SRM VALLIAMMAI ENGEINEERING COLLEGE

S.R.M.NAGAR, KATTANKULATHUR-603 203

DEPARTMENT OF MATHEMATICS

QUESTION BANK

SUBJECT : **MA8551 / ALGEBRA AND NUMBER THEORY**
SEMESTER / YEAR : **V Semester / III year**

UNIT I - GROUPS AND RINGS

Groups : Definition - Properties - Homomorphism - Isomorphism - Cyclic groups - Cosets - Lagrange's theorem. Rings: Definition - Sub rings - Integral domain - Field - Integer modulo n - Ring homomorphism

PART-A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Define group and State any two properties of a group.	BTL -1	Remembering
2.	Prove that identity element in a group is unique.	BTL -4	Analyzing
3.	Find the idempotent elements of $G = \{1, -1, i, -i\}$ under the binary operation multiplication	BTL -2	Understanding
4.	Let Z be a group of integers with binary operation * defined by $a * b = a + b - 2$ for all $a, b \in Z$. Find the identity element of the group $\langle Z, * \rangle$.	BTL -2	Understanding
5.	Prove that if G is abelian group, then for all $a, b \in G, (a * b)^2 = a^2 * b^2$	BTL -4	Analyzing
6.	Show that every cyclic group is abelian.	BTL -2	Understanding
7.	If a is a generator of a cyclic group$G$, then show that a^{-1} is also a generator of G.	BTL -6	Creating
8.	Show that $(Z_5, +_5)$ is a cyclic group.	BTL -6	Creating
9.	Find the left cosets of $\{[0], [3]\}$ in the addition modulo group $(Z_6, +_6)$.	BTL -3	Applying
10.	State Lagrange's theorem.	BTL -1	Remembering
11.	Prove that the order of an element a of a group G is the same as that of its inverse (a^{-1})	BTL -1	Remembering
12.	Define a ring and give an example	BTL -1	Remembering
13.	Define ring homomorphism and Ring Isomorphism	BTL -1	Remembering
14.	Define sub ring with example	BTL -1	Remembering
15.	Define ideal with example	BTL -1	Remembering
16.	In Z_{72} , find 25^{-1}	BTL -2	Understanding

17.	Find x such that $25x \equiv 3mod72$	**BTL -2**	Understanding
18.	Give an example of a ring which is not a field.	**BTL -6**	Creating
19.	Define integral domain and give an example.	**BTL -4**	Analyzing
20.	Find $n > 1$ such that $401 \equiv 323(modn)$	**BTL -2**	Understanding
	PART – B		
1.(a)	Show that group homomorphism preserves identity, inverse, and sub group.	**BTL -1**	Remembering
1. (b)	Prove that, if (G,*) is a finite cyclic group generated by an element $a\epsilon G$ and is of order n then $a^n = e$ so that $G = \{a, a^2, \ldots a^n(= e)\}$. Also n is the least positive integer for which $a^n = e$.	**BTL -2**	Understanding
2. (a)	Prove that the intersection of two subgroups of a group G is again a subgroup of G	**BTL -2**	Understanding
2.(b)	Let G be a group and $a \in G$. Let $f: G \to G$ be given by $f(x) = axa^{-1}, \forall x \in G$. Prove that f is an isomorphism of G onto G	**BTL -2**	Understanding
3. (a)	Show that M_2, the set of all $2X2$ non singular matrices over R is a group under usual matrix multiplication. Is it abelian?	**BTL -6**	Creating
3.(b)	Show that the union of two subgroups of a group G is again a subgroup of G if and only if one is contained in the other.	**BTL -4**	Analyzing
4. (a)	State and prove Lagrange's theorem	**BTL -4**	Analyzing
4.(b)	In any ring $(R, +, .)$ (a) The zero element of R is unique (b) The additive inverse of each element is unique	**BTL -5**	Evaluating
5. (a)	Prove that the set $Z_4 = \{0,1,2,3\}$ is a commutative ring with respct ot the binary operation $+_4$ *and* $\times_4$	**BTL -1**	Remembering
5.(b)	Determine whether $H_1 = \{0, 5, 10\}$ *and* $H_2 = \{0, 4, 8, 12\}$ are subgroups of Z_{15}.	**BTL -5**	Evaluating
6.(a)	Discuss Ring and Fields with suitable examples.	**BTL -4**	Analyzing
6.(b)	State and prove the fundamental theorem of group homomorphism	**BTL -1**	Remembering
7. (a)	Find the left cosets of the subgroup $H = \{[0], [3]\}$ of the group [Z_6, $+_6$].	**BTL -5**	Evaluating
7. (b)	If G is a group of prime order, then G has no proper subgroups.	**BTL -6**	Creating
8. (a)	Prove that the necessary and sufficient condition for a non-empty subset H of a group $(G,*)$to be a subgroup is $a, b \in H \Rightarrow a * b^{-1} \in H$	**BTL -1**	Remembering
8.(b)	If $(F, +, .)$ is a filed then it is an integral domain	**BTL -4**	Analyzing
9. (a)	Prove that every subgroup of a cyclic group is cyclic.	**BTL -5**	Evaluating
9.(b)	If $(G,*)$ is an abelian group and if $\forall\, a, b \in G$. Show that $(a * b)^n = a^n * b^n, for\ every\ integer\ n$	**BTL -5**	Evaluating
10.(a)	Define a cyclic group. Prove that any group of prime order is cyclic.	**BTL -6**	Creating
10.(b)	Let f and g be the permutations of the elements of $\{1, 2, 3, 4, 5\}$ Given $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ then find $gf^2g^{-1} and\ g^{-1}fgf^{-1}$.	**BTL -2**	Understanding

