ME8594 DYNAMICS OF MACHINES

UNIT I FORCE ANALYSIS

Dynamic force analysis – Inertia force and Inertia torque– D Alembert's principle – Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod– Bearing loads – Crank shaft torque – Turning moment diagrams –Fly Wheels – Flywheels of punching presses- Dynamics of Cam- follower mechanism.

UNIT II BALANCING

Static and dynamic balancing – Balancing of rotating masses – Balancing a single cylinder engine – Balancing of Multi-cylinder inline, V-engines – Partial balancing in engines – Balancing of linkages – Balancing machines-Field balancing of discs and rotors.

UNIT III FREE VIBRATION

Basic features of vibratory systems – Degrees of freedom – single degree of freedom – Free vibration– Equations of motion – Natural frequency – Types of Damping – Damped vibration– Torsional vibration of shaft – Critical speeds of shafts – Torsional vibration – Two and three rotor torsional systems.

UNIT IV FORCED VIBRATION

Response of one degree freedom systems to periodic forcing – Harmonic disturbances –Disturbance caused by unbalance – Support motion –transmissibility – Vibration isolation vibration measurement.

UNIT V MECHANISM FOR CONTROL

Governors – Types – Centrifugal governors – Gravity controlled and spring controlled centrifugal governors – Characteristics – Effect of friction – Controlling force curves. Gyroscopes –Gyroscopic forces and torques – Gyroscopic stabilization – Gyroscopic effects in Automobiles, ships and airplanes.

TOTAL : 60 PERIODS

TEXT BOOKS:

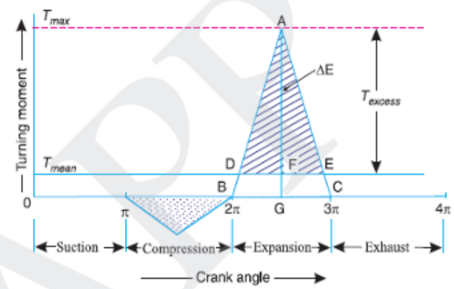
1. F. B. Sayyad, "Dynamics of Machinery", McMillan Publishers India Ltd., Tech-Max Educational resources, 2011.
2. Rattan, S.S, "Theory of Machines", 4th Edition, Tata McGraw-Hill, 2014.
3. Uicker, J.J., Pennock G.R and Shigley, J.E., "Theory of Machines and Mechanisms", 4th Edition, Oxford University Press, 2014.\

REFERENCES:

1. Cleghorn. W. L, "Mechanisms of Machines", Oxford University Press, 2014
2. Ghosh. A and Mallick, A.K., "Theory of Mechanisms and Machines", 3rd Edition Affiliated East-West Pvt. Ltd., New Delhi, 2006.
3. Khurmi, R.S.,"Theory of Machines", 14th Edition, S Chand Publications, 2005.
4. Rao.J.S. and Dukkipati.R.V. "Mechanisms and Machine Theory", Wiley-Eastern Ltd., New Delhi, 1992.
5. Robert L. Norton, "Kinematics and Dynamics of Machinery", Tata McGraw-Hill, 2009.
6. V.Ramamurthi, "Mechanics of Machines", Narosa Publishing House, 2002.

PART - B
Unit – I Force Analysis

1. **A single cylinder, single acting, four stroke gas engine develops 20 kW at 300 r.p.m. The work done by the gases during the expansion stroke is three times the work done on the gases during the compression stroke, the work done during the suction and exhaust strokes being negligible. If the total fluctuation of speed is not to exceed ± 2 per cent of the mean speed and the turning moment diagram during compression and expansion is assumed to be triangular in shape. Find the moment of inertia of the flywheel.**



<u>**Given**</u>: P=20kW;N=300rpm; $\omega=2\Pi N/60=31.442$rad/s

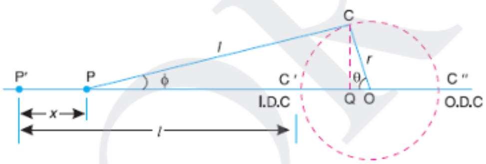
since the total fluctuation of speed (ω_1-ω_2) is not to exceed ±2 percent of the mean speed (ω),

Therefore, $\omega_1-\omega_2=4\%\omega$; $C_S=\omega_1-\omega_2/\omega=0.04$

n=N/2=300/2=150; ∴Workdone/cycle=P*60/n=8000N-m

<u>Max. Fluctuation Energy $\Delta E=I\omega^2 C_S=255.2$Kg-m^2</u>

2. (a) **Derive the equation of forces on the reciprocating parts of an engine, neglecting weight of the connecting rod.**



Motion of a crank and connecting rod of a reciprocating steam engine.

Let
l = Length of connecting rod between the centres,
r = Radius of crank or crank pin circle,
ϕ = Inclination of connecting rod to the line of stroke PO, and
n = Ratio of length of connecting rod to the radius of crank = l/r.

Velocity of the piston: $v_{PO} = v_P = \omega r\left(\sin\theta + \dfrac{\sin 2\theta}{2n}\right)$

Acceleration of the piston: $a_P = \omega . r\left[\cos\theta + \dfrac{\cos 2\theta}{n}\right]\times\omega = \omega^2 . r\left[\cos\theta + \dfrac{\cos 2\theta}{n}\right]$

∴Accelerating force or inertia force of the reciprocating parts: $F_I = m_R . a_R = m_R . \omega^2 . r\left(\cos\theta + \dfrac{\cos 2\theta}{n}\right)$

(b) **What is turning moment diagram and draw it's for four stroke IC engine?**
A turning moment diagram for a 4 stroke cycle IC engine is shown in figure. We know that in a 4 stroke cycle IC engine, there is one working stroke after the crank has turned through two revolutions i.e.720^0 (or 4Π radians)

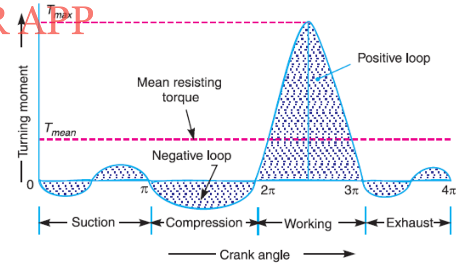


Fig. Turning moment diagram for a four stroke cycle internal combustion engine.

3. **The torque delivered by a two-stroke engine is represented by T = (1000+300 sin 2θ – 500 cos 2 θ) N-m.where θ is the angle turned by the crank from the inner dead centre. the engine speed is 250 rpm.the mass of the flywheel is 400 kg and radius of gyration 400 mm. determine (i) the power developed,(ii) the total percentage fluctuation of speed,(iii) the angular acceleration of flywheel when the crank has rotated through an angle of 60° from the inner dead centre. (iv) The maximum angular acceleration and retardation of the flywheel.**

   Given: T = (1000+300 sin 2θ – 500 cos 2 θ) N-m: N=250rpm; m=400kg; k=0.4m; $\theta=60^0$; ω=2π*250/60=26.18rad/s;

   Hint: T_{mean}=Workdone per cycle/Crank angle per rev=976.13N-m; $\theta_1=29.51^0 \theta_2=119.5^0$;
   Power Developed P= $T_{mean*}\omega$ =25.56kW; Max. Fluctuation of Energy=$\Delta E=mk^2\omega^2 c_s$;
   c_s=1.33%; Angular acceleration'α' when $\theta=60^0$; $\alpha=7.965rad/s^2$; when $2\theta=149.04^0$; $T-T_{mean=}$583N-m; 2θ=329.04; $T-T_{mean=}$583N-m=-583.1N-m; α_{Max} or α_{Min}= $T-T_{mean}/T=9.11rad/s^2$

4. **A vertical petrol engine 150 mm diameter and 200 mm stroke has a connecting rod 350 mm long. The mass of the piston is 1.6 kg and the engine speed is 1800 rpm.on the expansion stroke with crank angle 30° from the top dead centre, the gas pressure is 750 kN/m². Determine the net thrust on the engine.**

   Given: D=150=0.15m; L=200mm=0.2m; Radius of the crank; r=L/2=0.1m; Connecting rod length l=0.35m
   m=1.6kg; p=750kN/m²
   N=1800rpm; Angular velocity ω=188.49rad/s, Crank angle $\theta=30^0$; Gas pressure p=750kN/m²
   Piston Force F_p=p*area of the piston= 13253.59N; F_i=Inertia force= -(mass of piston)*Accleration of piston
   Net thrust for vertical engine is given by $F=F_P+F_i\pm W$= 7534.396N

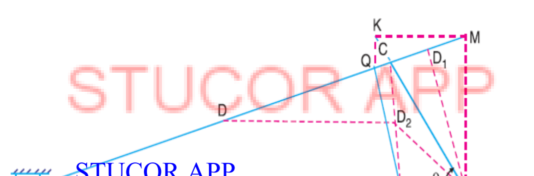
5. **In a slider crank mechanism, the length of the crank and connecting rod are150 mm and 600 mm respectively. The crank position is 60° from inner dead centre. The crank shaft speed is 450 r.p.m. clockwise. Determine 1. Velocity and acceleration of the slider, 2. Velocity and acceleration of point D on the connecting rod which is 150 mm from crank pin C, and 3. angular velocity and angular acceleration of the connecting rod.**

   Given : OC = 150 mm = 0.15m ; PC = 600 mm = 0.6 m ; CD = 150 mm = 0.15 m ; N = 450 r.p.m. or ω = 2π × 450/60 = 47.13 rad/s

   Hint:

   1. Velocity & Accceleration of the slider:
      $V_p=\omega*OM=6.834m/s$; $a_p=\omega^2*NO=124.4m/s^2$

   l acce leration of point D on the connecting rod:
      $V_D=\omega*OD_1=6.834m/s$; $a_D=\omega$ $*OD_2$ =266.55m/s



3. Angular velocity and angular acceleration of the connecting rod:
$\omega_{PC}= V_{pc}/PC=6.127rad/s$; $\alpha_{PC}=a^t_{PC}/PC=481.27rad/s^2$

6. **A vertical double acting steam engine develops 75 kW at 250 rpm.the maximum fluctuation of energy is 30 percent of the work done per stroke. The maximum and minimum speeds are not to vary more than 1% on either side of the mean speed. Find the mass of the flywheel required if the radius of gyration is 0.6 meters.**
<u>Given</u>: Power=75kW; N=250rpm; ω_1-ω_2=1% ω=0.01ω; k=0.6
C_s= ω_1-$\omega_{2/}$ ω=0.01; Maximum fluctuation of energy,Δ E = Work done per cycle × C_E
We know that Δ E =mk^2 ω^2 C_s; Mass of the flywheel=547kg

7. **The lengths of crank and connecting rod of a horizontal reciprocating engine are 200 mm and 1 meter respectively. The crank is rotating at 400 rpm.when the crank has turned through 30° from the inner dead centre. The difference of pressure between cover and piston rod is 0.4 N/mm^2.if the mass of the reciprocating parts is 100 kg and cylinder bore is 0.4 meters, then calculate: (i) inertia force, (ii) force on piston, (iii) piston effort, (iv) thrust on the sides of the cylinder walls, (v) thrust in the connecting rod, and (vi) crank effort.**
<u>Given</u>: r=0.2m; l=1m;N=400rpm or ω=41.88rad/s: $\theta=30^0$; p_1-p_2=0.4N/mm^2; m=100kg; D=0.4m;n=l/r=5

(i)Inertia Force(F_i): F_i=-m*a [-ve sign is due to the fact that inertia force opposes the accelerating force]

a=acceleration of the piston whish is given as:

$$= r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) \text{ where } \theta = 30°$$

=338.86m/s^2 ; Therefore F_i=-m*a=-33886N

(ii) Force of the piston: F_P=p*area of the piston; p=p_1-p_2=0.4 N/mm^2; F_P=50265N

