

**ME6502**

**HEAT AND MASS TRANSFER**

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**OBJECTIVES:**

- To understand the mechanisms of heat transfer under steady and transient conditions.
- To understand the concepts of heat transfer through extended surfaces.
- To learn the thermal analysis and sizing of heat exchangers and to understand the basic concepts of mass transfer.

(Use of standard HMT data book permitted)

**UNIT I CONDUCTION 9**

General Differential equation of Heat Conduction– Cartesian and Polar Coordinates – One Dimensional Steady State Heat Conduction — plane and Composite Systems – Conduction with Internal Heat Generation – Extended Surfaces – Unsteady Heat Conduction – Lumped Analysis – Semi Infinite and Infinite Solids –Use of Heisler's charts.

**UNIT II CONVECTION 9**

Free and Forced Convection - Hydrodynamic and Thermal Boundary Layer. Free and Forced Convection during external flow over Plates and Cylinders and Internal flow through tubes .

**UNIT III PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS 9**

Nusselt's theory of condensation - Regimes of Pool boiling and Flow boiling. Correlations in boiling and condensation. Heat Exchanger Types - Overall Heat Transfer Coefficient – Fouling Factors - Analysis – LMTD method - NTU method.

**UNIT IV RADIATION 9**

Black Body Radiation – Grey body radiation - Shape Factor – Electrical Analogy – Radiation Shields. Radiation through gases.

**UNIT V MASS TRANSFER 9**

Basic Concepts – Diffusion Mass Transfer – Fick's Law of Diffusion – Steady state Molecular Diffusion – Convective Mass Transfer – Momentum, Heat and Mass Transfer Analogy –Convective Mass Transfer Correlations.

**TOTAL : 45 PERIODS**

**OUTCOMES:**

- Upon completion of this course, the students can able to understand and apply different heat and mass transfer principles of different applications.

**TEXT BOOK:**

1. Yunus A. Cengel, "Heat Transfer A Practical Approach", Tata McGraw Hill, 2010

**REFERENCE BOOKS:**

1. Frank P. Incropera and David P. Dewitt, "Fundamentals of Heat and Mass Transfer", John Wiley & Sons, 1998.
2. Venkateshan. S.P., "Heat Transfer", Ane Books, New Delhi, 2004.
3. Ghoshdastidar, P.S, "Heat Transfer", Oxford, 2004,
4. Nag, P.K., "Heat Transfer", Tata McGraw Hill, New Delhi, 2002
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6. Ozisik, M.N., "Heat Transfer", McGraw Hill Book Co., 1994.
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8. Yadav, R., "Heat and Mass Transfer", Central Publishing House, 1995.
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| UNIT NO | CONTENT  | PAGE NO |
|---------|--|---------|
| 1       | <b>CONDUCTION</b>  |         |
|         | <b>1.1 GENERAL HEAT CONDUCTION IN CARTESIAN COORDINATES ,POLAR</b> | 5       |
|         | <b>1.2 DERIVATION ON PLANE WALL</b>                                | 8       |
|         | <b>1.3 DERIVATION ON COMPOSITE SYSTEMS</b>                         | 11      |
|         | <b>1.4 CONDUCTION WITH INTERNAL HEAT</b>                           | 15      |
|         | <b>1.5 EXTENDED SURFACES</b>                                       | 17      |
|         | <b>1.6 UNSTEADY STATE HEAT TRANSFER</b>                            | 20      |
|         | <b>1.7 LUMPED HEAT ANALYSIS</b>                                    | 23      |
|         | <b>1.8 SEMI INFINITE SOLIDS</b>                                    | 27      |
| 2       | <b>CONVECTION</b>  |         |
|         | <b>2.1 FREE AND FORCED CONVECTION</b>                              | 30      |
|         | <b>2.2 FORMATION OF THERMAL AND HYDRODYNAMIC LAYER</b>             | 32      |
|         | <b>2.3 FREE AND FORCED CONVECTION OVER PLATES</b>                  | 34      |
|         | <b>2.4 FLOW OVER CYLINDERS</b>                                     | 37      |
|         | <b>2.5 INTERNAL FLOW THROUGH TUBES</b>                             | 44      |
| 3       | <b>PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS</b>              |         |
|         | <b>3.1. NUSSELT S THEORY OF CONDENSATION</b>                       | 67      |
|         | <b>3.2 REGIMES OF POOL BOILING</b>                                 | 72      |

|   |   |     |
|---|---|-----|
|   | <b>3.3 CORRELATIONS IN BOILING AND CONDENSATION</b> | 75  |
|   | <b>3.4 HEAT EXCHANGER TYPES</b>                     | 76  |
|   | <b>3.5 FOULING FACTORS AND NTU METHOD</b>           | 82  |
| 4 | <b>RADIATION</b>                                    |     |
|   | <b>4.1 BLACK BODY AND GREY BODY RADIATION</b>       | 105 |
|   | <b>4.2 SHAPE FACTOR</b>                             | 106 |
|   | <b>4.3 RADIATION SHIELDS</b>                        | 109 |
|   | <b>4.4 RADIATION THROUGH GASES</b>                  | 114 |
| 5 | <b>MASS TRANSFER</b>                                |     |
|   | <b>5.1 BASIC CONCEPTS</b>                           | 136 |
|   | <b>5.2 STEADY STATE MOLECULAR DIFFUSION</b>         | 137 |
|   | <b>5.3 CONVECTIVE MASS TRANSFER</b>                 | 137 |
|   | <b>5.4 HEAT AND MASS TRANSFER ANALOGY</b>           | 140 |
|   | <b>5.5 CONVECTIVE MASS TRANSFER</b>                 | 143 |

CHAPTER 1: CONDUCTION

GENERAL DIFFERENTIAL EQUATION OF HEAT CONDUCTION

1.1 The General Heat Conduction Equation in Cartesian coordinates and Polar coordinates

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Any physical phenomenon is generally accompanied by a change in space and time of its physical properties. The heat transfer by conduction in solids can only take place when there is a variation of temperature, in both space and time. Let us consider a small volume of a solid element as shown in Fig. 1.2. The dimensions are:  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$  along the X-, Y-, and Z- coordinates.

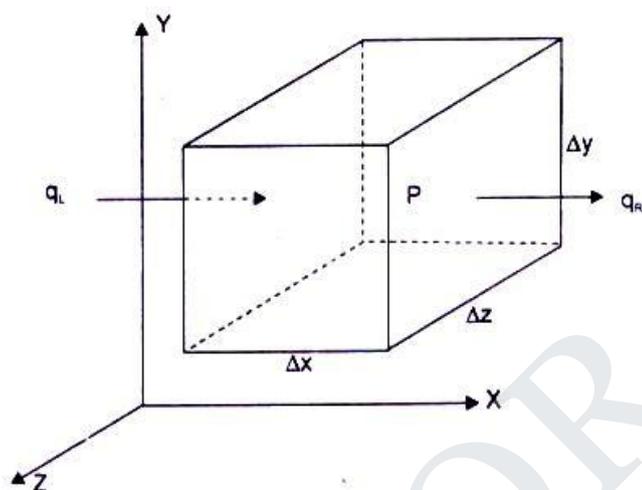


Fig 1.1 Elemental volume in Cartesian coordinates

First we consider heat conduction the X-direction. Let T denote the temperature at the point P (x, y, z) located at the geometric centre of the element. The temperature gradient at the left hand face (x -  $\Delta x/2$ ) and at the right hand face (x +  $\Delta x/2$ ), using the Taylor's series, can be written as:

$$\frac{\partial T}{\partial x} \Big|_L = \frac{\partial T}{\partial x} - \frac{\partial^2 T}{\partial x^2} \cdot \frac{\Delta x}{2} + \text{higher order terms.}$$

$$\frac{\partial T}{\partial x} \Big|_R = \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \cdot \frac{\Delta x}{2} + \text{higher order terms.}$$

The net rate at which heat is conducted out of the element 10 X-direction assuming k as constant and neglecting the higher order terms,

$$\text{we get } -k\Delta y\Delta z \left[ \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \frac{\Delta x}{2} - \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \frac{\Delta x}{2} \right] = -k\Delta y\Delta z\Delta x \left( \frac{\partial^2 T}{\partial x^2} \right)$$

Similarly for Y- and Z-direction,

$$\text{We have } -k\Delta x\Delta y\Delta z \frac{\partial^2 T}{\partial y^2} \text{ and } -k\Delta x\Delta y\Delta z \frac{\partial^2 T}{\partial z^2}.$$

If there is heat generation within the element as Q, per unit volume and the internal energy of

the element changes with time, by making an energy balance, we write

|                                      |   |  |
|--------------------------------------|---|--|
| Heat generated within<br>the element | Heat conducted away<br>from the element | Rate of change of internal<br>energy within with the element |
|--------------------------------------|---|--|

$$\text{or, } \dot{Q}_v (\Delta x \Delta y \Delta z) + k (\Delta x \Delta y \Delta z) \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

$$= \rho c (\Delta x \Delta y \Delta z) \frac{\partial T}{\partial t}$$

$$\text{Upon simplification, } \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{Q}_v}{k} = \frac{\rho c}{k} \frac{\partial T}{\partial t}$$

$$\text{or, } \nabla^2 T + \frac{\dot{Q}_v}{k} = 1/\alpha \left( \frac{\partial T}{\partial t} \right)$$

where  $\alpha = k/\rho \cdot c$ , is called the thermal diffusivity and is seen to be a physical property of the material of which the solid is composed.

The Eq. (2.1a) is the general heat conduction equation for an isotropic solid with a constant thermal conductivity. The equation in cylindrical (radius  $r$ , axis  $Z$  and longitude  $\theta$ ) coordinates is written as: Fig. 2.1(b),

$$\frac{\partial^2 T}{\partial r^2} + (1/r) \frac{\partial T}{\partial r} + \left( \frac{1}{r^2} \right) \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{Q}_v}{k} = 1/\alpha \frac{\partial T}{\partial t} \quad (2.1b)$$

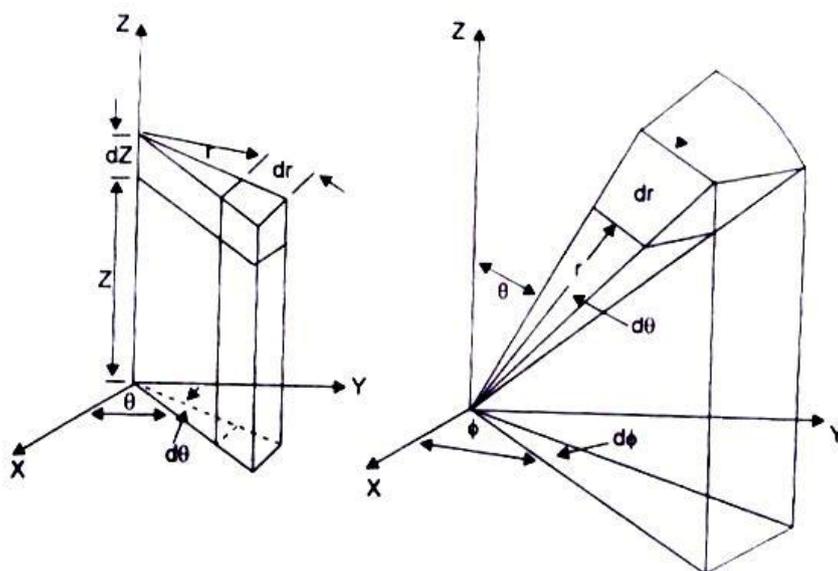
And, in spherical polar coordinates Fig. 2.1(c) (radius,  $\phi$  longitude,  $\theta$  colatitudes) is

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} + \frac{\dot{Q}_v}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (2.1c)$$

Under steady state or stationary condition, the temperature of a body does not vary with time, i.e.  $\frac{\partial T}{\partial t} = 0$ . And, with no internal generation, the equation (2.1) reduces to

$$\nabla^2 T = 0$$

It should be noted that Fourier law can always be used to compute the rate of heat transfer by conduction from the knowledge of temperature distribution even for unsteady condition and with internal heat generation.



**2.1 ONE DIMENSIONAL STEADY STATE EQUATION PLANE WALL :**

The term 'one-dimensional' is applied to heat conduction problem when:

- (i) Only one space coordinate is required to describe the temperature distribution within a heat conducting body;
- (ii) Edge effects are neglected;
- (iii) The flow of heat energy takes place along the coordinate measured normal to the surface.

A plane wall is considered to be made out of a constant thermal conductivity material and extends to infinity in the Y- and Z-direction. The wall is assumed to be homogeneous and isotropic, heat flow is one-dimensional, under steady state conditions and losing negligible energy through the edges of the wall under the above mentioned assumptions the Eq. (2.2) reduces to

$$d^2T / dx^2 = 0; \text{ the boundary conditions are: at } x = 0, T = T_1$$

$$\text{Integrating the above equation, } x = L, T = T_2$$

$$T = C_1x + C_2, \text{ where } C_1 \text{ and } C_2 \text{ are two constants.}$$

Substituting the boundary conditions, we get  $C_2 = T_1$  and  $C_1 = (T_2 - T_1)/L$  The temperature distribution in the plane wall is given by

$$T = T_1 - (T_1 - T_2) x/L \tag{2.3}$$

which is linear and is independent of the material.

Further, the heat flow rate,  $\dot{Q} / A = -k dT/dx = (T_1 - T_2)k/L$ , and therefore the temperature distribution can also be written as

$$T - T_1 = (\dot{Q} / A)(x/k) \tag{2.4}$$

i.e., "the temperature drop within the wall will increase with greater heat flow rate or when  $k$  is small for the same heat flow rate,"

## 2.2 A Cylindrical Shell-Expression for Temperature Distribution

In the cylindrical system, when the temperature is a function of radial distance only and is independent of azimuth angle or axial distance, the differential equation (2.2) would be, (Fig. 1.4)

$$d^2T/dr^2 + (1/r) dT/dr = 0$$

with boundary conditions: at  $r = r_1$ ,  $T = T_1$  and at  $r = r_2$ ,  $T = T_2$ .

The differential equation can be written as:

$$\frac{1}{r} \frac{d}{dr}(r dT/dr) = 0, \text{ or, } \frac{d}{dr}(r dT/dr) = 0$$

upon integration,  $T = C_1 \ln(r) + C_2$ , where  $C_1$  and  $C_2$  are the arbitrary constants.

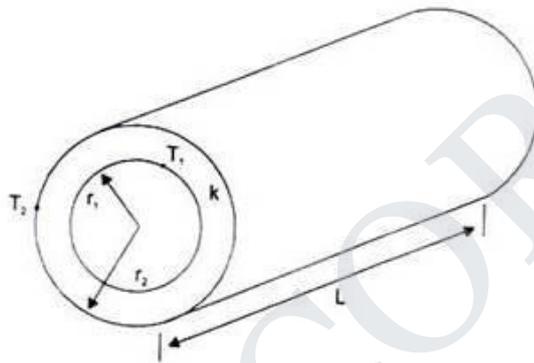


Fig 1.3: A Cylindrical shell

By applying the boundary conditions,

$$C_1 = (T_2 - T_1) / \ln(r_2/r_1)$$

$$\text{and } C_2 = T_1 - \ln(r_1) \cdot (T_2 - T_1) / \ln(r_2/r_1)$$

The temperature distribution is given by

$$T = T_1 + (T_2 - T_1) \cdot \ln(r/r_1) / \ln(r_2/r_1) \text{ and}$$

$$\dot{Q}/L = -kA dT/dr = 2\pi k(T_1 - T_2) / \ln(r_2/r_1) \quad (2.5)$$

From Eq (2.5) It can be seen that the temperature varies logarithmically through the cylinder wall in contrast with the linear variation in the plane wall .

If we write Eq. (2.5) as  $\dot{Q} = kA_m(T_1 - T_2) / (r_2 - r_1)$  , where

$$A_m = 2\pi(r_2 - r_1)L / \ln(r_2/r_1) = (A_2 - A_1) / \ln(A_2/A_1)$$

where  $A_2$  and  $A_1$  are the outside and inside surface areas respectively. The term  $A_m$  is called 'Logarithmic Mean Area' and the expression for the heat flow through a cylindrical wall has the same form as that for a plane wall.

**3.1 ONE DIMENSIONAL STEADY STATE HEAT CONDUCTION COMPOSITE SYSTEMS:**

**3.2 Composite Surfaces**

There are many practical situations where different materials are placed in layers to form composite surfaces, such as the wall of a building, cylindrical pipes or spherical shells having different layers of insulation. Composite surfaces may involve any number of series and parallel thermal circuits.

**3.3 Heat Transfer Rate through a Composite Wall**

Let us consider a general case of a composite wall as shown in Fig. 1.5. There are 'n' layers of different materials of thicknesses  $L_1, L_2,$  etc and having thermal conductivities  $k_1, k_2,$  etc. On one side of the composite wall, there is a fluid A at temperature  $T_A$  and on the other side of the wall there is a fluid B at temperature  $T_B$ . The convective heat transfer coefficients on the two sides of the wall are  $h_A$  and  $h_B$  respectively.

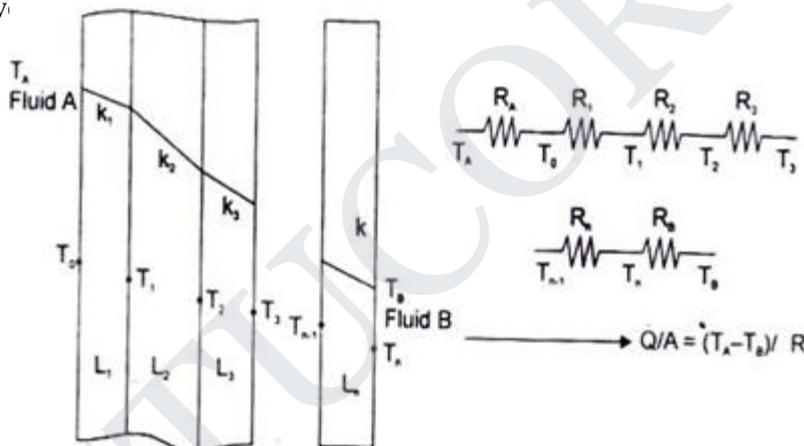


Fig 1.4 Heat transfer through a composite wall

**The Equivalent Thermal Conductivity**

The process of heat transfer through composite and plane walls can be more conveniently compared by introducing the concept of 'equivalent thermal conductivity',  $k_{eq}$ . It is defined as:

$$k_{eq} = \left( \sum_{i=1}^n L_i \right) / \sum_{i=1}^n (L_i / k_i) \tag{2.8}$$

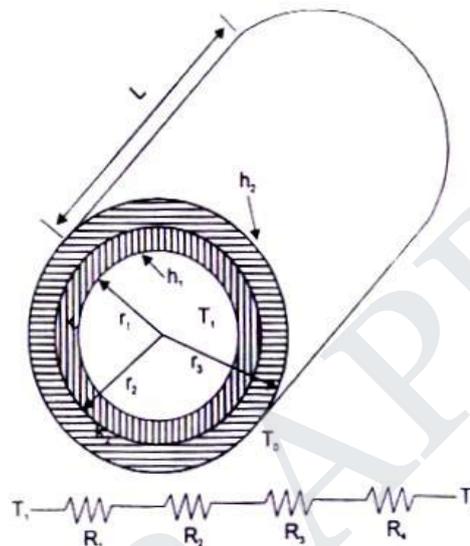
$$= \frac{\text{Total thickness of the composite wall}}{\text{Total thermal resistance of the composite wall}}$$

And, its value depends on the thermal and physical properties and the thickness of each

constituent of the composite structure.

**3.4 An Expression for the Heat Transfer Rate through a Composite Cylindrical System**

Let us consider a composite cylindrical system consisting of two coaxial cylinders, radii  $r_1$ ,  $r_2$  and  $r_3$ , thermal conductivities  $k_1$  and  $k_2$  the convective heat transfer coefficients at the inside and outside surfaces  $h_1$  and  $h_2$  as shown in the figure. Assuming radial conduction under steady state



conditions we have:

Fig 1.5

$$R_1 = 1/h_1 A_1 = 1/2 \pi r_1 L h_1$$

$$R_2 = \ln(r_2 / r_1) / 2\pi L k_1$$

$$R_3 = \ln(r_3 / r_2) / 2\pi L k_2$$

$$R_4 = 1/h_2 A_2 = 1/2\pi r_3 L h_2$$

And  $\dot{Q} / 2\pi L = (T_1 - T_0) / \Sigma R$

$$= (T_1 - T_0) / \left[ \left( 1/h_1 r_1 + \ln(r_2 / r_1) / k_1 + \ln(r_3 / r_2) / k_2 + 1/h_2 r_3 \right) \right]$$

**4.1 CONDUCTION WITH INTERNAL HEAT GENERATION:**

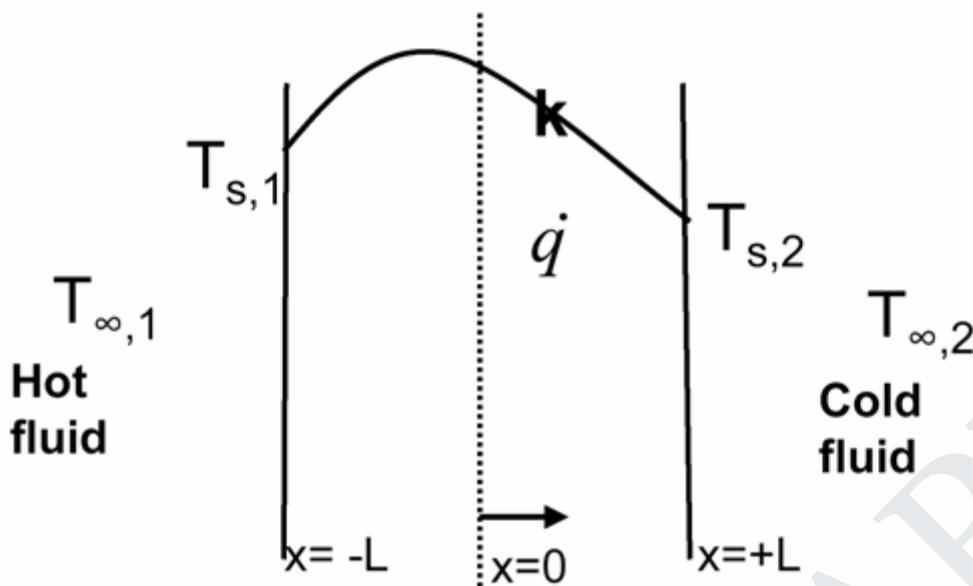
Applications: current carrying conductor, chemically reacting systems, nuclear reactors.

Energy generated per unit volume is given by  $V Eq$ .

Plane wall with heat source: Assumptions: 1D, steady state, constant k, uniform

Consider one-dimensional, steady-state conduction in a plane wall of constant k, with uniform generation, and asymmetric surface conditions:

Heat diffusion equation.



A medium through which heat is conducted may involve the conversion of mechanical, electrical, nuclear, or chemical energy into heat (or thermal energy). In heat conduction analysis, such conversion processes are characterized as **heat generation**.

For example, the temperature of a resistance wire rises rapidly when electric current passes through it as a result of the electrical energy being converted to heat at a rate of  $I^2R$ , where  $I$  is the current and  $R$  is the electrical resistance of the wire. The safe and effective removal of this heat away from the sites of heat generation (the electronic circuits) is the subject of *electronics cooling*, which is one of the modern application areas of heat transfer.

Likewise, a large amount of heat is generated in the fuel elements of nuclear reactors as a result of nuclear fission that serves as the *heat source* for the nuclear power plants. The natural disintegration of radioactive elements in nuclear waste or other radioactive material also results in the generation of heat throughout the body. The heat generated in the sun as a result of the fusion of hydrogen into helium makes the sun a large nuclear reactor that supplies heat to the earth.

Another source of heat generation in a medium is exothermic chemical reactions that may occur throughout the medium. The chemical reaction in this case serves as a *heat source* for the medium. In the case of endothermic reactions, however, heat is absorbed instead of being released during reaction, and thus the chemical reaction serves as a *heat sink*. The heat generation term becomes a negative quantity in this case.

Often it is also convenient to model the absorption of radiation such as solar energy or gamma rays as heat generation when these rays penetrate deep into the body while being absorbed gradually. For example, the absorption of solar energy in large bodies of water can be treated as heat generation

throughout the water at a rate equal to the rate of absorption, which varies with depth. But the absorption of solar energy by an opaque body occurs within a few microns of the surface, and the solar energy that penetrates into the medium in this case can be treated as specified heat flux on the surface.

Note that heat generation is a *volumetric phenomenon*. That is, it occurs throughout the body of a medium. Therefore, the rate of heat generation in a medium is usually specified *per unit volume*.

The rate of heat generation in a medium may vary with time as well as position within the medium. When the variation of heat generation with position is known, the *total* rate of heat generation in a medium of volume  $V$  can be determined from In the special case of *uniform* heat generation, as in the case of electric resistance heating throughout a homogeneous material, the relation in reduces to  $E \cdot \text{gen} = e \cdot \text{gen} V$ , where  $E_{\text{gen}}$  is the constant rate of heat generation per unit volume.

### 5.1 EXTENDED SURFACES :

Newton's cooling law:  $q = hA(T_s - T_\infty)$ . Therefore, to increase the convective heat transfer,

Convection heat transfer between a solid surface (Face  $T_s$ ) and between the solid surface and the fluid

Increase the convection coefficient  $h$ . This can be accomplished by increasing the fluid flow over the surface since  $h$  is a function of the flow velocity and the higher the velocity, the higher the  $h$ . Example: a cooling fan.

Increase the contact surface area  $A$ . Example: a heat sink with fin.

$A_c$  : the fin cross-sectional area.

$P$ : the fin perimeter.

Many times, when the first option is not in our control and the second option (i.e. increasing  $h$ ) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces. Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid. Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers) and elsewhere.

The fin efficiency is defined as the ratio of the energy transferred through a real fin to that transferred through an ideal fin. An ideal fin is thought to be one made of a perfect or infinite conductor material. A perfect conductor has an infinite thermal conductivity so that the entire fin is at the base material temperature.