11. (a)	Prove that in a group G the equations $a * x = b$ and $y * a = b$ have unique solutions for the unknowns x and y as $x = a^{-1} * b, y = b * a^{-1}$ when $a, b \in G$.	**BTL -3**	Applying
11. (b)	Prove that the set of all matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ forms an abelian group with respect to matrix multiplication.	**BTL -1**	Remembering
12. (a)	Find $[777]^{-1}$ *in the ring* Z_{1009}.	**BTL -2**	Understanding
12. (b)	Show that $(Q^+, *)$ is an abelian group where $*$ is defined as $a * b = ab/2, \forall a, b \in Q^+$	**BTL -1**	Remembering
13. (a)	Show that the group $(\{1, 2, 3, 4\}, X_5)$ is cyclic.	**BTL -6**	Creating
13.(b)	Prove that $G = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$ forms an abelian group under matrix multiplication.	**BTL -2**	Understanding
14. (a)	Prove that the set $\{1, -1, i, -i\}$ is a finite abelian group with respect to the multiplication of complex numbers.	**BTL -1**	Remembering
14. (b)	Prove that Z_n is a field if and only if n is prime.	**BTL -1**	Remembering

UNIT II - FINITE FIELDS AND POLYNOMIALS

Rings - Polynomial rings - Irreducible polynomials over finite fields - Factorization of polynomials over finite fields.

PART- A

Q.No.	**Question**	**Bloom's Taxonomy Level**	**Domain**
1	Define polynomial.	**BTL -1**	Remembering
2	Determine whether $x^2 + 1$ is an irreducible polynomial over the field $\{0,1\}$.	**BTL-3**	Applying
3	Does the set $F = \{0,1,2,3)$ form a filed with respect to addition modulo 4 and multiplication modulo 4? Why?	**BTL-4**	Analyzing
4	Define irreducible polynomial.	**BTL -1**	Remembering
5	Show that $x^2 + x + 1$ is irreducible over Z_5.	**BTL-4**	Analyzing
6	Find the roots for the function $f(x) = x^2 + 3x + 2 \in \mathbb{Z}_6[x]$.	**BTL-2**	Understanding
7	What are the roots of $f(x) = x^2 - 6x + 9 \in \mathbb{R}[x]$.	**BTL-2**	Understanding
8	Find the roots for the function $f(x) = x^2 - 2$ *in* $R[x]$ *and* $Q[x]$.	**BTL-2**	Understanding
9	Obtain reducible polynomial of degree six with no roots in $\mathbb{Z}_2$.	**BTL-5**	Evaluating
10	How many polynomials in $\mathbb{Z}_5$ has degree 3?	**BTL-4**	Analyzing
11	How many polynomials in $\mathbb{Z}_7$ has degree 5?	**BTL-4**	Analyzing
12	What is the remainder when $f(x) = x^7 - 6x^5 + 4x^4 - x^2 + 3x - 7 \in \mathbb{Q}[x]$ is divided by $x - 2$.	**BTL-3**	Applying
13	Define root of a polynomial.	**BTL -1**	Remembering