(iii) Piston effort: F=F_i+F_P=16379N

(iv) Thrust on the sides of the cylinder walls:

The thrust on the sides of cylinder walls(or normal reaction), F_N is given as; F_N=Ftanϕ

ϕ=Angle made by connecting rod with line of stroke, the value of ϕ in terms of θ is given as

$$\sin\phi = \frac{r}{l}\sin\theta = 0.1$$

$$\phi = \sin^{-1} 0.1 = 5.739°$$

$$F_N = F\tan\phi = 16379 \times \tan 5.739°$$

$$= 16379 \times 0.1005 = 1646.1 \text{ N.}$$

(v) Thrust in the connecting rod:F_C=F/cosϕ=16461.5N

(vi) Crank effort (F_T) or Tangential Force: F_T=$F_C$sin(θ+ϕ)=9615N

8. **The radius of gyration of a fly wheel is 1 meter and the fluctuation of speed is not to exceed 1% of the mean speed of the flywheel. If the mass of the flywheel is 3340 kg and the steam engine develops 150 kW at 135 rpm, then find (i) maximum fluctuation of energy and (ii) coefficient of fluctuation of energy.**

uctuat ion of speed=1% of mean speed or ω_2- ω_1=1% of ω or ω_2- $\omega_{1/}$ ω=0.01 or coefficient of fluctuation of speed, K_S=0.01; m=3340kg; P=150Kw; N=135rpm; ω=14.137rad/s

$\Delta E = mk^2 \times \omega^2 \times K_s$ =6675.13Nm; K_E=Max.Fluctuation of energy/Workdone per cycle

Workdone per cycle=$T_{mean}*\theta= T_{mean}*2\pi$; $T_{mean}= P/\omega$=10610.45Nm

Workdone per cycle=10610.45*2π=66667.42Nm/cycle

$$K_e = \frac{\Delta E}{66667.42} = \frac{6675.13}{66667.42} = 0.1001 \approx 0.1$$

9. **(i) Deduce the expression for the inertia force in the reciprocating force neglecting the weight of the connecting rod. Solution: Refer 2(a) Problem in Part-B**

(ii) A vertical petrol engine with cylinder of 150 mm diameter and 200 mm stroke has a connecting rod of 350 mm long. The mass of the piston is 1.6 kg and the engine speed is 1800 rpm.on the expansion stroke with crank angle 30° from TDC, the gas pressure is 750 kPa.Determine the net thrust on the piston.

Given: D=0.15m; r=l/2=0.1m; l=0.35m; m_k=1.6kg; N=1800rpm; $\theta=30^0$; $F_{gas\ Pr}=750*10^3 N/m^2$

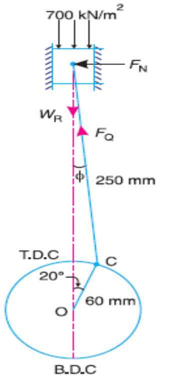
By Analytical Method: Net Thrust on the piston $F_P=F_{gas\ pr}+W_R-F_I$

$$F_I = m_R . a_R = m_R . \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n}\right) = 5736N$$

$F_{gas\ pressure\ force} = F_{gas\ pr\ *\ Area}$ = 13,254N; Weight of piston W_R=15.7N

Net Thrust on the piston F_P=7534N

10. **A single cylinder vertical engine has a bore of 100 mm and a stroke of 120 mm has a connecting rod of 250mm long. The mass of the piston is 1.1kg. The speed is 2000rpm. On the expansion stroke, with a crank at 20° from top dead center, the gas pressure is 700kN/mm². Determine (i) Net force acting on the piston (ii) Resultant load on the gudgeon pin (iii) Thrust on the cylinder walls, and (iv) Speed above which, other things remaining the same, the gudgeon pin load would be reversed in direction.**



Given: D=0.1m; L=0.12m; r=L/2=0.06m; l=0.25m; m_R=1.1kg; N=2000rpm; ω=209.5rad/s; $\theta=20^0$; p=700kN/m²

1. Net force on the piston: F_L=5.5kN; 3.Net Force=2256.8N
2. Inertia Force $F_I = m_R . \omega^2 . r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = 3254$ N
3. Resultant load on the gudgeon pin: $F_Q=F_P/\cos\phi$=2265N
4. Thrust on the cylinder walls: $F_N= F_P \tan\phi$=185.5N
5. Speed, above which the gudgeon pin load would be reversed in the direction

   ∴Corresponding speed in rpm….N_1>2606rpm

11. **The slider of a slider crank mechanism is subjected to a force of 3000 N.the crank is at 60° position. The length of the crank and connecting rod are 100 mm and 300 mm respectively. Determine the driving torque on the crank.**

Given: F=3000N; r=length of the crank=0.1m; l=length of the connecting rod=0.3m; n=l/r=0.3/0.1=3; $\sin\phi=\sin\theta/n$;

Torque on the crank $T = F_c \times r\sin(\theta + \phi)$

12. **In a reciprocating engine mechanism, if the crank and the connecting rod are 300 mm and 1**

m long respectively and the crank rotates at a constant speed of 200 rpm.determine analytically: (i) the crank angle at which the maximum velocity occurs, and (ii) the maximum velocity of the piston (iii) derive the relevant equations.

Given : r = 300 mm = 0.3 m ; l = 1 m ; N = 200 r.p.m. or ω= 2 Π× 200/60 = 20.95 rad/s

1. Crank angle at which the maximum velocity occurs, n=l/r=3.33

$$v_p = \omega r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \quad \frac{dv_p}{d\theta} = 0 \quad \theta = 75^\circ$$

2. Maximum velocity of the piston; $v_{p(max)}$=6.54m/s

13. A vertical double acting steam engine has a cylinder 300 mm diameter and 450 mm stroke and runs at 200 rpm.the reciprocating parts has a mass of 225 kg and the piston rod is 50 mm diameter. The connecting rod is 1.2 m long. When the crank has turned through 125° from the top dead centre the steam pressure above the piston is 30 kN/m² and below the piston is 1.45 kN/m².Calculate: (i) Crank pin effort, and (ii) Effective turning moment on the crank shaft.

Given: D=0.3m; L=0.45m; r=L/2=0.225m; N=200rpm; ω=20.95rad/s; m_R=225kg; d=0.05m; l=1.2m; θ=125^0

p_1=30kN/m²; p_2=1.5kN/m²

∴ Force on the piston due to steam pressure,

$$F_L = p_1.A_1 - p_2(A_1 - a) = 2121 - 103 = 2018 \text{ N}$$

inertia force on the reciprocating parts,

$$F_1 = m_R.\omega^2.r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = -14\ 172 \text{ N}$$

$$\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 125^\circ}{5.33} = \frac{0.8191}{5.33} = 0.1537 \quad \phi = 8.84^\circ$$

Effective turning moment of the crankshaft $T = \dfrac{F_P \times \sin(\theta+\phi)}{\cos\phi} \times r = 3021.6$ N-m

14. The turning moment diagram for a petrol engine is drawn to the following scales : Turning moment, 1 mm = 5 N-m crank angle, 1 mm = 1°. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm². The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. Determine the coefficient of fluctuation of speed when the engine runs at 1800 r.p.m.

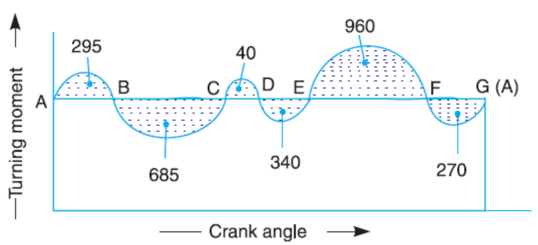
Given: m=36kg; k=0.15m; N=1800rpm; ω=188.52rad/s

ΔE = Maximum Energy – Minimum Energy=985mm²

$\Delta E = mk^2\omega^2 C_S$

$985 = 36*0.1^2*188.52^2* C_S$

C_S= 0.3%



od of an I.C. engine has a mass of 2 kg and the distance between the centre of

gudgeon pin along the line of centres. The radius of gyration about an axis through the C.G. perpendicular to the plane of rotation is 110 mm. Find the equivalent dynamical system if only one of the masses is located at gudgeon pin. If the connecting rod is replaced by two masses, one at the gudgeon pin and the other at the crank pin and the angular acceleration of the rod is 23 000 rad/s² clockwise. Determine the correction couple applied to the system to reduce it to a dynamically equivalent system.

<u>Given</u>: $m=2kg$; $l=0.25m$; $l_1=0.1m$; $k_G=0.11m$; $\alpha=23000 rad/s^2$

(a) Equivalent Dynamical System: $m_1=l_2m/l_1+l_2=1.1kg$; $m_2=l_1m/l_1+l_2=0.9kg$

(b) Correction Couple: $T'=m(k_1^2-k_g^2)\ \alpha=133.4N\text{-}m$

Unit – II Balancing of Masses

1. **Four masses A, B, C and D revolve at equal radii and are equally spaced along a shaft. The mass B is 7 kg and the radii of C and D make angles of 90° and 240° respectively with the radius of B. Find the magnitude of the masses A, C and D and the angular position of A so that the system may be completely balanced. (Nov / Dec 2012)**

Key : Given: m_A = 7 kg ; C = 90° with B; D = 240° with B

Plane	Mass (m) kg	Radius (r) m	Cent.force /ω^2 (m.r) kg-m	Distance from R.P (l) m	Couple /ω^2 (m.r.l) kg-m^2
A	m_A	r_A	$m_A\ r_A$	1	$m_A\ r_A$
B	m_B	r_B	$m_B\ r_B$	1	0
C	m_C	r_C	$m_C\ r_C$	1	$m_C\ r_C$
D	m_D	r_D	$m_D\ r_D$	1	$m_D\ r_D$

Ans. 5 kg ; 6 kg ; 4.67 kg ; 205° from mass B in anticlockwise direction

2. **The following particulars relate to an outside cylinder of uncoupled locomotive: Revolving mass per cylinder = 300kg; Reciprocating mass per cylinder = 450 kg; Length of each crank = 350 mm; Distance between wheels = 1.6 m; Distance between cylinder centers = 1.9 m; Diameter of driving wheels = 2m; Radius of balancing mass = 0.8m; angle between the cranks = 90°. If the whole of the revolving mass and 2/3 of the reciprocating masses are to be balanced in planes of driving wheels, determine;**
Magnitude and direction of the balance masses, speed at which the wheel will lift off the rails when the load on each driving wheel is 35 KN, and Swaying couple at speed arrived in (ii) above. (Dec 2013)

<u>Key:</u>

parts to be balanced per cylinder at the crank pin, $m = m_B = m_C = m_1 + c.m_2$

Plane	Mass (m) kg	Radius (r) m	Cent.force /ω^2 (m.r) kg-m	Distance from R.P (l) m	Couple /ω^2 (m.r.l) kg-m^2
A	m_A	r_A	$m_A\ r_A$	1	$m_A\ r_A$
B	m_B	r_B	$m_B\ r_B$	1	0
C	m_C	r_C	$m_C\ r_C$	1	$m_C\ r_C$
D	m_D	r_D	$m_D\ r_D$	1	$m_D\ r_D$

Fluctuation in rail pressure or hammer blow = B.ω□.b

We know that maximum variation of tractive effort $= \pm\sqrt{2}(1-c)m_2.\omega^2.r$

We know that maximum swaying couple = a(1-c)/(2)$^{1/2}$ x m_2 ω□□r

3. **The cranks are 3 cylinder locomotive are set at 120°. The reciprocating masses are 450 kg for the inside cylinder and 390 kg for each outside cylinder. The pitch of the cylinder is 1.2**

960 mm from the inside cylinder. If 40% of the reciprocating masses are to be balanced,

determine: The magnitude and the position of the balancing masses required at a radial distance of 500 mm; and The hammer blow per wheel when the axle rotates at 350 rpm.

