BTL -1	Remembering
6.	Define Markov chain and one – step transition probability.	BTL -1	Remembering
7.	Give the four types of a stochastic process.	BTL -2	Understanding
8.	Give an example for a continuous time random process.	BTL -2	Understanding
9.	State the postulates of a Poisson process.	BTL -2	Understanding
10.	State any two properties of Poisson process.	BTL -2	Understanding
11.	Prove that a first order stationary random process has a constant mean.	BTL -3	Applying
12.	Show that the random process $X(t) = A \cos(\omega_o t + \theta)$ is not stationary if A and ω_o are constants and θ is uniformly distributed in $(0, \pi)$.	BTL -3	Applying
13.	Give an example of evolutionary random process.	BTL -2	Understanding
14.	Write a detailed note on Sine wave process.	BTL -4	Analyzing
15.	When is a Markov chain, called homogeneous?	BTL -4	Analyzing
16.	When is a Random process said to be ergodic. Give an example of an ergodic process.	BTL -4	Analyzing
17.	If the TPM of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, find the steady state distribution of the chain.	BTL -3	Applying
18.	For the sine wave process $X(t) = Y \cos\omega t$, $-\infty < t < \infty$ ω is a constant, the amplitude Y is a random variable with uniform distribution (0,1). Judge whether the process is stationary or not.	BTL -5	Evaluating
19.	A gambler has Rs.2. He bets Rs.1 at a time and wins Rs.1 with probability ½ . He stops playing if he loses Rs. 2 or wins Rs.4. Formulate the transition probability matrix for the Markov chain.	BTL -6	Creating
20.	Let X(t) be a Poisson process with rate λ. Formulate $E(X(t)\, X(t+\tau))$, where $\tau > 0$.	BTL -6	Creating

PART – B

1(a).	If $X(t) = Y \cos wt + Z \sin wt$, where Y and Z are two independent normal random variables with $E(Y) = 0 = E(Z)$, $E(Y^2) = E(Z^2) = \sigma^2$ and w is a constant, Examine that $\{X(t)\}$ is a strict sense Stationary process of order 2	BTL -1	Remembering
1(b).	A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Estimate the probability that 10 particles are recorded in 4 minute period.	BTL -2	Understanding

2(a).	Estimate the mean, variance, auto correlation function of a Poisson process.	**BTL -1**	Remembering
2(b).	Examine that the process $X(t) = A\, cos\lambda t + Bsin\lambda t$ where A nad B are random variables, is wide sense stationary process if $E(A) = E(B) = E(AB) = 0, E(A^2) = E(B^2)$	**BTL -2**	Understanding
3(a).	A random process X (t) is defined by $X(t) = A\, cos\lambda t + Bsin\lambda t, -\infty < t < \infty$ where A, B are independent RV each of which has the value -2 with probability 1/3 and a value 1 with probability 2/3. Examine that X (t) is a wide sense stationary process.	**BTL -1**	Remembering
3(b).	Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Calculate the probability that during a time interval of 2 minutes (i) exactly 4 customers arrive and (ii) more than 4 customers arrive.	**BTL -3**	Applying
4.	The probability distribution of the process {X(t)} is given by $P(X(t) = n) = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3... \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$ .Show that it is not stationary.	**BTL -2**	Understanding
5.	A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Calculate (i) the probability that he takes a train on the third day and (ii) the probability that he drives to work in the long run.	**BTL -3**	Applying
6.	A fair die is tossed repeatedly. The maximum of the first 'n' outcomes is denoted by X_n. Is $\{X_n, n = 1,2,\ldots\}$ a Markov chain? If so, Find its transition probability matrix, also find $P\{X_2 = 6\} and\ P^2$.	**BTL -3**	Applying
7.	There are 2 white marbles in Urn A and 3 red marbles in Urn B. At each step of the process, a marble is selected from each urn and the two marbles selected are interchanged. The state of the related Markov chain is the number of red marbles in Urn A after the interchange. Point out the probability that there are 2 red marbles in Urn A after 3 steps? In the long run, what is the probability that there are 2 red marbles in Urn A?	**BTL -4**	Analyzing
8.	Given a RV Y with characteristic function $\varphi(\omega) = E(e^{i\omega Y})$ and a random process defined by $X(t) = \cos(\lambda t + Y)$. Judge that X(t) is stationary in wide sense if $\varphi(1) = \varphi(2) = 0$.	**BTL -5**	Evaluating
9.	Prove that the random processes X(t) and Y(t) defined by $X(t) = A\, coswt + B\, sinwt$, $Y(t) = B\, cos\, wt - A\, sin\, wt$ are jointly wide sense stationary if A and B are uncorrelated zero mean random variables with the same variance.	**BTL -6**	Creating
10(a).	The tpm of a Markov Chain $\{X_n\}$, three states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = [0.7, 0.2, 0.1]$. Identify (i) $P(X_2 = 3)$ (ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$	**BTL -1**	Remembering
10(b).	On the average , a submarine on patrol sights 6 enemy ships per hour. Assuming that the no. of ships sighted in a given length of time is a Poisson variate, Point out the probability of sighting (i) 6 ships in the next half-an-hour, (ii) 4 ships in the next 2 hours (iii) at least 1 ship in the next 15 min and (iv) at least 2 ships in the next 20 minutes.	**BTL -4**	Analyzing