14	State division algorithm for polynomials.	**BTL -1**	Remembering
15	State Remainder theorem.	**BTL -1**	Remembering
16	If $f(x) = 7x^4 + 4x^3 + 3x^2 + x + 4$ & $g(x) = 3x^3 + 5x^2 + 6x + 1$, $f(x), g(x) \in Z_7[x]$, then find $f(x) + g(x)$ & $\deg(f(x) + g(x))$.	**BTL-2**	Understanding
17	Define characteristics of a field.	**BTL -1**	Remembering
18	State Euclidean algorithm.	**BTL -1**	Remembering
19	Show that $x^2 + 1$ is irreducible in $\mathbb{Z}_3[x]$.	**BTL-3**	Applying
20	Determine all polynomials of degree 2 in $\mathbb{Z}_2[x]$.	**BTL-5**	Evaluating
	PART-B		
1.	Let $\mathbb{R}[x]$ be a polynomial ring, then Prove the following (a) If $\mathbb{R}$ is commutative then $\mathbb{R}[x]$ is commutative. (b) If $\mathbb{R}$ is a ring with unity then $\mathbb{R}[x]$ is a ring with unity. (c) $\mathbb{R}[x]$ is an integral domain if and only if $\mathbb{R}$ is an integral domain.	**BTL-3**	Applying
2. a)	If F is a field and $f(x) \in F[x]$ has degree ≥ 1 , then prove that $f(x)$ has at most n roots in F.	**BTL-3**	Applying
2. b)	If $f(x) = 3x^5 - 8x^4 + x^3 - x^2 + 4x - 7, g(x) = x + 9$ and $f(x), g(x) \in \mathbb{Z}[x]$, find the remainder when $f(x)$ is divided by $g(x)$.	**BTL-2**	Understanding
3	If $\mathbb{R}$ is a ring then prove that $(\mathbb{R}[x], +, \ .)$ is a ring called a polynomial ring over $\mathbb{R}$.	**BTL-3**	Applying
4. a)	Let $(\mathbb{R}, +, .)$ be a commutative ring with unity u. Then $\mathbb{R}$ is an integral domain iff for all $f(x), g(x) \in \mathbb{R}[x]$, if neither $f(x)$ nor $g(x)$ is the zero polynomial, then prove that degree of $f(x)g(x) = degree f(x) + degree g(x)$.	**BTL-3**	Applying
4. b)	Find the remainder when $g(x) = 7x^3 - 2x^2 + 5x - 2$ is divided by $f(x) = x - 3$.	**BTL-2**	Understanding
5. a)	Find all roots of $f(x) = x^2 + 4x$ if $f(x) \in Z_{12}$.	**BTL-2**	Understanding
5. b)	If $g(x) = x^5 - 2x^2 + 5x - 3$ & $f(x) = x^4 - 5x^3 + 7x$ Find $q(x), r(x)$ $such\ that$ $g(x) = f(x)q(x) + r(x)$.	**BTL-2**	Understanding
6. a)	Give an example of polynomial $f(x) \in F(x)$, where $f(x)$ has degree 8 and degree 6, it is reducible but it has no real roots.	**BTL-2**	Understanding
6. b)	Discuss whether $x^4 - 2$ is reducible over $\mathbb{Q}, \mathbb{R}, \mathbb{C}$.	**BTL-4**	Analyzing
7. a)	State and Prove Factor Theorem.	**BTL-3**	Applying
7. b)	Determine whether the given polynomial is irreducible or not? $f(x) = x^2 + x + 1$ over Z_3, Z_5, Z_7	**BTL-5**	Evaluating
8.	Show that: A finite field F has order p^t where p is a prime $t \in z^+$.	**BTL-3**	Applying