### 6.1 UNSTEADY HEAT CONDUCTION :

#### 6. 1 Transient State Systems-Defined

The process of heat transfer by conduction where the temperature varies with time and with space coordinates, is called 'unsteady or transient'. All transient state systems may be broadly classified into two categories:

(a) Non-periodic Heat Flow System - the temperature at any point within the system changes as a non-linear function of time.

(b) Periodic Heat Flow System - the temperature within the system undergoes periodic changes which may be regular or irregular but definitely cyclic.

There are numerous problems where changes in conditions result in transient temperature distributions and they are quite significant. Such conditions are encountered in - manufacture of ceramics, bricks, glass and heat flow to boiler tubes, metal forming, heat treatment, etc.

Let us consider an initially heated long cylinder ( $L \gg R$ ) placed in a moving stream of fluid at  $T_\infty < T_s$ , as shown in Fig. 3.1(a). The convective heat transfer coefficient at the surface is  $h$ , where,

## 6.2. Biot and Fourier Modulus-Definition and Significance

$$Q = hA (T_s - T_\infty)$$

This energy must be conducted to the surface, and therefore,

$$Q = -kA(dT / dr)_{r=R}$$

$$\text{or, } h(T_s - T_\infty) = -k(dT/dr)_{r=R} \approx -k(T_c - T_s)/R$$

where  $T_c$  is the temperature at the axis of the cylinder

$$\text{By rearranging, } (T_s - T_c) / (T_s - T_\infty) = hR/k \quad (3.1)$$

The term,  $hR/k$ , is called the 'BIOT MODULUS'. It is a dimensionless number and is the ratio of internal heat flow resistance to external heat flow resistance and plays a fundamental role in transient conduction problems involving surface convection effects. It provides a measure of the temperature drop in the solid relative to the temperature difference between the surface and the fluid.

For  $Bi \ll 1$ , it is reasonable to assume a uniform temperature distribution across a solid at any time during a transient process.

Founer Modulus - It is also a dimensionless number and is defined as

$$Fo = \alpha t / L^2 \quad (3.2)$$

where  $L$  is the characteristic length of the body,  $\alpha$  is the thermal diffusivity, and  $t$  is the time

The Fourier modulus measures the magnitude of the rate of conduction relative to the change in temperature, i.e., the unsteady effect. If  $Fo \ll 1$ , the change in temperature will be experienced by a

region very close to the surface.

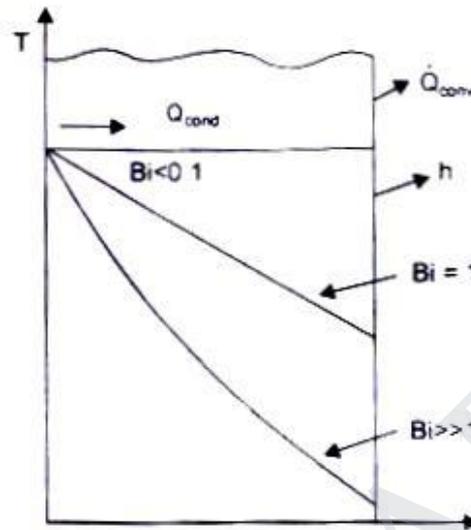


Fig. 1.7 Effect of Biot Modulus on steady state temperature distribution in a plane wall with surface convection.

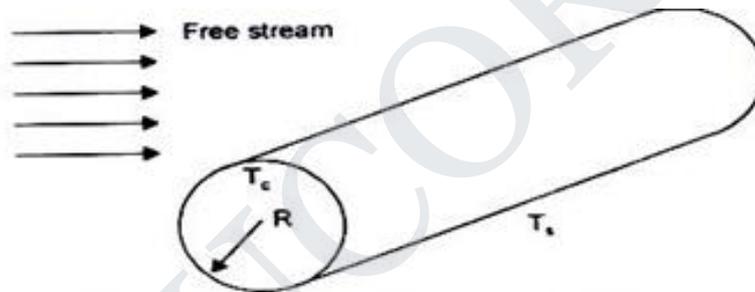


Fig. 1.8 (a) Nomenclature for Biot Modulus

## 7.1 LUMPED HEAT ANALYSIS:

### Lumped Capacity System-Necessary Physical Assumptions

We know that a temperature gradient must exist in a material if heat energy is to be conducted into or out of the body. When  $Bi < 0.1$ , it is assumed that the internal thermal resistance of the body is very small in comparison with the external resistance and the transfer of heat energy is primarily controlled by the convective heat transfer at the surface. That is, the temperature within the body is approximately uniform. This idealised assumption is possible, if

- the physical size of the body is very small,
- the thermal conductivity of the material is very large, and
- the convective heat transfer coefficient at the surface is very small and there is a large

ME8693 HEAT AND MASS TRANSFER temperature difference across the fluid layer at the interface.

## 7.2 An Expression for Evaluating the Temperature Variation in a Solid Using Lumped Capacity Analysis

Let us consider a small metallic object which has been suddenly immersed in a fluid during a heat treatment operation. By applying the first law of

Heat flowing out of the body = Decrease in the internal thermal energy of  
during a time  $dt$  the body during that time  $dt$

$$\text{or, } hA_s(T - T_\infty)dt = -\rho CVdT$$

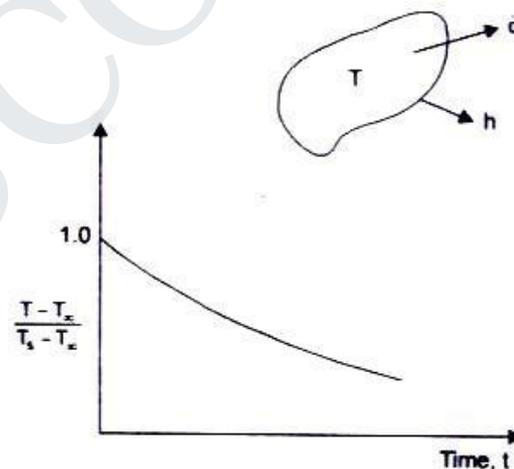
where  $A_s$  is the surface area of the body,  $V$  is the volume of the body and  $C$  is the specific heat capacity.

$$\text{or, } (hA / \rho CV)dt = -dT / (T - T_\infty)$$

with the initial condition being: at  $t = 0$ ,  $T = T_s$

The solution is :  $(T - T_\infty) / (T_s - T_\infty) = \exp(-hA / \rho CV)t$

Fig. depicts the cooling of a body (temperature distribution  $\square$  time) using lumped thermal capacity system. The temperature history is seen to be an exponential decay.



We can express

$$Bi \times Fo = (hL/k) \times (\alpha t/L^2) = (hL/k)(k/\rho C)(t/L^2) = (hA/\rho CV)t,$$

where  $V/A$  is the characteristic length  $L$ .

And, the solution describing the temperature variation of the object with respect to time is given

$$(T - T_\infty) / (T_s - T_\infty) = \exp(-Bi \cdot Fo)$$

## 8 .SEMI INFINITE AND INFINITE SOLIDS:

A semi-infinite solid is an idealized body that has a *single plane surface* and extends to infinity in all directions, as shown in .This idealized body is used to indicate that the temperature change in the part of the body in which we are interested (the region close to the surface) is due to the thermal conditions on a single surface. The earth, for example, can be considered to be a semi-infinite medium in determining the variation of temperature near its surface. Also, a thick wall can be modeled as a semi-infinite medium if all we are interested in is the variation of temperature in the region near one of the surfaces, and the other surface is too far to have any impact on the region of interest during the time of observation. The temperature in the core region of the wall remains unchanged in this case.

For short periods of time, most bodies can be modeled as semi-infinite solids since heat does not have sufficient time to penetrate deep into the body, and the thickness of the body does not enter into the heat transfer analysis. A steel piece of any shape, for example, can be treated as a semi-infinite solid when it is quenched rapidly to harden its surface. A body whose surface is heated by a laser pulse can be treated the same way.

Consider a semi-infinite solid with constant thermo physical properties ,no internal heat generation, uniform thermal conditions on its exposed surface, and initially a uniform temperature of  $T_i$  throughout. Heat transfer in this case occurs only in the direction normal to the surface (the  $x$  direction), and thus it is one-dimensional. Differential equations are independent of the boundary or initial conditions, and thus for one-dimensional transient conduction in Cartesian coordinates applies. The depth of the solid is large ( $x \rightarrow \infty$ ) compared to the depth that heat can penetrate, and these phenomena can be expressed mathematically as a boundary condition as  $T(x \rightarrow \infty, t) = T_i$ .

Heat conduction in a semi-infinite solid is governed by the thermal condition imposed on the exposed surface, and thus the solution depends strongly on the boundary condition at  $x = 0$ . Below we present a detailed analytical solution for the case of constant temperature  $T_s$  on the surface, and give the results for other more complicated boundary conditions. When the surface temperature is changed to  $T_s$  at  $t = 0$  and held constant at that value at all times, the formulation of the problem The separation of variables technique does not work in this case since the medium is infinite. But another clever approach that converts the partial differential equation into an ordinary differential equation by combining the two independent variables  $x$  and  $t$  into a single variable  $\eta$ , called the **similarity variable**, works well. For transient conduction in a semi-infinite medium, it is defined as Similarity variable.

### 9.USE OF HEISLER CHARTS :

There are three charts, associated with different geometries. For a plate/wall (Cartesian geometry) the Heisler chart

The first chart is to determine the temperature at the center  $0 T$  at a given time.

By having the temperature at the center  $0 T$  at a given time, the second chart is to determine the

The third chart is to determine the total amount of heat transfer up to the time.

STUCOR APP

## PART A

**1. Define Heat Transfer.****(AU 2010)**

Heat transfer can be defined as transmission of energy from one region to another region due to temperature difference.

**2. What are the modes of heat transfer?**

Conduction,

Convection, Radiation.

**3. State Fourier law of conduction.****(AU2014)**

The rate of heat conduction is proportional to the area measured normal to the direction of heat flow and to the temperature gradient in that direction.

$$\dot{Q} = -kAdT/dx$$

**4. Define Thermal Conductivity.****(AU2013)**

Thermal conductivity is defined as the ability of a substance to conduct heat.

**5. Write down the three dimensional heat conduction equations in cylindrical coordinates.**

The general three dimensional heat conduction equation in cylindrical coordinates

$$\partial^2 T / \partial r^2 + 1/r \partial T / \partial r + 1/r^2 \partial^2 T / \partial \phi^2 + \partial^2 T / \partial z^2 + q/k = 1/\alpha \partial T / \partial \theta.$$

**6. List down the three types of boundary conditions.****(AU 2011)**

1. Prescribed temperature.
2. Prescribed heat flux.
3. Convection boundary conditions.

**7. State Newtons law of cooling.****(AU2010)**

Heat transfer by convection is given by Newtons law of cooling

$$Q = h A (T_s - T_\infty)$$

Where A- Area exposed to heat transfer in  $m^2$

h- Heat transfer coefficient in  $W/m^2 K$

T- Temperature of the surface and fluid in K.

**8. What is meant by lumped heat analysis?****(AU 2013)**

In a Newtonian heating or cooling process the temperature throughout the solid is considered to be uniform at a given time. Such an analysis is called lumped heat capacity analysis.

**9. Define fin efficiency and fin effectiveness.****(AU 2012)**

The efficiency of a fin is defined as the ratio of actual heat transfer to the maximum possible heat transferred by the fin.

$$\eta_{\text{fin}} = Q_{\text{fin}} / Q_{\text{max}}$$

Fin effectiveness is the ratio of heat transfer with fin to that without fin.

$$\text{Fin effectiveness} = Q_{\text{with fin}} / Q_{\text{without fin}}$$

**10. What is critical radius of insulation? (AU 2010)**

Addition of insulating material on a surface does not reduce the amount of heat transfer rate always. In fact under certain circumstances it actually increases the heat loss up to certain thickness of insulation. The radius of insulation for which the heat transfer is maximum is called critical radius of insulation.

**11. Write the Poisson's equation for heat conduction.**

When the temperature is not varying with respect to time, then the conduction is called as steady state condition.

$$\partial T / \partial \tau = 0.$$

**12. What is heat generation in solids? Give examples.**

The rate of energy generation per unit volume is known as internal heat generation in solids.

Examples: 1. Electric coils 2. Resistance heater 3. Nuclear reactor.

**13. A 3 mm wire of thermal conductivity 19 W / m K at a steady heat generation of 500 MW / m<sup>3</sup>. Determine the centre temperature is maintained at 25°C. (AU 2013)**

Solution;

Critical / Centre temperature,

$$T_c = T_{\infty} + qr^2/4K$$

$$= 298 + 500 \times 10^6 \times (0.015)^2$$

$$= 298 + 14.8$$

$$T_c = 312.8 \text{ K.}$$

**14. What are the factors affecting the thermal conductivity?**

Moisture,

Density of material,

Pressure,

Temperatures.

**15. What are Heislercharts ?****(AU 2009)**

In Heisler chart, the solutions for temperature distributions and heat flow in plane walls, long cylinders and spheres with finite internal and surface resistance are presented. Heisler chart nothing but a analytical solution in the form of graphs.

**Part B****ANSWER THE FOLLOWING:**

1. A composite wall consists of three layers of thicknesses 300 mm, 200mm and 100mm with thermal conductivities 1.5, 3.5 and is W/m K respectively. The inside surface is exposed to gases at 1200°C with convection heat transfer coefficient as 30W/m<sup>2</sup>K. The temperature of air on the other side of the wall is 30°C with convective heat transfer coefficient 10 Wm<sup>2</sup>K. If the temperature at the outside surface of the wall is 180°C, calculate the temperature at other surface of the wall, the rate of heat transfer and the overall heat transfer coefficient.

**Solution:** The composite wall and its equivalent thermal circuits is shown in the figure.

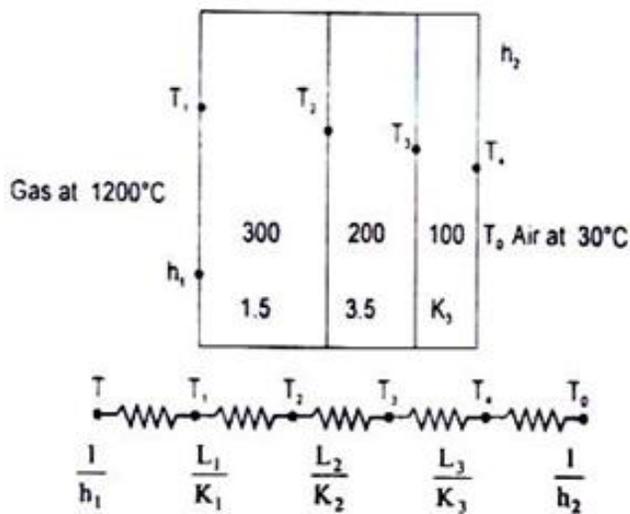


Fig 1.1

The heat energy will flow from hot gases to the cold air through the wall.

From the electric Circuit, we have

$$\dot{Q}/A = h_2(T_4 - T_0) = 10 \times (180 - 30) = 1500 \text{ W/m}^2$$

also,  $\dot{Q}/A = h_1(1200 - T_1)$

$$T_1 = 1200 - 1500/30 = 1150^\circ\text{C}$$

$$\dot{Q}/A = (T_1 - T_2)/L_1/k_1$$

$$T_2 = T_1 - 1500 \times 0.3/1.5 = 850$$

Similarly,  $\dot{Q}/A = (T_2 - T_3)/(L_2/k_2)$

$$T_3 = T_2 - 1500 \times 0.2/3.5 = 764.3^\circ\text{C}$$

And  $\dot{Q}/A = (T_3 - T_4)/(L_3/k_3)$

$$L_3/k_3 = (764.3 - 180)/1500 \text{ And } k_3 = 0.256 \text{ W/mK}$$

$$\dot{Q}/A = (1200 - 30)/\Sigma R;$$

Where  $\Sigma R = 1/h_1 + L_1/k_1 + L_2/k_2 + L_3/k_3 + 1/h_2$

$$\Sigma R = 1/30 + 0.3/1.5 + 0.2/3.5 + 0.1/0.256 + 1/10 = 0.75$$

$$\text{And } \dot{Q}/A = 1170/0.78 = 1500 \text{ W/m}^2$$

The overall heat transfer coefficient,  $U = 1/\Sigma R = 1/0.78 = 1.282 \text{ W/m}^2\text{K}$

Since the gas temperature is very high, we should consider the effects of radiation also. Assuming the heat transfer coefficient due to radiation =  $3.0 \text{ W/m}^2\text{K}$  the electric circuit would be:

The combined resistance due to convection and radiation would be

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{\frac{1}{h_c}} + \frac{1}{\frac{1}{h_r}} = h_c + h_r = 60 \text{ W/m}^2\text{C}$$

$$\therefore \dot{Q}/A = 1500 = 60(T - T_1) = 60(1200 - T_1)$$

$$\therefore T_1 = 1200 - \frac{1500}{60} = 1175^\circ\text{C}$$

$$\text{Again, } \therefore \dot{Q}/A = (T_1 - T_2)/L_1/k_1 \Rightarrow T_2 = T_1 - 1500 \times 0.3/1.5 = 875^\circ\text{C}$$

$$\text{And } T_3 = T_2 - 1500 \times 0.2/3.5 = 789.3^\circ\text{C}$$

$$L_3/k_3 = (789.3 - 180)/1500; \therefore k_3 = 0.246 \text{ W/mK}$$

$$\Sigma R = \frac{1}{60} + \frac{0.3}{1.5} + \frac{0.2}{1.5} + \frac{0.2}{3.5} + \frac{0.1}{0.246} + \frac{1}{10} = 0.78$$

$$\text{And } U = 1/\Sigma R = 1.282 \text{ W/m}^2\text{K}.$$

## 2. Derive the General Heat Conduction Equation for an Isotropic Solid with Constant Thermal Conductivity in Cartesian coordinates. (AU 2014, 2013)

Any physical phenomenon is generally accompanied by a change in space and time of its physical properties. The heat transfer by conduction in solids can only take place when there is a variation of temperature, in both space and time. Let us consider a small volume of a solid

.2. The dimensions are:  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$  along the X-, Y-, and Z- element as shown in Fig. 1 coordinates.

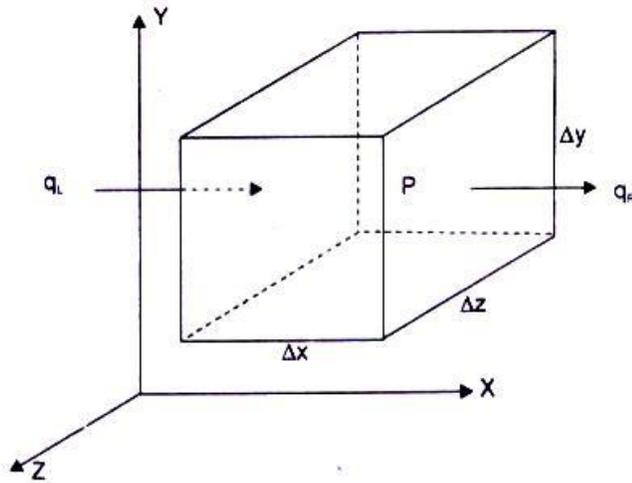


Fig 1.2 Elemental volume in Cartesian coordinates

First we consider heat conduction the X-direction. Let T denote the temperature at the point P (x, y, z) located at the geometric centre of the element. The temperature gradient at the left hand face (x -  $\Delta x/2$ ) and at the right hand face (x +  $\Delta x/2$ ), using the Taylor's series, can be written as:

$$\frac{\partial T}{\partial x} \Big|_L = \frac{\partial T}{\partial x} - \frac{\partial^2 T}{\partial x^2} \cdot \frac{\Delta x}{2} + \text{Higher order terms.}$$

$$\frac{\partial T}{\partial x} \Big|_R = \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \cdot \frac{\Delta x}{2} + \text{Higher order terms.}$$

The net rate at which heat is conducted out of the element in X-direction assuming k as constant and neglecting the higher order terms,

$$\text{We get } -k\Delta y\Delta z \left[ \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \frac{\Delta x}{2} - \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \frac{\Delta x}{2} \right] = -k\Delta y\Delta z\Delta x \left( \frac{\partial^2 T}{\partial x^2} \right)$$

Similarly for Y- and Z-direction,

$$\text{We have } -k\Delta x\Delta y\Delta z \frac{\partial^2 T}{\partial y^2} \text{ and } -k\Delta x\Delta y\Delta z \frac{\partial^2 T}{\partial z^2}.$$

If there is heat generation within the element as Q, per unit volume and the internal energy of the element changes with time, by making an energy balance, we write

Heat generated within Heat conducted away. Rate of change of internal

The element from the element energy within with the element

$$\text{Or, } \dot{Q}_v (\Delta x \Delta y \Delta z) + k (\Delta x \Delta y \Delta z) (\partial^2 T / \partial x^2 + \partial^2 T / \partial y^2 + \partial^2 T / \partial z^2) \\ = \rho c (\Delta x \Delta y \Delta z) \partial T / \partial t$$

Upon simplification,  $\partial^2 T / \partial x^2 + \partial^2 T / \partial y^2 + \partial^2 T / \partial z^2 + \dot{Q}_v / k = \frac{\rho c}{k} \partial T / \partial t$

Or,  $\nabla^2 T + \dot{Q}_v / k = 1 / \alpha (\partial T / \partial t)$

Where  $\alpha = k / \rho . c$ , is called the thermal diffusivity and is seen to be a physical property of the material of which the solid is composed.

The Eq. (2.1a) is the general heat conduction equation for an isotropic solid with a constant thermal conductivity. The equation in cylindrical (radius r, axis Z and longitude  $\theta$ ) coordinates is written as:

$$\partial^2 T / \partial r^2 + (1/r) \partial T / \partial r + (1/r^2) \partial^2 T / \partial \theta^2 + \partial^2 T / \partial z^2 + \dot{Q}_v / k = 1 / \alpha \partial T / \partial t \quad (2.1b)$$

And, in spherical polar coordinates Fig. 2.1(c) (radius,  $\theta$  longitude,  $\phi$  colatitudes) is

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} + \frac{\dot{Q}_v}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (2.1c)$$

Under steady state or stationary condition, the temperature of a body does not vary with time, i.e.  $\partial T / \partial t = 0$ . And, with no internal generation, the equation (2.1) reduces to

$$\nabla^2 T = 0$$

It should be noted that Fourier law can always be used to compute the rate of heat transfer by conduction from the knowledge of temperature distribution even for unsteady condition and with internal heat generation.

**3. A 20 cm thick slab of aluminums ( $k = 230 \text{ W/mK}$ ) is placed in contact with a 15 cm thick stainless steel plate ( $k = 15 \text{ W/mK}$ ). Due to roughness, 40 percent of the area is in direct contact and the gap (0.0002 m) is filled with air ( $k = 0.032 \text{ W/mK}$ ). The difference in temperature between the two outside surfaces of the plate is  $200^\circ\text{C}$  Estimate (i) the heat flow rate, (ii) the contact resistance, and (iii) the drop in temperature at the interface.**

(AU 2010)

**Solution:** Let us assume that out of 40% area in direct contact, half the surface area is occupied by steel and half is occupied by aluminums.

The physical system and its analogous electric circuits is shown in Fig. 1.3.

$$R_1 = \frac{0.2}{230 \times 1} = 0.00087, \quad R_2 = \frac{0.0002}{230 \times 0.2} = 4.348 \times 10^{-6}$$

$$R_3 = \frac{0.0002}{0.032 \times 0.6} = 1.04 \times 10^{-2}, \quad R_4 = \frac{0.0002}{15 \times 0.2} = 6.667 \times 10^{-5}$$

And  $R_5 = \frac{0.15}{(15 \times 1)} = 0.01$

Again  $1/R_{2,3,4} = 1/R_2 + 1/R_3 + 1/R_4$

$$= 2.3 \times 10^5 + 96.15 + 1.5 \times 10^4 = 24.5 \times 10^4$$

Therefore,  $R_{2,3,4} = 4.08 \times 10^{-6}$

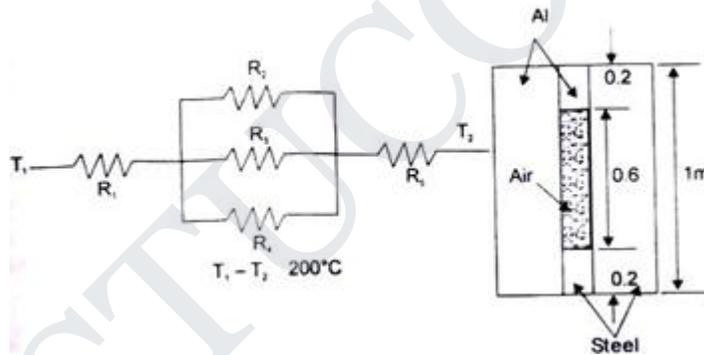


Fig 1.3

Total resistance,  $\Sigma R = R_1 + R_{2,3,4} + R_5$

$$= 870 \times 10^{-6} + 4.08 \times 10^{-6} + 1000 \times 10^{-6} = 1.0874 \times 10^{-2}$$

Heat flow rate,  $\dot{Q} = 200 / 1.087 \times 10^{-2} = 18.392 \text{ kW per unit depth of the plate.}$

Contact resistance,  $R_{R_{2,3,4}} = 4.08 \times 10^{-6} \text{ mK/W}$

Drop in temperature at the interface,  $\Delta T = 4.08 \times 10^{-6} \times 18392 = 0.075^\circ\text{C}$

**4. A steel pipe. Inside diameter 100 mm, outside diameter 120 mm (k 50 W/m K) IS Insulated with a 40mm thick high temperature Insulation (k = 0.09 W/m K) and another Insulation 60 mm thick (k = 0.07 W/m K). The ambient temperature IS 25°C. The heat transfer coefficient for the inside and outside surfaces are 550 and 15 W/m<sup>2</sup>K respectively. The pipe carries steam at 300°C. Calculate (1) the rate of heat loss by steam per unit length of the pipe (2) the temperature of the outside surface .**

**Solution:** A cross-section of the pipe with two layers of insulation is shown Fig. 1.4 with its analogous electrical circuit.