11(a).	Classify the Random Processes with suitable Examples.	**BTL-3**	Applying
11(b).	The TPM of a Markov chain with three states 0,1,2 is $P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial state distribution of the chain is $P(X_0 = i) = \frac{1}{3}, i = 0,1,2,$ Estimate $(i) P(X_2 = 2) and \ (ii) P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$.	**BTL -4**	Analyzing
12(a).	Consider the Markov chain with tpm given by $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \end{bmatrix}$. Show that it is ergodic.	**BTL -5**	Evaluating
12(b).	The probability of a dry day following a rainy day is $\frac{1}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$. Given that May 1^{st} is a dry day. Find the probability that May 3^{rd} is a dry day and also May 5^{th} is a dry day.	**BTL -6**	Creating
13(a).	Radha bought three kinds of cereals A , B and C. she never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However if she buys B or C the next week she is 3 times as likely to buy A as the other cereal. How often she buys each of the 3 cereals?	**BTL -4**	Analyzing
13(b).	Three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What fraction of the vehicles on the road are trucks?	**BTL -6**	Creating
14.	Check if a random telegraph signal process is wide sense stationary.	**BTL -6**	Creating

UNIT -IV-CORRELATION AND SPECTRAL DENSITIES

Auto correlation functions – Cross correlation functions – Properties – Power spectral density – Cross spectral density – Properties.

PART – A

Q. No.	**Question**	**BT Level**	**Competence**
1.	Define autocorrelation function and prove that for a WSS Process{X(t)}, $R_{xx}(-\tau) = R_{xx}(\tau)$	**BTL -1**	Remembering
2.	Define Cross correlation function and state any two of its properties	**BTL -1**	Remembering
3.	State any two properties of an auto correlation function.	**BTL -1**	Remembering
4.	Define the power spectral density and cross power spectral density of a random process	**BTL -1**	Remembering
5.	Give an example of cross – spectral density.	**BTL -1**	Remembering
6.	State and prove any one of the properties of cross – spectral density function.	**BTL -1**	Remembering
7.	Estimate the variance of the stationary process { X (t)} whose auto correlation function is given by $R(\tau) = 2+4e^{-2\lambda\|\tau\|}$	**BTL -2**	Understanding
8.	Estimate the variance of the stationary process {X(t)}, whose auto correlation function is given by $Rxx(\tau) = 16 + (\frac{9}{1+6\tau^2})$.	**BTL -2**	Understanding
9.	Given that the autocorrelation function for a stationary ergodic process with no periodic components is $Rxx(\tau) = 25 + (\frac{4}{1+6\tau^2})$. Estimate the mean and variance of the process {X(t)}.	**BTL -2**	Understanding