9. a)	Find the remainder when $g(x) = 6x^4 + 4x^3 + 5x^2 + 3x + 1$ is divided by $f(x) = 3x^2 + 4x + 2$ over polynomials in $\mathbb{Z}_7[x]$.	**BTL-2**	Understanding
9. b)	Let $(F, +, .)$ be a field. If Char (F) > 0, then prove that Char (F) must be Prime.	**BTL-3**	Applying
10. a)	Detemine whether the following polynomial is irreducible or not? $f(x) = x^2 + 3x - 1$ in $\mathbb{R}[x]$, $\mathbb{Q}[x]$, $\mathbb{C}[x]$.	**BTL-5**	Evaluating
10. b)	If $f(x) = x^3 + 5x^2 + 2x + 6 \in \mathbb{Z}_7[x]$, then write $f(x)$ as a product of first degree polynomials.	**BTL-2**	Understanding
11. a)	Write $f(x) = (2x^2 + 1)(5x^3 - 5x + 3)(4x - 3) \in Z_7[x]$ as a product of the unit and three Monic polynomial.	**BTL-2**	Understanding
11. b)	State and Prove Remainder Theorem.	**BTL-3**	Applying
12. a)	If $f(x) = 4x^2 + 1, g(x) = 2x + 3, f(x), g(x) \in Z_8[x]$. Then show that $\deg f(x)g(x) = deg f(x) + deg g(x)$.	**BTL-3**	Applying
12. b)	If $f(x) = 2x^4 + 5x^2 + 2, g(x) = 6x^2 + 4$, then determine $q(x) and\ r(x) in\ \mathbb{Z}_7[x]$, where $f(x)$ is divided by $g(x)$.	**BTL-5**	Evaluating
13. a)	Discuss whether $x^4 + x^3 + 1$ is reducible over Z_2.	**BTL-4**	Analyzing
13. b)	Find the g.c.d of $x^4 + x^3 + 2x^2 + x + 1$ and $x^3 - 1$ over Q.	**BTL-2**	Understanding
14. a)	Check whether $f(x) = x^4 + x^3 + x^2 + x + 1 \in Z_2[x]$ is irreducible or not?	**BTL-4**	Analyzing
14. b)	Find four distinct linear polynomials $g(x), h(x), s(x), t(x) \in Z_{12}[x]$ so that $f(x) = g(x)h(x) = s(x)t(x)$.	**BTL-2**	Understanding

UNIT III - DIVISIBILITY THEORY AND CANONICAL DECOMPOSITIONS

Division algorithm – Base - b representations – Number patterns – Prime and composite numbers – GCD – Euclidean algorithm – Fundamental theorem of arithmetic – LCM.

PART –A

1	State divisible algorithm	**BTL-1**	Remembering
2	State pigeon hole principle	**BTL-1**	Remembering
3	State principle of inclusion and exclusion	**BTL-1**	Remembering
4	Find the number of positive integers ≤ 2076 that are divisible by 19	**BTL-2**	Understanding
5	Find the number of positive integers ≤ 3076 that are not divisible by 17	**BTL-2**	Understanding
6	Express $(10110)_2$ in base 10 and express $(1076)_{10}$ in base two	**BTL-4**	Analyzing
7	Express $(1776)_8$ in base 10 and express $(676)_{10}$ as octagonal	**BTL-4**	Analyzing
8	Express $(1976)_{16}$ in base 10 and express $(2076)_{10}$ as hexadecimal	**BTL-4**	Analyzing

9	Find the six consecutive integers that are composite	**BTL-2**	Understanding
10	Express (12,15,21) as a linear combination of 12,15,and 21	**BTL-4**	Analyzing
11	Prove that the product of any two integers of the form 4n+1 is also the same form	**BTL-3**	Applying
12	Use canonical decomposition to Evaluate the GCD of 168 and 180	**BTL-5**	Evaluating
13	Use canonical decomposition to evaluate LCM of 1050 and 2574	**BTL-5**	Evaluating
14	Find the canonical decomposition of 2520	**BTL-2**	Understanding
15	Find the prime factorization of 420, 135, 1925	**BTL-2**	Understanding
16	Using (252,360) construct [252,360]	**BTL-6**	Creating
17	Using recursion evaluate [24,28,36,40]	**BTL-5**	Evaluating
18	Using recursion evaluate (18,30,60,75,132)	**BTL-5**	Evaluating
19	Find the GCD (414,662) using Euclidean algorithm	**BTL-2**	Understanding
20	Find the LCM (120.500)	**BTL-2**	Understanding
PART -B			
1. a)	If $a, b, c \in Z$ then (i) $a/a\ , for\ all\ a \neq 0\ \in Z$ (ii) $a/b\ \ and\ b/c\ \ then\ a/c\ , \forall\ a, b \neq 0, c \neq 0\ \ \in Z$ (iii) $a/b\ then\ \ a/b\,c, \forall a \neq 0, b \in Z$ $(iv)\ a/b\ \ and\ a/c\ \ then\ a/(xb + yc)\ , \forall\ x, y \in Z, a \neq 0 \in Z$	**BTL-1**	Remembering
1. b)	Find the number of positive integers ≤ 2076 divisible by neither 4 nor 5	**BTL-2**	Understanding
2. a)	State and Prove Euclidean algorithm	**BTL-1**	Remembering
2. b)	Find the number of positive integers ≤ 3000 divisible by 3, 5 or 7	**BTL-2**	Understanding
3. a)	Prove that the GCD of two positive integers a and b is a linear combination of a and b	**BTL-3**	Applying
3. b)	Find the number of positive integers in the range 1976 through 3776 that are divisible by 13 and not divisible by 17	**BTL-2**	Understanding
4. a)	Prove that (i) If p is a prime and p/ab then p/a or p/b (ii) If p is a prime and $p/a_1a_2a_3 \cdots a_n$, where $a_{1,}a_{2,}a_{3,} \cdots , a_n$ are positive integers then p/a_i for some i, $1 \leq i \leq n$	**BTL-3**	Applying
4. b)	Find the number of integers from 1 to 250 that are divisible by any of the integers 2,3,5,7	**BTL-2**	Understanding
5. a)	Prove that $(a, a - b) = 1$ if and only if $(a, b)\ = 1$	**BTL-3**	Applying
5. b)	Prove by induction that $2n^3 + 3n^2 + n$ is divisible by 6 for all integers $n \geq 0$	**BTL-3**	Applying