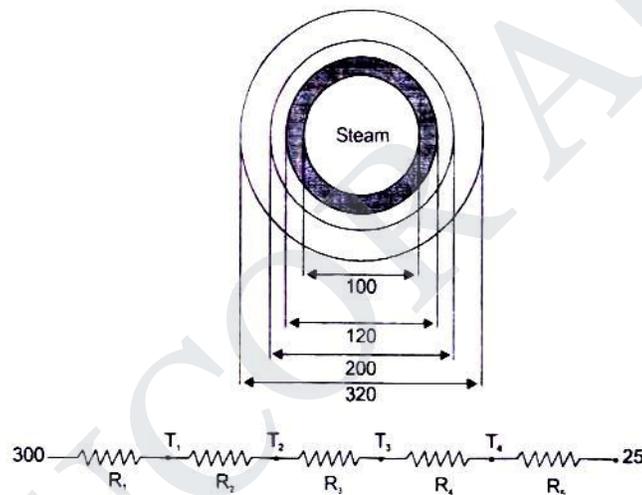


Fig 1.4A cross section through an insulated cylinder, thermal resistances in series.

For L = 1.0 m. we have

$$R_1, \text{ the resistance of steam film} = \frac{1}{h_1 A} = \frac{1}{(550 \times 2\pi \times 50 \times 10^{-3})} = 0.00579$$

$$R_2, \text{ the resistance of steel pipe} = \frac{\ln(r_2/r_1)}{2\pi k}$$

$$= \frac{\ln(60/50)}{2\pi \times 50} = 0.00058$$

R<sub>3</sub>, resistance of high temperature Insulation

$$\frac{\ln(r_3/r_2)}{2\pi k} = \frac{\ln(100/60)}{2\pi \times 0.09} = 0.903$$

$$R_4 = \frac{\ln(r_4/r_3)}{2\pi k} = \frac{\ln(160/100)}{2\pi \times 0.07} = 1.068$$

$$R_5 = \text{resistance of the air film} = \frac{1}{(15 \times 2\pi \times 160 \times 10^{-3})} = 0.0663$$

The total resistance = 2.04367

And  $\dot{Q} = \Delta T / \Sigma R = (300 - 25) / 204367 = 134.56$  W per meter length of pipe.

Temperature at the outside surface.  $T_4 = 25 + R_5 \dot{Q} = 25 + 134.56 \times 0.0663 = 33.92^\circ \text{C}$

When the better insulating material ( $k = 0.07$ , thickness 60 mm) is placed first on the steel pipe, the new value of  $R_3$  would be

$$R_3 = \ln(120/60) / 2\pi \times 0.07 = 1.576 ; \text{ and the new value of } R_4 \text{ will be}$$

$$R_4 = \ln(160/120) / 2\pi \times 0.09 = 0.5087$$

The total resistance = 2.15737 and  $Q = 275/2.15737 = 127.47$  W per m length (Thus the better insulating material be applied first to reduce the heat loss.) The overall heat transfer coefficient,  $U$ , is obtained as  $U = \dot{Q} / A \Delta T$

The outer surface area =  $\pi \times 320 \times 10^{-3} \times 1 = 1.0054$

$$\text{And } U = 134.56 / (275 \times 1.0054) = 0.487 \text{ W/m}^2 \text{ K.}$$

**5. Steel balls 10 mm in diameter ( $k = 48$  W/mK), ( $C = 600$  J/kgK) are cooled in air at temperature  $35^\circ\text{C}$  from an initial temperature of  $750^\circ\text{C}$ . Calculate the time required for the temperature to drop to  $150^\circ\text{C}$  when  $h = 25$  W/m<sup>2</sup>K and density  $\rho = 7800$  kg/m<sup>3</sup>. (AU 2012).**

**Solution:** Characteristic length,  $L = V/A = 4/3 \pi r^3 / 4 \pi r^2 = r/3 = 5 \times 10^{-3} / 3 \text{ m}$

$$Bi = hL/k = 25 \times 5 \times 10^{-3} / (3 \times 48) = 8.68 \times 10^{-4} \ll 0.1,$$

Since the internal resistance is negligible, we make use of lumped capacity analysis: Eq. (3.4),

$$(T - T_\infty) / (T_s - T_\infty) = \exp(-Bi Fo) ; (150 - 35) / (750 - 35) = 0.16084$$

$$\therefore Bi \times Fo = 1.827 ; Fo = 1.827 / (8.68 \times 10^{-4}) = 2.1 \times 10^3$$

$$\text{Or, } \alpha t / L^2 = k / (\rho C L^2) t = 2100 \text{ and } t = 568 = 0.158 \text{ hour}$$

We can also compute the change in the internal energy of the object as:

$$\begin{aligned} U_0 - U_t &= -\int_0^1 \rho C V dT = \int_0^1 \rho C V (T_s - T_\infty) (-hA / \rho C V) \exp t (-hAt / \rho C V) dt \\ &= -\rho C V (T_s - T_\infty) [\exp(-hAt / \rho C V) - 1] \end{aligned} \quad (3.5)$$

$$= -7800 \times 600 \times (4/3) \pi (5 \times 10^{-3})^3 (750-35) (0.16084 - 1)$$

$$= 1.47 \times 10^3 \text{ J} = 1.47 \text{ kJ.}$$

If we allow the time 't' to go to infinity, we would have a situation that corresponds to steady state in the new environment. The change in internal energy will be  $U_0 - U_\infty = [\rho CV(T_s - T_\infty) \exp(-\infty) - 1] = [\rho CV(T_s - T_\infty)]$ .

We can also compute the instantaneous heat transfer rate at any time.

$$\text{Or. } Q = -\rho VC dT/dt = -\rho VC d/dt [T_\infty + (T_s - T_\infty) \exp(-hAt/\rho CV)]$$

$$= hA(T_s - T_\infty) [\exp(-hAt/\rho CV)] \text{ and for } t = 60\text{s,}$$

$$Q = 25 \times 4 \times 3.142 (5 \times 10^{-3})^2 (750 - 35) [\exp(-25 \times 3 \times 60 / 5 \times 10^{-3} \times 7800 \times 600)]$$

$$= 4.63 \text{ W.}$$

**6. Aluminums fins 1.5 cm wide and 10 mm thick are placed on a 2.5 cm diameter tube to dissipate the heat. The tube surface temperature is 170°C ambient temperatures is 20°C. calculate the heat loss per fin. Take  $h = 130 \text{ W/m}^2 \text{ C}$  and  $K = 200 \text{ W/m}^2 \text{ C}$  for aluminums.**

### Given

$$\text{Wide of the fin } b = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}$$

$$\text{Thickness } t = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$$

$$\text{Diameter of the tube } d = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$$

$$\text{Surface temperature } T_b = 170^\circ\text{C} + 273 = 443 \text{ K}$$

$$\text{Ambient temperature } T_\infty = 20^\circ\text{C} + 273 = 293 \text{ K}$$

$$\text{Heat transfer co-efficient } h = 130 \text{ W/m}^2\text{C}$$

$$\text{Thermal conductivity } K = 200 \text{ W/m}^2\text{C}$$

### Solution

Assume fin end is insulated, so this is short fin end insulated type problem.

Heat transfer [short fin, end insulated]

$$Q = (hPKA)^{1/2} (T_b - T_\infty) \tan h(mL) \dots\dots(1) \text{ [From HMT data book Page No.41]}$$

Where

A – Area = Breadth  $\times$  thickness

$$= 1.5 \times 10^{-2} \times 10 \times 10^{-3}$$

$$\boxed{A = 1.5 \times 10^{-4} \text{ m}^2}$$

P – Perimeter = 2(b + t)

$$= 2[(1.5 \times 10^{-2}) + (10 \times 10^{-3})]$$

$$\boxed{P = 0.05 \text{ m}}$$

$$m = \sqrt{\frac{hP}{KA}}$$

$$= \sqrt{\frac{130 \times 0.05}{200 \times 1.5 \times 10^{-4}}}$$

$$\boxed{m = 14.7 \text{ m}^{-1}}$$

$$(1) \Rightarrow Q = [130 \times 0.05 \times 200 \times 1.5 \times 10^{-4}]^{1/2} \\ \times (443 - 293) \times \tanh (14.7 \times 1.5 \times 10^{-2})$$

$$\boxed{Q = 14.3 \text{ W}}$$

**UNIT 2****CONVECTION****2.1 Convection Heat Transfer-Requirements**

The heat transfer by convection requires a solid-fluid interface, a temperature difference between the solid surface and the surrounding fluid and a motion of the fluid. The process of heat transfer by convection would occur when there is a movement of macro-particles of the fluid in space from a region of higher temperature to lower temperature.

**2.2. Convection Heat Transfer Mechanism**

Let us imagine a heated solid surface, say a plane wall at a temperature  $T_w$  placed in an atmosphere at temperature  $T_\infty$ , Fig. 2.1 Since all real fluids are viscous, the fluid particles adjacent to the solid surface will stick to the surface. The fluid particle at A, which is at a lower temperature, will receive heat energy from the plate by conduction. The internal energy of the particle would increase and when the particle moves away from the solid surface (wall or plate) and collides with another fluid particle at B which is at the ambient temperature, it will transfer a part of its stored energy to B. And, the temperature of the fluid particle at B would increase. This way the heat energy is transferred from the heated plate to the surrounding fluid. Therefore the process of heat transfer by convection involves a combined action of heat conduction, energy storage and transfer of energy by mixing motion of fluid particles.

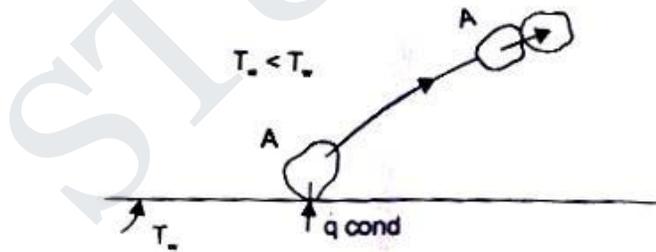


Fig. 2.1 Principle of heat transfer by convection

**2.3. Free and Forced Convection**

When the mixing motion of the fluid particles is the result of the density difference

caused by a temperature gradient, the process of heat transfer is called natural or free convection. When the mixing motion is created by an artificial means (by some external agent), the process of heat transfer is called forced convection. Since the effectiveness of heat transfer by convection depends largely on the mixing motion of the fluid particles, it is essential to have a knowledge of the characteristics of fluid flow.

## 2.4. Basic Difference between Laminar and Turbulent Flow

In laminar or streamline flow, the fluid particles move in layers such that each fluid particle follows a smooth and continuous path. There is no macroscopic mixing of fluid particles between successive layers, and the order is maintained even when there is a turn around a corner or an obstacle is to be crossed. If a time dependent fluctuating motion is observed in directions which are parallel and transverse to the main flow, i.e., there is a random macroscopic mixing of fluid particles across successive layers of fluid flow, the motion of the fluid is called 'turbulent flow'. The path of a fluid particle would then be zigzag and irregular, but on a statistical basis, the overall motion of the macroparticles would be regular and predictable.

## 2.5. Formation of a Boundary Layer

When a fluid flow, over a surface, irrespective of whether the flow is laminar or turbulent, the fluid particles adjacent to the solid surface will always stick to it and their velocity at the solid surface will be zero, because of the viscosity of the fluid. Due to the shearing action of one fluid layer over the adjacent layer moving at the faster rate, there would be a velocity gradient in a direction normal to the flow.

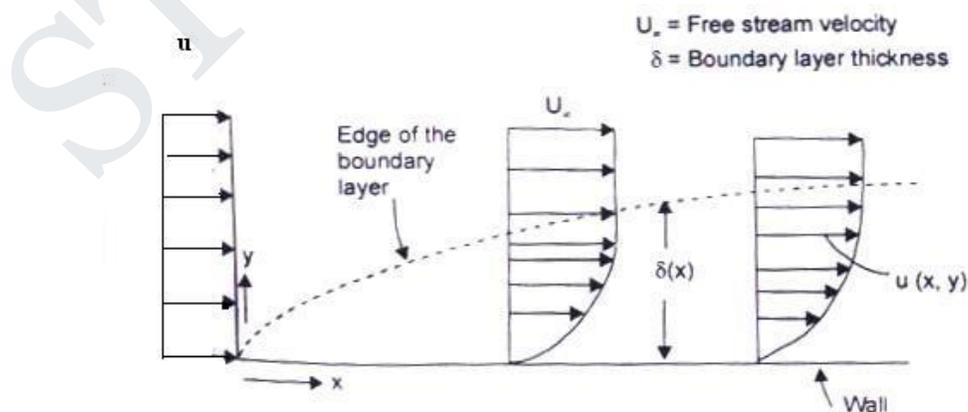


Fig 2.2: sketch of a boundary layer on a wall

Let us consider a two-dimensional flow of a real fluid about a solid (slender in cross-section) as shown in Fig. 2.2. Detailed investigations have revealed that the velocity of the fluid particles at the surface of the solid is zero. The transition from zero velocity at the surface of the solid to the free stream velocity at some distance away from the solid surface in the V-direction (normal to the direction of flow) takes place in a very thin layer called 'momentum or hydrodynamic boundary layer'. The flow field can thus be divided in two regions:

(i) A very thin layer in the vicinity of the body where a velocity gradient normal to the direction of flow exists, the velocity gradient  $du/dy$  being large. In this thin region, even a very small Viscosity  $\mu$  of the fluid exerts a substantial Influence and the shearing stress  $\tau = \mu du/dy$  may assume large values. The thickness of the boundary layer is very small and decreases with decreasing viscosity.

(ii) In the remaining region, no such large velocity gradients exist and the Influence of viscosity is unimportant. The flow can be considered frictionless and potential.

## 2.6. Thermal Boundary Layer

Since the heat transfer by convection involves the motion of fluid particles, we must superimpose the temperature field on the physical motion of fluid and the two fields are bound to interact. It is intuitively evident that the temperature distribution around a hot body in a fluid stream will often have the same character as the velocity distribution in the boundary layer flow. When a heated solid body is placed in a fluid stream, the temperature of the fluid stream will also vary within a thin layer in the immediate neighbourhood of the solid body. The variation in temperature of the fluid stream also takes place in a thin layer in the neighbourhood of the body and is termed 'thermal boundary layer'. Fig. shows the temperature profiles inside a thermal boundary layer.

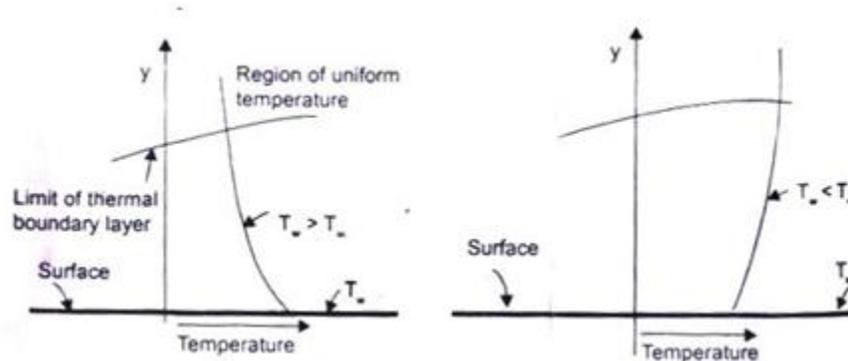


Fig: The thermal boundary layer

## 2.7 HYDRODYNAMIC BOUNDARY LAYER :

One of the most important concepts in understanding the external flows is the boundary layer development. For simplicity, we are going to analyze a boundary layer flow over a flat plate with no curvature and no external pressure variation.

- Boundary layer thickness ( $\delta$ ): defined as the distance away from the surface where the local velocity reaches to 99% of the free-stream velocity, that is  $u(y=\delta)=0.99U_\infty$ . Somewhat an easy to understand but arbitrary definition.
- Boundary layer is usually very thin:  $\delta/x$  usually  $\ll 1$ .
- As we have seen earlier, the hydrodynamic boundary layer is a region of a fluid flow, near a solid surface, where the flow patterns are directly influenced by viscous drag from the surface wall.
- $0 < u < U$ ,  $0 < y < \delta$
- The Thermal Boundary Layer is a region of a fluid flow, near a solid surface, where the fluid temperatures are directly influenced by heating or cooling from the surface wall.
- $0 < t < T$ ,  $0 < y < \delta_t$
- *The two boundary layers may be expected to have similar characteristics but do not normally coincide.* Liquid metals tend to conduct heat from the wall easily and temperature changes are observed well outside the dynamic boundary layer. Other materials tend to show velocity changes well outside the thermal layer.

**2.8 FREE AND FORCED CONVECTION DURING EXTERNAL FLOW OVER PLATES**

:

**Dimensionless Parameters and their Significance**

The following dimensionless parameters are significant in evaluating the convection heat transfer coefficient:

(a) *The Nusselt Number (Nu)*-It is a dimensionless quantity defined as  $hL/k$ , where  $h$  = convective heat transfer coefficient,  $L$  is the characteristic length and  $k$  is the thermal conductivity of the fluid. The Nusselt number could be interpreted physically as the ratio of the temperature gradient in the fluid immediately in contact with the surface to a reference temperature gradient  $(T_s - T_\infty)/L$ . The convective heat transfer coefficient can easily be obtained if the Nusselt number, the thermal conductivity of the fluid in that temperature range and the characteristic dimension of the object is known.

Let us consider a hot flat plate (temperature  $T_w$ ) placed in a free stream (temperature  $T_\infty < T_w$ ). The temperature distribution is shown in Fig. 2.4. Newton's Law of Cooling says that the rate of heat transfer per unit area by convection is given by

$$\dot{Q}/A = h(T_w - T_\infty)$$

$$\frac{\dot{Q}}{A} = h(T_w - T_\infty)$$

$$= k \frac{T_w - T_\infty}{\delta_t}$$

$$h = \frac{k}{\delta_t}$$

$$Nu = \frac{hL}{k} = \frac{L}{\delta_t}$$

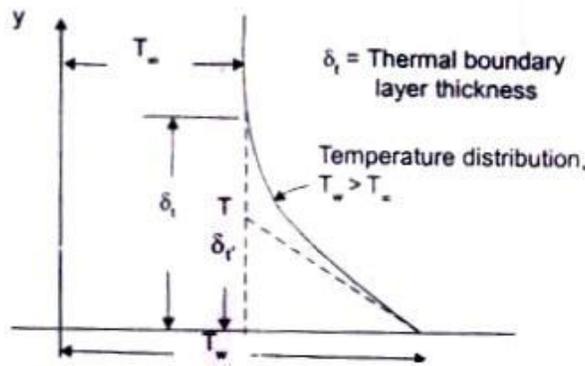


Fig. 2.4 Temperature distribution in a boundary layer: Nusselt modulus

The heat transfer by convection involves conduction and mixing motion of fluid particles. At the solid fluid interface ( $y = 0$ ), the heat flows by conduction only, and is given by

$$\frac{\dot{Q}}{A} = -k \left( \frac{dT}{dy} \right)_{y=0} \quad \therefore h = \frac{\left( -k \frac{dT}{dy} \right)_{y=0}}{(T_w - T_\infty)}$$

Since the magnitude of the temperature gradient in the fluid will remain the same, irrespective of the reference temperature, we can write  $dT = d(T - T_w)$  and by introducing a characteristic length dimension  $L$  to indicate the geometry of the object from which the heat flows, we get

$$\frac{hL}{k} = \frac{\left( \frac{dT}{dy} \right)_{y=0}}{(T_w - T_\infty)/L}, \text{ and in dimensionless form,}$$

$$= \left( \frac{d(T_w - T)/(T_w - T_\infty)}{d(y/L)} \right)_{y=0}$$

(b) *The Grashof Number (Gr)*-In natural or free convection heat transfer, the motion of fluid particles is created due to buoyancy effects. The driving force for fluid motion is the body force arising from the temperature gradient. If a body with a constant wall temperature  $T_w$  is exposed to a quiescent ambient fluid at  $T_\infty$ , the force per unit volume can be written as  $\rho g \beta (T_w - T_\infty)$  where  $\rho$  = mass density of the fluid,  $\beta$  = volume coefficient of expansion and  $g$  is the acceleration due to gravity.

The ratio of inertia force  $\times$  Buoyancy force/(viscous force)<sup>2</sup> can be written as

$$\text{Gr} = \frac{(\rho V^2 L^2) \times \rho g \beta (T_w - T_\infty) L^3}{(\mu V L)^2}$$

$$= \frac{\rho^2 g \beta (T_w - T_\infty) L^3}{\mu^2} = g \beta L^3 (T_w - T_\infty) / \nu^2$$

The magnitude of Grashof number indicates whether the flow is laminar or turbulent. If the Grashof number is greater than  $10^9$ , the flow is turbulent and for Grashof number less than  $10^8$ , the flow is laminar. For  $10^8 < \text{Gr} < 10^9$ , It is the transition range.

(c) *The Prandtl Number (Pr)* - It is a dimensionless parameter defined as

$$\text{Pr} = \mu C_p / k = \nu / \alpha$$

where  $\mu$  is the dynamic viscosity of the fluid,  $\nu$  = kinematic viscosity and  $\alpha$  = thermal diffusivity.

This number assumes significance when both momentum and energy are propagated through the system. It is a physical parameter depending upon the properties of the medium. It is a measure of the relative magnitudes of momentum and thermal diffusion in the fluid: That is, for  $\text{Pr} = 1$ , the rate of diffusion of momentum and energy are equal which means that the calculated temperature and velocity fields will be similar, the thickness of the momentum and thermal boundary layers will be equal. For  $\text{Pr} \ll 1$  (in case of liquid metals), the thickness of the thermal boundary layer will be much more than the thickness of the momentum boundary layer and vice versa. The product of Grashof and Prandtl number is called Rayleigh number. Or,  $\text{Ra} = \text{Gr} \times \text{Pr}$ .

## 2.9 Evaluation of Convective Heat Transfer Coefficient

The convective heat transfer coefficient in free or natural convection can be evaluated by two methods:

- (a) Dimensional Analysis combined with experimental investigations
- (b) Analytical solution of momentum and energy equations in the boundary layer.

### *Dimensional Analysis and Its Limitations*

Since the evaluation of convective heat transfer coefficient is quite complex, it is based on a combination of physical analysis and experimental studies. Experimental observations become necessary to study the influence of pertinent variables on the physical phenomena.

Dimensional analysis is a mathematical technique used in reducing the number of experiments to a minimum by determining an empirical relation connecting the relevant variables and in grouping the variables together in terms of dimensionless numbers. And, the method can only be applied after the pertinent variables controlling the phenomenon are identified and expressed in terms of the primary dimensions. (Table 1.1)

In natural convection heat transfer, the pertinent variables are:  $h$ ,  $\rho$ ,  $k$ ,  $\mu$ ,  $C_p$ ,  $L$ ,  $(\Delta T)$ ,  $\beta$  and  $g$ . Buckingham  $\pi$ 's method provides a systematic technique for arranging the variables in dimensionless numbers. It states that the number of dimensionless groups,  $\pi$ 's, required to describe a phenomenon involving 'n' variables is equal to the number of variables minus the number of primary dimensions 'm' in the problem.

In SI system of units, the number of primary dimensions are 4 and the number of variables for free convection heat transfer phenomenon are 9 and therefore, we should expect  $(9 - 4) = 5$  dimensionless numbers. Since the dimension of the coefficient of volume expansion,  $\beta$ , is  $\theta^{-1}$ , one dimensionless number is obviously  $\beta(\Delta T)$ . The remaining variables are written in a functional form:

$$\phi(h, \rho, k, \mu, C_p, L, g) = 0.$$

Since the number of primary dimensions are 4, we arbitrarily choose 4 independent variables as primary variables such that all the four dimensions are represented. The selected primary variables are:  $\rho, g, k, L$ . Thus the dimensionless group,

$$\pi_1 = \rho^a g^b k^c L^d h = (ML^{-3})^a (LT^{-2})^b (MLT^{-3}\theta^{-1}) = M^0 L^0 T^0 \theta^0$$

Equating the powers of M, L, T,  $\theta$  on both sides, we have

$$\left. \begin{aligned} M : a + c + 1 = 0 \\ L : -3a + b + c + d = 0 \\ T : -2b - 3c - 3 = 0 \\ \theta : -c - 1 = 0 \end{aligned} \right\} \text{Upon solving them,}$$

$$c = 1, b = a = 0 \text{ and } d = 1.$$

and  $\pi_1 = hL/k$ , the Nusselt number.

The other dimensionless number

$\pi_2 = \rho^a g^b k^c L^d C_p = (ML^{-3})^a (LT^{-2})^b (MLT^{-3} \theta^{-1})^c (L)^d (MT^{-1} \theta^{-1}) = M^0 L^0 T^0 \theta^0$  Equating the powers of M, L, T and  $\theta$  and upon solving, we get

$$\pi_3 = \mu^2 / \rho^2 g L^3$$

By combining  $\pi_2$  and  $\pi_3$ , we write  $\pi_4 = [\pi_2 \times \pi_3]^{1/2}$

$$= \left[ \rho^2 g L^3 C_p^2 / k^2 \times \mu^2 / g L^3 \right]^{1/2} = \frac{\mu C_p}{k} \text{ (the Prandtl number)}$$

By combining  $\pi_3$  with  $(\beta \Delta T)$ , we have  $\pi_5 = (\beta \Delta T) * \frac{1}{\pi_3}$

$$= \beta(\Delta T) \times \frac{\rho^2 g L^3}{\mu^2} = g\beta(\Delta T)L^3 / \nu^2 \text{ (the Grashof number)}$$

Therefore, the functional relationship is expressed as:

$$\phi(\text{Nu}, \text{Pr}, \text{Gr}) = 0; \text{ Or, } \text{Nu} = \phi_1(\text{Gr Pr}) = \text{Const} \times (\text{Gr} \times \text{Pr})^m \quad (2.1)$$

and values of the constant and 'm' are determined experimentally.

Table 2.1 gives the values of constants for use with Eq. (2.1) for isothermal surfaces.