10.	Prove that $R_{xy}(\tau) = R_{yx}(-\tau)$.	**BTL -1**	Remembering
11.	The random process X(t) has an autocorrelation function $R_{xx}(\tau) = 18 + \frac{2}{6+\tau^2}$ Calculate E(X(t)) and E(X^2 (t)).	**BTL -3**	Applying
12.	If a random process $X(t)$ is defined as $X(t) = \begin{cases} A, 0 \le t \le 1 \\ 0, \ otherwise \end{cases}$ where A is a r.v uniformly distributed from $-\theta$ to θ. P.T the autocorrelation function of $X(t)$ is $\frac{\theta^2}{3}$.	**BTL -1**	Remembering
13.	Check whether the following functions are valid auto correlation $\frac{1}{1+9\tau^2}$.	**BTL -4**	Analyzing
14.	If $R(\tau) = e^{-2\lambda\|\tau\|}$ is the auto correlation function of a random process {X(t)}. Point out the spectral density of {X(t)}.	**BTL -4**	Analyzing
15.	The autocorrelation function of the random telegraph signal process is given by $R_{xx}(\tau) = a^2 e^{-2r\|\tau\|}$. Point out the power density spectrum of the random telegraph signal.	**BTL -4**	Analyzing
16.	Point out the auto correlation function whose spectral density is $S(\omega) = \begin{cases} \pi, & \|\omega\| \le 1 \\ 0, & otherwise \end{cases}$	**BTL -4**	Analyzing
17.	Evaluate the power spectral density of a random signal with autocorrelation function $e^{-\lambda\|\tau\|}$.	**BTL -5**	Evaluating
18.	Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-\sigma\tau^2}$	**BTL -5**	Evaluating
19.	Given the power spectral density: $Sxx(\omega) = \left(\frac{1}{4+\omega^2}\right)$ formulate the average power of the process .	**BTL -6**	Creating
20.	Formulate a valid power density spectrum for $\frac{\omega+4}{\omega^2+5}$.	**BTL -6**	Creating
	PART-B		
1.	Consider two random processes X(t) = 3 cos(ωt + θ) and Y(t) = 2 cos (ωt + θ - π/2) where θ is a random variable uniformly distributed in (0 , 2 π). Give the proof for $\sqrt{R_{xx}(0)R_{yy}(0)} \ge \|R_{xy}(\tau)\|$.	**BTL -2**	Understanding
2.	Identify the power spectral density of a random binary transmission process where auto correlation function is $R(\tau) = 1 - \frac{\|\tau\|}{T}; \|\tau\| \le T$.	**BTL -1**	Remembering
3.	If the power spectral density of a continuous process is $S_{xx}(\omega) = \frac{\omega^2+9}{\omega^4+5\omega^2+4}$, Give the mean value, mean- square value of the process.	**BTL -2**	Understanding
4(a).	The power spectrum of a wide sense stationary process X(t) is given by $S_{xx}(\omega) = \frac{1}{(1+\omega^2)^2}$ .Calculate the auto correlation function.	**BTL -2**	Understanding
4(b).	Point out the auto correlation function of the process {X(t)}, if its power spectral density is given by $S(\omega) = \begin{cases} 1+\omega^2, for \ \|\omega\| \le 1 \\ 0, for \ \ \ \|\omega\| \ge 1 \end{cases}$	**BTL -4**	Analyzing
5(a).	A random process {X(t)} is given by $X(t) = Acospt + B\ sinpt$, where A and B are independent RV's such that E(A)=E(B)= 0 and $E(A^2) = E(B^2) = \sigma^2$. Calculate the power spectral density of the process.	**BTL -3**	Applying
5(b).	If the power spectral density of a WSS process is given by $S(\omega) = \begin{cases} \frac{b}{a}(a - \|\omega\|), \|\omega\| \le a \\ 0, \quad \|\omega\| > a \end{cases}$ Evaluate auto correlation function.	**BTL -5**	Evaluating

6.	Find the mean-square value of the Processes whose power spectral density is $\frac{\omega^2+2}{\omega^4+13\omega^2+36}$.	**BTL -2**	Understanding
7.	If the cross correlation of two processes {X(t)} and {Y(t)} is $R_{XY}(t,t+\tau)=\frac{AB}{2}\{sin\omega_0\tau + cos\omega_0(2t+\tau)\}$, where A,B and ω are constants. Point out the cross power spectrum.	**BTL -4**	Analyzing
8.	Consider the random process X(t) = Ycosωt, t ≥ 0, where ω is a constant and Y is a uniform random variable over (0 , 1) . Identify auto correlation function R_{xx}(t, s) of X(t)and auto covariance C_{xx} (t , s) of X(t).	**BTL -1**	Remembering
9.	State and prove Wiener Khinchine theorem	**BTL -1**	Remembering
10.	If $Y(t) = X(t+a) - X(t-a)$, Examine $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau+2a) - R_{XX}(\tau-2a)$. Hence examine $S_{YY}(\omega) = 4sin^2 a\omega S_{XX}(\omega)$.	**BTL -4**	Analyzing
11(a).	Show that the Random Process$X(t) = A\sin(\omega t + \phi)$, where A and ω are constants, $\emptyset$ is a Random variable uniformly distributed in $(0, 2\pi)$.Find the autocorrelation function of the process.	**BTL -3**	Applying
11(b).	Two Random Process $\{X(t)\}\&\{Y(t)\}$are given by $X(t) = A\cos(\omega t + \theta), Y(t) = A\sin(\omega t + \theta)$ where A and ω are constants and θ is a uniform random variable over 0 to 2π. Find the cross – correlation function.	**BTL -5**	Evaluating
12(a).	Find the power spectral density function whose auto correlation function is given by $R_{XX}(\tau) = \frac{A^2}{2}\cos(\omega_0\tau)$.	**BTL -3**	Applying
12(b)	Given the power density spectrum $S_{XX}(\omega) = \frac{157+12\,\omega^2}{(\omega^2+16)(\omega^2+9)}$.Find the auto correlation function.	**BTL -3**	Applying
13.	Show that the power spectrum of the Auto correlation function $e^{-\alpha|\tau|}[1+\alpha|\tau|]$ is$\frac{4\alpha^3}{(\alpha^2+\omega^2)^2}$.	**BTL -3**	Applying
14.	If $\{x(t)\}\&\{y(t)\}$ are two random processes with auto correlation functions $R_{xx}(\tau)$ and $R_{yy}(\tau)$ respectively and jointly WSS, then prove that $\lvert R_{xy}(\tau)\rvert \le \sqrt{R_{xx}(0)R_{yy}(0)}$. Establish any two properties of autocorrelation function $R_{xx}(\tau)$.	**BTL -6**	Creating