6.	State and prove Fundamental Theorem of Arithmetic.	**BTL-1**	Remembering
7. a)	Prove that every integer $n \geq 2$ has a prime factor.	**BTL-3**	Applying
7. b)	Use Euclidean algorithm to find the GCD of $(1819, 3587)$. Also express the GCD as a linear combination of the given numbers	**BTL-4**	Analyzing
8. a)	Prove that: (Euclid) There are infinitely many primes.	**BTL-3**	Applying
8. b)	Evaluate $(625,1000)$ by using canonical decomposition	**BTL-5**	Evaluating
9. a)	Prove that there are infinitely many primes of the form $4n + 3$	**BTL-3**	Applying
9. b)	Use Euclidean algorithm to find the GCD of $(12345,54321)$. Also express the GCD as a linear combination of the given numbers	**BTL-4**	Analyzing
10. a)	Prove that every composite number n has prime factor $\leq [\sqrt{n}]$	**BTL-3**	Applying
10. b)	Use Euclidean algorithm to find the GCD of (2076,1776). Also express the GCD as a linear combination of the given numbers	**BTL-4**	Analyzing
11. a)	Prove that for every positive integer n there are n consecutive integers that are composite numbers	**BTL-3**	Applying
11. b)	Use Euclidean algorithm to find the GCD of (4076,1024). Also express the GCD as a linear combination of the given numbers	**BTL-4**	Analyzing
12. a)	Construct the canonical decomposition of 23!	**BTL-6**	Creating
12. b)	Use Euclidean algorithm to find the GCD of (3076,1976). Also express the GCD as a linear combination of the given numbers	**BTL-4**	Analyzing
13. a)	Prove that two positive integers a and b are relatively prime iff $[a, b] = ab$	**BTL-3**	Applying
13. b)	Use Euclidean algorithm to evaluate the GCD of (2024,1024). Also express the GCD as a linear combination of the given numbers	**BTL-5**	Evaluating
14. a)	If a and b are positive integers then prove that $[a, b] = \frac{a.b}{(a,b)}$	**BTL-3**	Applying
14. b)	For any positive integer , Prove that $8n + 3$and $5n + 2$ are relatively prime	**BTL-3**	Applying

UNIT –IV DIOPHANTINE EQUATIONS AND CONGRUENCES

Linear Diophantine equations – Congruence‘s – Linear Congruence‘s - Applications: Divisibility tests - Modular exponentiation-Chinese remainder theorem – 2 x 2 linear systems

PART – A

1	Define linear Diophantine Equation in two variables.	**BTL-1**	Remembering
2	Prove that $a \equiv b(mod\ m)$ if and only if $a = b + km\ for\ some\ integer\ k.$	**BTL-3**	Applying
3	Find the least residue of 23 modulo 5, -3 modulo 5.	**BTL-2**	Understanding
4	Define complete sets of residues modulo m.	**BTL-1**	Remembering