Key:

1. Since 40% of the reciprocating masses are to be balanced, therefore mass of the reciprocating
2. parts to be balanced for each outside cylinder, $m_A = m_C = c \times m_O$
3. mass of the reciprocating parts to be balanced for inside cylinder, $m_B = c \times m_1$
4. Table
5. hammer blow = $B.\omega^{\square}.b$

4. **A 4 cylinder engine has the two outer cranks as 120° to each other and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 400mm, 700mm, 700mm and 500mm. Find the reciprocating mass and the relative angular position for each of the inner cranks, if the engine is to be in completely balance. Also find the maximum unbalanced secondary force, if the length of each crank is 350 mm, the length of each connecting rod 1.7m and the engine speed 500 rpm. (Nov / Dec 2012)**

Key: Given : $m_1 = m_4 = 400$ kg ; r = 300 mm = 0.3 m ; l = 1.2 m ; N = 240 r.p.m.

Plane	Mass (m) kg	Radius (r) m	Cent.force $/\omega^2$ (m.r) kg-m	Distance from R.P (l) m	Couple $/\omega^2$ (m.r.l) kg-m^2
A	m_A	r_A	$m_A\ r_A$	1	$m_A\ r_A$
B	m_B	r_B	$m_B\ r_B$	1	0
C	m_C	r_C	$m_C\ r_C$	1	$m_C\ r_C$
D	m_D	r_D	$m_D\ r_D$	1	$m_D\ r_D$

5. **A 4 cylinder vertical engine has cranks 150 mm long. The planes of rotation of first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their respective masses are 50kg, 60kg, and 50 kg respectively. Find the mass of the reciprocating mass for the third cylinder and the relative angular positions of the cranks in order that the engine may be in computer primary balance.**

Key: Given $r_1 = r_2 = r_3 = r_4 = 150$ mm = 0.15 m ; $m_1 = 50$ kg ; $m_2 = 60$ kg ; $m_4 = 50$ kg

Plane	Mass (m) kg	Radius (r) m	Cent.force $/\omega^2$ (m.r) kg-m	Distance from R.P (l) m	Couple $/\omega^2$ (m.r.l) kg-m^2
A	m_A	r_A	$m_A\ r_A$	1	$m_A\ r_A$
B	m_B	r_B	$m_B\ r_B$	1	$m_B\ r_B$
C (R.P)	m_C	r_C	$m_C\ r_C$	0	0
D	m_D	r_D	$m_D\ r_D$	1	$m_D\ r_D$

Ans: $\theta_2 = 160°$, $\theta_4 = 26°$ $m_3 = 60$ kg

6. **A 3 cylinder radial engine driven by a common crank has the cylinders spaced at 120°. The stroke is 125 mm; the length of the connecting rod is 225 mm and the reciprocating mass per cylinder 2 kg. Calculate the primary and secondary forces at crank shaft speed of 1200 rpm. (Dec 2013)**

Key: Given : L = 125 mm ; l = 225 mm; m = 2 kg ; N = 1200 r.p.m.

1. Maximum Primary Force = $3m/2 \times \omega^2 r$
2. Maximum Secondary force = $2m/2(2\ \omega^2)(r/4n)$.

7. **The reciprocating mass per cylinder in a 60° V-twin engine is 1.5 kg. The stroke is 100 mm for each cylinder. If the engine runs at 1800 rpm, determine the maximum and minimum values of the primary forces and find out the corresponding crank position.**

Key: θ= 30°, m = 1.5 kg, l = 100 mm ; N = 1800 r.p.m.

Maximum and minimum values of primary forces = $m/2 \times \omega^2 r\ (9\cos^2\theta+\sin^2\theta)^{1/2}$

$$= \frac{\sqrt{3}}{2}\times\frac{m}{2}\times\omega^2 r$$

Maximum and minimum values of secondary forces

8. **The firing order of a six cylinder, vertical, four stroke, in-line engine is 1-4-2-6-3-5. The piston stroke is 80 mm and length of each connecting rod is 180 mm. the pitch distances between the cylinder centre lines are 80 mm, 80 mm, 120 mm, 80 mm and 80 mm respectively. The reciprocating mass per cylinder is 1.2 kg and the engine speed is 2400 rpm. Determine the out-of-balance primary and secondary forces and couples on the engine taking a plane mid-way between the cylinders 3 and 4 as the reference plane.**

Key: Given : L = 80 mm or r = L / 2 = 40 mm = 0.04 m ; l = 180 mm ; m = 1.2 kg ; N = 2400 r.p.m.

Plane	Mass (m) kg	Radius (r) m	Cent.force $/\omega^2$ (m.r) kg-m	Distance from R.P (l) m	Couple $/\omega^2$ (m.r.l) kg-m^2
1	1.2	0.04	0.04	l1	0.04l1
2	1.2	0.04	0.04	l2	0.04l2
3	1.2	0.04	0.04	l3	0.04l3
4	1.2	0.04	0.04	l4	0.04l4
5	1.2	0.04	0.04	l5	0.04l5
6	1.2	0.04	0.04	l6	0.04l6

Draw force polygon and couple polygon.

UNIT III – FREE VIBRATIONS

PART B

1. **(a) A machine of weighs 18 kg and is supported on springs and dashpots. The total stiffness of the springs is 12 N/mm and damping is 0.2 N/mm/s the system is initially at rest and a velocity of 120 mm/s is imparted to the mass. Determine (1) the displacement and velocity of mass as a function of time (2) the displacement and velocity after 0.4s.**
(b) Describe the types of vibrations with simple sketch. (Dec 2013)

HINT:
(a) Given Data:
Mass m = 18 kg,
Initial Velocity = 120 mm/s = 0.12 m/s
Stiffness s = 12 N/mm = 12000 N/m
Damping coefficient c = 0.2 N/mm/s = 200 N/m/s

Step 1:
Find the Circular Natural frequency ω_n = Sqrt [s/m]
Find the Damped Natural frequency ω_d = Sqrt [$\omega_n^2 - a^2$] where a = c/2m

Step 2:
The General Equation for Displacement of Damped vibration in terms of time is
$X = C e^{-at} \sin(\omega_d t + \theta)$
Intially at rest t =0, Displacement X=0
Substitute X = 0 and t = 0 in the above equation and thus θ = 0,
Hence X becomes
$X = C e^{-at} \sin \omega_d t$

Step 3:

Differentiate above equation to get velocity V

$V = dX/dt = C\,[e^{-at}\omega_d \cos\omega_d t - ae^{-at} \sin \omega_d t]$ .....(differentiate by u.v method)

Now Substitute Initial velocity of V = 0.12 m/s at time t = 0

Hence Find the constant C = 4.76 mm.

Step 4:

a)Displacement and Velocity in terms of t:

Substitute C, a, ω_d in

$X = C\, e^{-at} \sin \omega_d t$ and

$V = C\,[e^{-at}\omega_d \cos\omega_d t - ae^{-at} \sin \omega_d t]$

Step 5:

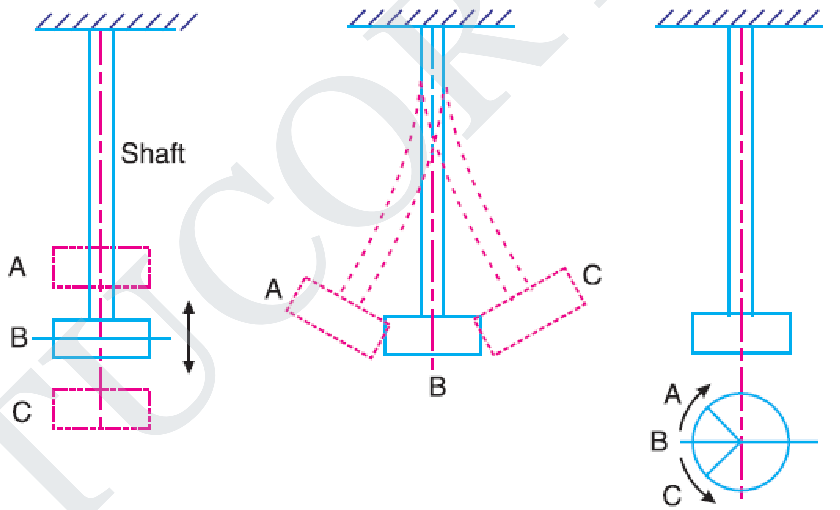
a)Displacement and Velocity after 0.4 seconds:

Substitute t=0.4 sec in above equation to get X and V

b) Types of Free Vibrations

The following three types of free vibrations are important from the subject point of view :

1. Longitudinal vibrations, **2.** Transverse vibrations, and **3.** Torsional vibrations.



B = Mean position ; *A* and *C* = Extreme positions.

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

2. Derive the expression for the natural frequency of free transverse or longitudinal vibrations by using any two methods.

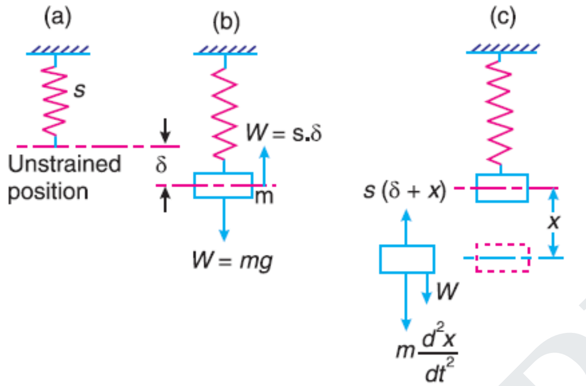
HINT:

1. Equilibrium Method

Consider a constraint (i.e. spring) of negligible mass in an unstrained position, as shown in Figure

Let s = Stiffness of the constraint. It is the force required to produce unit displacement in the direction of vibration. It is usually expressed in N/m.

W = Weight of the body in newtons = m.g,
δ= Static deflection of the spring in metres due to weight W newtons, and
x = Displacement given to the body by the external force, in metres.



Restoring force =W - s (δ + x) =W - s.δ - s.x
= s.δ- s.δ - s. x = -s.x (since W = s.δ) . . . **(i)** . . . (Taking upward force as negative)

And

Accelerating force = Mass × Acceleration

Accelerating force = Mass × Acceleration

$$= m \times \frac{d^2x}{dt^2} \quad \text{. . . (Taking downward force as positive) .}$$

Equating Restoring force and accelerating force

$$m \times \frac{d^2x}{dt^2} = -s.x \quad \text{or} \quad m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0 \qquad \text{---(iii)}$$

$$\frac{d^2x}{dt^2} + \omega^2.x = 0 \qquad \text{----(iv)}$$

Comparing equations *(iii)* and *(iv)*, we have

$$\omega = \sqrt{\frac{s}{m}}$$

$$\therefore \quad \text{Time period,} \quad t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$$

natural frequency,
$$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

Taking the value of g as 9.81 m/s^2 and δ in metres,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{Hz}$$

2. **Energy Method:**

$$\frac{d}{dt}(K.E. + P.E.) = 0$$

$$K.E. = \frac{1}{2}\times m\left(\frac{dx}{dt}\right)^2 \qquad P.E. = \left(\frac{0+s.x}{2}\right)x = \frac{1}{2}\times s.x^2$$

$$\frac{d}{dt}\left[\frac{1}{2}\times m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\times s.x^2\right] = 0$$

$$\frac{1}{2}\times m\times 2\times\frac{dx}{dt}\times\frac{d^2x}{dt^2} + \frac{1}{2}\times s\times 2x\times\frac{dx}{dt} = 0$$

$$m\times\frac{d^2x}{dt^2} + s.x = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} + \frac{s}{m}\times x = 0$$

The time period and the natural frequency may be obtained as discussed in the previous method.