**Table 2.1 Constants for use with Eq. 2.1 for Isothermal Surfaces**

| Geometry                      | $G_{r_f} Pr_f$ | C    | m   |
|-------------------------------|----------------|------|-----|
| Vertical planes and cylinders | $10^4 - 10^9$  | 0.59 | 1/4 |

|   |                                 |       |       |
|---|---------------------------------|-------|-------|
|   | $10^9 - 10^{13}$                | 0.021 | 2/5   |
|   | $10^9 - 10^{13}$                | 0.10  | 1/3   |
| Horizontal cylinders  | $0 - 10^{-5}$                   | 0.4   | 0     |
|   | $10^4 - 10^9$                   | 0.53  | 1/4   |
|   | $10^9 - 10^{12}$                | 0.13  | 1/3   |
|   | $10^{10} - 10^{-2}$             | 0.675 | 0.058 |
|   | $10^{-2} - 10^2$                | 1.02  | 0.148 |
|   | $10^2 - 10^4$                   | 0.85  | 0.188 |
|   | $10^4 - 10^7$                   | 0.48  | 1/4   |
|   | $10^7 - 10^{12}$                | 0.125 | 1/3   |
| Upper surface of heated plates or lower surface of cooled plates                                | $8 \times 10^6 - 10^{11}$       | 0.15  | 1/3   |
| - do -  | $2 \times 10^4 - 8 \times 10^6$ | 0.54  | 1/4   |
| Lower surface of heated plates or upper surface of cooled plates                                | $10^5 - 10^{11}$                | 0.27  | 1/4   |
| Vertical cylinder height = diameter characteristic length = diameter                            | $10^4 - 10^6$                   | 0.775 | 0.21  |
| Irregular solids, characteristic length = distance the fluid particle travels in boundary layer | $10^4 - 10^9$                   | 0.52  | 1/4   |

*Analytical Solution-Flow over a Heated Vertical Plate in Air*

Let us consider a heated vertical plate in air, shown in Fig. 2.5. The plate is maintained at uniform temperature  $T_w$ . The coordinates are chosen in such a way that  $x$  - is in the streamwise direction and  $y$  - is in the transverse direction. There will be a thin layer of fluid adjacent to the hot surface of the vertical plate. Within

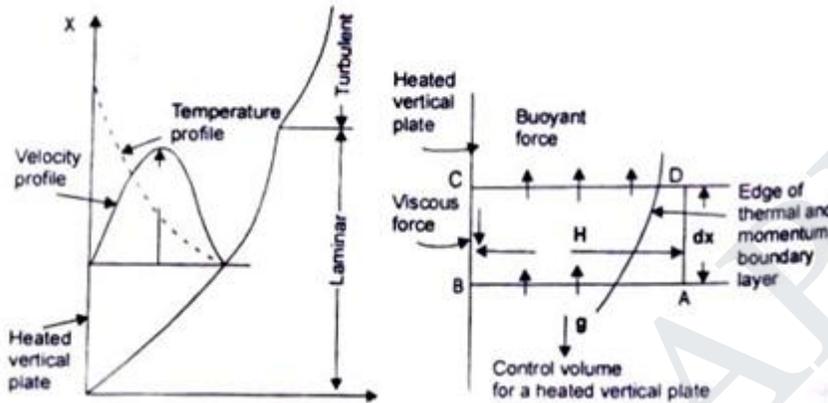


Fig. 2.5 Boundary layer on a heated vertical plate

which the variations in velocity and temperature would remain confined. The relative thickness of the momentum and the thermal boundary layer strongly depends upon the Prandtl number. Since in natural convection heat transfer, the motion of the fluid particles is caused by the temperature difference between the temperatures of the wall and the ambient fluid, the thickness of the two boundary layers are expected to be equal. When the temperature of the vertical plate is less than the fluid temperature, the boundary layer will form from top to bottom but the mathematical analysis will remain the same.

The boundary layer will remain laminar upto a certain length of the plate ( $Gr < 10^8$ ) and beyond which it will become turbulent ( $Gr > 10^9$ ). In order to obtain the analytical solution, the integral approach, suggested by von-Karman is preferred.

We choose a control volume ABCD, having a height  $H$ , length  $dx$  and unit thickness normal to the plane of paper, as shown in Fig. 25. We have:

(b) Conservation of Mass:

$$\text{Mass of fluid entering through face AB} = \dot{m}_{AB} = \int_0^H \rho u dy$$

$$\text{Mass of fluid leaving face CD} = \dot{m}_{CD} = \int_0^H \rho u dy + \frac{d}{dx} \left[ \int_0^H \rho u dy \right] dx$$

$$\therefore \text{Mass of fluid entering the face DA} = \frac{d}{dx} \left[ \int_0^H \rho u dy \right] dx$$

(ii) Conservation of Momentum :

$$\text{Momentum entering face AB} = \int_0^H \rho u^2 dy$$

$$\text{Momentum leaving face CD} = \int_0^H \rho u^2 dy + \frac{d}{dx} \left[ \int_0^H \rho u^2 dy \right] dx$$

$$\therefore \text{Net efflux of momentum in the + x-direction} = \frac{d}{dx} \left[ \int_0^H \rho u^2 dy \right] dx$$

The external forces acting on the control volume are:

$$(a) \text{Viscous force} = \mu \left. \frac{du}{dy} \right|_{y=0} dx \text{ acting in the -ve x-direction}$$

$$(b) \text{Buoyant force approximated as} \left[ \int_0^H \rho g \beta (T - T_\infty) dy \right] dx$$

From Newton's law, the equation of motion can be written as:

$$\frac{d}{dx} \left[ \int_0^\delta \rho u^2 dy \right] = -\mu \left. \frac{du}{dy} \right|_{y=0} + \int_0^\delta \rho g \beta (T - T_\infty) dy \quad (2.2)$$

because the value of the integrand between  $\delta$  and  $H$  would be zero.

(iii) Conservation of Energy:

$$\dot{Q}_{AB, \text{convection}} + \dot{Q}_{AD, \text{convection}} + \dot{Q}_{BC, \text{conduction}} = \dot{Q}_{CD, \text{convection}}$$

$$\text{or, } \int_0^H \rho u C T dy + C T_\infty \left[ \frac{d}{dx} \int_0^H \rho u dy \right] dx - k \left. \frac{dT}{dy} \right|_{y=0} dx$$

$$= \int_0^H \rho u C T dy + \frac{d}{dx} \left[ \int_0^H \rho u T C dy \right] dx$$

$$\text{or } \frac{d}{dx} \int_0^\delta \rho u (T_\infty - T) dy \frac{k}{\rho C} \frac{dT}{dy} \Big|_{y=0} = \alpha \frac{dT}{dy} \Big|_{y=0} \quad (2.3)$$

The boundary conditions are:

or,

(2.3)

Velocity profile

$$u = 0 \text{ at } y = 0$$

$$u = 0 \text{ at } y = \delta$$

$$du/dy = 0 \text{ at } y = \delta$$

Temperature profile

$$T = T_w \text{ at } y = 0$$

$$T = T_\infty \text{ at } y = \delta_1 \equiv \delta$$

$$dT/dy \equiv 0 \text{ at } y = \delta_1 \equiv \delta$$

Since the equations (2.2) and (2.3) are coupled equations, it is essential that the functional form of both the velocity and temperature distribution are known in order to arrive at a solution.

The functional relationship for velocity and temperature profiles which satisfy the above boundary conditions are assumed of the form:

$$\frac{u}{u_*} = \frac{y}{\delta} \left( 1 - \frac{y}{\delta} \right)^2 \quad (2.4)$$

where  $u_*$  is a fictitious velocity which is a function of  $x$ ; and

$$\frac{(T - T_\infty)}{(T_w - T_\infty)} = \left( 1 - \frac{y}{\delta} \right)^2 \quad (2.5)$$

After the Eqs. (5.4) and (5.5) are inserted in Eqs. (5.2) and (5.3) and the operations are performed (details of the solution are given in Chapman, A.J. Heat Transfer, Macmillan Company, New York), we get the expression for boundary layer thickness as:

$$\delta/x = 3.93 \text{Pr}^{-0.5} (0.952 + \text{Pr})^{0.25} \text{Gr}_x^{-0.25}$$

where  $\text{Gr}$ , is the local Grashof number =  $g\beta x^3 (T_w - T_\infty)/\nu^2$

The heat transfer coefficient can be evaluated from:

$$\dot{q}_w = -k \left. \frac{dT}{dy} \right|_{y=0} = h(T_w - T_\infty)$$

Using Eq. (5.5) which gives the temperature distribution, we have

$$h = 2k/\delta \quad \text{or, } hx/k = Nu_x = 2x/\delta$$

The non-dimensional equation for the heat transfer coefficient is

$$Nu_x = 0.508 Pr^{0.5} (0.952 + Pr)^{-0.25} Gr_x^{0.25} \quad (2.7)$$

The average heat transfer coefficient,  $\bar{h} = \frac{1}{L} \int_0^L h_x dx = 4/3 h_{x=L}$

$$Nu_L = 0.677 Pr^{0.5} (0.952 + Pr)^{-0.25} Gr^{0.25} \quad (2.8)$$

*Limitations of Analytical Solution:* Except for the analytical solution for flow over a flat plate, experimental measurements are required to evaluate the heat transfer coefficient. Since in free convection systems, the velocity at the surface of the wall and at the edge of the boundary layer is zero and its magnitude within the boundary layer is so small. It is very difficult to measure them. Therefore, velocity measurements require hydrogen-bubble technique or sensitive hot wire anemometers. The temperature field measurement is obtained by interferometer.

#### *Expression for 'h' for a Heated Vertical Cylinder in Air*

The characteristic length used in evaluating the Nusselt number and Grashof number for vertical surfaces is the height of the surface. If the boundary layer thickness is not too large compared with the diameter of the cylinder, the convective heat transfer coefficient can be evaluated by the equation used for vertical plane surfaces. That is, when  $D/L \geq 25/(Gr_L)^{0.25}$

### **FLOW OVER CYLINDERS:**

#### Laminar Flow Forced Convection Heat Transfer

### **2.10 Forced Convection Heat Transfer Principles**

The mechanism of heat transfer by convection requires mixing of one portion of fluid with another portion due to gross movement of the mass of the fluid. The transfer of heat energy

from one fluid particle or a molecule to another one is still by conduction but the energy is transported from one point in space to another by the displacement of fluid.

When the motion of fluid is created by the imposition of external forces in the form of pressure differences, the process of heat transfer is called 'forced convection'. And, the motion of fluid particles may be either laminar or turbulent and that depends upon the relative magnitude of inertia and viscous forces, determined by the dimensionless parameter Reynolds number. In free convection, the velocity of fluid particle is very small in comparison with the velocity of fluid particles in forced convection, whether laminar or turbulent. In forced convection heat transfer,  $Gr/Re^2 \ll 1$ , in free convection heat transfer,  $Gr/Re^2 \gg 1$  and we have combined free and forced convection when  $Gr/Re^2 \approx 1$ .

### Methods for Determining Heat Transfer Coefficient

The convective heat transfer coefficient in forced flow can be evaluated by: (a)  
Dimensional Analysis combined with experiments;

(b) Reynolds Analogy – an analogy between heat and momentum transfer; (c)  
Analytical Methods – exact and approximate analyses of boundary layer  
equations.

### 2.11 Method of Dimensional Analysis

As pointed out in Chapter 5, dimensional analysis does not yield equations which can be solved. It simply combines the pertinent variables into non-dimensional numbers which facilitate the interpretation and extend the range of application of experimental data. The relevant variables for forced convection heat transfer phenomenon whether laminar or turbulent, are

(b) the properties of the fluid – density  $\rho$ , specific heat capacity  $C_p$ , dynamic or absolute viscosity  $\mu$ , thermal conductivity  $k$ .

(ii) the properties of flow – flow velocity  $V$ , and the characteristic dimension of the system  $L$ .

As such, the convective heat transfer coefficient,  $h$ , is written as  $h = f(\rho, V, L, \mu, C_p, k) = 0$  (5.14)

Since there are seven variables and four primary dimensions, we would expect three

dimensionless numbers. As before, we choose four independent or core variables as  $\rho, V, L, k$ , and calculate the dimensionless numbers by applying Buckingham  $\pi$ 's method:

$$\begin{aligned}\pi_1 &= \rho^a V^b L^c K^d h = (ML^{-3})^a (LT^{-1})^b (L)^c (MLT^{-3}\theta^{-1})^d (MT^{-3}\theta^{-1}) \\ &= M^0 L^0 T^0 \theta^0\end{aligned}$$

Equating the powers of M, L, T and  $\theta$  on both sides, we get

$$M : a + d + 1 = 0$$

$$L : -3a + b + c + d = 0$$

$$T : -b - 3d - 3 = 0$$

$$\theta : -d - 1 = 0.$$

By solving them, we have

$$D = -1, a = 0, b = 0, c = 1.$$

Therefore,  $\pi_1 = hL/k$  is the Nusselt number.

$$\begin{aligned}\pi_2 &= \rho^a V^b L^c K^d \mu = (ML^{-3})^a (LT^{-1})^b (L)^c (MLT^{-3}\theta^{-1})^d (ML^{-1}T^{-1}) \\ &= M^0 L^0 T^0 \theta^0\end{aligned}$$

Equating the powers of M, L, T and on both sides, we get

$$M : a + d + 1 = 0$$

$$L : -3a + b + c + d = 1 = 0$$

$$T : -b - 3d - 1 = 0$$

$$\theta : -d = 0.$$

By solving them,  $d = 0, b = -1, a = -1, c = -1$

$$\text{and } \pi_2 = \mu/\rho VL; \text{ or, } \pi_3 = \frac{1}{\pi_2} = \frac{\rho VL}{\mu}$$

(Reynolds number is a flow parameter of greatest significance. It is the ratio of inertia forces to viscous forces and is of prime importance to ascertain the conditions under which a flow is laminar or turbulent. It also compares one flow with another provided the corresponding length and velocities are comparable in two flows. There would be a similarity in flow between

two flows when the Reynolds numbers are equal and the geometrical similarities are taken into consideration.)

$$\pi_4 = \rho^a V^b L^c k^d C_p = (\text{ML}^{-3})^a (\text{LT}^{-1})^b (\text{L})^c (\text{MLT}^{-3}\theta^{-1})^d (\text{L}^2\text{T}^{-2}\theta^{-1})$$

$$\text{M}^0 \text{L}^0 \text{T}^0 \theta^0$$

Equating the powers of M, L, T, on both Sides, we get

$$\text{M} : a + d = 0;$$

$$\text{L} : -3a + b + c + d + 2 = 0$$

$$\text{T} : -b - 3d - 2 = 0;$$

$$\theta : -d - 1 = 0$$

By solving them,

$$d = -1, a = 1, b = 1, c = 1,$$

$$\pi_4 = \frac{\rho V L}{k} C_p; \quad \pi_5 = \pi_4 \times \pi_2$$

$$= \frac{\rho V L}{k} C_p \times \frac{\mu}{\rho V L} = \frac{\mu C_p}{k}$$

$\therefore \pi_5$  is Prandtl number.

Therefore, the functional relationship is expressed as:

$$\text{Nu} = f(\text{Re}, \text{Pr}); \text{ or } \text{Nu} = C \text{Re}^m \text{Pr}^n \quad (5.15)$$

where the values of c, m and n are determined experimentally.

## 2.12 Principles of Reynolds Analogy

Reynolds was the first person to observe that there exists a similarity between the exchange of momentum and the exchange of heat energy in laminar motion and for that reason it has been termed 'Reynolds analogy'. Let us consider the motion of a fluid where the fluid is flowing over a plane wall. The X-coordinate is measured parallel to the surface and the V-coordinate is measured normal to it. Since all fluids are real and viscous, there would be a thin layer, called momentum boundary layer, in the vicinity of the wall where a velocity gradient normal to the direction of flow exists. When the temperature of the surface of the wall is different than the temperature of the fluid stream, there would also be a thin layer, called thermal

boundary layer, where there is a variation in temperature normal to the direction of flow. Fig. 2.6 depicts the velocity distribution and temperature profile for the laminar motion of the fluid flowing past a plane wall.

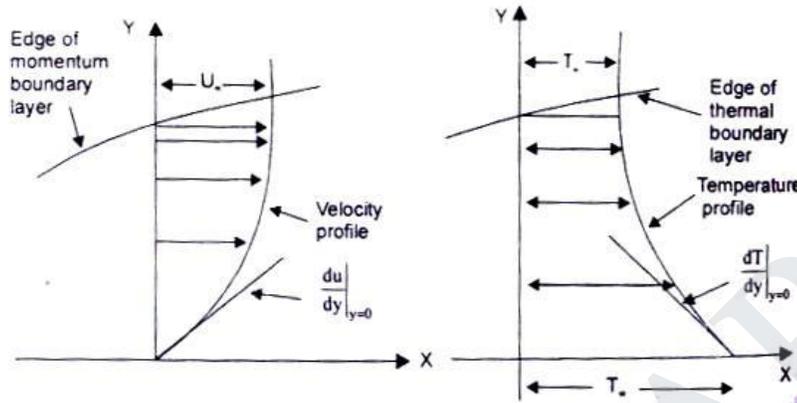


Fig. velocity distribution and temperature profile for laminar motion of the fluid over a plane surface

In a two-dimensional flow, the shearing stress is given by  $\tau_w = \mu \left. \frac{du}{dy} \right|_{y=0}$

and the rate of heat transfer per unit area is given by  $\frac{\dot{Q}}{A} = \frac{\tau_w k}{\mu} \frac{dT}{du}$

For  $Pr = \mu C_p/k = 1$ , we have  $k/\mu = C_p$  and therefore, we can write after separating the variables,

$$\frac{\dot{Q}}{A\tau_w C_p} du = -dT \tag{5.16}$$

Assuming that  $\dot{Q}$  and  $\tau_w$  are constant at any station  $x$ , we integrate equation (5.16) between the limits:  $u = 0$  when  $T = T_w$ , and  $u = U_\infty$  when  $T = T_\infty$ , and we get,

$$\dot{Q}/(A\tau_w C_p) \times U_\infty = (T_w - T_\infty)$$

Since by definition,  $\dot{Q}/A = h_x (T_w - T_\infty)$ , and  $\tau_w = C_{fx} \times \rho U_\infty^2 / 2$ ,

where  $C_{fx}$ , is the skin friction coefficient at the station  $x$ . We have

$$C_{fx} / 2 = h_x / (C_p \rho U_\infty) \quad (5.17)$$

$$\text{Since } h_x / C_p \rho U_\infty = (h_x \cdot x / k) \times (\mu / \rho \times U_\infty) \times (k / \mu \cdot C_p) = Nu_x / (Re \cdot Pr),$$

$$Nu_x / Re \cdot Pr = C_{fx} / 2 = \text{ Stanton number, } St. \quad (5.18)$$

Equation (5.18) is satisfactory for gases in which Pr is approximately equal to unity. Colburn has shown that Eq. (5.18) can also be used for fluids having Prandtl numbers ranging from 0.6 to about 50 if it is modified in accordance with experimental results.

$$\text{Or, } \frac{Nu_x}{Re_x \cdot Pr} \cdot Pr^{2/3} = St_x \cdot Pr^{2/3} = C_{fx} / 2 \quad (5.19)$$

Eq. (5.19) expresses the relation between fluid friction and heat transfer for laminar flow over a plane wall. The heat transfer coefficient could thus be determined by making measurements of the frictional drag on a plate under conditions in which no heat transfer is involved.

### 2.13 Analytical Evaluation of 'h' for Laminar Flow over a Flat Plat – Assumptions

As pointed out earlier, when the motion of the fluid is caused by the imposition of external forces, such as pressure differences, and the fluid flows over a solid surface, at a temperature different from the temperature of the fluid, the mechanism of heat transfer is called 'forced convection'. Therefore, any analytical approach to determine the convective heat transfer coefficient would require the temperature distribution in the flow field surrounding the body. That is, the theoretical analysis would require the use of the equation of motion of the viscous fluid flowing over the body along with the application of the principles of conservation of mass and energy in order to relate the heat energy that is convected away by the fluid from the solid surface.

For the sake of simplicity, we will consider the motion of the fluid in 2 space dimension, and a steady flow. Further, the fluid properties like viscosity, density, specific heat, etc are constant in the flow field, the viscous shear forces in the Y –direction is negligible and there are no variations in pressure also in the Y –direction.

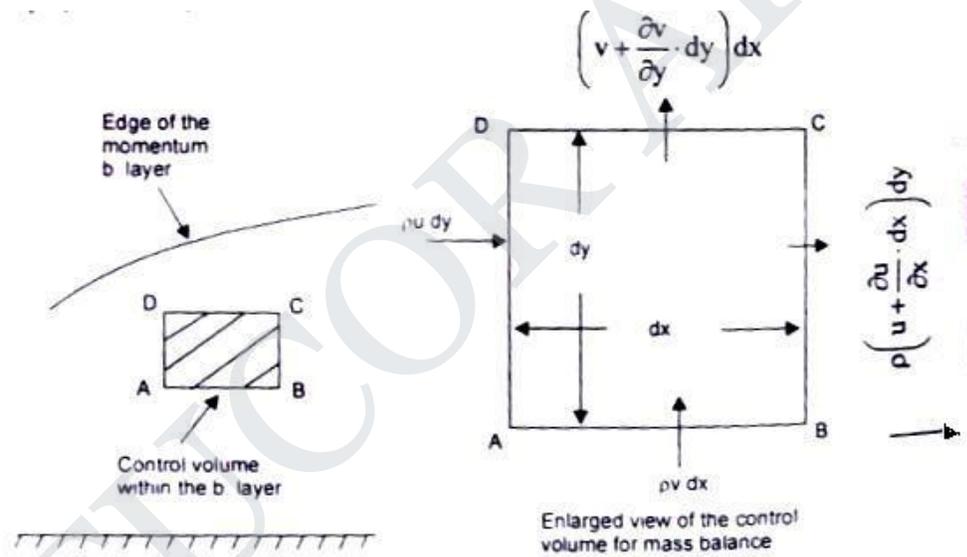
**2.14 Derivation of the Equation of Continuity–Conservation of Mass**

We choose a control volume within the laminar boundary layer as shown in Fig. 6.2. The mass will enter the control volume from the left and bottom face and will leave the control volume from the right and top face. As such, for unit depth in the Z-direction,

$$\dot{m}_{AD} = \rho u dy ; \quad \dot{m}_{BC} = \rho \left( u + \frac{\partial u}{\partial x} \cdot dx \right) dy;$$

$$\dot{m}_{AB} = \rho v dx ; \quad \dot{m}_{CD} = \rho \left( v + \frac{\partial v}{\partial y} \cdot dy \right) dx;$$

For steady flow conditions, the net efflux of mass from the control volume is zero, therefore,



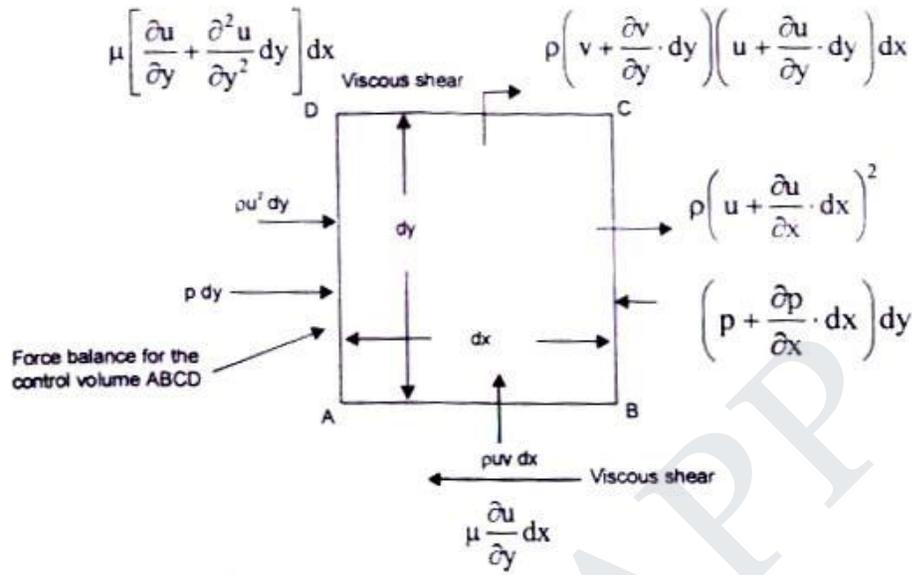


Fig. 2.7 a differential control volume within the boundary layer for laminar flow over a plane wall

$$\rho u dy + \rho x dx = \rho u dy + \rho \frac{\partial u}{\partial x} dx dy + \rho v dx + \rho \frac{\partial v}{\partial x} dx dy$$

or,  $\partial u / \partial x + \partial v / \partial y = 0$ , the equation of continuity. (2.20)

### 2.15 Concept of Critical Thickness of Insulation

The addition of insulation at the outside surface of small pipes may not reduce the rate of heat transfer. When an insulation is added on the outer surface of a bare pipe, its outer radius,  $r_0$  increases and this increases the thermal resistance due to conduction logarithmically whereas the thermal resistance to heat flow due to fluid film on the outer surface decreases linearly with increasing radius,  $r_0$ . Since the total thermal resistance is proportional to the sum of these two resistances, the rate of heat flow may not decrease as insulation is added to the bare pipe.

Fig. 2.7 shows a plot of heat loss against the insulation radius for two different cases. For small pipes or wires, the radius  $r_1$  may be less than  $r_c$  and in that case, addition of insulation to the bare pipe will increase the heat loss until the critical radius is reached. Further addition of insulation will decrease the heat loss rate from this peak value. The insulation thickness  $(r^* - r_1)$  must be added to reduce the heat loss below the uninsulated rate. If the outer pipe radius  $r_1$  is greater than the critical radius  $r_c$  any insulation added will decrease the heat loss.