5	Find the Congruence classes modulo 5.	**BTL-2**	Understanding
6	Find the remainder when $1! + 2! + \cdots + 100!$ is divided by 15.	**BTL-2**	Understanding
7	Find the remainder when $1! + 2! + \cdots + 1000!$ is divided by 10.	**BTL-2**	Understanding
8	Find the remainder when $1! + 2! + \cdots + 1000!$ is divided by 12.	**BTL-2**	Understanding
9	If $a \equiv b(mod\ m)$, then prove that $a^n \equiv b^n(mod\ m)$ for any positive integer n.	**BTL-4**	Analyzing
10	If $ac \equiv bc(mod\ m)$ and $(c\ ,m) = 1\ , then\ \ a \equiv b(mod\ m)$.	**BTL-3**	Applying
11	If $ac \equiv bc(mod\ m)$ and $(c\ ,m) = d\ , then\ \ a \equiv b\left(mod\ \frac{m}{d}\right)$.	**BTL-3**	Applying
12	Determine if the congruence $8x \equiv 10(mod\ 6)$ is solvable.	**BTL-4**	Analyzing
13	Determine if the congruence $2x \equiv 3(mod\ 4)$ is solvable.	**BTL-4**	Analyzing
14	Determine if the congruence $4x \equiv 7(mod\ 5)$ is solvable.	**BTL-4**	Analyzing
15	Using Chinese Remainder theorem, determine whether the linear system is solvable $x \equiv 7(mod\ 9)\ ,\ \ x \equiv 11(mod\ 12)$.	**BTL-2**	Understanding
16	Using Chinese Remainder theorem, determine whether the linear system is solvable $x \equiv 3(mod\ 6)\ ,\ \ x \equiv 5(mod\ 8)$.	**BTL-2**	Understanding
17	Using Chinese Remainder theorem, determine whether the linear system is solvable $x \equiv 2(mod\ 10)\ ,\ \ x \equiv 7(mod\ 15)$.	**BTL-2**	Understanding
18	Define 2x2 linear system.	**BTL-1**	Remembering
19	State Chinese Remainder Theorem.	**BTL-1**	Remembering
20	Define Congruence and incongruence solution.	**BTL-1**	Remembering
	PART -B		
1. a)	Solve Mahaveer's puzzle	**BTL-3**	Applying
1. b)	Find the remainder when 16^{53} is divided by 7	**BTL-2**	Understanding
2. a)	Solve Hundred fowls puzzle	**BTL-3**	Applying
2. b)	Find the remainder when $(n^2 + n + 41)$ is divided by 12	**BTL-2**	Understanding
3. a)	Determine if each linear Diophantine equation is solvable $(i)12x + 16y = 18\ (ii)28x + 91y = 119$, $(iii)1776x + 1976y = 4152\ (iv)1076x + 2076y = 1155$	**BTL -5**	Evaluating
3. b)	Solve $x \equiv 1(mod\ 3)\ ,\ \ x \equiv 2(mod\ 5), x \equiv 3(mod\ 7)$.	**BTL-3**	Applying
4. a)	Prove that the linear Diophantine equation $ax + by = c$ is solvable iff $d/c\ where\ d = (a,b)$	**BTL-3**	Applying
4. b)	Find the least positive integer that leaves the remainder 3 when divided by 7,4 when divided by 9 and 8 when divided by 11	**BTL-2**	Understanding
5. a)	Prove that the relation $'\equiv'$ (congruence) is an equivalence relation	**BTL-3**	Applying