UNIT 5- LINEAR SYSTEMS WITH RANDOM INPUTS

Linear time invariant system – System transfer function – Linear systems with random inputs – Auto correlation and cross correlation functions of input and output.

PART-A

Q. No.	Question	BT Level	Competence
1.	Define a linear system with random input	**BTL -1**	Remembering
2.	State White Noise.	**BTL -1**	Remembering
3.	Define Band –Limited white noise.	**BTL -1**	Remembering
4.	Define system weighting function.	**BTL -1**	Remembering
5.	Define a system when is it called memory less system.	**BTL -1**	Remembering
6.	State stable system.	**BTL -1**	Remembering
7.	Give an example for a linear system.	**BTL -2**	Understanding
8.	Give any two properties of a linear time invariant system.	**BTL -2**	Understanding
9.	Give the properties of a linear system.	**BTL -2**	Understanding

10.	Give the relation between input and out put of a linear time invariant system.	**BTL -2**	Understanding
11.	Show that Y(t) = t X(t) is linear.	**BTL -3**	Applying
12.	Find the autocorrelation function of the white noise.	**BTL -3**	Applying
13.	A wide sense stationary noise process N(t) has an autocorrelation function $R_{NN}(t) = P e^{-3\|\tau\|}$, $-\infty<\tau<\infty$, with P as a constant. Calculate its power density spectrum.	**BTL -3**	Applying
14.	If {X (t)}& {Y(t) } in the system Y(t) = $\int_{-\infty}^{\infty} h(u)X(t-u)du$ are WSS process explains how the auto correlation function related.	**BTL -4**	Analyzing
15.	Define a system when is it called linear system?	**BTL -1**	Remembering
16.	If the input of a linear filter is a Gaussian random process, comment about the output random process.	**BTL -4**	Analyzing
17.	If Y (t) is the output of an linear time – invariant system with impulse response h(t) then Evaluate the cross correlation of the input function X (t) and output function Y(t).	**BTL -5**	Evaluating
18.	Evaluate the system Transfer function ,if a Linear Time Invariant system has an impulse function H(t) = $\begin{cases} \frac{1}{2c}, & \|t\| \le c \\ 0, & \|t\| \ge c \end{cases}$	**BTL -5**	Evaluating
19.	State any two properties of cross power density spectrum.	**BTL -6**	Creating
20.	What is unit impulse response of a system? Why is it so called?	**BTL -6**	Creating
	PART – B		
1.	If the input to a time- invariant, stable linear system is a WSS process, Examine that the output will also be a WSS process.	**BTL -1**	Remembering
2(a).	Identify the output power density spectrum and output correlation function for a system $h(t) = e^{-t}, t \ge 0$, for an input power density system $\frac{h_0}{2}, -\infty < f < \infty$.	**BTL -1**	Remembering
2(b).	Let Y(t)= X(t)+N(t) be a wide sense stationary process where X(t) is the actual signal and N(t)is the zero mean noise process with variance σ_N^2, and independent of X(t). Estimate the power spectral density of Y(t).	**BTL -2**	Understanding
3.	Show that $S_{yy}(\omega) = \|H(\omega)\|^2 S_{xx}(\omega)$ where $S_{xx}(\omega)$ and $S_{yy}(\omega)$ are the power spectral density functions of the input X(t) , output Y(t) and H(ω) is the system transfer function.	**BTL -3**	Applying
4.	Assume a random process X(t) is given as input to a system with transfer function $H(\omega) = 1$ for $-\omega_0 < \omega < \omega_0$. If the autocorrelation function of the input process is $\frac{N_0}{2}\delta(\tau)$, Point out the autocorrelation function of the output process.	**BTL -4**	Analyzing
5.	Let X(t) be a stationary process with mean 0 and autocorrelation function $e^{-2\|\tau\|}$. If X(t) isthe input to a linear system and Y(t) is the output process , Calculate (i) E[Y(t)] (ii) $S_{YY}(\omega)$ and (iii) $R_{YY}(\|\tau\|)$, if the system function $H(\omega) = \frac{1}{\omega+2i}$.	**BTL -3**	Applying
6.	A wide sense stationary random process {X(t)} with autocorrelation $R_{XX}(\tau) = Ae^{-a\|\tau\|}$, where A and a are positive constants, is applied to the input of a linear transmission input system with impulse response $h(t) = e^{-bt}\omega(t)$, Where b is a real positive constant. Give the autocorrelation of the output Y(t) of the system.	**BTL -2**	Understanding