3. **A shaft of 100 mm diameter and 1 m long is fixed at one end and other end carries a flywheel of mass 1 tonne. Taking young's modulus for the shaft material as 200 GN/m^2; find the natural frequency of longitudinal and transverse vibrations.**

HINT:
Step 1:
Determine the cross sectional area of the shaft A = (π/4) d^2
Determine the Moment of Inertia I = (π/64) d^4

Step 2:
Natural Frequency of Longitudinal vibrations
Using the below formula, find f_n – which is natural frequency of longitudinal vibration
Static deflection of cantilever beam

$$\delta = \frac{W.l}{A.E}$$

Natural frequency of longitudinal vibration

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$

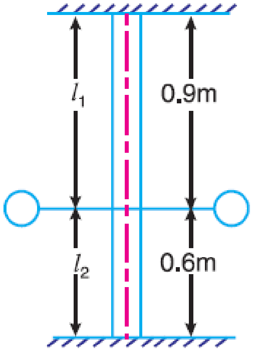
Step 3:
Natural Frequency of Transverse vibrations
Using the below formula, find f_n – which is natural frequency of transverse vibration
Static deflection of cantilever beam for transverse vibrations is

$$\delta = \frac{W.l^3}{3E.I}$$

Natural frequency of transverse vibration

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$

4. **A flywheel is mounted on a vertical shaft as shown in figure. The both ends of the shaft are fixed and its diameter is 50 mm. The flywheel has a mass of 500 kg. Find the natural frequency of free longitudinal and transverse vibrations. Take E = 200GN/m^2.**



HINT
Step 1:
Determine the cross sectional area of the shaft A = $(\pi/4)\, d^2$
Determine the Moment of Inertia I = $(\pi/64)\, d^4$
Step 2:
Let m_1 be the mass of flywheel carried by length l_1 and
$m – m_1$ be the mass of flywheel carried by length l_2
Extension of length l_1 is

$$= \frac{W_1.l_1}{A.E} = \frac{m_1.g.l_1}{A.E}$$

Compression of length 1_2 is

$$= \frac{(W - W_1)\, l_2}{A.E} = \frac{(m - m_1)\, g.l_2}{A.E}$$

Equating Extension length and compression length find m_1.
Step 3:
Static Deflection from extension length l_1 is

$$\delta = \frac{m_1.g.l_1}{A.E}$$

frequ ency using

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$

Step 4:

To determine natural frequency of transverse vibration, find δ using the below formula.

$$\delta = \frac{Wa^3b^3}{3EIl^3}$$ where a and b are the distance of flywheel from ends

Find the natural frequency using

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$

5. A shaft 1.5 m long is supported by two short bearings and carries two wheels each of 50 kg mass. One wheel is situated at the centre of the shaft and other at a distance of 0.4 m from the centre towards right. The shaft is hollow of external diameter 75 mm and inner diameter 37.5 mm. The density of the shaft material is 8000 kg/m³. The young's modulus for the shaft material is 200 GN/m². Find the frequency of free transverse vibration.

Step 1

Determine the Moment of Inertia

$$I = \frac{\pi}{64}\left[(d_1)^4 - (d_2)^4\right]$$

Step 2

Density of given material is 8000 kg/m³

Find mass of shaft per unit length using

$$m_S = \text{Area} \times \text{length} \times \text{density}$$

Step 3

General expression for Static deflection δ [for simply supported beam] due to weight W is given by

$$= \frac{Wa^2b^2}{3EIl}$$

Determine the static deflection due load W_1 and W_2. (Both 50 kg located at different places as specified in the questions) using

$$\delta_1 = \frac{m_1 g a^2 b^2}{3EIl}$$

$$\delta_2 = \frac{m_1 g a^2 b^2}{3EIl}$$

Note: The value of a and b changes for both δ_1 and δ_2 as per the position given in questions

Step 4

General expression for Static deflection δ [for simply supported beam] due to self weight (uniformly

$$\delta_S = \frac{5}{384} \times \frac{wl^4}{EI}$$

Find w by using $w = m_s g$

Step 5
Determine Natural frequency of transverse vibration using

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \dfrac{\delta_S}{1.27}}}$$

6. A Steel bar 25 mm wide and 50 mm deep is freely supported at two points 1m apart and carries a mass of 200 kg in the middle of the bar. Neglecting the mass of the bar, find the frequency of free transverse vibration. If an additional mass of 200 kg is distributed uniformly over the length of the shaft, what will be the frequency of free transverse vibration? Take E = 200 GN/m^2.

Step 1:
Determine the Moment of Inertia using $I = bd^3/12$

Step 2:
Determine the Static Deflection δ [for simply supported beam with pt load at centre] using

$$\delta = \frac{W\,l^3}{48\,EI}$$

Step 3:
Find the natural frequency of transverse vibration neglecting the self weight using

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$

Step 4:
General expression for Static deflection δ [for simply supported beam] due to self weight (uniformly distributed load)

$$\delta_S = \frac{5}{384} \times \frac{wl^4}{EI}$$

Determine w by using $w = m_s g$. [m_s = Area X Length X Density] [Take Density of steel = 7700 kg/m^3

Step 5:
Find the Natural frequency of transverse vibration considering the self weight using

$f_n = 0.4985 / \text{Sqrt}\,[\delta + \delta_s/1.27]$

shaft 15 mm diameter is held in long bearings 100 cm apart and carries at its middle a disc weighing 147.15N. The eccentricity of the centre of gravity of the disc from the

centre of the rotor is 0.3mm. The modulus of elasticity for the shaft material is 19.6 X 10^6 N/cm^2, and the permissible stress is 6867 N/cm^2. Determine :(i) the critical speed of the shaft (ii) The range of speed over which it is unsafe to run the shaft. Neglect the weight of the shaft.

HINT:

Step 1:

Determine the moment of Inertia

$$I = \frac{\pi}{64} \times d^4$$

Step 2:

Critical speed of shaft N_c

Determine Static deflection δ with point load at middle [consider as fixed at both ends – for long bearings]

$$\delta = \frac{Wl^3}{192\,EI}$$

Determine natural frequency f_n as

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$

Critical speed Nc in r.p.s is same as natural frequency in Hz, therefore

Nc = f_n X60 rpm

Step 3:

To determine range of speed:

`

Additional Dynamic Load during shaft rotation can be determined by relations

$$\frac{M}{I} = \frac{\sigma}{y_1} \quad \text{or} \quad M = \frac{\sigma . I}{y_1}$$

Also

$$M = \frac{W_1 . l}{8} = \frac{m_1 . g . l}{8} \quad \text{and} \quad y_1 = \frac{d}{2}$$

σ is the permissible stress (given in the question)

Find m_1 using above relation [Equating both M values]

Additional deflection due to load [W_1= m_1g] is

$$y = \frac{W_1}{W} \times \delta$$

Also

$$y = \frac{\pm e}{\left(\frac{\omega_c}{\omega}\right)^2 - 1} \quad \text{or} \quad \pm\frac{y}{e} = \frac{1}{\left(\frac{N_c}{N}\right)^2 - 1}$$

N_c is known to us, Equate the value of y and find N value [taking positive and negative equations] to get the range of speed N_1 and N_2.

8. A vertical shaft 25 mm diameter and 0.75 m long, is mounted in long bearings and carries a pulley of mass 10 kg midway between the bearings. The centre of the pulley is 0.5 mm from the axis of the shaft. Find (a) the Whirling Speed (b) the bending stress in the shaft, when it is rotating at 1700 rpm

HINT:

Step 1:
Determine the Moment of Inertia of shaft

$$I = \frac{\pi}{64} \times d^4$$

Step 2:
Determine the static deflection δ of the shaft [considered as fixed on both ends – for long bearings]

$$\delta = \frac{Wl^3}{192\,EI}$$

Determine the natural frequency

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$

Natural frequency of transverse vibration in Hz is same as Critical speed in rpm, Hence

$N_c = f_n$ X60 rpm

Step 3:
To find Max Bending Stress if N = 1700 rpm

$$\frac{M}{I} = \frac{\sigma}{y_1} \quad \text{or} \quad M = \frac{\sigma . I}{y_1}$$

$$M = \frac{W_1 . l}{8} \quad \text{and} \quad y_1 = \frac{d}{2}$$

Find W_1 in terms of σ from the above expression.

Step 4:
Determine y using [in terms of σ, as W_1 is in terms of σ]

$$y = \frac{W_1}{W} \times \delta$$

Also find y using

$$y = \frac{\pm e}{\left(\frac{\omega_c}{\omega}\right)^2 - 1} = \frac{\pm e}{\left(\frac{N_c}{N}\right) - 1}$$

[the value of e is the distance of pulley from centre]

Equating both y, we get Bending Stress σ.

9. The disc of a torsional pendulum has a moment of inertia of 600 kg-cm^2 and is immersed in a viscous fluid. The brass shaft attached to it is of 10 cm diameter and 40 cm long. When the pendulum is vibrating, the observed on the same side of the rest position for successive cycles are 9 degree, 6 degree and 4 degree. Determine (a) Logarithmic decrement (b) damping force at unit velocity and (c) periodic time of vibration. Assume for the brass shaft, G = 4.4 X 10^{10} N/m^2. What would the frequency be if the disc is removed from the viscous fluid?

HINT:

Step 1:
Determine the Circular Natural frequency

$$\omega_n = \sqrt{\frac{s}{m}}$$

Step 2:
Determine the Damping Coefficient C using below method
x_1 = 9 deg, x_2= 6 deg and x_3 = 4 deg

Convert x_1,x_2 and x_3 in radians
We know that

$$\frac{x_1}{x_2} = \frac{x_2}{x_3}$$

Also
$[x_1/x_3] = [x_1/x_2][x_2/x_3] = [x_1/x_2]^2$
or
$[x_1/x_2] = [x_1/x_3]^{1/2}$

Also we know that

$$\log_e\left(\frac{x_1}{x_2}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

Where a = c/2m

From the above expression,
Determine the value of 'c'
'c' is the Damping force per unit velocity [in N/m/s]

Determine c_c using $c_c = 2m\omega_n$

Determine Logarithmic Decrement using

$$\delta = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}}$$

Step 4:
Time period $t_p = 1/f_n$

For **Damped vibration $f_n = \omega_d/2\pi$ where ω_d = Sqrt $[\omega_n^2 - a^2]$ and $a = c/2m$**

Determine Periodic time of vibration t_p from the above expression

Step 5:
For Undamped vibration [if disc is removed from viscous fluid]
Determine natural frequency using

$f_n = \omega_n/2\pi$

10. A vibrating system consists of a mass 0f 8 kg, spring of stiffness 5.6 N/mm and a dashpot of damping coefficient of 40 N/m/s. Find (a) damping factor (b) logarithmic decrement (c) ratio of two consecutive amplitudes.

HINT:
Step 1:
a) Determine the critical damping coefficient c_c

$$c_c = 2m.\omega_n = 2m \times \sqrt{\frac{s}{m}}$$

Damping factor = c/c_c [c is given as damping coefficient]

Step 2:
b) Determine the Logarithmic Decrement

$$\delta = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}}$$

Step 3:
Determine the ratio of two consecutive amplitudes

$$\delta = \log_e\left[\frac{x_n}{x_{n+1}}\right] \text{ or } \frac{x_n}{x_{n+1}} = e^{\delta}$$

11. A centrifugal pump is driven through a pair of spur wheels from an oil engine. The pump runs at 4 times the speed of the engine. The shaft from the engine flywheel to the gear is 75 mm diameter and 1.2 m long, while that from the pinion to pump is 50 mm diameter and 400 mm long. The moments of inertia are as follows: flywheel = 1000kg-m^2; pinion = 10kg-

m^2; and pump impeller = 40kg-m^2. Find the natural frequencies of torsional oscillations. Take C = 84 GN/m^2.