5. b)	Solve $x \equiv 1(mod\ 3)$, $x \equiv 3(mod\ 4), x \equiv 4(mod\ 7), x \equiv 7(mod\ 11)$.	**BTL-3**	Applying
6. a)	Prove that $a \equiv b(modm)$ iff a and b leave the same remainder when divided by m.	**BTL-3**	Applying
6. b)	Solve $3x + 13y \equiv 8(mod\ 55), 5x + 21y \equiv 34(mod\ 55)$P	**BTL-3**	Applying
7. a)	Prove that the integer r is the remainder when a is divided by m iff $a \equiv r(modm)where\ 0 \leq r < m$	**BTL-3**	Applying
7. b)	Solve $2x + 3y \equiv 4(mod\ 13),\ 3x + 4y \equiv 5(mod\ 13)$	**BTL-3**	Applying
8. a)	Show that every integer is congruent to exactly one of the least residues $0,1,2, \cdots, (m-1) modulo\ m$	**BTL-4**	Analyzing
8. b)	A fruit basket contains apples and oranges. Each apple cost 65 Rs. Each orange cost 45Rs. For a total of 810 Rs. Find the minimum possible numbers of apple in the basket.	**BTL-2**	Understanding
9. a)	Verify that whether the number of prime of the form $4n + 3$ be expressed as the sum of two squares	**BTL-4**	Analyzing
9. b)	Compute the remainder when 3^{247} is divided by 25	**BTL-4**	Analyzing
10. a)	Prove that , let $a \equiv b(modm) and\ c \equiv d(modm)$ then $(i) a + c \equiv b + d(modm)$ $(ii) ac \equiv bd(modm)$	**BTL-4**	Analyzing
10. b)	Compute the remainder when 5^{31} is divided by 12	**BTL-4**	Analyzing
11. a)	Verify that whether the number of integer of the form $8n + 7$ be expressed as the sum of three squares	**BTL-4**	Analyzing
11. b)	Compute the remainder when 23^{1001} is divided by 17	**BTL-4**	Analyzing
12. a)	If n is any integer then show that $(i) n^2 + n \equiv 0(mod2)$ $(ii) n^4 + 2n^3 + n^2 \equiv 0(mod4)(iii) 2n^3 + 3n^2 + n \equiv 0(mod6)$	**BTL-2**	Understanding
12. b)	(i) Prove that an integer is divisible by 10 iff its unit digit iszero. (ii) Prove that an integer is divisible by 5 iff its unit digit is zero or five (iii) Prove that an integer 'n' is divisible by 2^i iff the number formed by the last 'i' digits in 'n' is divisible by 2^i	**BTL-2**	Understanding
13	State and prove Chinese remainder theorem	**BTL-1**	Remembering
14. a)	The linear congruence $ax \equiv b(modm)$ is solvable if and only if d/b, where $d = (a, m)$. If d/b, then it has d incongruent solutions	**BTL-2**	Understanding
14. b)	(i) Prove that an integer is divisible by 3 if the sum of its digits id divisible by 3. (ii) Prove that an integer is divisible by 9 if the sum of its digits id divisible by 9.	**BTL-2**	Understanding

UNIT-V . CLASSICAL THEOREMS AND MULTIPLICATIVE FUNCTIONS Wilson's theorem – Fermat's little theorem – Euler's theorem – Euler's Phi functions – Tau and Sigma functions			
PART -A			
1	State Wilsons Theorem	**BTL-1**	Remembering
2	State Fermat's Theorem	**BTL-1**	Remembering
3	State Euler's Theorem	**BTL-1**	Remembering
4	Define Euler Phi Function	**BTL-1**	Remembering
5	Define Tau Function	**BTL-1**	Remembering
6	Define Sigma Function	**BTL-1**	Remembering
7	Show that 11 is self invertible.	**BTL-4**	Analyzing
8	Evaluate $\frac{(np)!}{n!\,p^n}$ if n=46, p=5	**BTL-5**	Evaluating
9	How many primes are there of the form m! + 1 when m≤ 100?	**BTL-2**	Understanding
10	Find the self invertible least residue modulo each prime 7 and 19	**BTL-2**	Understanding
11	Solve $x^2 \equiv 1\ mod(6)$	**BTL-2**	Understanding
12	Find the least residues of $1,2,\dots,p-1\ modulo\ 7$	**BTL-3**	Applying
13	Let p be a prime number and a any integer such that $p \times a$ then prove that a^{p-2} is an inverse of a modulo p	**BTL-3**	Applying
14	Evaluate the inverse of 12 modulo 7	**BTL-5**	Evaluating
15	Solve the linear congruence of 12x ≡ 6 (mod 7)	**BTL-2**	Understanding
16	Solve the linear congruence of 24x ≡ 11 (mod 17)	**BTL-2**	Understanding
17	Create $\emptyset\,(11)\ and\ \emptyset(18)$	**BTL-6**	Creating
18	Solve the linear congruence of 35x ≡ 47 (mod 24)	**BTL-2**	Understanding
19	Define Multiplication Theorem	**BTL-1**	Remembering
20	Compute $\emptyset$ (15,625)	**BTL-5**	Evaluating
PART-B			
1. a)	Prove that a positive integer a is invertible modulo p iff $a \equiv \pm 1\ (mod\ P)$	**BTL-3**	Applying
1. b)	Find the remainder when 24^{1947} is divided by 17	**BTL-2**	Understanding
2. a)	State and Prove Wilson's Theorem.	**BTL-1**	Remembering
2. b)	Find the remainder when 7^{1001} is divided by 17	**BTL-2**	Understanding
3. a)	State and Prove Fermat's Little Theorem	**BTL-1**	Remembering