### 2.16 Expression for Critical Thickness of Insulation for a Cylindrical Pipe

Let us consider a pipe, outer radius  $r_1$  as shown in Fig. 2.18. An insulation is added such that the outermost radius is  $r$  a variable and the insulation thickness is  $(r - r_1)$ . We assume that the thermal conductivity,  $k$ , for the insulating material is very small in comparison with the thermal conductivity of the pipe material and as such the temperature  $T_1$ , at the inside surface of the insulation is constant. It is further assumed that both  $h$  and  $k$  are constant. The rate of heat flow, per unit length of pipe, through the insulation is then,

$$\dot{Q}/L = 2\pi(T_1 - T_\infty) / (\ln(r/r_1)/k + 1/hr), \text{ where } T_\infty \text{ is the ambient temperature.}$$

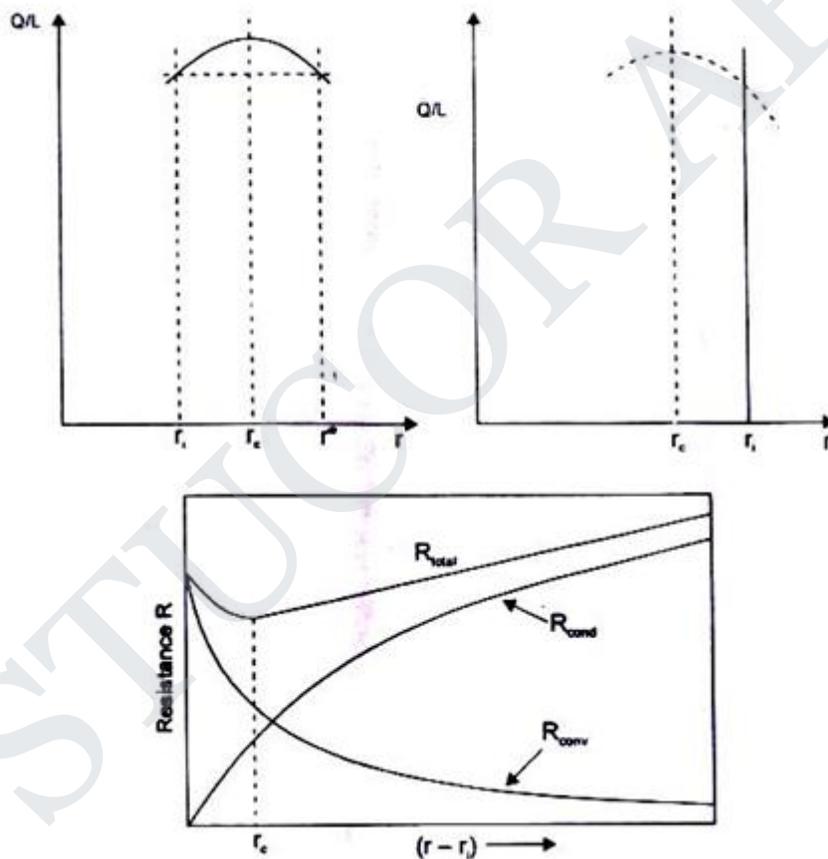


Fig 2.8 Critical thickness for pipe insulation

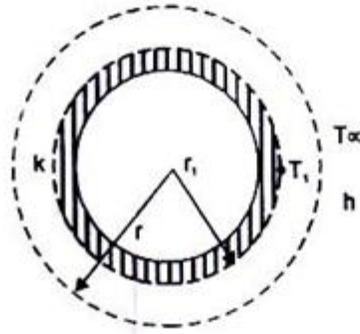


Fig 2.9 critical thickness of insulation for a pipe

An optimum value of the heat loss is found by setting  $\frac{d(\dot{Q}/L)}{dr} = 0$ .

$$\text{or, } \frac{d(\dot{Q}/L)}{dr} = 0 = -\frac{2\pi(T_1 - T_\infty)(1/kr - 1/hr^2)}{(\ln(r/r_1)/k + 1/hr^2)}$$

$$\text{or, } (1/kr) - (1/hr^2) = 0 \text{ and } r = r_c = k/h \tag{2.21}$$

where  $r_c$  denote the ‘critical radius’ and depends only on thermal quantities  $k$  and  $h$ .

If we evaluate the second derivative of  $(Q/L)$  at  $r = r_c$ , we get

$$\left. \frac{d^2(Q/L)}{dr^2} \right|_{r=r_c} = -2\pi(T_1 - T_\infty) \left[ \frac{\frac{k}{hr} + \ln\left(\frac{r}{r_1}\right)\left(\frac{2k}{hr} - 1\right) - 2\left(1 - \frac{k}{hr}\right)^2}{\frac{1}{kr}\left(\frac{k}{h} + r \ln\left(\frac{r}{r_1}\right)\right)} \right]_{r=r_c}$$

$$= -\left[ 2\pi(T_1 - T_\infty)h^2/k \right] / \left[ 1 + \ln r_c/r_1 \right]^2$$

which is always a negative quantity. Thus, the optimum radius,  $r_c = k/h$  will always give a maximum heat loss and not a minimum.

### 2.17. An Expression for the Critical Thickness of Insulation for a Spherical Shell

Let us consider a spherical shell having an outer radius  $r_1$  and the temperature at that surface  $T_1$ . Insulation is added such that the outermost radius of the shell is  $r$ , a variable. The thermal conductivity of the insulating material,  $k$ , the convective heat transfer coefficient at the

outer surface,  $h$ , and the ambient temperature  $T_\infty$  is constant. The rate of heat transfer through the insulation on the spherical shell is given by

$$\dot{Q} = \frac{(T_1 - T_\infty)}{(r - r_1)/4\pi k r r_1 + 1/h 4\pi r^2}$$

$$\frac{d\dot{Q}}{dr} = 0 = \frac{4\pi(T_1 - T_\infty)(1/kr^2 - 2/hr^3)}{\left[ (r - r_1)/k r r_1 + 1/hr^2 \right]^2}$$

which gives,  $1/Kr^2 - 2/hr^3 = 0$ ;

or  $r = r_c = 2 k/h$

## 2.18 INTERNAL FLOW THROUGH TUBES :

Thermal conditions Laminar or turbulent entrance flow and fully developed thermal condition

For laminar flows the thermal entrance length is a function of the Reynolds number and the Prandtl number:  $x_{fd,t}/D \approx 0.05 Re D Pr$ , where the Prandtl number is defined as  $Pr = \frac{\mu}{\rho \alpha}$  and  $\alpha$  is the thermal diffusivity.

For turbulent flow,  $x_{fd,t} \approx 10D$ .

**PART A****TWO MARK QUESTIONS:****1. Define convection. (AU2013)**

Convection is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperatures.

**2. What is meant by free or natural convection? (AU 2011)**

If the fluid motion is produced due to change in density resulting from temperature gradients, the mode of heat transfer is said to be free or natural convection.

**3. What is forced convection? (AU 2010)**

If the fluid motion is artificially created by means of an external force like a blower or fan, that type of heat transfer is known as forced convection.

**4. What are the dimensionless parameters used in forced convection?**

1. Reynolds number (Re)
2. Nusselt number (Nu)
3. Prandtl number (Pr)

**5. Define boundary layer thickness. (AU2014)**

The thickness of the boundary layer has been defined as the distance from the surface at which the local velocity or temperature reaches 99% of the external velocity or temperature.

**6. What is meant by laminar flow and turbulent flow? (AU 2012)**

**Laminar flow:** Laminar flow is sometimes called stream line flow. In this type of flow, the fluid moves in layers and each fluid particle follows a smooth continuous path. The fluid particles in each layer remain in an orderly sequence without mixing with each other.

**Turbulent flow:** In addition to the laminar type of flow, a distinct irregular flow is frequently observed in nature. This type of flow is called turbulent flow. The path of any individual particle is zig-zag and irregular. Fig. shows the instantaneous velocity in laminar and turbulent flow.

**7. What is hydrodynamic boundary layer? (AU2012)**

In hydrodynamic boundary layer, velocity of the fluid is less than 99% of free stream velocity.

**8. What is thermal boundary layer? (AU 2012)**

In thermal boundary layer, temperature of the fluid is less than 99% of free stream velocity.

**9. Define Grashof number (Gr). (AU 2009)**

It is defined as the ratio of product of inertia force and buoyancy force to the square of viscous force.

$$Gr = \frac{\text{Inertia force} \times \text{Buoyancy force}}{(\text{Viscous force})^2}$$

**10. Define Reynolds number (Re). (AU2010)**

It is defined as the ratio of inertia force to viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

**11. Define prandtl number (Pr).**

It is the ratio of the momentum diffusivity of the thermal diffusivity.

$$Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

**12. Which mode heat transfer is the convection heat transfer coefficient usually higher, natural or forced convection? why?**

Convection heat transfer coefficient is usually higher in forced convection than in natural convection, because it mainly depends upon the factors such as fluid density, velocity and viscosity.

**PART B**

**ANSWER THE FOLLOWING:**

**1. Air at 20°C at atmospheric pressure flows over a flat plate at a velocity of 3 m/s. if the plate is 1 m wide and 80°C, calculate the following at x = 300 mm. (AU2010)**

- 1. Hydrodynamic boundary layer thickness,**
- 2. Thermal boundary layer thickness,**
- 3. Local friction coefficient,**
- 4. Average friction coefficient,**

**5. Local heat transfer coefficient****6. Average heat transfer coefficient,****7. Heat transfer.**

**Given:** Fluid temperature  $T_{\infty} = 20^{\circ}\text{C}$  , Velocity  $U = 3 \text{ m/s}$

Wide  $W = 1 \text{ m}$

Surface temperature  $T_w = 80^{\circ}\text{C}$

Distance  $x = 300 \text{ mm} = 0.3 \text{ m}$

**Solution:** We know

$$\text{Film temperature } T_f = \frac{T_w + T_{\infty}}{2}$$

$$= \frac{80 + 20}{2}$$

$$T_f = 50^{\circ}\text{C}$$

Properties of air at  $50^{\circ}\text{C}$

Density  $\rho = 1.093 \text{ kg/m}^3$

Kinematic viscosity  $\nu = 17.95 \times 10^{-6} \text{ m}^2/\text{s}$

Prandtl number  $Pr = 0.698$

Thermal conductivity  $K = 28.26 \times 10^{-3} \text{ W/mK}$

We know,

$$\text{Reynolds number } Re = \frac{UL}{\nu}$$

$$= \frac{3 \times 0.3}{17.95 \times 10^{-6}}$$

$$Re = 5.01 \times 10^4 < 5 \times 10^5$$

Since  $Re < 5 \times 10^5$ , flow is laminar

For Flat plate, laminar flow,

**1. Hydrodynamic boundary layer thickness:**

$$\begin{aligned}\delta_{hx} &= 5 \times x \times (\text{Re})^{-0.5} \\ &= 5 \times 0.3 \times (5.01 \times 10^4)^{-0.5} \\ \delta_{hx} &= 6.7 \times 10^{-3} \text{ m}\end{aligned}$$

**2. Thermal boundary layer thickness:**

$$\begin{aligned}\delta_{TX} &= \delta_{hx} (\text{Pr})^{-0.333} \\ \Rightarrow \delta_{TX} &= (6.7 \times 10^{-3}) (0.698)^{-0.333} \\ \delta_{TX} &= 7.5 \times 10^{-3} \text{ m}\end{aligned}$$

**3. Local Friction coefficient:**

$$\begin{aligned}C_{fx} &= 0.664 (\text{Re})^{-0.5} \\ &= 0.664 (5.01 \times 10^4)^{-0.5} \\ C_{fx} &= 2.96 \times 10^{-3}\end{aligned}$$

**4. Average friction coefficient:**

$$\begin{aligned}\overline{C_{fL}} &= 1.328 (\text{Re})^{-0.5} \\ &= 1.328 (5.01 \times 10^4)^{-0.5} \\ &= 5.9 \times 10^{-3} \\ \overline{C_{fL}} &= 5.9 \times 10^{-3}\end{aligned}$$

**5. Local heat transfer coefficient ( $h_x$ ):**

Local Nusselt Number

$$\begin{aligned}\text{Nu}_x &= 0.332 (\text{Re})^{0.5} (\text{Pr})^{0.333} \\ &= 0.332 (5.01 \times 10^4) (0.698)^{0.333} \\ \text{Nu}_x &= 65.9\end{aligned}$$

We know

Local Nusselt Number

$$Nu_x = \frac{h_x \times L}{K}$$

$$65.9 = \frac{h_x \times 0.3}{23.26 \times 10^{-3}} \quad [ \because x = L = 0.3\text{m} ]$$

$$\Rightarrow h_x = 6.20 \text{ W/m}^2\text{K}$$

Local heat transfer coefficient  $h_x = 6.20 \text{ W/m}^2\text{K}$

### 6. Average heat transfer coefficient (h):

$$h = 2 \times h_x$$

$$= 2 \times 6.20$$

$$h = 12.41 \text{ W/m}^2\text{K}$$

### 7. Heat transfer:

We know that,

$$Q = h A (T_w - T_\infty)$$

$$= 12.41 \times (1 \times 0.3) (80 - 20)$$

$$Q = 23.38 \text{ Watts}$$

2. Air at 40°C is flows over a flat plate of 0.9 m at a velocity of 3 m/s. Calculate the following: (AU 2012)

1. Overall drag coefficient
2. Average shear stress,
3. Compare the average shear stress with local shear stress (shear stress at the trailing edge)

Given:

Fluid temperature  $T_\infty = 40^\circ\text{C}$

Length  $L = 0.9 \text{ m}$

Velocity  $U = 3 \text{ m/s}$ .

Solution:

Properties of air at 40°C:

$$\rho = 1.128 \text{ Kg/m}^3$$

$$\nu = 16.96 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Pr = 0.699$$

$$K = 26.56 \times 10^{-3} \text{ W/mK}$$

We know,

$$\text{Reynolds number } Re = \frac{UL}{\nu}$$

$$= \frac{3 \times 0.9}{16.96 \times 10^{-6}}$$

$$Re = 1.59 \times 10^5 < 5 \times 10^5$$

Since  $Re < 5 \times 10^5$ , flow is laminar

For plate, laminar flow,

Drag coefficient (or) Average skin friction coefficient

$$\overline{C}_{fL} = 1.328 \times (Re)^{-0.5}$$

$$= 1.328 \times (1.59 \times 10^5)^{-0.5}$$

$$\overline{C}_{fL} = 3.3 \times 10^{-3}$$

We know

$$\text{Drag friction coefficient } \overline{C}_{fL} = \frac{\tau}{\frac{\rho U^2}{2}}$$

$$\tau = \overline{C}_{fL} \times \frac{\rho U^2}{2}$$

$$= \frac{3.3 \times 10^{-3} \times 1.128 \times (3)^2}{2}$$

Average shear stress  $\tau = 0.016 \text{ N/m}^2$

We know,

Local skin friction coefficient

$$\overline{C}_{fx} = 0.664 \times (Re)^{-0.5}$$

$$= 0.664 \times (1.59 \times 10^5)^{-0.5}$$

$$\overline{C}_{fx} = 1.66 \times 10^{-3}$$

We know

$$\text{Local skin friction coefficient } \overline{C_{fx}} = \frac{\tau}{\frac{\rho U^2}{2}}$$

$$\Rightarrow 1.66 \times 10^{-3} = \frac{\tau}{\frac{1.128 \times (3)^2}{2}}$$

$$\tau_x = 8.4 \times 10^{-3} \text{ N/m}^2$$

$$\text{Local shear stress } \tau_x = 8.4 \times 10^{-3} \text{ N/m}^2$$

$$\frac{\text{Local shear stress } \tau_x}{\text{Average shear stress } \tau} = \frac{8.4 \times 10^{-3} \text{ N/m}^2}{0.016 \text{ N/m}^2}$$

$$= 0.52$$

**3. Air at 20°C flows over a flat plate at 60°C with a free stream velocity of 6 m/s. Determine the value of the average convective heat transfer coefficient up to a length of 1 m in the flow direction. (AU2013)**

**Given:**

$$\text{Fluid temperature } T_\infty = 20^\circ\text{C}$$

$$\text{Plate temperature } T_w = 60^\circ\text{C}$$

$$\text{Velocity } U = 6 \text{ m/s}$$

$$\text{Length } L = 1 \text{ m}$$

**To find:** Average heat transfer coefficient

**Solution:** We know

$$\text{Film temperature } T_f = \frac{T_w + T_\infty}{2}$$

$$= \frac{60 + 20}{2}$$

$$T_f = 40^\circ\text{C}$$

Properties of air at 40°C:

$$\text{Density } \rho = 1.128 \text{ Kg/m}^3$$

$$\text{Thermal conductivity } K = 26.56 \times 10^{-3} \text{ W/mK}$$

$$\text{Kinematic viscosity } \nu = 16.96 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Prandtl number } Pr = 0.699$$

We know

$$\text{Reynolds number } Re = \frac{UL}{\nu}$$

$$= \frac{6 \times 1}{16.96 \times 10^{-6}}$$

$$Re = 3.53 \times 10^5 < 5 \times 10^5$$

Since  $Re < 5 \times 10^5$ , flow is laminar

For flat plate, laminar flow

$$\text{Local nusselt number } Nu_x = 0.332 \times (Re)^{0.5} \times (Pr)^{0.333}$$

$$= 0.332 \times (3.53 \times 10^5)^{0.5} \times (0.699)^{0.333}$$

$$NU_x = 175.27$$

We know,

$$\text{Local Nusselt number } NU_x = \frac{h_x \times L}{K}$$

$$175.27 = \frac{h_x \times 1}{26.56 \times 10^{-3}}$$

$$\text{Local nusselt number } NU_x = 4.65 \text{ W/m}^2\text{K}$$

$$\text{Average heat transfer coefficient } h = 2 \times h_x$$

$$= 2 \times 4.65$$

$$h = 9.31 \text{ W/m}^2\text{K}$$

**4. Engine oil flows through a 50 mm diameter tube at an average temperature of 147°C. The flow velocity is 80 cm/s. Calculate the average heat transfer coefficient if the tube wall is maintained at a temperature of 200°C and it is 2 m long.**

**Given:**

$$\text{Diameter } D = 50 \text{ mm} = 0.050 \text{ m}$$

$$\text{Average temperature } T_m = 147^\circ\text{C}$$

$$\text{Velocity } U = 80 \text{ cm/s} = 0.80 \text{ m/s}$$

$$\text{Tube wall temperature } T_w = 200^\circ\text{C}$$

$$\text{Length } L = 2 \text{ m}$$

**To find:** Average heat transfer coefficient (h)

**Solution:** Properties of engine oil at 147°C

$$\rho = 816 \text{ Kg/m}^3$$

$$\nu = 7 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 116$$

$$K = 133.8 \times 10^{-3} \text{ W/mK}$$

We know

$$\text{Reynolds number } \text{Re} = \frac{UD}{\nu}$$

$$= \frac{0.8 \times 0.05}{7 \times 10^{-6}}$$

$$\text{Re} = 5714.2$$

Since  $\text{Re} < 2300$  flow is turbulent

$$\frac{L}{D} = \frac{2}{0.050} = 40$$

$$10 < \frac{L}{D} < 400$$

For turbulent flow, ( $\text{Re} < 10000$ )

$$\text{Nusselt number } \text{Nu} = 0.036 (\text{Re})^{0.8} (\text{Pr})^{0.33} \left(\frac{D}{L}\right)^{0.055}$$

$$\text{Nu} = 0.036 (5714.2)^{0.8} \times (116)^{0.33} \times \left(\frac{0.050}{2}\right)^{0.055}$$

$$\text{Nu} = 142.8$$

$$\text{We know } \text{Nu} = \frac{hD}{K}$$

$$\Rightarrow 142.8 = \frac{h \times 0.050}{133.8 \times 10^{-3}}$$

$$\Rightarrow h = 382.3 \text{ W/m}^2\text{K}$$

**5. A thin 100 cm long and 10 cm wide horizontal plate is maintained at a uniform temperature of 150°C in a large tank full of water at 75°C. Estimate the rate of heat to be supplied to the plate to maintain constant plate temperature as heat is dissipated from either side of plate. (AU 2014)**

**Given:**

Length of horizontal plate  $L = 100 \text{ cm} = 1 \text{ m}$

Wide  $W = 10 \text{ cm} = 0.10 \text{ m}$

Plate temperature  $T_w = 150^\circ\text{C}$

Fluid temperature  $T_\infty = 75^\circ\text{C}$

**To find:** Heat loss (Q) from either side of plate

**Solution:**

$$\text{Coefficient of thermal expansion } \beta = \frac{1}{T_f \text{ in K}} = \frac{1}{112.5 + 273}$$

$$\beta = 2.59 \times 10^{-3} \text{ K}^{-1}$$

$$\text{Grashof Number } Gr = \frac{g \times \beta \times L^3 \times \Delta T}{\nu^2}$$

For horizontal plate,

$$\text{Characteristic length } L_c = \frac{W}{2} = \frac{0.10}{2}$$

$$L_c = 0.05 \text{ m}$$

$$(1) \Rightarrow Gr = \frac{9.81 \times 2.59 \times 10^{-3} \times (0.05)^3 \times (150 - 75)}{(0.264 \times 10^{-6})^2}$$

$$Gr = 3.41 \times 10^9$$

$$Gr Pr = 3.41 \times 10^9 \times 1.55$$

$$Gr Pr = 5.29 \times 10^9$$

Gr Pr value is in between  $8 \times 10^6$  and  $10^{11}$

$$\text{i.e., } 8 \times 10^6 < Gr Pr < 10^{11}$$

**For horizontal plate, upper surface heated:**

$$\text{Nusselt number } Nu = 0.15 (Gr Pr)^{0.333}$$

$$\Rightarrow Nu = 0.15 [5.29 \times 10^9]^{0.333}$$

$$\Rightarrow Nu = 259.41$$

We know that,

$$\text{Nusselt number } Nu = \frac{h_u L_c}{K}$$

$$259.41 = \frac{h_u \times 0.05}{683 \times 10^{-3}}$$

$$h_u = 3543.6 \text{ W/m}^2\text{K}$$

Upper surface heated, heat transfer coefficient  $h_u = 3543.6 \text{ W/m}^2\text{K}$

**For horizontal plate, lower surface heated:**

$$\text{Nusselt number } Nu = 0.27 [\text{Gr Pr}]^{0.25}$$

$$\Rightarrow Nu = 0.27 [5.29 \times 10^9]^{0.25}$$

$$Nu = 72.8$$

We know that,

$$\text{Nusselt number } Nu = \frac{h_1 L_c}{K}$$

$$72.8 = \frac{h_1 L_c}{K}$$

$$72.8 = \frac{h_1 \times 0.05}{683 \times 10^{-3}}$$

$$h_1 = 994.6 \text{ W/m}^2\text{K}$$

Lower surface heated, heat transfer coefficient  $h_1 = 994.6 \text{ W/m}^2\text{K}$

$$\text{Total heat transfer } Q = (h_u + h_1) \times A \times \Delta T$$

$$= (h_u + h_1) \times W \times L \times (T_w - T_\infty)$$

$$= (3543.6 + 994.6) \times 0.10 \times (150 - 75)$$

$$Q = 34036.5 \text{ W}$$

**6. For a particular engine, the underside of the crank case can be idealized as a flat plate measuring 80 cm × 20 cm. The engine runs at 80 km/hr and the crank case is cooled by air flowing past it at the same speed. Calculate the loss of heat from the crank case surface of temperature 75°C to the ambient air temperature 25°C. Assume the boundary layer becomes turbulent from the leading edge itself.**

**Given:** Area  $A = 80 \text{ cm} \times 20 \text{ cm}$

$$= 1600 \text{ cm}^2 = 0.16 \text{ m}^2$$

Velocity  $U = 80 \text{ Km/hr}$

$$= \frac{80 \times 10^3 \text{ m}}{3600 \text{ s}}$$

$$= 22.22 \text{ m/s}$$

Surface temperature  $T_w = 75^\circ\text{C}$

Ambient air temperature  $T_\infty = 25^\circ\text{C}$

Flow is turbulent from the leading edge, flow is fully turbulent.

**To find:**

1. Heat loss

$$\text{Film temperature } T_f = \frac{T_w + T_\infty}{2} = \frac{75 + 25}{2}$$

$$T_f = 50^\circ\text{C}$$

Properties of air at  $50^\circ\text{C}$ :

$$\rho = 1.093 \text{ Kg/m}^3$$

$$\nu = 17.95 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.698$$

$$K = 28.26 \times 10^{-3} \text{ W/mK}$$

$$\text{Reynolds number } \text{Re} = \frac{UL}{\nu}$$

$$= \frac{22.22 \times 0.8}{17.95 \times 10^{-6}} \quad [ \because L = 0.8\text{m} ]$$

$$\text{Re} = 9 \times 10^5$$

$$\text{Re} = 9 \times 10^5 > 5 \times 10^5$$

Since  $\text{Re} > 5 \times 10^5$ , Flow is turbulent

For flat plate, turbulent flow,

[Fully turbulent from leading edge – given]

$$\text{Local Nusselt number } \text{NU}_x = 0.0296 (\text{Re})^{0.8} (\text{Pr})^{0.333}$$

$$= 0.0296 [9 \times 10^5]^{0.8} (0.698)^{0.33}$$

$$\text{NU}_x = 1524.6$$

We know that,  $NU_x = \frac{h_x L}{K}$

$$1524.6 = \frac{h_x \times 0.8}{28.26 \times 10^{-3}} \quad [\because L = 0.8\text{m}]$$

$$h_x = 53.85 \text{ W/m}^2\text{K}$$

Local heat transfer coefficient }  $h_x = 53.85 \text{ W/m}^2\text{K}$

For turbulent flow, flat plate

Average heat transfer coefficient }  $h = 1.24 h_x$

$$h = 1.24 \times 53.85$$

$$h = 66.78 \text{ W/m}^2\text{K}$$

We know,

$$\text{Heat loss } Q = h A (T_w - T_\infty)$$

$$= 66.78 \times 0.16 (75 - 25)$$

$$Q = 534.2 \text{ watts.}$$

**UNIT 3 PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS :****NUSSELT THEORY OF CONDENSATION :**

Condensation is a rather complicated process. It was Wilhelm Nusselt's idea to reduce the complexity of the real process to a rather simple model, namely that the only resistance for the removal of the heat released during condensation occurs in the condensate film. The following gives an explanation of the Nusselt theory at the example of condensation on a vertical wall. Condensation occurs if a vapor is cooled below its (pressure dependent) saturation temperature. The heat of evaporation which is released during condensation must be removed by heat transfer, e.g. at a cooled wall. Figure shows how saturated vapor at temperature  $T_s$  is condensing on a vertical wall whose temperature  $T_w$  is constant and lower than the saturation temperature.