HINT:

Step 1:
Determine the mass moment of equivalent rotor B

$I_B' = I_B/G^2$

Additional length of equivalent shaft is

$l_3 = G^2.l_2\ [d_1/d_2]^4$

Total length of equivalent shaft $l = l_1 + l_3$

Step 2:

$l_A I_A = l_B I_B'$

Determine l_A from the above expression

Polar Moment of Inertia for equivalent shaft is

$J = [\pi/32]d_1^4$

Step 3:

Determine the Natural frequency of torsional vibration

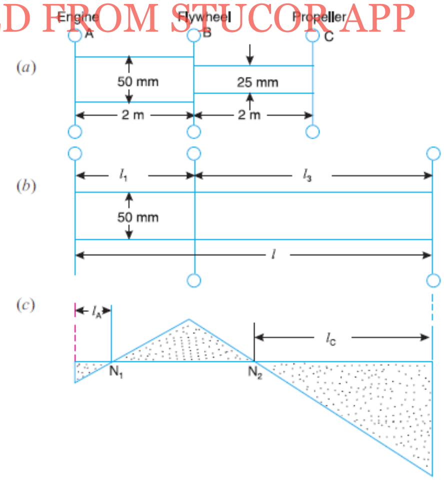
$\mathbf{f_n = [1/2\pi]sqrt[C.J/l_A I_A]}$

12. A 4 cylinder engine and flywheel coupled to a propeller are approximated to a 3 rotor system in which the engine is equivalent to a rotor of moment of inertia 800 kg-m^2, the flywheel to a second rotor of 320 kg-m^2 and the propeller to a third rotor of 20 kg-m^2. The first and second rotors are being connected by a 25 mm diameter and 2m long shaft. Neglecting the inertia of the shaft and taking its modulus of rigidity as 80 GN/m^2, determine; (i) natural frequencies of free torsional vibrations and (ii) the positions of the nodes.

HINT:
Step 1:
Determine the torsionaly equivalent shaft using

$l = l_1 + l_2\ [d1/d2]^4$

Step 2:
Determine l_A and l_C using
$l_A I_A = l_C I_C$
and
$1/l_C I_C = 1/I_B\ [\ 1/(l_1 - l_A) + 1/(l_3 - l_C)\ ]$



Locate l_A and l_C in the drawing as shown, l_A and l_C will have two values [First value for single node and second value of two node system]

Step 3:
Determine the Polar Moment of Inertia J = [π / 32]d^4

Natural frequency of torsional vibration for single node/two node system is

fn1 = [1/2π]. Sqrt [CJ/l_AI_A] {Substituting two diff values of l_A]

Step 4:
Position of Nodes:
From the value of l_C of two node system, the corresponding value of l_C from original system of propeller is

= l_C $[d_2/d_1]^4$, where l_C { Value of two node system of l_C]

13. The mass of a single degree damped vibrating system is 7.5 kg and makes 24 free oscillations in 14 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Determine: 1. stiffness of the spring, 2. logarithmic decrement, and 3. damping factor, i.e. the ratio of the system damping to critical damping.

Step 1:
From 24 oscillations in 14 seconds, determine f_n – Natural frequency
f_n = 24/14

Circular Natural frequency $\omega_n = 2\pi f_n$

From the above, Stiffness of spring can be determined using

$$\omega_n = \sqrt{\frac{s}{m}}$$

[since m and ω_n are known]

x_1 = Initial amplitude,

x_6 = Final amplitude after five oscillations = $0.25\, x_1$

$$\frac{x_1}{x_6} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6} = \left(\frac{x_1}{x_2}\right)^5 \quad \ldots \left[\because \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6}\right]$$

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_6}\right)^{1/5} = \left(\frac{x_1}{0.25\, x_1}\right)^{1/5}$$

Logarithmic Decrement is

$$\delta = \log_e \left(\frac{x_1}{x_2}\right)$$

Step 3:
Damping factor: = c/c_c

Determine critical damping coefficient from below expression

$$c_c = 2m.\omega_n$$

Determine Damping coefficient c from below expression (as δ is known)

$$\delta = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}}$$

Find damping factor using c/c_c

14. Derive an expression for the frequency of free torsional vibrations for a shaft fixed at one end and carrying a load on the free end.

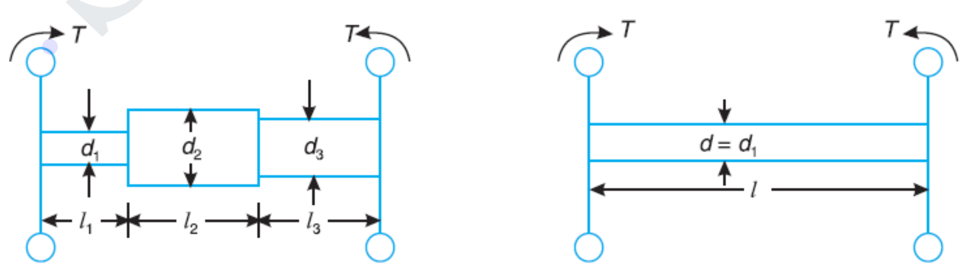
Let

d_1, d_2 and d_3 = Diameters for the lengths l_1, l_2 and l_3 respectively,

$_1$, $_2$ and $_3$ = Angle of twist for the lengths l_1, l_2 and l_3 respectively,

= Total angle of twist, and

J_1, J_2 and J_3 = Polar moment of inertia for the shafts of diameters d_1, d_2 and d_3 respectively.



(a) Shaft of varying diameters. (b) Torsionally equivalent shaft.

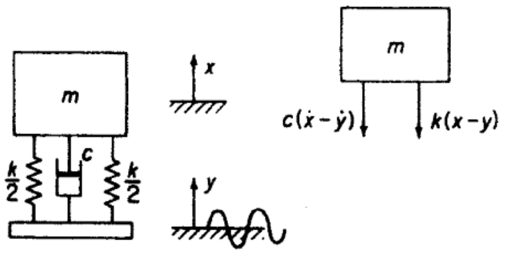
From this concept

T.l/C.J = [T.l_1/C.J_1] + [T.l_2/C.J2] + [T.l3/C.J_3]
Substituting value of J, J1, J2 and J3 in all equation and simplifying we get
Torsional equivalent shaft is

$$l = l_1 + l_2 [d_1/d_2]^4 + l_3 [d_1/d_3]^4$$

Unit – IV Forced Vibration

1. **(a) Discuss the forcing due to support motion.**
   (b) What is meant by magnification factor in case of forced vibrations?
   HINT:
   a) Force due to support motion
   The excitation of at the location of support is commonly known as base excitation



With the motion of the base denoted as y and the motion of the mass relative to the intertial reference frame as x, the differential equation becomes

$$m\ddot{x} = -k(x-y) - c(\dot{x}-\dot{y})$$

Substitute

$$z = x - y$$

into the equations to give

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} = m\omega^2 Y \sin \omega t$$

The solution can be written by

$$Z = \frac{m\omega^2 Y}{\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2}}$$

$$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$$

Steady state amplitude and phase form are given by

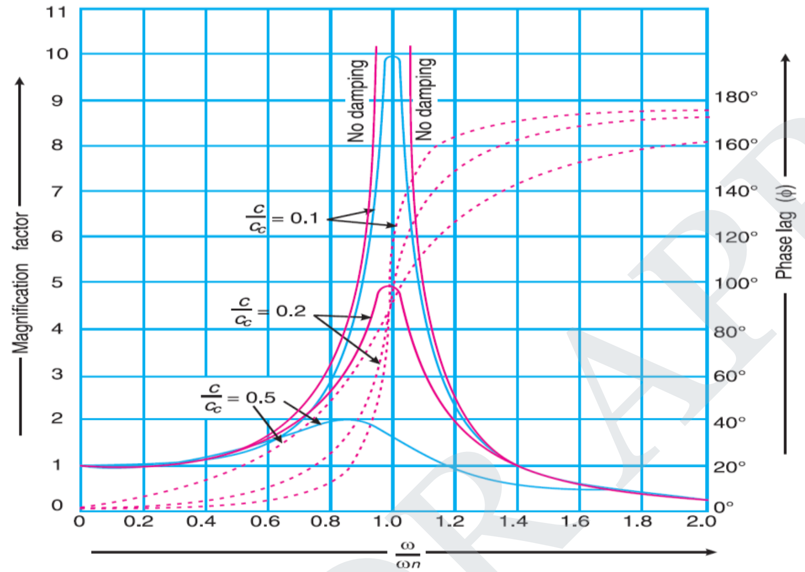
$$\left|\frac{X}{Y}\right| = \sqrt{\frac{k^2 + (c\omega)^2}{\left(k - m\omega^2\right)^2 + (c\omega)^2}}$$

$$\tan \phi = \left(\frac{mc\omega^3}{k\left(k - m\omega^2\right) - (c\omega)^2}\right)$$

b) Magnification factor

It is the ratio of maximum displacement of the forced vibration (x_{max}) to the deflection due to the static force F(xo). We have proved in the previous article that the maximum displacement or the amplitude of forced vibration,

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$



∴ Magnification factor or dynamic magnifier,

$$D = \frac{x_{max}}{x_o} = \frac{1}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}} \quad \text{... } (i)$$

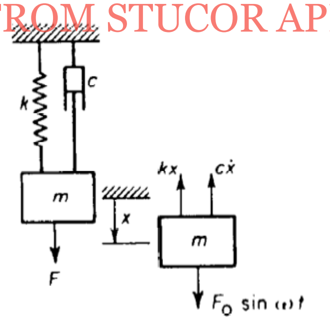
$$= \frac{1}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$$\ldots \left[\because \frac{c.\omega}{s} = \frac{2c.\omega}{2m \times \frac{s}{m}} = \frac{2c.\omega}{2m(\omega_n)^2} = \frac{2c.\omega}{c_c.\omega_n}\right]$$

2. **(a) Derive the relation for the displacement of mass from the equilibrium position of a damped vibration system with harmonic forcing.**
(b) **Define the term vibration isolation**.

HINT:

a) Harmonic forcing



Diff equation is given by

$$m\ddot{x} + c\dot{x} + kx = F_o Sin\omega\ t$$

Solution is

$$x = x_c + x_p$$

Complementary function is

$$x_c = A_2 e^{-\zeta\omega_n t} Sin\left|\sqrt{1-\xi^2}\,\omega_n t + \varphi_2\right|$$

Particular Integral is

$$x_p = XSin(\omega t - \varphi)$$

The complete solution with harmonic function is

$$x = A_2 e^{-\zeta\omega_n t} Sin\left|\sqrt{1-\xi^2}\,\omega_n t + \varphi_2\right| + \frac{X_{st} Sin(\omega t - \varphi)}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\xi(\omega/\omega_n)\right]^2}}$$

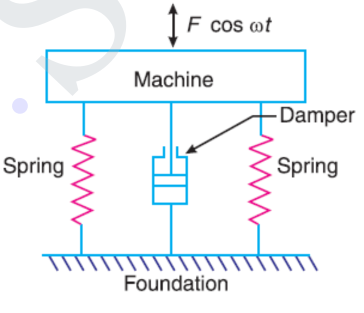
where

$$X_{st} = F_o/k$$

and

$$\varphi = tan^{-1}\left[\frac{2\xi(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right]$$

b) Vibration Isolation



The ratio of the force transmitted (F_T) to the force applied (F) is known as the ***isolation factor*** or ***transmissibility ratio*** of the spring support.

We have discussed above that the force transmitted to the foundation consists of the following two forces :

1. Spring force or elastic force which is equal to $s.\,x_{max}$, and
2. Damping force which is equal to $c.\,\omega.x_{max}$.

$$F_T = \sqrt{(s.x_{max})^2 + (c.\omega.x_{max})^2}$$

$$= x_{max}\sqrt{s^2 + c^2.\omega^2}$$

Transmissibility Ratio

$$\varepsilon = \frac{F_T}{F}$$

3. A machine has a mass of 125 kg and unbalanced reciprocating mass 3 kg which moves through a vertical stroke of 90 mm with SHM. The machine is mounted upon 5 springs. Neglecting damping, calculate the combined stiffness of the spring in order that force transmitted is 1/20th of the applied force, when the speed of the machine crank shaft is 1200 rpm. When the machine is actually supported on the springs, it is found that damping reduces the amplitude of successive free vibration by 30%

Determine; (1) Force transmitted to the foundation at 1200 rpm
(2) Force transmitted to the foundation at resonance.