7.	A linear system is described by the impulse response $h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$. Assume an input process whose auto correlation function is B$\delta(\tau)$. Point out the mean and the autocorrelation function of the output function.	**BTL -4**	Analyzing
8(a).	If $\{N(t)\}$ is a band limited white noise centered at a carrier frequency ω_0 such that $S_{NN}(\omega) = \begin{cases} \frac{N_o}{2}, & for\ \lvert\omega - \omega_0\rvert < \omega_B \\ 0, & elsewhere \end{cases}$ $Identify\ the\ auto\ correlation\ function\ of\ \{N(t)\}.$	**BTL -1**	Remembering
8(b).	Consider a Gaussian white noise of zero mean and power spectral density $\frac{N_0}{2}$ applied to a low pass filter whose transfer function is $H(f) = \frac{1}{1+i2\pi fRC}$. Evaluate the auto correlation function.	**BTL -5**	Evaluating
9.	Let X(t) be the input voltage to a circuit system and Y(t) be the output voltage.If X(t)is a stationary random process with mean 0 and autocorrelation function $R_{XX}(\tau) = e^{-\alpha\lvert\tau\rvert}$. Identify (i) E[Y(t)] (ii) $S_{XX}(\omega)$ and (iii) the spectral density of Y(t) if the power transfer function $H(\omega)=\frac{R}{R+iL\omega}$.	**BTL -1**	Remembering
10(a).	A random process X(t) is the input to a linear system whose impulse function is h(t) = $2e^{-t}$, $t \geq 0$ .The auto correlation function of the process is $R_{XX}(\tau) = e^{-2\lvert\tau\rvert}$, Identify the power spectral density of the output process Y(t).	**BTL -1**	Remembering
10(b).	Analyze the mean of the output of a linear system is given by $\mu_Y = H(0)\mu_X$ where X(t) is WSS.	**BTL -4**	Analyzing
11.	Find the power spectral density of the random telegraph signal.	**BTL -3**	Applying
12(a).	If $\{x(t)\}$ is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$, then we have the following i. $R_{YY}(\tau) = R_{XX}(\tau) * h(\tau) * h(-\tau)$ ii. $S_{XY}(\omega) = S_{XX}(\omega)H(\omega)$ iii. $S_{YY}(\omega) = S_{XX}(\omega)\lvert H(\omega)\rvert^2$	**BTL -6**	Creating
12(b).	If $x(t)$ is the input voltage to a circuit and $y(t)$is the output voltage, $\{X(t)\}$ is a stationary random process with $E(X) = 0$ and $R_{XX}(\tau) = e^{-2\lvert\tau\rvert}$. Find $E(Y), S_{XX}(\omega)\ and\ S_{YY}(\omega)$, if the system function is given by $H(\omega) = \frac{1}{\omega^2+2^2}$.	**BTL -1**	Remembering
13.	If $Y(t) = A\cos(\omega_o t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in (-π , π) and{N(t)} is a band limited Gaussian whit noise with a power spectral density $S_{NN}(\omega) = \begin{cases} \frac{N_o}{2}, & for\ \lvert\omega - \omega_0\rvert < \omega_B \\ 0, & elsewhere \end{cases}$ Calculate the power spectral density of {Y(t)}. Assume that N(t) and θ are independent.	**BTL -3**	Applying
14.	If $X(t)$ is the input and $Y(t)$is the output of the system. The autocorrelation of $X(t)$is $R_{XX}(\tau) = 3.\delta(\tau)$. Find the power spectral density, autocorrelation function and mean-square value of the output $Y(t)$ with $H(\omega) = \frac{1}{6+j\omega}$.	**BTL -3**	Applying

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