**3.1. Condensation and Boiling**

Heat energy is being converted into electrical energy with the help of water as a working fluid. Water is first converted into steam when heated in a heat exchanger and then the exhaust steam coming out of the steam turbine/engine is condensed in a condenser so that the condensate (water) is recycled again for power generation. Therefore, the condensation and boiling processes involve heat transfer with change of phase. When a fluid changes its phase, the magnitude of its properties like density, viscosity, thermal conductivity, specific heat capacity, etc., change appreciably and the processes taking place are greatly influenced by them. Thus, the condensation and boiling processes must be well understood for an effective design of different types of heat exchangers being used in thermal and nuclear power plants, and in process cooling and heating systems.

**3.2 Condensation-Filmwise and Dropwise**

Condensation is the process of transition from a vapour to the liquid or solid state. The process is accompanied by liberation of heat energy due to the change of phase. When a vapour comes in contact with a surface maintained at a temperature lower than the saturation temperature of the vapour corresponding to the pressure at which it exists, the vapour condenses on the surface and the heat energy thus released has to be removed. The efficiency of the condensing unit is determined by the mode of condensation that takes place:

Filmwise - the condensing vapour forms a continuous film covering the entire surface,

Dropwise - the vapour condenses into small liquid droplets of various sizes. The dropwise condensation has a much higher rate of heat transfer than filmwise condensation because the condensate in dropwise condensation gets removed at a faster rate leading to better heat transfer between the vapour and the bare surface. .

It is therefore desirable to maintain a condition of dropwise condensation in commercial application. Dropwise condensation can only occur either on highly polished surfaces or on surfaces contaminated with certain chemicals. Filmwise condensation is expected to occur in most instances because the formation of dropwise condensation is greatly influenced by the presence of non-condensable gases, the nature and composition of surfaces and the velocity of vapour past the surface.

### **Filmwise Condensation Mechanism on a Vertical Plane Surface--**

#### **Assumption**

Let us consider a plane vertical surface at a constant temperature,  $T_s$  on which a pure vapour at saturation temperature,  $T_g$  ( $T_g > T_s$ ) is condensing. The coordinates are: X-axis along the plane surface with its origin at the top edge and Y-axis is normal to the plane surface as shown in Fig. 11.1. The condensing liquid would wet the solid surface, spread out and form a continuous film over the entire condensing surface. It is further assumed that

(i) the continuous film of liquid will flow downward (positive X-axis) under the action of gravity and its thickness would increase as more and more vapour condenses at the liquid - vapour interface,

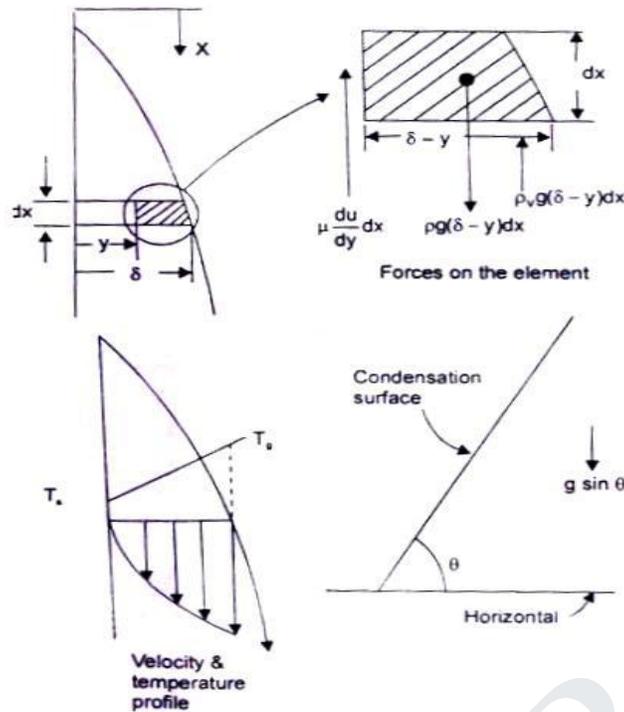


Fig. 5.11 Filmwise condensation on a vertical and Inclined surface

- (ii) the continuous film so formed would offer a thermal resistance between the vapour and the surface and would reduce the heat transfer rates,
- (iii) the flow in the film would be laminar,
- (iv) there would be no shear stress exerted at the liquid vapour interface,
- (v) the temperature profile would be linear, and
- (vi) the weight of the liquid film would be balanced by the viscous shear in the liquid film and the buoyant force due to the displaced vapour.

### 3.3 An Expression for the Liquid Film Thickness and the Heat Transfer Coefficient Laminar Filmwise Condensation on a Vertical Plate

We choose a small element, as shown in Fig. 11.1 and by making a force balance, we write

$$\rho g(\delta - y)dx = \mu(du / dy)dx + \rho_v g(\delta - y)dx \tag{5.44}$$

where  $\rho$  is the density of the liquid,  $\rho_v$  is the density of vapour,  $\mu$  is the viscosity of the

liquid,  $\delta$  is the thickness of the liquid film at any  $x$ , and  $du/dy$  is the velocity gradient at  $x$ .

Since the no-slip condition requires  $u = 0$  at  $y = 0$ , by integration we get:

$$u = (\rho - \rho_v)g(\delta y - y^2/2) / \mu \quad (5.45)$$

And the mass flow rate of condensate through any  $x$  position of the film would be

$$\begin{aligned} \dot{m} &= \int_0^{\delta} \rho u \, dy = \int_0^{\delta} \left[ \rho(\rho - \rho_v)(g/\mu)(\delta y - y^2/2) \right] dy \\ &= \rho(\rho - \rho_v) g \delta^3 / 3\mu \end{aligned} \quad (5.46)$$

The rate of heat transfer at the wall in the area  $dx$  is, for unit width,

$$\dot{Q} = -kA(dt/dy)_{y=0} = k(dx \times 1)(T_g - T_s) / \delta,$$

(temperature distribution is linear)

Since the thickness of the film increases in the positive  $X$ -direction, an additional mass of vapour will condense between  $x$  and  $x + dx$ , i.e.,

$$\begin{aligned} \frac{d}{dx} \left( \frac{\rho(\rho - \rho_v)g\delta^3}{3\mu} \right) dx &= \frac{d}{d\delta} \left( \frac{\rho(\rho - \rho_v)g\delta^3}{3\mu} \right) \frac{d\delta}{dx} dx \\ &= \frac{\rho(\rho - \rho_v)g\delta^2 d\delta}{\mu} \end{aligned}$$

This additional mass of condensing vapour will release heat energy and that has to be removed by conduction through the wall, or,

$$\therefore \frac{\rho(\rho - \rho_v)g\delta^2 d\delta}{\mu} \times h_{fg} = k \, dx (T_g - T_s) / \delta \quad (5.47)$$

We can, therefore, determine the thickness,  $\delta$ , of the liquid film by integrating Eq. (11.4) with the boundary condition: at  $x = 0$ ,  $\delta = 0$ ,

$$\text{or, } \delta = \left( \frac{4\mu k x (T_g - T_s)}{g h_{fg} \rho (\rho - \rho_v)} \right)^{0.25} \quad (5.48)$$

The rate of heat transfer is also related by the relation,

$$h \, dx (T_g - T_s) = k \, dx (T_g - T_s) / \delta; \text{ or, } h = k / \delta$$

which can be expressed In dimensionless form in terms of Nusselt number,

$$Nu = hx / k = \left[ \frac{\rho(\rho - \rho_v)gh_{fg}x^3}{4\mu k(T_g - T_s)} \right]^{0.25} \tag{5.49}$$

The average value of the heat transfer coefficient is obtained by integrating over the length of the plate:

$$\bar{h} = (1/L) \int_0^L h_x \, dx = (4/3)h_x = L$$

$$Nu_L = 0.943 \left[ \frac{\rho(\rho - \rho_v)gh_{fg}L^3}{k\mu(T_g - T_s)} \right]^{0.25} \tag{5.50}$$

The properties of the liquid in Eq. (5.50) and Eq. (5.49) should be evaluated at the mean temperature,  $T = (T_g + T_s)/2$ .

The above analysis is also applicable to a plane surface inclined at angle  $\theta$  with the horizontal, If  $g$  is everywhere replaced by  $g \cdot \sin\theta$ .

Thus:

$$\text{Local } Nu_x = 0.707 \left[ \frac{\rho(\rho - \rho_v)h_{fg}x^3 g \sin \theta}{\mu k (T_g - T_s)} \right]^{0.25}$$

$$\text{and the average } Nu_L = 0.943 \left[ \frac{\rho(\rho - \rho_v)h_{fg}L^3 g \sin \theta}{\mu k (T_g - T_s)} \right] \tag{5.51}$$

These relations should be used with caution for small values of  $\theta$  because some of the assumptions made in deriving these relations become invalid; for example, when  $\theta$  is equal to zero, (a horizontal surface) we would get an absurd result. But these equations are valid for condensation on the outside surface of vertical tubes as long as the curvature of the tube surface

is not too great.

### 3.4 REGIMES OF POOL BOILING AND FLOW BOILING ;

#### Regimes of Boiling

Let us consider a heating surface (a wire or a flat plate) submerged in a pool of water which is at its saturation temperature. If the temperature of the heated surface exceeds the temperature of the liquid, heat energy will be transferred from the solid surface to the liquid. From Newton's law of cooling, we have

$$\dot{Q}/A = \dot{q} = h(T_w - T_s)$$

where  $\dot{Q}/A$  is the heat flux,  $T_w$  is the temperature of the heated surface and  $T_s$  is the temperature of the liquid, and the boiling process will start.

(i) Pool Boiling - Pool boiling occurs only when the temperature of the heated surface exceeds the saturation temperature of the liquid. The liquid above the hot surface is quiescent and its motion near the surface is due to free convection.

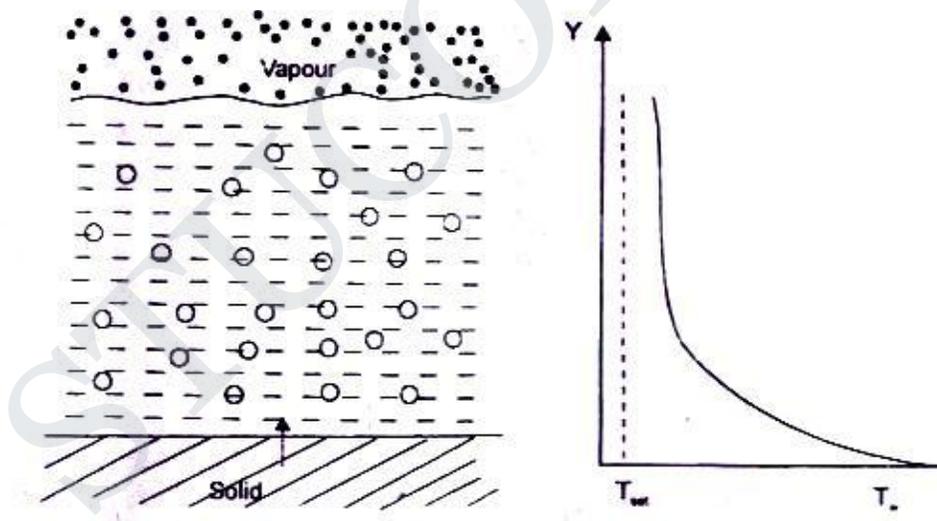


Fig. 5.13 Temperature distribution in pool boiling at liquid-vapour interface

Bubbles grow at the heated surface, get detached and move upward toward the free surface due to buoyancy effect. If the temperature of the liquid is lower than the saturation temperature, the process is called 'subcooled or local boiling'. If the temperature of the liquid is equal to the saturation temperature, the process is known as 'saturated or bulk boiling'. The

temperature distribution in saturated pool boiling is shown in Fig5.13. When  $T_w$  exceeds  $T_s$  by a few degrees, the convection currents circulate in the superheated liquid and the evaporation takes place at the free surface of the liquid.

(ii) Nucleate Boiling - Fig. I 1.5 illustrates the different regimes of boiling where the heat flux ( $\dot{Q}/A$ ) is plotted against the temperature difference ( $T_w - T_s$ ). When the temperature  $T_w$  increases a little more, vapour bubbles are formed at a number of favoured spots on the heating surface. The vapour bubbles are initially small and condense before they reach the free surface. When the temperature is raised further, their number increases and they grow bigger and finally rise to the free surface. This phenomenon is called 'nucleate boiling'. It can be seen from the figure (5.14) that in nucleate boiling regime, the heat flux increases rapidly with increasing surface temperature. In the latter part of the nucleate boiling, (regime 3), heat transfer by evaporation is more important and predominating. The point A on the curve represents 'critical heat flux'.

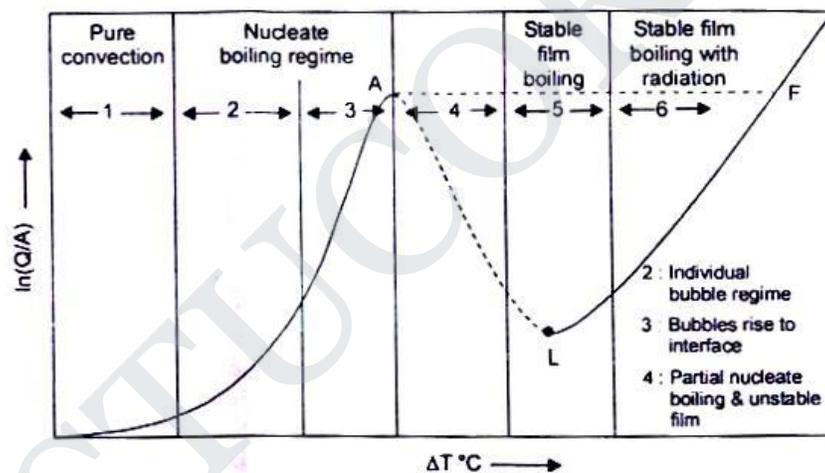


Fig. 5.14 Heat Flux - Temperature difference curve for boiling water heated by a wire (Nukiyama's boiling curve for saturated water at atmospheric pressure) (L is the Laidenrost Point)

(iii) Film Boiling - When the excess temperature,  $\Delta T = (T_w - T_s)$  increases beyond the point A, a vapour film forms and covers the entire heating surface. The heat transfer takes place through the vapour which is a poor conductor and this increased thermal resistance causes a drop in the heat flux. This phase is film boiling'. The transition from the nucleate boiling regime to the

film boiling regime is not a sharp one and the vapour film under the action of circulating currents collapses and rapidly reforms. In regime 5, the film is stable and the heat flow rate is the lowest.

(iv) Critical Heat Flux and Burnout Point - For  $\Delta T$  beyond  $550^{\circ}\text{C}$  (regime 6) the temperature of the heating metallic surface is very high and the heat transfer occurs predominantly by radiation, thereby, increasing the heat flux. And finally, a point is reached at which the heating surface melts - point F in Fig. 11.5. It can be observed from the boiling curve that the whole boiling process remains in the unstable state between A and F. Any increase in the heat flux beyond point A will cause a departure from the boiling curve and there would be a large increase in surface temperature.

### 3.5 Boiling Curve - Operating Constraints

The boiling curve, shown in Fig. 11.5, is based on the assumption that the temperature of the heated surface can be maintained at the desired value. In that case, it would be possible to operate the vapour producing system at the point of maximum flux with nucleate boiling. If the heat flux instead of the surface temperature, is the independent variable and it is desired to operate the system at the point of maximum flux, it is just possible that a slight increase in the heat flux will increase the surface temperature substantially. And, the equilibrium will be established at point F. If the material of the heating element has its melting point temperature lower than the temperature at the equilibrium point F, the heating element will melt.

### 3.6 Factors Affecting Nucleate Boiling

Since high heat transfer rates and convection coefficients are associated with small values of the excess temperature, it is desirable that many engineering devices operate in the nucleate boiling regime. It is possible to get heat transfer coefficients in excess of  $10^4 \text{ W/m}^2$  in nucleate boiling regime and these values are substantially larger than those normally obtained in convection processes with no phase change. The factors which affect the nucleate boiling are:

(a) Pressure - Pressure controls the rate of bubble growth and therefore affects the temperature difference causing the heat energy to flow. The maximum allowable heat flux for a boiling liquid first increases with pressure until critical pressure is reached and then decreases.

(b) Heating Surface Characteristics - The material of the heating element has a significant effect on the boiling heat transfer coefficient. Copper has a higher value than

chromium, steel and zinc. Further, a rough surface gives a better heat transfer rate than a smooth or coated surface, because a rough surface gets wet more easily than a smooth one.

(c) Thermo-mechanical Properties of Liquids - A higher thermal conductivity of the liquid will cause higher heat transfer rates and the viscosity and surface tension will have a marked effect on the bubble size and their rate of formation which affects the rate of heat transfer.

(d) Mechanical Agitation - The rate of heat transfer will increase with the increasing degree of mechanical agitation. Forced convection increases mixing of bubbles and the rate of heat transfer.

### 3.7 HEAT EXCHANGERS TYPES :

#### Heat Exchangers: Regenerators and Recuperators

A heat exchanger is an equipment where heat energy is transferred from a hot fluid to a colder fluid. The transfer of heat energy between the two fluids could be carried out (i) either by direct mixing of the two fluids and the mixed fluids leave at an intermediate temperature determined from the principles of conservation of energy, (ii) or by transmission through a wall separating the two fluids. The former types are called direct contact heat exchangers such as water cooling towers and jet condensers. The latter types are called regenerators, recuperator surface exchangers.

In a regenerator, hot and cold fluids alternately flow over a surface which provides alternately a sink and source for heat flow. Fig. 10.1 (a) shows a cylinder containing a matrix that rotates in such a way that it passes alternately through cold and hot gas streams which are sealed

from each other. Fig. 10.1 (b) shows a stationary matrix regenerator in which hot and cold gases flow through them alternately.

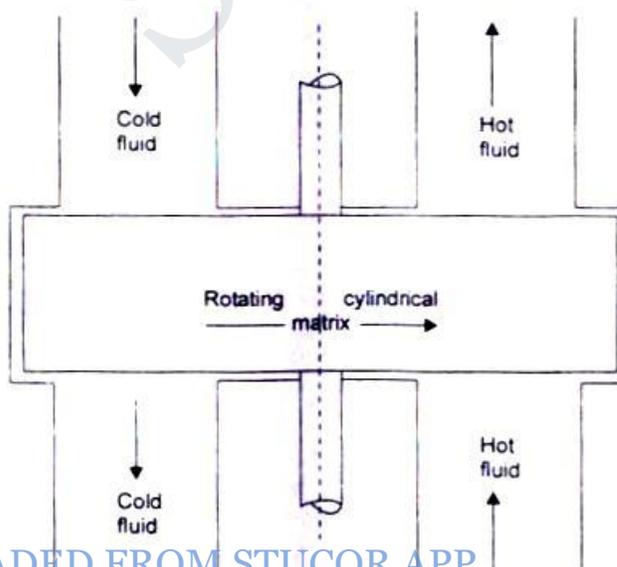


Fig. 3.1 (a) Rotating matrix regenerator

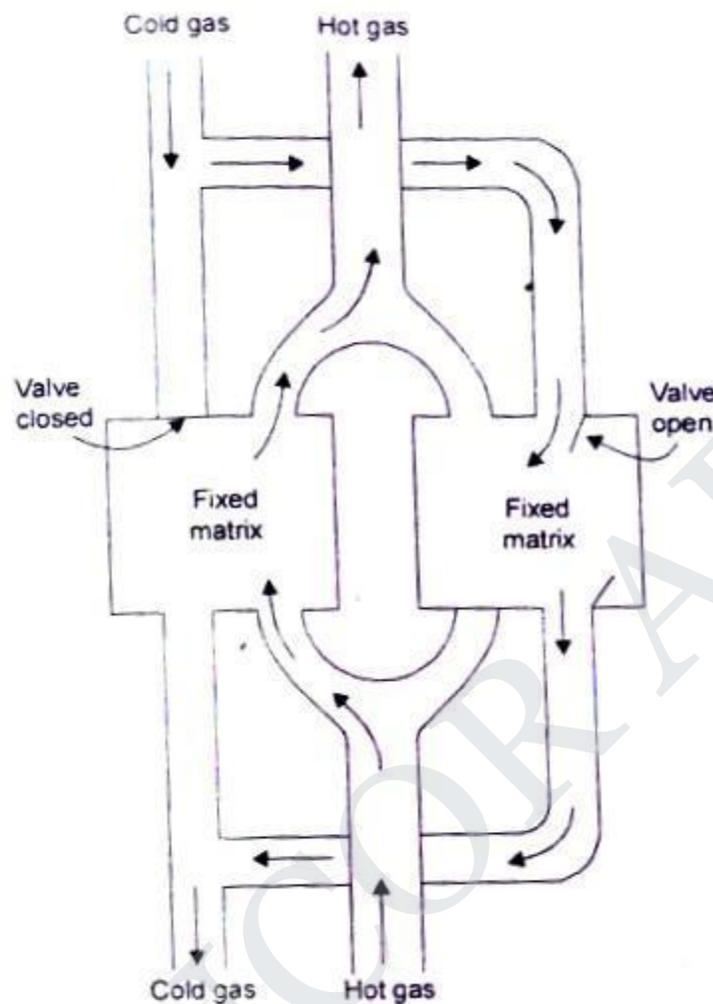


Fig. 3.1 (b) Stationary matrix regenerator

In a recuperator, hot and cold fluids flow continuously following the same path. The heat transfer process consists of convection between the fluid and the separating wall, conduction through the wall and convection between the wall and the other fluid. Most common heat exchangers are of recuperative type having a wide variety of geometries:

### 3.8 Classification of Heat Exchangers

Heat exchangers are generally classified according to the relative directions of hot and cold fluids:

(a) Parallel Flow – the hot and cold fluids flow in the same direction. Fig 3.2 depicts such a heat exchanger where one fluid (say hot) flows through the pipe and the other fluid (cold) flows

through the annulus.

(b) Counter Flow – the two fluids flow through the pipe but in opposite directions. A common type of such a heat exchanger is shown in Fig. 3.3. By comparing the temperature distribution of the two types of heat exchanger

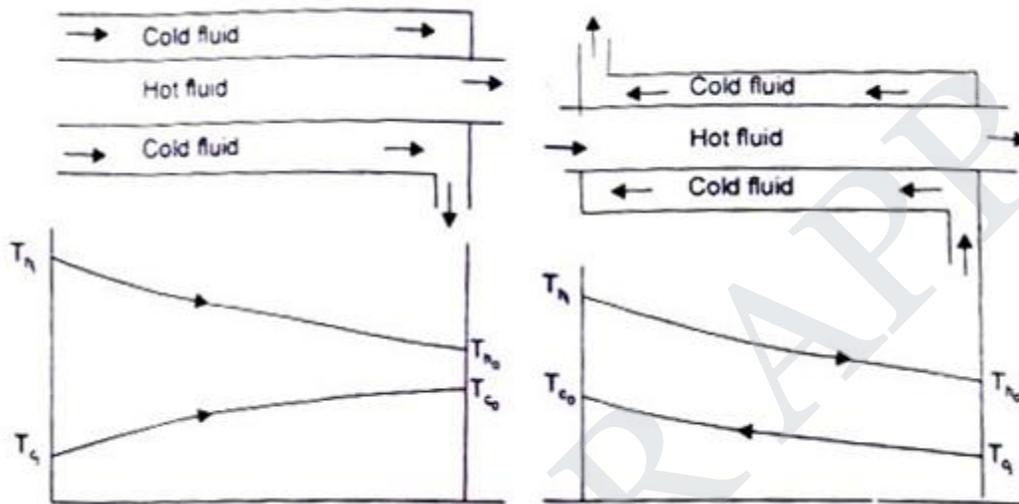


Fig 3.2 Parallel flow heat exchanger with temperature distribution

Fig 3.3 Counter-flow heat exchanger with temperature distribution

we find that the temperature difference between the two fluids is more uniform in counter flow than in the parallel flow. Counter flow exchangers give the maximum heat transfer rate and are the most favoured devices for heating or cooling of fluids.

When the two fluids flow through the heat exchanger only once, it is called one-shell-pass and one-tube-pass as shown in Fig. 3.2 and 3.3. If the fluid flowing through the tube makes one pass through half of the tube, reverses its direction of flow, and makes a second pass through the remaining half of the tube, it is called 'one-shell-pass, two-tube-pass' heat exchanger, fig 3.4. Many other possible flow arrangements exist and are being used. Fig. 10.5 depicts a 'two-shell-pass, four-tube-pass' exchanger.

(c) Cross-flow - A cross-flow heat exchanger has the two fluid streams flowing at right angles to each other. Fig. 3.6 illustrates such an arrangement An automobile radiator is a good example of cross-flow exchanger. These exchangers are 'mixed' or 'unmixed' depending upon the

mixing or not mixing of either fluid in the direction transverse to the direction of the flow stream and the analysis of this type of heat exchanger is extremely complex because of the variation in the temperature of the fluid in and normal to the direction of flow.

(d) Condenser and Evaporator - In a condenser, the condensing fluid temperature remains almost constant throughout the exchanger and temperature of the colder fluid gradually increases from the inlet to the exit, Fig. 3.7 (a). In an evaporator, the temperature of the hot fluid gradually decreases from the inlet to the outlet whereas the temperature of the colder fluid remains the same during the evaporation process, Fig. 3.7(b). Since the temperature of one of the fluids can be treated as constant, it is immaterial whether the exchanger is parallel flow or counter flow.

(e) Compact Heat Exchangers - these devices have close arrays of finned tubes or plates and are typically used when atleast one of the fluids is a gas. The tubes are either flat or circular as shown in Fig. 10.8 and the fins may be flat or circular. Such heat exchangers are used to achieve a very large ( $\geq 700 \text{ m}^2/\text{mJ}$ ) heat transfer surface area per unit volume. Flow passages are typically small and the flow is usually laminar.