HINT:

Step 1:
Determine the angular velocity [circular frequency] using the equation
$\omega = 2\pi N/60$
and
Determine the Eccentricity e = Stroke / 2

Step 2:
Determine circular natural frequency using transmissibility ratio.

Transmissibility ratio **ε = +/- [1/ (1-r^2)]**

where **$r = \omega/\omega_n$**

Since force transmitted is 1/20th of applied force $\varepsilon = 1/20$

In the transmissibility ratio equation put **(1-r^2) as (r^2-1) to get positive root**

Find combined stiffness using **ω_n = Sqrt [s/m]**

Step 3:
To determine the Force transmitted to the foundation at 1200 rpm

Given that percentage of successive amplitude is 30%, hence $X_1 = 0.70 X_o$

Logarithmic Decrement is
$\ln[X_o/X_1] = 2\pi c/\text{Sqrt}[c_c^2 - c^2]$

Find critical damping coefficient from $c_c = 2m\omega_n$,

Find the value of damping coefficient c from the above expression.

Actual value of transmissibility is

$$\varepsilon = \frac{\sqrt{1+\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2+\left(1-\frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

Step 4
The maximum unbalance force due to reciprocating parts is given by
$F = m_u\omega^2 r$ where m_u is mass of reciprocating part = 3 kg
Force transmitted to the foundation is
$F_T = \varepsilon F$

Step 5
Force transmitted to the foundation at resonance,

At resonance **$\omega = \omega_n$**

Hence the expression is reduced to

$$\varepsilon = \frac{\sqrt{1+\left(\frac{2c}{c_c}\right)^2}}{\sqrt{\left(\frac{2c}{c_c}\right)^2}}$$

Find F_T in same manner as above
[$F = m_u\omega^2 r$ where m_u is mass of reciprocating part = 3 kg
Force transmitted to the foundation is
$F_T = \varepsilon F$]

4. **A vibrating system having a mass of 1.5 kg is suspended by a spring of stiffness 1200N/m and it is put to harmonic excitation of 12 N. Assuming viscous damping, Determine, (1) Resonant Frequency (2) Phase angle at resonance (3) Amplitude at resonance (4) Damped frequency; Take c = 48 NS/m**

Step 1

Determine circular natural frequency
ω_n = Sqrt [s/m] [s & m are given]

At resonance $\omega = \omega_n$

i) Find Resonant frequency using **$f_n = \omega_n/2\pi$**

Step 2
ii)Determine the phase angle φ at resonance using
[**At resonance $\omega = \omega_n$**]

$$\phi = \tan^{-1}\left(\frac{c.\omega}{s - m.\omega^2}\right)$$

[c- given in the question]
Step 3
(iii) Amplitude at resonance
Max amplitude can be determined using below expression [Force F=12N,$\omega=\omega_n$ at resonance]

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

Step 4:
Damped frequency
$f_d = \omega_d/2\pi$
where
ω_d = sqrt [ω_n^2- a^2] and a = c/2m

Substitute and find frequency of damped vibration

5. **A machine supported symmetrically on five springs, has a mass of 90 kg. The mass of the reciprocating parts is 3 kg which moves through a vertical stroke of 90 mm with SHM. Neglecting damping determine the combined stiffness of the springs so that force transmitted to the foundation is 1/30th of impressed force. The machine crank shaft rotates at 750 rpm. If the under actual working conditions the damping reduces the amplitude of successive vibration by 25%, find:**
   (i) Force transmitted to the foundation at 900 rpm
   (ii)Force transmitted to the foundation at resonance.
   (iii)The amplitude of vibration at resonance

[PROCEDURE OF THIS PROBLEM AS SAME AS PROBLEM NO 3 EXCEPT AMPLITUDE DETERMINATION]
Step 1:
Determine the angular velocity [circular frequency] using the equation
$\omega = 2\pi N/60$
and
Determine the Eccentricity e = Stroke / 2

Determine circular natural frequency using transmissibility ratio.

Transmissibility ratio **$\varepsilon = +/- [\ 1/ (1-r^2)]$**

where **$r = \omega/\omega_n$**

Since force transmitted is 1/30th of applied force $\varepsilon = 1/30$

In the transmissibility ratio equation put **(1-r) as (r^2-1) to get positive root**

Find combined stiffness using **ω_n = Sqrt [s/m]**

Step 3:
To determine the Force transmitted to the foundation at 900 rpm

Find **frequency ratio $r = \omega/\omega_n$**

Given that percentage of successive amplitude is 25%, hence $X_1 = 0.75\ X_o$

Logarithmic Decrement is
$\ln[X_o/X_1] = 2\pi c/\text{Sqrt}\ [c_c^2-c^2]$

Find critical damping coefficient from $c_c = 2m\omega_n$,

Find the value of damping coefficient c from the above expression.

Actual value of transmissibility is

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2+\left(1-\dfrac{\omega^2}{(\omega_n)^2}\right)^2}}$$

Step 4
The maximum unbalance force due to reciprocating parts is given by
$F = m_u\omega^2 r$ where m_u is mass of reciprocating part = 3 kg
Force transmitted to the foundation is
$F_T = \varepsilon F$

Step 5
Force transmitted to the foundation at resonance,

At resonance **$\omega = \omega_n$**

ssion is reduced to

$$\varepsilon = \frac{\sqrt{1+\left(\frac{2c}{c_c}\right)^2}}{\sqrt{\left(\frac{2c}{c_c}\right)^2}}$$

Find F_T in same manner as above
[$F = m_u\omega^2 r$ where m_u is mass of reciprocating part = 3 kg
Force transmitted to the foundation is
$F_T = \varepsilon F$]

Step 6:
Determine the Amplitude using :
Amplitude = Force transmitted at the resonance / Combined Stiffness

6. **A 75 kg machine is mounted on springs of stiffness K= 11.76 X 10 ^{5} N/m with an assumed damping factor of 0.2. A 2 kg piston within the machine has a reciprocating motion with a stroke of 0.08 m and a speed of 3000 rpm. Assuming the motion of the piston to be harmonic, determine the amplitude of vibration of the machine and the vibratory force transmitted to the foundation.**

Step 1:
Determine the angular velocity of unbalance force using
$\omega = 2\pi N/60$
Circular Natural frequency **ω_n = Sqrt [s/m]** [s is stiffness which is denoted as k in question]

Eccentricity e = stroke / 2
Frequency ration $r = \omega/\omega_n$

Step 2
Determine the exciting force $F = m_u.\omega^2.e$ [where m_u – mass of reciprocating part – piston]

Max amplitude of vibration will be given by

$$x_{max} = \frac{F/s}{\sqrt{\frac{c^2.\omega^2}{s^2} + \frac{(s - m.\omega^2)^2}{s^2}}}$$

Damping factor c/c_c is given in question.

$c_c = 2m\omega_n$

From this find c = [c_c.][Damping factor]

Step 3:
Determine transmissibility ratio ε from the below expression

$$\varepsilon = \frac{\sqrt{1+\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2+\left(1-\frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

Also $\varepsilon = F_T/F$

Since F is known, determine F_T from the above expression, which is force transmitted to the foundation.

7. **A body of mass 70 kg is suspended from a spring which deflects 2 cm under the load. It is subjected to a damping effect adjusted to a value of 0.23 times that required for critical damping. Find the natural frequency of the un-damped and damped vibrations and ratio of successive amplitudes of damped vibrations. If the body is subjected to a periodic disturbing force of 700 N and of frequency equal to 0.78 times the natural frequency, find the amplitude of forced vibrations and the phase difference with respect to the disturbing force.**

HINT
Step 1
Determine the natural frequency of free vibration [without damping] using

$f_n = 0.4985/\mathrm{Sqrt}[\delta]$ where δ – Deflection of spring due to load = 20cm = 0.2 m[given]

Damping coefficient $c = 0.23\ c_c$ [given- damping coeff is 0.23 times of critical damping coeff]

Circular natural frequency $\omega_n = 2\pi f_n$

Critical Damping coefficient $c_c = 2m\omega_n$

Determine damping coefficient c from the above expressions

Step 2

Natural frequency of undamped vibration is f_n.

Natural frequency of damped vibration

$f_d = \omega_d/2\pi$

where $\omega_d = \mathrm{Sqrt}\ [\omega_n^2 - a^2]$ and $a = c/2m$

Step 3
For forced vibration [excitation] in the same setup
Frequency $f = 0.78f_n$ [given in question as frequency is 0.78 times of natural frequency]

Max amplitude of forced vibration can be determined by

$$x_{max} = \frac{F/s}{\sqrt{\frac{c^2.\omega^2}{s^2} + \frac{(s - m.\omega^2)^2}{s^2}}}$$

Phase difference φ can be determined by

$$\phi = \tan^{-1}\left(\frac{c.\omega}{s - m.\omega^2}\right)$$

Determine s by using **ω_n = sqrt [s/m],** [since **ω_n and m** is known, s can be determined]

8. The support of a spring mass system is vibrating with amplitude of 6 mm and a frequency of 1200 cycles/min. If a mass is 95 kg and the spring has a stiffness of 1950 N/m, determine the amplitude of vibration of the mass. If a damping factor of 0.2 is include, what would be the amplitude?

Step 1:
Determine the frequency of the support $\omega = 2\pi f$

Determine the natural frequency ω_n = Sqrt [s/m]
Step 2:
Amplitude of Support Y = 6mm = 0.006m

Amplitude of forced vibration due to excitation of support is

X_{max} = Y. [sqrt{s^2+($c\omega^2$)}^{2}] / [sqrt { (s – $m\omega^2$)^2 +($c\omega$)^2]

Intially without damping, damping coefficient c = 0

Step 3:
If a damping factor of 0.2 is included, $c/c_c = 0.2$

Determine $c_c = 2m\omega_n$ and find x_{max} with same above expression

X_{max} = Y. [sqrt{s^2+($c\omega^2$)}^{2}] / [sqrt { (s – $m\omega^2$)^2 +($c\omega$)^2]

9. A machine has a mass of 100 kg and unbalanced reciprocating parts of mass 2 kg which move through a vertical stroke of 80 mm with SHM. The machine is mounted on 4 springs, symmetrically arranged with respect to center of the mass, in such a way that the machine has one degree of freedom and can undergo vertical displacements only. Neglecting damping, calculate the combined stiffness of the spring in order that the force transmitted to the foundation is 1/25th of the applied force, when the speed of the rotation of machine crank shaft is 1000 rpm. When the machine is actually supported on the springs it is found ng re duces the amplitude of successive five vibrations by 25%. Find:(i)The

force transmitted to the foundation at 1000 rpm; (ii)The force transmitted to the foundation at resonance; (iii) The amplitude of the forced vibration of the machine at resonance.

[SIMILAR MODEL TO PROBLEM NO 3 and 5…. Refer Hint of Problem No. 5]
Step 1:
Determine the angular velocity [circular frequency] using the equation
$\omega = 2\pi N/60$
and
Determine the Eccentricity e = Stroke / 2

Step 2:
Determine circular natural frequency using transmissibility ratio.