3. b)	Find the remainder when 15^{1976} is divided by 23	**BTL-2**	Understanding
4.a)	Prove that $\frac{(np)!}{n!\,p^n} \equiv (-1)^n \ (mod\ p)$, where p is a prime number and n any positive integer	**BTL-1**	Remembering
4. b)	Verify that $\sum_{\frac{d}{n}} \emptyset(d) = n$ for n=28	**BTL-4**	Analyzing
5.a)	Let p be a prime and a any integer such that $p \nmid a$ then prove that the solution of the linear congruence $ax \equiv b\ (mod\ p)$ is given by $x \equiv a^{p-2}\, b\ (modp)$	**BTL-4**	Analyzing
5. b)	Let p be a prime and e any positive integer then prove that $\emptyset(pe) = p^e - p^{e-1}$ hence find $\emptyset\ (6125)$	**BTL-4**	Analyzing
6.a)	Prove that if n is a positive integer such that $(n-1)! \equiv -1\ (mod\ n)$, then n is prime.	**BTL-4**	Analyzing
6.b)	Evaluate $\tau\ (n) and\ \sigma\ (n)$ for each $n = 43, 1560, 44982\ and\ 496$	**BTL-5**	Evaluating
7. a)	Let p be a prime and a any positive integer then show that $a^p = a\ (mod\ P)$	**BTL-3**	Applying
7. b)	Create the remainder when 245^{1040} is divided by 18 and the remainder when 7^{1020} is divided by 15	**BTL-6**	Creating
8. a)	State and Prove Euler's Theorem.	**BTL-1**	Remembering
8. b)	Evaluate the remainder when 199^{2020} is divided by 28 and the remainder when 79^{1776} is divided by 24	**BTL-5**	Evaluating
9. a)	Let m and n be relatively prime positive integers and r any integers then prove that the integer $r, m + r, 2m + r, \ldots,$ $(n-1)m + r$ are congruent modulo n to $0,1,2, \ldots, n-1$ in some order.	**BTL-3**	Applying
9. b)	Using Euler's Theorem, evaluate the ones digit in the decimal value of each (i) 17^{666} $(ii) 23^{7777}$	**BTL5**	Evaluating
10. a)	State and prove fundamental theorem for multiplicative function	**BTL-1**	Remembering
10. b)	Let n be a positive integer with canonical decomposition $n = {p_1}^{e_1}{p_2}^{e_2}, \ldots, {p_k}^{e_k}$ then show that $\tau(n) = (e_1 + 1)(e_2 + 1), \ldots, (e_k + 1)$ $\sigma(n) = \frac{{p_1}^{e_1+1}-1}{p_1-1} \cdot \frac{{p_2}^{e_2+1}-1}{p_1-1}, \ldots, \frac{{p_k}^{e_1+1}-1}{p_1-1}$. Also compute $\tau(6120)\ \ and\ \sigma\ (6120)$.	**BTL-4**	Analyzing
11. a)	Show that the Euler's φ function is multiplicative function. Hence find $\varphi\ (1976)$	**BTL-4**	Analyzing
11. b)	Let p be a prime and e any positive integer then prove that	**BTL-3**	Applying

	$\tau(p^e) = e + 1$ and $\sigma(p^e) = \frac{p^{e+1}-1}{p-1}$. Also find $\tau(49)$		
12. a)	Let n be a positive integer then Prove that $\sum_{\frac{d}{n}} \emptyset(d) = n$	**BTL-3**	Applying
12. b)	Show that the Tau and Sigma functions are multiplicative function. Also compute $\tau(36)$ and $\sigma(36)$	**BTL-4**	Analyzing
13. a)	Let $n = p_1^{e_1} p_2^{e_2}, \ldots, p_k^{e_k}$ be the canonical decomposition of a positive integer n. Then Prove that $\emptyset(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right), \ldots, \left(1-\frac{1}{p_k}\right)$, Hence find $\emptyset(666)$ and $\emptyset(1976)$	**BTL-3**	Applying
13. b)	Let $p_1, p_2, \ldots, p_k$ be any distinct primes, a any positive integer and $l = [p_1 - 1, p_2 - 1, \ldots, p_k - 1]$ Then prove that $a^{l+1} \equiv a \ (mod p_1, p_2, \ldots, p_k)$	**BTL-3**	Applying
14. a)	If f is a multiplicative function. Then show that $F(n) = \sum_d F(d)$ is also multiplicative.	**BTL-4**	Analyzing
14. b)	Let p be a prime and a is any integer such that $p \nmid a$ then prove that the least residues of the integers $a, 2a, 3a, \ldots, (p-1)a$ modulo p are permutation of integers $1,2,3, \ldots, p-1$.	**BTL-3**	Applying

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