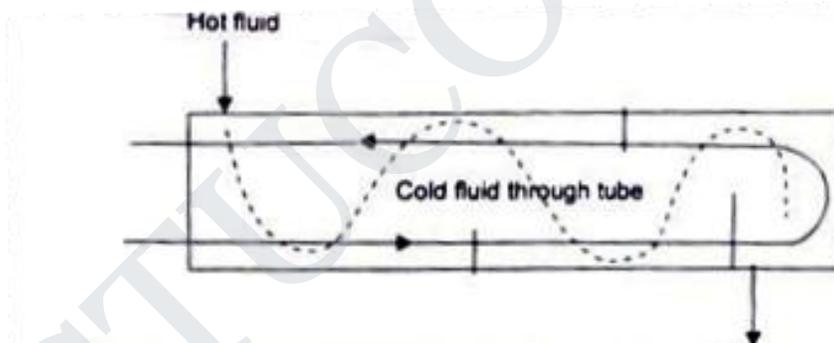


Fig 3.4: multi pass exchanger one shell pass, two shell pass

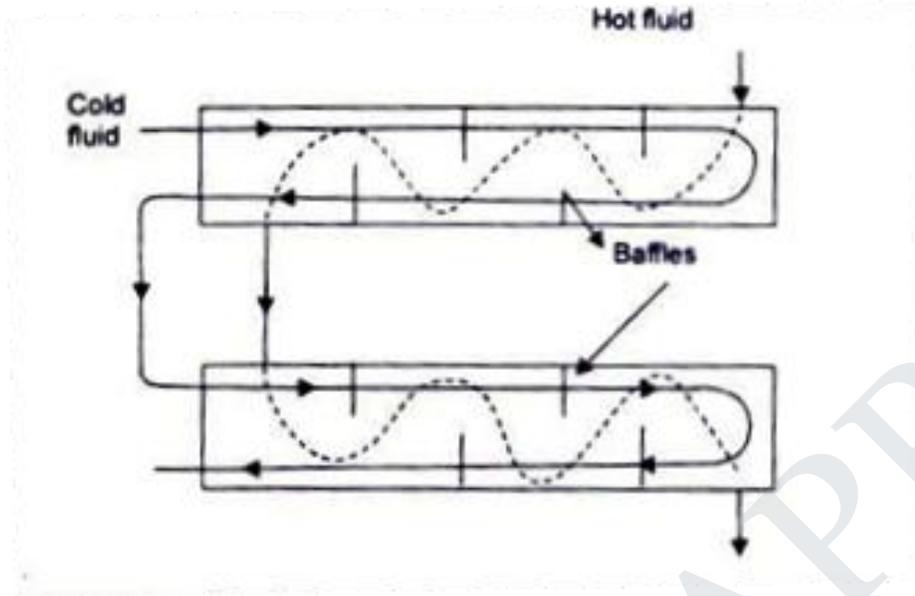


Fig 3.5: Two shell passes, four-tube passes heat exchanger (baffles increases the convection coefficient of the shell side fluid by inducing turbulence and a cross flow velocity component)

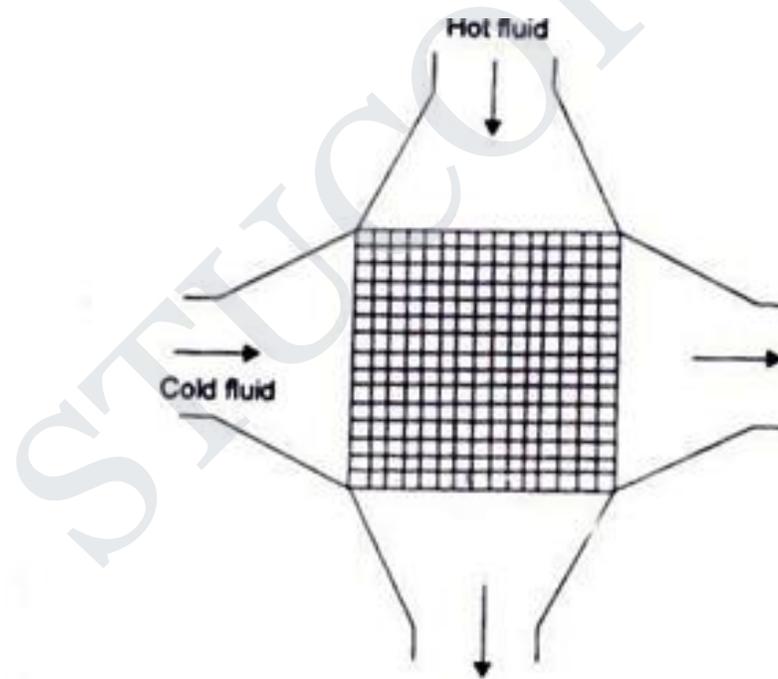


Fig 3.6: A cross-flow exchanger

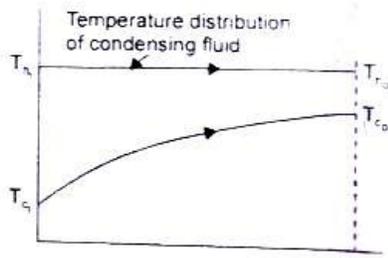


Fig. 10.7 (a) A condenser

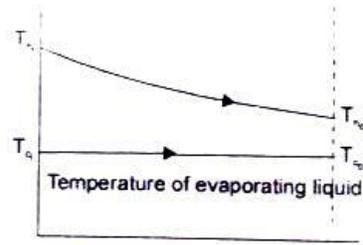


Fig. 10.7 (b) An evaporator

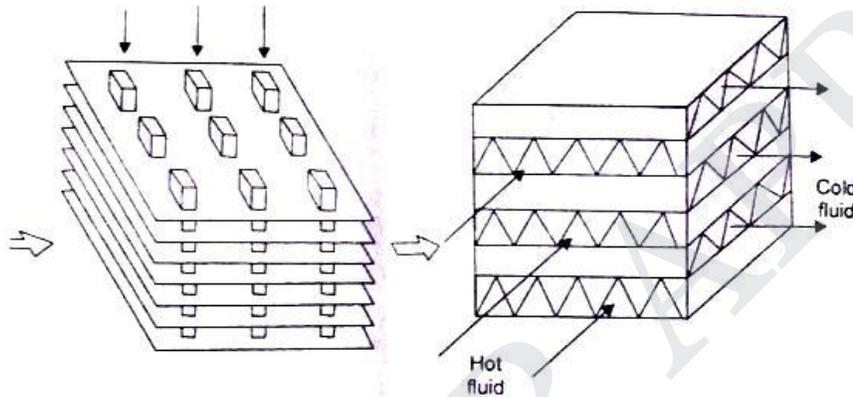


Fig. 3.8 Compact heat exchangers: (a) flat tubes, continuous plate fins, (b) plate fin (single pass)

### 3.9 The Overall Heat Transfer Coefficient

The determination of the overall heat transfer coefficient is an essential, and often the most uncertain, part of any heat exchanger analysis. We have seen that if the two fluids are separated by a plane composite wall the overall heat transfer coefficient is given by:

$$1/U = (1/h_i) + (L_1/k_1) + (L_2/k_2) + (1/h_o) \tag{3.8}$$

If the two fluids are separated by a cylindrical tube (inner radius \$r\_i\$, outer radius \$r\_o\$), the overall heat transfer coefficient is obtained as:

$$1/U_i = (1/h_i) + (r_i/k) \ln(r_o/r_i) + (r_i/r_o)(1/h_o) \tag{3.9}$$

where \$h\_i\$, and \$h\_o\$ are the convective heat transfer coefficients at the inside and outside surfaces and \$U\_i\$ is the overall heat transfer coefficient based on the inside surface area. Similarly, for the outer surface area, we have:

$$1/U_o = (1/h_o) + (r_o/k) \ln(r_o/r_i) + (r_o/r_i)(1/h_i) \tag{3.10}$$

and  $U_i A_i$  will be equal to  $U_o A_o$ ; or,  $U_i r_i = U_o r_o$ .

The effect of scale formation on the inside and outside surfaces of the tubes of a heat exchanger would be to introduce two additional thermal resistances to the heat flow path. If  $h_{si}$  and  $h_{so}$  are the two heat transfer coefficients due to scale formation on the inside and outside surface of the inner pipe, the rate of heat transfer is given by

$$Q = (T_i - T_o) / \left[ \left( \frac{1}{h_i A_i} \right) + \frac{1}{h_{si} A_i} + \frac{\ln(r_o / r_i)}{2\pi L k} + \frac{1}{h_{so} A_o} + \left( \frac{1}{h_o A_o} \right) \right] \quad (3.11)$$

where  $T_i$ , and  $T_o$  are the temperature of the fluid at the inside and outside of the tube. Thus, the overall heat transfer coefficient based on the inside and outside surface area of the tube would be:

$$1/U_i = 1/h_i + 1/h_{si} + (r_i/k) \ln(r_o/r_i) + (r_i/r_o)(1/h_{so}) + (r_i/r_o)(1/h_o); \quad (3.12)$$

and

$$1/U_o = (r_o/r_i)(1/h_i) + (r_o/r_i)(1/h_{si}) + \ln(r_o/r_i)(r_o/k) + 1/h_{so} + 1/h_o \text{ K}$$

### 3.10 FOULING FACTORS ANALYSIS :

Heat exchanger walls are usually made of single materials. Sometimes the walls are bimetallic (steel with aluminium cladding) or coated with a plastic as a protection against corrosion, because, during normal operation surfaces are subjected to fouling by fluid impurities, rust formation, or other reactions between the fluid and the wall material. The deposition of a film or scale on the surface greatly increases the resistance to heat transfer between the hot and cold fluids. And, a scale coefficient of heat transfer  $h_s$ , is defined as:

$$R_s = 1/h_s A, \text{ } ^\circ\text{C/W or K/W}$$

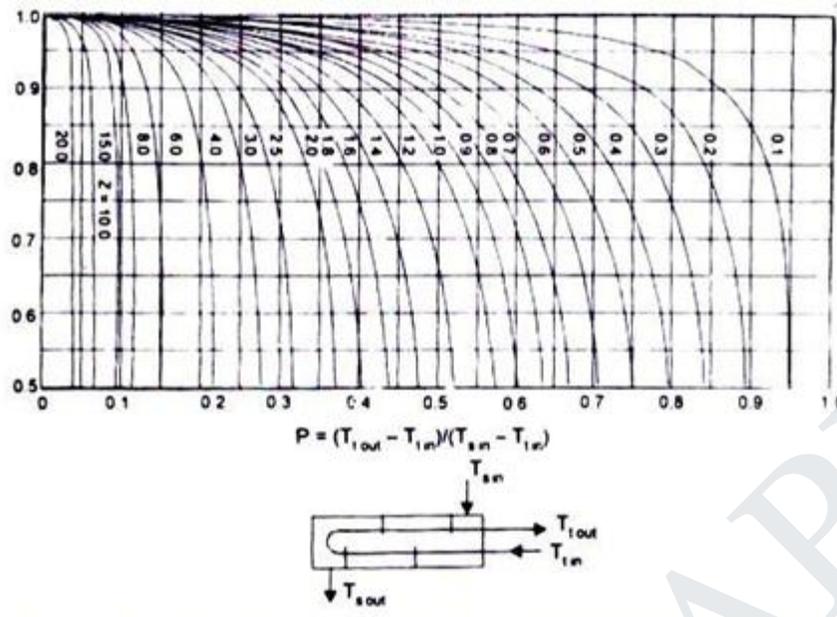


Fig (a) correctio factor to counter flow LMTD for heat exchanger with one shell pass andtwo, or a muple of two,tube passes

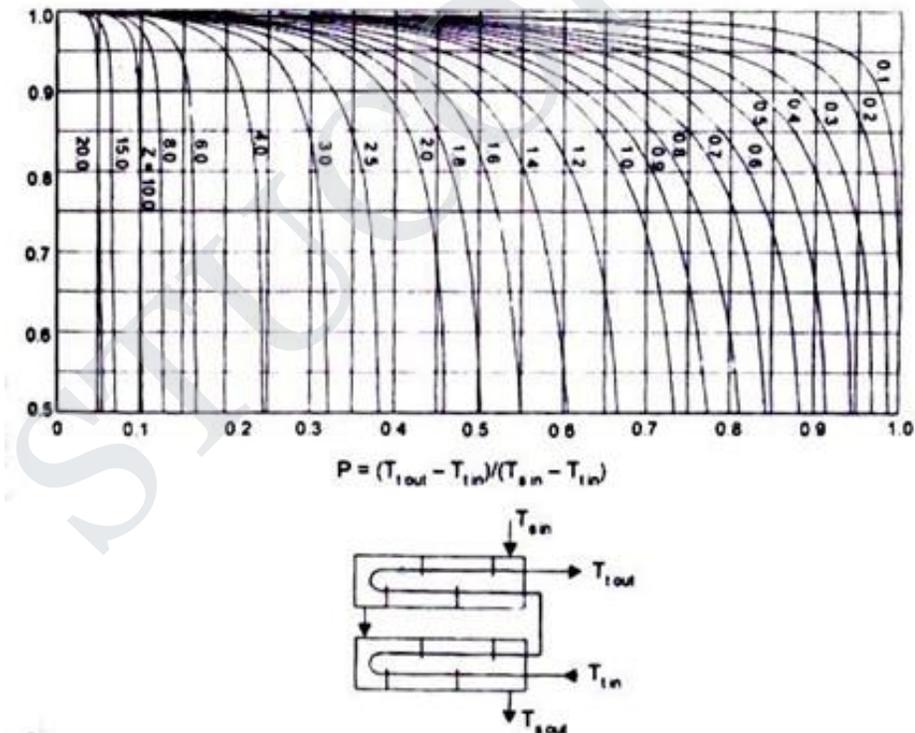
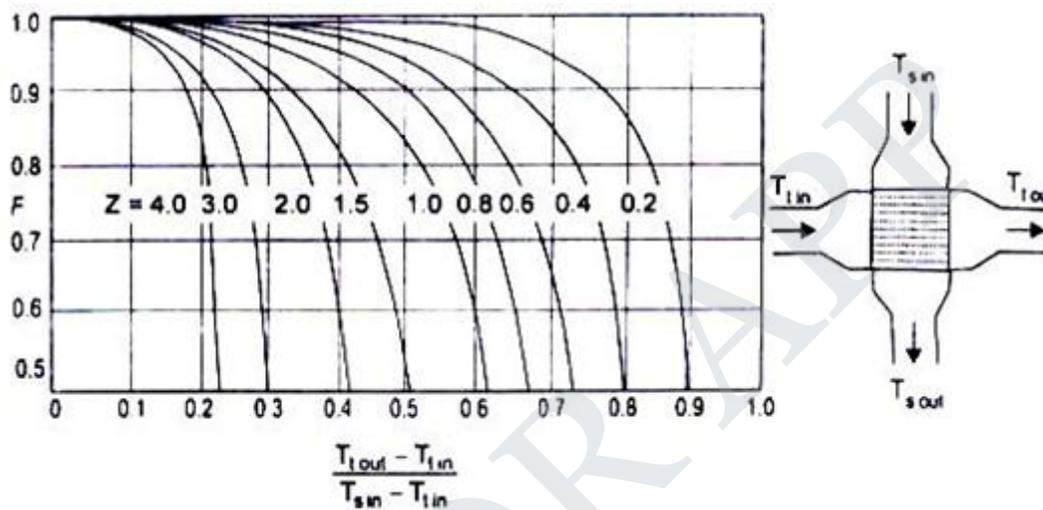


Fig Correction factor to counter flow LMTD for heat exchanger with two shell passes and a multiple of two tube passes

where  $A$  is the area of the surface before scaling began and  $1/h_s$ , is called 'Fouling Factor'. Its value depends upon the operating temperature, fluid velocity, and length of service of the heat exchanger. Table 10.1 gives the magnitude of  $1/h$ , recommended for inclusion in the overall heat transfer coefficient for calculating the required surface area of the exchanger



Fig(c) Correction factor to counter flow LMTD for cross flow heat exchangers, fluid on shell side mixed, other fluid unmixed one tube pass..

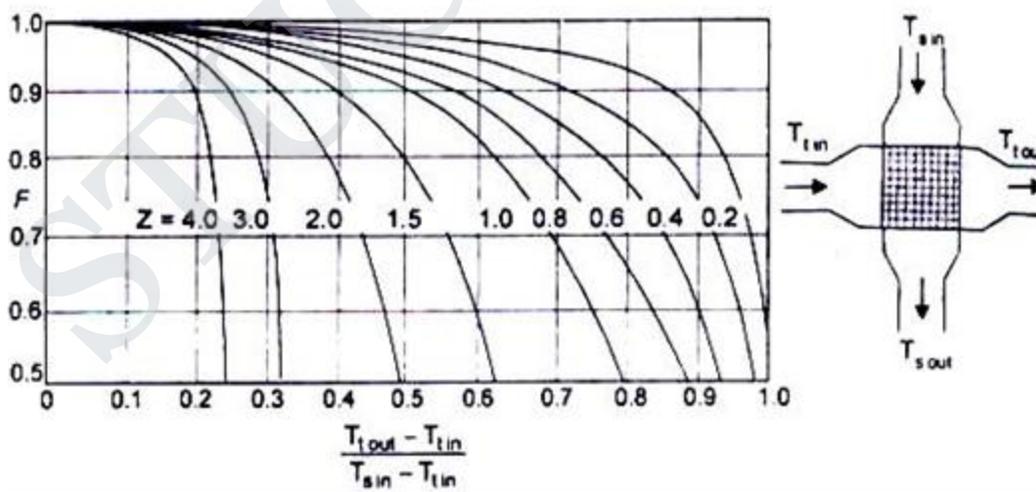


Fig. (d) Correction factor to counter flow LMTD for cross flow heat exchangers, both fluids unmixed, one tube pass.

**3.11 LMTD METHOD :**

**Expression for Log Mean Temperature Difference - Its Characteristics**

Fig. 10.9 represents a typical temperature distribution which is obtained in heat exchangers. The rate of heat transfer through any short section of heat exchanger tube of surface area  $dA$  is:  $dQ = U dA(T_h - T_c) = U dA \Delta T$ . For a parallel flow heat exchanger, the hot fluid cools and the cold fluid is heated in the direction of increasing area. therefore, we may write

$d\dot{Q} = -\dot{m}_h c_h dT_h = \dot{m}_c c_c dT_c$  and  $d\dot{Q} = -\dot{C}_h dT_h = \dot{C}_c dT_c$  where  $\dot{C} = \dot{m} \times c$ , and is called the 'heat capacity rate.'

Thus,  $d(\Delta T) = d(T_h - T_c) = dT_h - dT_c = -(1/C_h + 1/C_c) d\dot{Q}$  (3.1)

For a counter flow heat exchanger, the temperature of both hot and cold fluid decreases in the direction of increasing area, hence

$d\dot{Q} = -\dot{m}_h c_h dT_h = -\dot{m}_c c_c dT_c$ , and  $d\dot{Q} = -C_h dT_h = -C_c dT_c$   
 or,  $d(\Delta T) = dT_h - dT_c = (1/C_h - 1/C_c) d\dot{Q}$  (3.2)

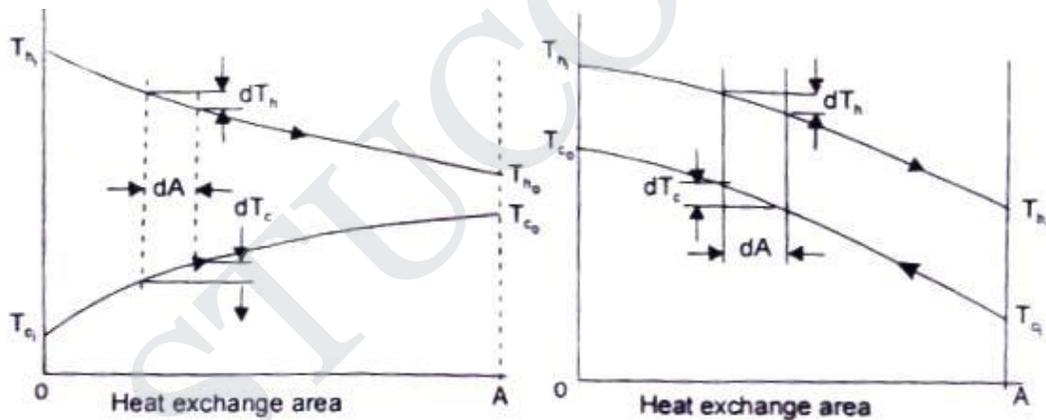


Fig. 3.9 Parallel flow and Counter flow heat exchangers and the temperature distribution with length

Integrating equations (3.1) and (3.2) between the inlet and outlet. and assuming that the specific heats are constant, we get

$-(1/C_h \pm 1/C_c) \dot{Q} = \Delta T_o - \Delta T_i$  (3.3)

The positive sign refers to parallel flow exchanger, and the negative sign to the counter

flow type. Also, substituting for  $dQ$  in equations (10.1) and (10.2) we get

$$-(1/C_h \pm 1/C_c)UdA = d(\Delta T)/\Delta T \quad (3.3a)$$

Upon integration between inlet  $i$  and outlet  $0$  and assuming  $U$  as a constant,

$$\text{We have } -(1/C_h \pm 1/C_c)U A = \ln(\Delta T_0/\Delta T_i)$$

By dividing (10.3) by (10.4), we get

$$\dot{Q} = UA[(\Delta T_0 - \Delta T_i)/\ln(\Delta T_0/\Delta T_i)] \quad (3.5)$$

Thus the mean temperature difference is written as

Log Mean Temperature Difference,

$$\text{LMTD} = (\Delta T_0 - \Delta T_i)/\ln(\Delta T_0/\Delta T_i) \quad (3.6)$$

(The assumption that  $U$  is constant along the heat exchanger is never strictly true but it may be a good approximation if at least one of the fluids is a gas. For a gas, the physical properties do not vary appreciably over moderate range of temperature and the resistance of the gas film is considerably higher than that of the metal wall or the liquid film, and the value of the gas film resistance effectively determines the value of the overall heat transfer coefficient  $U$ .)

It is evident from Fig.1 0.9 that for parallel flow exchangers, the final temperature of fluids lies between the initial values of each fluid whereas in counter flow exchanger, the temperature of the colder fluid at exit is higher than the temperature of the hot fluid at exit. Therefore, a counter flow exchanger provides a greater temperature range, and the LMTD for a counter flow exchanger will be higher than for a given rate of mass flow of the two fluids and for given temperature changes, a counter flow exchanger will require less surface area.

### 3.12 Special Operating Conditions for Heat Exchangers

(i) Fig. 3.7a shows temperature distributions for a heat exchanger (condenser) where the hot fluid has a much larger heat capacity rate,  $\dot{C}_h = \dot{m}_h c_h$  than that of cold fluid,  $\dot{C}_c = \dot{m}_c c_c$  and therefore, the temperature of the hot fluid remains almost constant throughout the exchanger and the temperature of the cold fluid increases. The LMTD, in this case is not affected by whether the exchanger is a parallel flow or counter flow.

(ii) Fig. 3.7b shows the temperature distribution for an evaporator. Here the cold fluid undergoes a change in phase and remains at a nearly uniform temperature ( $\dot{C}_c \rightarrow \infty$ ). The same effect would be achieved without phase change if  $\dot{C}_c \gg \dot{C}_h$ , and the LMTD will remain the same for both parallel flow and counter flow exchangers.

(iii) In a counter flow exchanger, when the heat capacity rate of both the fluids are equal,  $\dot{C}_c = \dot{C}_h$ , the temperature difference is the same all along the length of the tube. And in that case, LMTD should be replaced by  $\Delta T_a = \Delta T_b$ , and the temperature profiles of the two fluids along its length would be parallel straight lines.

(Since  $d\dot{Q} = -\dot{C}_c dT_c = -\dot{C}_h dT_h$ ;  $dT_c = -d\dot{Q}/\dot{C}_c$ , and  $dT_h = -d\dot{Q}/\dot{C}_h$

and,  $dT_c - dT_h = d\theta = -d\dot{Q}(1/\dot{C}_c - 1/\dot{C}_h) = 0$  (because  $\dot{C}_c = \dot{C}_h$ )

Or,  $d\theta = 0$ , gives  $\theta = \text{constant}$  and the temperature profiles of the two fluids along its length would be parallel straight lines.)

### 3.13 LMTD for Cross-flow Heat Exchangers :

LMTD given by Eq (10.6) is strictly applicable to either parallel flow or counter flow exchangers. When we have multipass parallel flow or counter flow or cross flow exchangers, LMTD is first calculated for single pass counter flow exchanger and the mean temperature difference is obtained by multiplying the LMTD with a correction factor  $F$  which takes care of the actual flow arrangement of the exchanger. Or,

$$\dot{Q} = U A F (\text{LMTD}) \quad (3.7)$$

The correction factor  $F$  for different flow arrangements are obtained from charts given.

### 3.14 NTU METHOD:

#### Heat Exchangers Effectiveness - Useful Parameters

In the design of heat exchangers, the efficiency of the heat transfer process is very important. The method suggested by Nusselt and developed by Kays and London is now being extensively used. The effectiveness of a heat exchanger is defined as the ratio of the actual heat transferred to the maximum possible heat transfer.

Let  $\dot{m}_h$  and  $\dot{m}_c$  be the mass flow rates of the hot and cold fluids,  $c_h$  and  $c_c$  be the respective specific heat capacities and the terminal temperatures be  $T_h$  and  $T_h$  for the hot fluid at inlet and outlet,  $T_{h_i}$  and  $T_{h_o}$  for the cold fluid at inlet and outlet. By making an energy balance and assuming that there is no loss of energy to the surroundings, we write

$$\begin{aligned}\dot{Q} &= \dot{m}_h c_h (T_{h_i} - T_{h_o}) = \dot{C}_h (T_{h_i} - T_{c_o}), \text{ and} \\ &= \dot{m}_c c_c (T_{c_o} - T_{c_i}) = \dot{C}_c (T_{c_o} - T_{c_i})\end{aligned}\quad (3.13)$$

From Eq. (10.13), it can be seen that the fluid with smaller thermal capacity,  $C$ , has the greater temperature change. Further, the maximum temperature change of any fluid would be  $(T_{h_i} - T_{c_i})$  and this Ideal temperature change can be obtained with the fluid which has the minimum heat capacity rate. Thus,

$$\text{Effectiveness, } \epsilon = \dot{Q} / C_{\min} (T_{h_i} - T_{c_i}) \quad (3.14)$$

Or, the effectiveness compares the actual heat transfer rate to the maximum heat transfer rate whose only limit is the second law of thermodynamics. An useful parameter which also measures the efficiency of the heat exchanger is the 'Number of Transfer Units', NTU, defined as

NTU = Temperature change of one fluid/LMTD.