Transmissibility ratio $\varepsilon = +/- [\, 1/ (1-r^2)]$

where $r = \omega/\omega_n$

Since force transmitted is $1/25^{th}$ of applied force $\varepsilon = 1/25$

In the transmissibility ratio equation put **(1-r) as (r^2-1) to get positive root**

Find combined stiffness using $\omega_n = \text{Sqrt}\, [s/m]$

Step 3:
To determine the Force transmitted to the foundation at 1000 rpm

Find **frequency ratio** $r = \omega/\omega_n$

Given that percentage of successive amplitude is 25%, hence $X_1 = 0.75\, X_o$

Logarithmic Decrement is
$\ln[X_o/X_1] = 2\pi c/\text{Sqrt}\,[c_c^2 - c^2]$

Find critical damping coefficient from $c_c = 2m\omega_n$,

Find the value of damping coefficient c from the above expression.

Actual value of transmissibility is

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2+\left(1-\dfrac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$F = m_u\omega^2 r$ where m_u is mass of reciprocating part = 3 kg
Force transmitted to the foundation is
$F_T = \varepsilon F$

Step 5
Force transmitted to the foundation at resonance,

At resonance **$\omega = \omega_n$**

Hence the expression is reduced to

$$\varepsilon = \frac{\sqrt{1+\left(\frac{2c}{c_c}\right)^2}}{\sqrt{\left(\frac{2c}{c_c}\right)^2}}$$

Find F_T in same manner as above
[$F = m_u\omega^2 r$ where m_u is mass of reciprocating part = 2 kg
Force transmitted to the foundation is
$F_T = \varepsilon F$]

Step 6:
Determine the Amplitude using :
Amplitude = Force transmitted at the resonance / Combined Stiffness

10. A machine of mass 75 kg is mounted on springs of stiffness 1200 kN/m and with an assumed damping factor of 0.2. A piston within the machine of mass 2 kg has a reciprocating motion with a stroke of 80 mm and a speed of 3000 cycles/min. Assuming the motion to be simple harmonic, find: 1. the amplitude of motion of the machine, 2. Its phase angle with respect to the exciting force, 3. the force transmitted to the foundation, and 4. the phase angle of transmitted force with respect to the exciting force.

Step 1
Determine the angular velocity of unbalanced force **$\omega = 2\pi N/60$**

Determine the circular natural frequency **ω_n = sqrt [s/m]**

Eccentricity e = stroke / s

Unbalanced excited force **$F = m_u.\omega^2.e$** [where m_u is the mass of the reciprocating parts]

Step 2

1) Amplitude of motion of machine

$$x_{max} = \frac{F/s}{\sqrt{\frac{c^2.\omega^2}{s^2}+\frac{(s-m.\omega^2)^2}{s^2}}}$$

Step 3

2) Phase lag φ with respect to excited force

$$\phi = \tan^{-1}\left(\frac{c.\omega}{s - m.\omega^2}\right)$$

Step 4

3) Force transmitted F_T to the foundation
Determine transmissibility ratio ε using

$$\varepsilon = \frac{\sqrt{1+\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2+\left(1-\frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

Find F_T from $\boldsymbol{\varepsilon = F_T/F}$

Step 5

4) Phase lag of transmitted force with respect to applied force

It can be determined by
$\alpha = \tan^{-1}[\,2\,(c/c_c)(\omega/\omega_n)]$

11. A single cylinder engine of total mass 200 kg is to be mounted on an elastic support which permits vibratory movement in vertical direction only. The mass of the piston is 3.5 kg and has a vertical reciprocating motion which may be assumed simple harmonic with a stroke of 150 mm. It is desired that the maximum vibratory force transmitted through the elastic support to the foundation shall be 600 N when the engine speed is 800 rpm and less than this at all high speeds. Find: (i) the necessary stiffness of the elastic support, and the amplitude of vibration at 800 rpm, and (ii) if the engine speed is reduced below 800 rpm at what speed will the transmitted force again becomes 600N.

Step 1

Determine the unbalanced force on the piston

$F = m_u\omega^2 e$ [where e = stroke /2 and m_u is the mass of reciprocating parts]

Step 2

Max vibratory force transmitted to the foundation will be given by

F_T = [Stiffness of elastic support].[Max amplitude of vibration]

$$= s \times x_{max} = s \times \frac{F}{m\left[\omega^2 - (\omega_n)^2\right]}$$

Since F_T is given in the question, determine s from the above expression

Step 3

Maximum Amplitude of vibration using

$$x_{max} = \frac{F}{m.\omega^2 - s}$$

Step 4
To find the speed at which the transmitted force again becomes 600 N

Find Disturbing force F = $m_u \omega_1^2 e$ [Keep the value in terms of ω_1]

Step 5

Since the engine speed is reduced, find amplitude with same above concept [as given formulae]

$$x_{max} = \frac{F}{m\left[(\omega_n)^2 - (\omega_1)^2\right]} = \frac{F}{m\left[\frac{s}{m} - (\omega_1)^2\right]} = \frac{F}{s - m(\omega_1)^2}$$

Force transmitted F_T = [Stiffness]. x_{max} [F_T = 600 N – known value]

$$= s \times \frac{F}{s - m(\omega_1)^2}$$

Determine ω_1 from the above expression and hence find N_1- that is speed required

12. An industrial machine weighing 445 kg is supported on a spring with a static deflection of 0.5 cm. If the machine has rotating imbalance of 25 kg-cm, determine the force transmitted at 1200 rpm and the dynamic amplitude at that speed.

Step 1
Determine the Angular velocity of the unbalance force
$\omega = 2\pi N/60$

Determine the natural frequency ω_n = sqrt [s/m] = sqrt [g/δ]
Since δ is known to us {given in the question as 0.5 cm}

Hence find the stiffness of spring s

Step 2
Determine the exciting force $\mathbf{F = [m_u e]\omega^2}$

{Rotating Imbalance in the machine is given as 0.25 kgcm which is the quantity $[m_u e]$

Step 3
Force transmitted at 1200 rpm, without damper

$$\boldsymbol{\varepsilon = F_T/F = +/- [1 / \{1- [\omega/\omega_n]^2\}]}$$

Since $\omega/\omega_n > 1$, the equation can be written as

$$\boldsymbol{\varepsilon = 1/[[\omega/\omega_n]^2 - 1]}$$

Since ε can be found using the above, find F_T – which is force transmitted without damper at 1200 rpm

Step 4
Amplitude of vibration without damper

Since $\omega/\omega_n > 1$

$$X_{max} = [F/s] / [1 - (\omega/\omega_n)^2]$$

13. What do you understand by transmissibility? Describe the method of finding the transmissibility ratio from unbalanced machine supported with foundation.

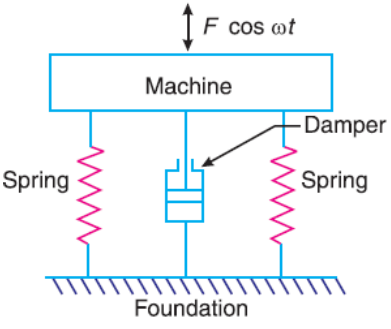
The ratio of the force transmitted (FT) to the force applied (F) is known as the isolation factor or transmissibility ratio of the spring support.
We have discussed above that the force transmitted to the foundation consists of the following two forces :
1. Spring force or elastic force which is equal to s. xmax, and
2. Damping force which is equal to c. ω xmax.

$$F_T = \sqrt{(s.x_{max})^2 + (c.\omega.x_{max})^2}$$

$$= x_{max}\sqrt{s^2 + c^2.\omega^2}$$

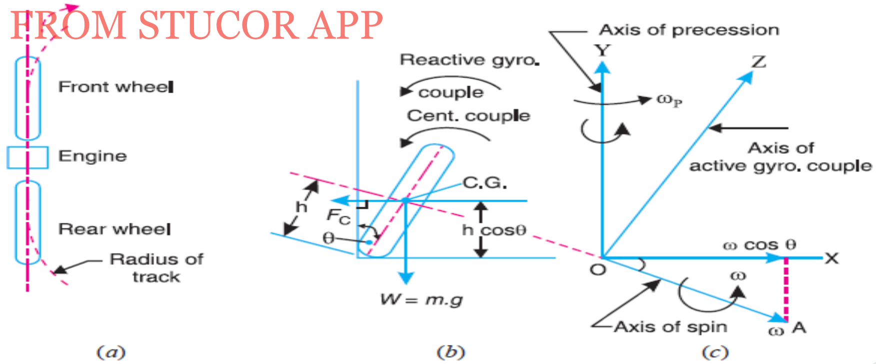
$$\varepsilon = \frac{F_T}{F} = \frac{x_{max}\sqrt{s^2 + c^2.\omega^2}}{F}$$

$$\varepsilon = \frac{\sqrt{1 + \left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \dfrac{\omega^2}{(\omega_n)^2}\right)^2}}$$



Unit – V Mechanisms for Control

1. **What is the effect of gyroscopic couple on the stability of a two wheel vehicle taking a turn? (Dec 2013)**

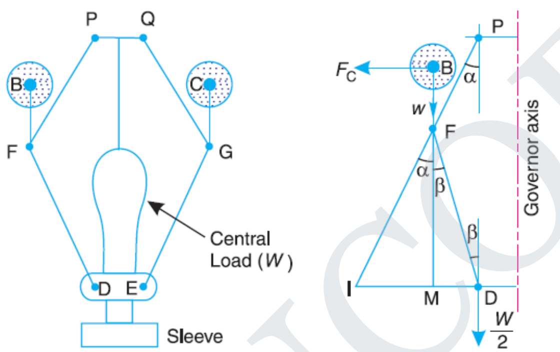


The balancing couple acts in clockwise direction when seen from the front of the vehicle. Therefore for stability, the overturning couple must be equal to the balancing couple, *i.e.*

$$\frac{v^2}{R}\left(\frac{2\,I_W + G.I_E}{r_W} + m.h\right)\cos\theta = m.g.h\sin\theta$$

From this expression, the value of the angle of heel (θ) may be determined, so that the vehicle does not skid.

2. (a) **Explain the function of a proell governor with the help of a neat sketch. Derive the relationship among the various forces acting on the link**.



$$F_C = \frac{FM}{BM}\left[m.g\times\frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM}+\frac{MD}{FM}\right)\right]$$

(b) **What are centrifugal governors? How do they differ from inertia governors? (Dec 2013)**
The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force***

3. **The mass of each ball of a proell governor is 7.5 kg and the load on the sleeve is 80 kg.each of the arms is 300 mm long. The upper arms are pivoted on the axis of rotation whereas the lower arms are pivoted to links of 40 mm from the axis of rotation. The extensions of the lower arms to which the balls are attached are 100 mm long and are parallel to the governor axis at the minimum radius. determine the equilibrium speeds corresponding to extreme radii of 180 mm and 240 mm**

Speed

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h}$$

N = 165.3 rpm

4. **Calculate the range of speed of a porter governor which has equal arms of each 200 mm long and pivoted on the axis of rotation. the mass of each ball is 4 kg and the central mass of the sleeve is 20 kg.the radius of rotation of the ball is 100 mm when the governor begins to lift and 130 mm when the governor is at maximum speed.**

Height of the governor $$h_1 = PG = \sqrt{(PB)^2 - (BG)^2}$$

Speed of the governor $$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2}$$

5. **Find the angle of inclination with respect to the vertical for a two wheeler having the following details negotiating a turn of radius 50 m: combined mass of vehicle with rider = 250 kg; Centre of gravity with rider in vertical position = 0.6 m; Moment of inertia of flywheel = 0.3 kgm^2; M.I of each road wheel = 1 kg-m^2; Speed of engine is five times that of road wheels and in same direction; Vehicle speed = 90km/h; wheel diameter = 600 mm.**

We know that gyroscopic couple,

$$C_1 = \frac{v^2}{R \times r_W}(2\,I_W + G.I_E)\cos\theta$$

and centrifugal couple, $$C_2 = \frac{m.v^2}{R} \times h\cos\theta$$

We know that balancing couple

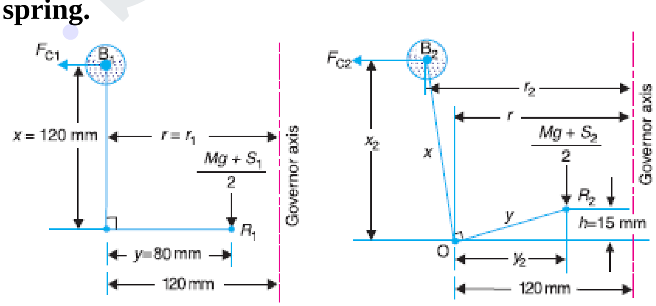
$$= m.g.h\sin\theta$$

6. **A porter governor has two balls each of mass 3 kg and a central load of mass 15 kg.the arms are all 200 mm long pivoted on the axis. The maximum and minimum radii of rotation are 160 mm and 120 mm respectively. Find the range of speed.**

Height of the governor $$h_1 = PG = \sqrt{(PB)^2 - (BG)^2}$$

Speed of the governor $$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2}$$

7. **A hartnell governor having a central sleeve spring and two right angled bell crank levers operates between 290 rpm and 310 rpm for a sleeve lift of 15 mm.the sleeve and ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg.the ball arms are parallel at lowest equilibrium speed. Determine (i) loads on the spring at maximum and minimum speeds, and (ii) stiffness of the spring.**



(a) Lowest position. (b) Highest position.

Centrifugal force at the maximum speed,

$$F_{C2} = m\,(\omega_2)^2\, r_2 = 2.5 \times (32.5)^2 \times 0.1425 = 376 \text{ N}$$

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 277 \times \frac{0.12}{0.08} = 831 \text{ N}$$

$$\therefore \quad S_2 = 831 \text{ N } \textbf{Ans.}$$

and for highest position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 376 \times \frac{0.12}{0.08} = 1128 \text{ N}$$

$$\therefore \quad S_1 = 1128 \text{ N } \textbf{Ans.}$$

We know that stiffness of the spring,

$$s = \frac{S_2 - S_1}{h}$$

8. In a Hartnell governor the lengths of ball and sleeve arms of a bell crank lever are 120 mm and 100 mm respectively. the fulcrum of the bell crank lever is located at 140 mm from the governor axis each governor ball is 4 kg.the governor runs at 5 rps with ball arms vertical and sleeve arms horizontal the sleeve movement is found to be 10 mm (upwards) for an increase of speed of 4%.find (i) maximum speed if the total sleeve movement is limited to 20 mm (ii) the spring stiffness (iii) sensitiveness of governor (iv) required spring stiffness for isochronous at 300 rpm. **(Nov/Dec 2012)**

We know that centrifugal force in the mean position,

$$F_C = m.\omega^2.r = 4\,(31.42)^2\, 0.14 = 553 \text{ N}$$

Centrifugal force in the minimum position,

$$F_{C1} = m\,(\omega_1)^2\, r_2 = 4\left(\frac{2\pi\, N_1}{60}\right)^2 0.128 = 0.0056\,(N_1)^2 \qquad \ldots (i)$$

and centrifugal force in the maximum position,

$$F_{C2} = m.(\omega_2)^2\, r_2 = 4\,(32.7)^2\, 0.152 = 650 \text{ N}$$

and for the maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad S_2 = 2\,F_{C2} \times \frac{x}{y} = 2 \times 650 \times \frac{0.12}{0.1} = 1560 \text{ N}$$

We know that spring stiffness,

$$s = \frac{S_2 - S_1}{h} = \frac{1560 - 1064.4}{20} = 23.28 \text{ N/mm } \textbf{Ans.}$$

3. Sensitiveness of the governor

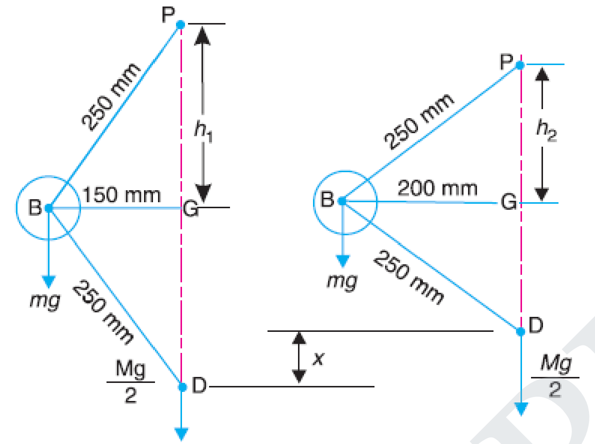
We know that sensitiveness of the governor

$$= \frac{2\,(N_2 - N_1)}{N_1 + N_2} = \frac{2\,(312 - 285.4)}{285.4 + 312} = 0.089 \quad \text{or} \quad 8.9\% \textbf{ Ans.}$$

$$\therefore \text{Spring stiffness, } s = \frac{S_2 - S_1}{h} = \frac{1440 - 1213}{20} = 11.35 \text{ N/mm } \textbf{Ans.}$$

9. A porter governor has equal arms each 250 mm long and pivoted on the axis of rotation each ball

ball is 150 mm and the governor begins to lift and 200 mm.when the governor is maximum speed. find the minimum and maximum speeds and range of speed of governor



(*a*) Minimum position. (*b*) Maximum position.

Range of speed

We know that range of speed

$$= N_2 - N_1 = 189 - 164 = 25 \text{ r.p.m.} \textbf{ Ans.}$$

Sleeve lift

We know that sleeve lift,

$$x = 2\,(h_1 - h_2) = 2\,(200 - 150) = 100 \text{ mm} = 0.1 \text{ m} \quad \textbf{Ans.}$$

Governor effort

We know that governor effort

$$P = c\,(m + M)\,g = 0.152\,(5 + 25)\,9.81 = 44.7 \text{ N} \textbf{ Ans.}$$

Power of the governor

We know that power of the governor

$$= Px = 44.7 \times 0.1 = 4.47 \text{ N-m} \textbf{ Ans.}$$

2. When the friction at the sleeve is taken into account

We know that $$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

Range of speed

We know that range of speed

$$= N_2 - N_1 = 192.4 - 161 = 31.4 \text{ r.p.m.} \textbf{ Ans.}$$

Sleeve lift

The sleeve lift (x) will be same as calculated above.

$\therefore$ Sleeve lift, $x = 100 \text{ mm} = 0.1 \text{ m}$ **Ans.**

We know that governor effort,

$$P = c\,(m.g + M.g + F) = 0.195\,(5 \times 9.81 + 25 \times 9.81 + 10)\text{ N}$$

$$= 57.4\text{ N } \textbf{Ans.}$$

Power of the governor

We know that power of the governor

$$= P x = 57.4 \times 0.1 = 5.74\text{ N-m } \textbf{Ans.}$$

10.

a. In a spring controlled governor, the curve of the controlling force is a straight line. When balls are 400 mm apart, the controlling force is 1200 N and when 200 mm apart, the controlling force is 450 N.at what speed will the governor run when the balls are 250 mm apart? What initial tension on the spring would be required for isochronisms and what would then be the speed? Take masses of each ball to be 10 kg.

We know that for the stability of the spring controlled governors, the controlling force (F_C) is expressed in the form

* $F_C = a.r - b$ . . . ***(i)***

When $r = r_1 = 100$ mm = 0.1 m, then

$450 = a \times 0.1 - b = 0.1\,a - b$ . . .***(ii)***

and when $r = r_2 = 200$ mm = 0.2 m, then

$1200 = a \times 0.2 - b = 0.2\,a - b$ . . . ***(iii)***

From equations ***(ii)*** and ***(iii)***, we find that

$a = 7500$, and $b = 300$

We know that $F_C = m.\omega^2.r = m\left(\dfrac{2\pi N}{60}\right)^2 r$

$$637.5 = 9\left(\frac{2\pi N}{60}\right)^2 0.125 = 0.012\,34\,N^2$$

$\therefore$ $N^2 = 637.5 / 0.012\,34 = 51\,661$ or $N = 227.3$ r.p.m.

11. A ship is propelled by a turbine rotor which has a mass of 5 tonnes and a speed of 2100 rpm.the rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effect in the following conditions: (i) the ship sails at a speed of 30 km/hr and steers to the left in curve having 60 m radius; (ii) the ship pitches 6° above and 6° below the horizontal position. The bow is descending with its maximum velocity. the motion due to pitching is simple harmonic and a periodic time is 20 seconds.(iii) the ship rolls and at a certain instant it has an angular velocity of 0.03 rad/sec clockwise when viewed from stern. **(Nov/Dec 2012)**

$\therefore$ Gyroscopic couple,

$$C = I.\omega.\omega_P = 1250 \times 220 \times 0.14 = 38\,500\text{ N-m} = 38.5\text{ kN-m}$$

$\therefore$ Maximum gyroscopic couple,

$$C_{max} = I.\omega.\omega_{Pmax} = 1250 \times 220 \times 0.033 = 9075\text{ N-m}$$

$\therefore$ Gyroscopic couple,

$$C = I.\omega.\omega_P = 1250 \times 220 \times 0.03 = 8250\text{ N-m}$$

We know that maximum angular acceleration during pitching.

$$\alpha_{max} = \phi\,(\omega_1)^2 = 0.105\,(0.3142)^2 = 0.01\text{ rad/s}^2$$

12. A four wheeled motor car weighing 2000 kg has height of C.G of 600 mm above ground level. the engine parts and transmission are equivalent to a flywheel of 80 kg with radius of gyration eir axis coincide with longitudinal axis of the vertical the car negotiates a curve (turning right) of 60 m radius at 72 kmph with over all gear ratio 4:1.the radius of road wheel is

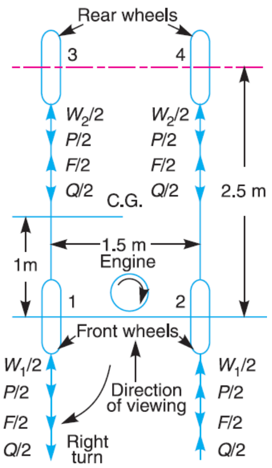
300 mm and moment of inertia is 3 kg-m^2.assuming wheel track as 1.5 m, weight distribution as 50:50.determine reaction at each wheel.

$\therefore$ Weight on each of the front wheels

$$= W_1 / 2 = 11\,772 / 2 = 5886 \text{ N}$$

and weight on each of the rear wheels

$$= W_2 / 2 = 7874 / 2 = 3924 \text{ N}$$



. Gyroscopic couple due to four wheels,

$$C_W = 4 I_W . \omega_W . \omega_P$$

$$= 4 \times 0.8 \times 41.675 \times 0.278 = 37.1 \text{ N-m}$$

$\therefore$ Gyroscopic couple due to rotating parts of the engine,

$$C_E = I_E . \omega_E . \omega_P = m_E (k_E)^2 G. \omega_W . \omega_P$$

$$= 75 (0.1)^2 4 \times 41.675 \times 0.278 = 34.7 \text{ N-m}$$

Load on the front wheel 1

$$= \frac{W_1}{2} - \frac{P}{2} - \frac{F}{2} - \frac{Q}{2} = 5886 - 12.37 - 6.94 - 1543.83 = 4322.86 \text{ N}$$

Load on the front wheel 2

$$= \frac{W_1}{2} + \frac{P}{2} - \frac{F}{2} + \frac{Q}{2} = 5886 + 12.37 - 6.94 + 1543.83 = 7435.26 \text{ N}$$

Load on the rear wheel 3

$$= \frac{W_2}{2} - \frac{P}{2} + \frac{F}{2} - \frac{Q}{2} = 3924 - 12.37 + 6.94 - 1543.83 = 2374.74 \text{ N}$$

Load on the rear wheel 4

$$= \frac{W_2}{2} + \frac{P}{2} + \frac{F}{2} + \frac{Q}{2} = 3924 + 12.37 + 6.94 + 1543.83 = 5487.14 \text{ N}$$