Thus, for the hot fluid:  $\text{NTU} = (T_{h_i} - T_{h_o}) / \text{LMTD}$ , and

for the cold fluid:  $\text{NTU} = (T_{c_o} - T_{c_i}) / \text{LMTD}$

Since  $\dot{Q} = UA(\text{LMTD}) = C_h (T_{h_i} - T_{h_o}) = \dot{C}_c (T_{c_o} - T_{c_i})$

we have  $\text{NTU}_h = UA / C_h$  and  $\text{NTU}_c = UA / C_c$

The heat exchanger would be more effective when the NTU is greater, and therefore,

$$\text{NTU} = AU / C_{\min} \quad (3.15)$$

An another useful parameter in the design of heat exchangers is the ratio the minimum to the maximum thermal capacity, i.e.,  $R = C_{\min} / C_{\max}$ ,

where  $R$  may vary between 1 (when both fluids have the same thermal capacity) and 0 (one of the fluids has infinite thermal capacity, e.g., a condensing vapour or a boiling liquid).

### 3.15 Effectiveness - NTU Relations

For any heat exchanger, we can write:  $\epsilon = f(\text{NTU}, C_{\min}/C_{\max})$ . In order to determine a specific form of the effectiveness-NTU relation, let us consider a parallel flow heat exchanger for which  $C_{\min} = C_h$ . From the definition of effectiveness (equation 10.14), we get

$$\epsilon = (T_{h_i} - T_{h_0}) / (T_{h_i} - T_{c_i})$$

and,  $C_{\min}/C_{\max} = C_h/C_c = (T_{c_0} - T_{c_i}) / (T_{h_i} - T_{h_0})$  for a parallel flow heat exchanger, from Equation 10.4,

$$\ln(T_{h_0} - T_{c_0}) / (T_{h_i} - T_{c_i}) = -UA(1/C_h + 1/C_c) = \frac{-UA}{C_{\min}}(1 + C_{\min}/C_{\max})$$

$$\text{or, } (T_{h_0} - T_{c_0}) / (T_{h_i} - T_{c_i}) = \exp[-\text{NTU}(1 + C_{\min}/C_{\max})]$$

$$\text{But, } (T_{h_0} - T_{c_0}) / (T_{h_i} - T_{c_i}) = (T_{h_0} - T_{h_i} + T_{h_i} - T_{c_0}) / (T_{h_i} - T_{c_i})$$

$$= \left[ (T_{h_0} - T_{h_i}) + (T_{h_i} - T_{c_i}) - R(T_{h_i} - T_{h_0}) \right] / (T_{h_i} - T_{c_i})$$

$$= \epsilon + 1 - R \Rightarrow \epsilon = 1 - \epsilon(1 + R)$$

$$\text{Therefore, } \epsilon = \left[ 1 - \exp\{-\text{NTU}(1 + R)\} \right] / (1 + R)$$

$$\text{NTU} = -\ln \left[ 1 - \epsilon(1 + R) \right] / (1 + R)$$

$$\text{Similarly, for a counter flow exchanger, } \epsilon = \frac{\left[ 1 - \exp\{-\text{NTU}(1 - R)\} \right]}{\left[ 1 - R \exp\{-\text{NTU}(1 - R)\} \right]};$$

$$\text{and, } \text{NTU} = \left[ 1 / (R - 1) \right] \ln \left[ (\epsilon - 1) / (\epsilon R - 1) \right]$$

**Heat Exchanger Effectiveness Relation**

Flow arrangement relationship

Concentric tube

$$\text{Parallel flow} \quad \epsilon = \frac{1 - \exp[-N(1+R)]}{(1+R)}; R = C_{\min} / C_{\max}$$

$$\text{Counter flow} \quad \epsilon = \frac{1 - \exp[-N(1-R)]}{1 - R \exp[-N(1-R)]}; R < 1$$

$$\epsilon = N / (1 + N) \text{ for } R = 1$$

Cross flow (single pass)

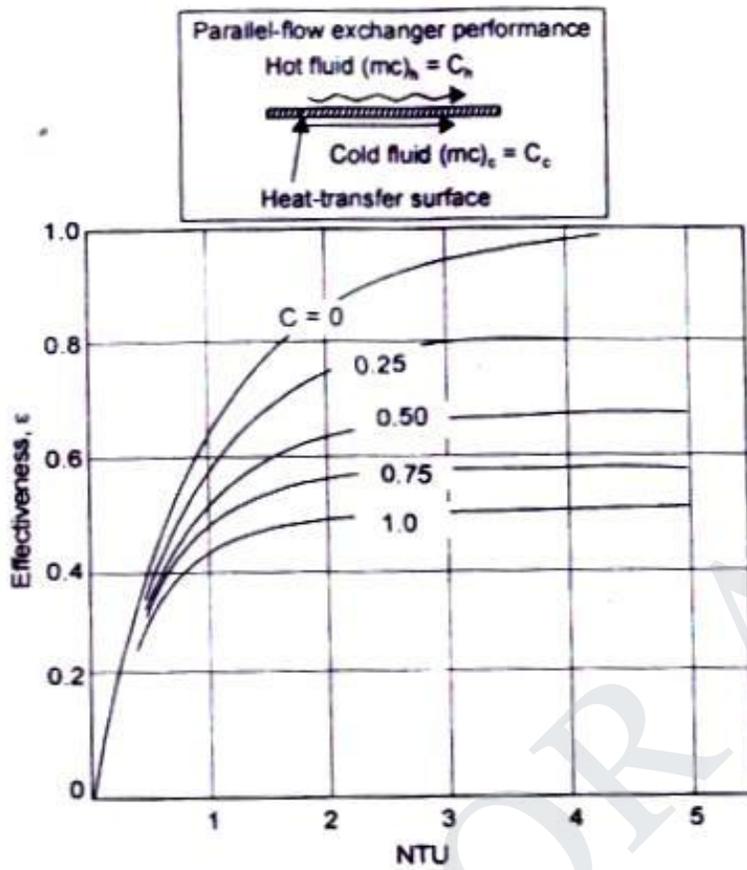
$$\text{Both fluids unmixed} \quad \epsilon = 1 - \exp\left[(1/R)(N)^{0.22} \left\{ \exp(-R(N)^{0.78}) - 1 \right\}\right]$$

$$C_{\max} \text{ mixed, } C_{\min} \text{ unmixed} \quad \epsilon = (1/R) \left[ 1 - \exp\{-R(1 - \exp(-N))\} \right]$$

$$C_{\min} \text{ mixed, } C_{\max} \text{ unmixed} \quad \epsilon = 1 - \exp\left[-R^{-1} \{1 - \exp(-RN)\}\right]$$

$$\text{All exchangers (} R = 0 \text{)} \quad \epsilon = 1 - \exp(-N)$$

Kays and London have presented graphs of effectiveness against NTU for Various values of R applicable to different heat exchanger arrangements,.



**UNIT 3: PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS****PART A****TWO MARK QUESTIONS:****(AU 2013)****1. What is meant by pool boiling?**

If heat is added to a liquid from a submerged solid surface, the boiling process is referred to as pool boiling. In the case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment.

**2. State the difference between the drop wise condensation and film wise condensation.**

Filmwise condensation:

The liquid condensate wets the solid surface, spreads out and forms a continuous film over the entire surface is known as film wise condensation.

Drop wise condensation:

In drop wise condensation, the vapor condenses into small liquid droplets of various sizes which fall down the surface in a random fashion.

**3. What is meant by LMTD ?****(AU2014)**

We know that the temperature difference between the hot and cold fluids in the heat exchanger varies from point to point. In addition various modes of heat transfer are involved. Therefore based on concept of appropriate mean temperature difference, also called logarithmic mean temperature difference, the total heat transfer rate in the heat exchanger is expressed as

$$Q = UA(\Delta T)_m,$$

Where  $U$  – Overall heat coefficient  $W/m^2K$

$A$  – Area,  $m^2$

$T$  – Temperature difference.

**4. What is meant by Fouling factor?****(AU 2012)**

We know the surface of a heat exchanger do not remain clean after it has been in use for some time. The surfaces become fouled with scaling or deposits. The effect of these deposits affecting the value of overall heat transfer coefficient. This effect is taken care of by introducing an additional thermal resistance called the fouling factor.

**5. What is meant by condensation?**

The change of phase from vapor to liquid state is known as condensation.

**6. What is compact heat exchanger?**

There are many special purpose heat exchangers called compact heat exchangers .They are generally employed when convective heat transfer coefficient associated with one of the fluids is much smaller than the associated with the other fluid.

**7. Define Effectiveness.**

**(AU 2013)**

The heat exchanger effectiveness is defined as the ratio of actual heat transfer to the maximum possible heat transfer.

Effectiveness  $\epsilon = \text{Actual heat transfer} / \text{maximum possible heat transfer}$ .

**8. What is meant by parallel flow heat exchangers and counter flow heat exchanger?**

Parallel flow heat exchanger:

In this type of heat exchanger, hot and cold fluids move in the same direction.

Counter flow heat exchanger:

In this type of heat exchanger, hot and cold fluids move in parallel but in opposite directions.

**9. What is heat exchanger?**

**(AU 2011)**

A heat exchanger is defined as an equipment which transfers the heat from a hot fluid to a cold fluid.

**10. Give the expression for NTU.**

**(AU2010)**

Number of Transfer Units (NTU) =  $UA / C_{min}$ .

**11. List the various promoters used for maintaining drop wise condensation.**

Oleic acid, benzyl, certain fats and waxes are effective promoters used for maintaining drop wise condensation.

## 12. How are heat exchangers classified based on flow arrangement?

Parallel flow, counter flow, compact and cross flow heat exchanger.

### PART B:

#### ANSWER THE FOLLOWING:

1. In a counter flow double pipe heat exchanger, oil is cooled from 85°C to 55°C by water entering at 25°C. The mass flow rate of oil is 9,800 kg/h and specific heat of oil is 2000 j/kg K. the mass flow rate of water is 8,000 kg/h and specific heat of water is 4180 j/kg K. Determine the heat exchanger area and heat transfer rate for an overall heat transfer coefficient of 280 W/m<sup>2</sup> K. (AU 2011)

#### Given:

Hot fluid – oil                      Cold fluid – water

T<sub>1</sub>, T<sub>2</sub>                                      t<sub>1</sub>, t<sub>2</sub>

Entry temperature of oil T<sub>1</sub> = 85°C

Exit temperature of oil T<sub>2</sub> = 55°C

Entry temperature of water t<sub>1</sub> = 25°C

Mass flow rate of oil (Hot fluid) m<sub>h</sub> = 9,800 kg/h

$$\frac{9,800}{3600} \text{ kg/s}$$

$$m_h = 2.72 \text{ kg/s}$$

Specific heat of oil C<sub>ph</sub> = 2000 j/kg K

Mass flow of water (Cold fluid m<sub>c</sub> = 8,000 kg/h)

$$\frac{8,000}{3600} \text{ kg/s}$$

$$m_e = 2.22 \text{ kg/s}$$

Specific heat of water  $C_{pc} = 4180 \text{ J/kg K}$

Overall heat transfer coefficient  $U = 280 \text{ W/m}^2\text{K}$

### To find

1. Heat exchanger area (A)
2. Heat transfer rate (Q)

### Solution

We know that

Heat lost by oil Hot fluid = Heat gained by water cold fluid

$$Q_h = Q_c$$

$$m_h C_{ph} (T_1 - T_2) = m_c C_{pc} (t_2 - t_1)$$

$$2.72 \times 2000 (85 - 55) = 2.22 \times 4180 \times (t_2 - 25)$$

$$163.2 \times 10^3 = 9279.6 t_2 - 231.9 \times 10^3$$

$$t_2 = 42.5^\circ\text{C}$$

$$\boxed{\text{Exit temperature of water } t_2 = 42.5^\circ\text{C}}$$

$$\text{Heat transfer } Q = m_c C_{pc} (t_2 - t_1) \text{ (or) } m_h C_{ph} (T_1 - T_2)$$

$$Q = 2.22 \times 4180 \times (42.5 - 25)$$

$$Q = 162 \times 10^3 \text{ W}$$

$$Q = UA (\Delta T)_m \dots\dots\dots 1$$

From HMT data book Page No.154

For counter flow

$$(\Delta T)_m = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln \left[ \frac{T_1 - t_1}{T_2 - t_2} \right]}$$

From HMT data book Page No.154

$$(\Delta T)_m = \frac{(85 - 42.5) - (55 - 25)}{\ln \left[ \frac{85 - 42.5}{55 - 25} \right]}$$

$$\boxed{(\Delta T)_m = 35.8^\circ\text{C}}$$

Substitute  $(\Delta T)_m$  U and Q values in Equation (1)

$$(1) \Rightarrow Q = UA (\Delta T)_m$$

$$\Rightarrow 162 \times 10^3 = 280 \times A \times 35.8$$

$$\Rightarrow A = 16.16 \text{ m}^2$$

2. An oil cooler of the form of tubular heat exchanger cools oil from a temperature of 90°C to 35°C by a large pool of stagnant water assumed constant temperature of 28°C. The tube length is 32 m and diameter is 28 mm. The specific heat and specific gravity of the oil are 2.45 kJ/kg K and 0.8 respectively. The velocity of the oil is 62 cm/s. Calculate the overall heat transfer coefficient. (AU 2013)

**Given:**

Hot fluid – Oil                      Cold fluid – Water

$T_1, T_2$      $t_1, t_2$

Entry temperature of oil  $T_1 = 90^\circ\text{C}$

Exit temperature of oil  $T_2 = 35^\circ\text{C}$

Entry and Exit temperature of water  $t_1=t_2=28^\circ\text{C}$

Tube length  $L = 32 \text{ m}$

Diameter  $D = 28 \text{ mm} = 0.028 \text{ m}$

Specific heat of oil  $C_{ph} = 2.45 \text{ kJ/kg/K}$

$C_{ph} = 2.45 \times 10^3 \text{ J/kg K}$

Specific gravity of oil = 0.8

Velocity of oil  $C = 62 \text{ cm/s} = 0.62 \text{ m/s}$ .

**To find**

$$\text{Specific gravity of oil} = \frac{\text{Density of oil}}{\text{Density of water}}$$

$$= \frac{\rho_o}{\rho_w}$$

We know

$$0.8 = \frac{\rho_o}{1000}$$

$$\text{Density of oil } \rho_o = 800 \text{ kg/m}^3$$

Mass flow rate of oil

$$m_h = \rho \times A \times C$$

$$= 800 \times \frac{\pi}{4} (D^2) \times 0.62$$

$$= 800 \times \frac{\pi}{4} (0.028)^2 \times 0.62$$

$$m_h = 0.305 \text{ kg/s}$$

We know

Heat transfer

$$Q = m_c C_{pc} (t_2 - t_1)$$

$$0.305 \times 2.45 \times 10^3 \times [90 - 35]$$

$$Q = 41 \times 10^3 \text{ W}$$

We know

$$\text{Heat transfer } Q = U A (\Delta T)_m \dots\dots\dots 1$$

(From HMT data book Page No.154)

Where

$(\Delta T)_m$  – Logarithmic Mean Temperature Difference LMTD.

For parallel flow

$$(\Delta T)_m = \frac{[(T_1 - t_1) - (T_2 - t_2)]}{\ln \left[ \frac{T_1 - t_1}{T_2 - t_2} \right]}$$

From HMT data book Page No.154

$$= \frac{(90-28)-(35-28)}{\ln \left[ \frac{90-28}{35-28} \right]}$$

$$(\Delta T)_m = 25.2^\circ\text{C}$$

Substitute  $(\Delta T)_m$  Q value in equation 1

$$(1) \Rightarrow q = U A (\Delta T)_m$$

$$41 \times 10^3 = U \times \pi D L \times (\Delta T)_m$$

$$41 \times 10^3 = U \times \pi \times 0.028 \times 32 \times 25.2$$

$$U = 577.9$$

$$\boxed{\text{Overall heat transfer coefficient } U = 577.9 \text{ W/m}^2\text{K}}$$

### Result

$$U = 577.9 \text{ W/m}^2\text{K}$$

3. A parallel flow heat exchanger is used to cool. 4.2 kg/min of hot liquid of specific heat 3.5 kJ/kgK at 130°C. A cooling water of specific heat 4.18 kJ/kg K is used for cooling purpose at a temperature of 15°C. The mass flow rate of cooling water is 17 kg/min calculate the following. (AU 2010)

1. Outlet temperature of liquid

2. Outlet temperature of water

3. Effectiveness of heat exchanger

Take care,

Overall heat transfer coefficient is 1100 W/m<sup>2</sup> K.

Heat exchanger area is 0.30 m<sup>2</sup>

Given:

Mass flow rate of hot liquid  $m_h = 4.2 \text{ kg/min}$

$M_h = 0.07 \text{ kg/s}$

Specific heat of hot liquid  $C_{ph} = 3.5 \text{ kJ/kg K}$

$$\boxed{C_{ph} = 3.5 \times 10^3 \text{ J/kg K}}$$

Inlet temperature of hot liquid  $T_1 = 130^\circ\text{C}$

Specific heat of water  $C_{ph} = 4.18 \text{ kJ/kg K}$

$$C_{ph} = 4.18 \times 10^3 \text{ J/kg K}$$

Inlet temperature of cooling water  $t_1 = 15^\circ\text{C}$

Mass flow rate of cooling water  $m_c = 17 \text{ kg/min}$

$$m_c = 0.28 \text{ kg/s}$$

Overall heat transfer coefficient  $U = 1100 \text{ w/m}^2\text{K}$

Area  $A = 0.30 \text{ m}^2$

**To find**

1. Outlet temperature of liquid ( $T_2$ )
2. Outlet temperature of water ( $t_2$ )
3. Effectiveness of heat exchanger ( $\epsilon$ )

**Solution:**

Capacity rate of hot liquid  $C = m_h \times C_{ph}$

$$0.07 \times 3.5 \times 10^3$$

$$C = 245 \text{ W/K} \dots\dots 1$$

Capacity rate of liquid  $C = m_h \times C_{ph}$

$$= 0.28 \times 4.18 \times 10^3$$

$$C = 1170.4 \text{ W/K} \dots\dots 2$$

From (1) and (2)

$$C_{\min} = 245 \text{ w/k}$$

$$C_{\max} = 1170.4 \text{ w/k}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{245}{1170.4} = 0.209$$

$$\frac{C_{\min}}{C_{\max}} = 0.209 \dots\dots 3$$

$$\text{Number of transfer units NTU} = \frac{UA}{C_{\min}}$$

From HMT data book Page No.155

$$\text{NTU} = \frac{1100 \times 0.30}{245}$$

$$\text{NTU} = 1.34 \dots\dots 4$$

To find effectiveness  $\epsilon$  refer HMT data book Page No.165

Parallel Flow heat exchanger

From graph

$$X_{\text{axis}} \rightarrow \text{NTU} = 1.34$$

$$\text{Curve} \rightarrow \frac{C_{\text{min}}}{C_{\text{max}}} = 0.209$$

Corresponding  $Y_{\text{axis}}$  value is 64%

$$\text{i.e. } \varepsilon = 0.64$$

We know

Maximum possible heat transfer

$$\begin{aligned} Q_{\text{max}} &= C_{\text{min}}(T_1 - t_1) \\ &= 245(130-15) \end{aligned}$$

$$Q_{\text{max}} = 28,175 \text{ W}$$

Actual heat transfer rate

$$\begin{aligned} Q &= \varepsilon \times Q_{\text{max}} \\ &= 0.64 \times 28.175 \end{aligned}$$

$$Q = 18,032 \text{ W}$$

We know that

Heat transfer

$$\begin{aligned} Q &= m_c C_{pc}(t_2 - t_1) \\ \Rightarrow 18,032 &= 0.28 \times 4.18 \times 10^3 (t_2 - 15) \\ \Rightarrow 18,032 &= 1170.4 t_2 - 17556 \\ \Rightarrow t_2 &= 30.40^\circ\text{C} \end{aligned}$$

$$\text{Outlet temperature of water } t_2 = 30.40^\circ\text{C}$$

We know that

Heat transfer

$$Q = m_c C_{pc} (T_1 - T_2)$$

$$\Rightarrow 18,032 = 0.07 \times 3.5 \times 10^3 (130 - T_2)$$

$$\Rightarrow 18,032 = 31850 - 245 T_2$$

$$\Rightarrow T_2 = 56.4^\circ\text{C}$$

Outlet temperature of liquid  $T_2 = 56.4^\circ\text{C}$

**4. A length square array of four hundred tubes, 1.5 cm outer diameter is used to condense steam at atmospheric pressure. The tube walls are maintained at  $88^\circ\text{C}$  by a coolant flowing inside the tubes. Calculate the amount of steam condensed per hour per unit length of the tubes.**

**Solution:** The properties at the mean temperature  $(88 + 100)/2 = 94^\circ\text{C}$  are:

$$\rho = 963 \text{ kg/m}^3, \mu = 3.06 \times 10^{-4} \text{ Pa-s}, k = 0.678 \text{ W/m K},$$

$$H_{fg} = 2255 \times 10^3 \text{ J/kg}$$

A square array of 400 tubes will have  $N = 20$ .

$$h = 0.725 \left[ \frac{(g \rho^2 k^3 h_{fg})}{N \mu D (T_g - T_s)} \right]^{0.25}$$

$$= 0.725 \left( \frac{9.81 \times (963)^2 \times (0.678)^3 \times 2255 \times 10^3}{20 \times 0.000306 \times 0.015 \times 12} \right) = 6.328 \text{ kW/m}^2\text{K}$$

$$\text{Surface area for 400 tubes} = 400 \times 3.142 \times 0.015 \times 1 \text{ (let } L = 1)$$

$$= 18.852 \text{ m}^2 \text{ per m length of the tube}$$

$$\dot{Q} = h A (\Delta T) = 6.328 \times 18.852 \times 12 = 1431.56 \text{ kW}$$

$$\dot{m} = \dot{Q} / h_{fg} = 1431.56 \times 3600 / 2255 = 2285.4 \text{ kg/hr per m}$$

**5. In a cross heat exchangers both fluids unmixed hot fluid with a specific heat of  $2300 \text{ J/kg K}$  enters at  $380^\circ\text{C}$  and leaves at  $300^\circ\text{C}$  cold fluids enters at  $25^\circ\text{C}$  and leaves at  $210^\circ\text{C}$ . Calculate the required surface of heat exchanger. Take overall heat transfer coefficient is  $750 \text{ W/m}^2 \text{ K}$ . Mass flow rate of hot fluid is  $1 \text{ kg/s}$ . (AU 2014)**

**Given:**

$$\text{Specific heat of hot fluid } C_{ph} = 23000 \text{ J/kg K}$$

ME8693 HEAT AND MASS  
TRANSFER

III/II

MECHANICAL

Entry temperature of hot fluid  $T_1 = 380^\circ\text{C}$ Exit temperature of heat fluid  $T_2 = 300^\circ\text{C}$ Entry temperature of cold fluid  $t_1 = 25^\circ\text{C}$ Exit temperature of cold fluid  $t_2 = 210^\circ\text{C}$ Overall heat transfer coefficient  $U = 750 \text{ W/m}^2\text{K}$ Mass flow rate of hot fluid  $m_h = 1 \text{ kg/s}$ **To find**

Heat exchanger area (A)

**Solution:**

This is cross flow both fluids unmixed type heat exchanger. For cross flow heat exchanger.

$$Q = FUA (\Delta T)_m \text{ (Counter flow)} \dots\dots\dots 1$$

From HMT data book Page No.154

Where

F – Correction factor

 $(\Delta T)_m$  – Logarithmic Mean Temperature Difference for Counter Flow

For Counter flow

$$(\Delta T)_m = \frac{[(T_1 - t_2) - (T_2 - t_1)]}{\ln \left[ \frac{T_1 - t_2}{T_2 - t_1} \right]}$$

$$= \frac{(380 - 210) - (300 - 25)}{\ln \left[ \frac{380 - 210}{300 - 25} \right]}$$

$$\boxed{(\Delta T)_m = 218.3^\circ\text{C}}$$

We know

Heat transfer

$$\Rightarrow Q = m_h C_{ph} (T_1 - T_2)$$

$$Q = 1 \times 1200 (380 - 300)$$

$$Q = 184 \times 10^3 \text{ W}$$

To find correction factor E refer HMT data book Page No.164.

Singlepass cross flow heat exchanger – Both fluids unmixed.

**From graph**

$$X_{\text{axis}} \text{ Value } P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{210 - 25}{380 - 25} = 0.52$$

$$\text{Curve value} = R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{380 - 300}{210 - 25} = 0.432$$

$X_{\text{axis}}$  Value is 0.52 curve

Value is 0.432 corresponding  $Y_{\text{axis}}$  value is 0.97

$$F = 0.07$$

Substitute Q, F  $(\Delta T)_m$  and U value in Equation (1)

$$(1) \Rightarrow Q = FU A (\Delta T)_m$$

$$184 \times 10^3 = 0.97 \times 750 \times A \times 218.3$$

$$A = 1.15 \text{ m}^2$$

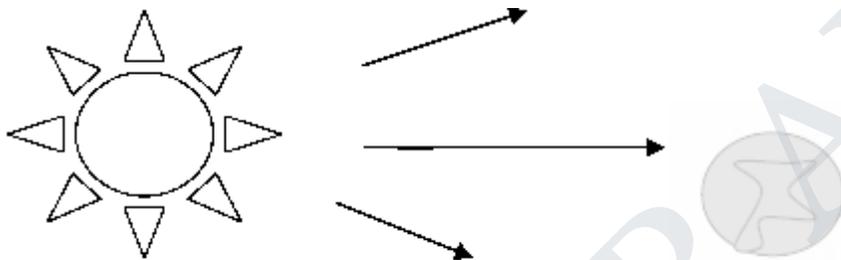
**UNIT 4 RADIATION :**

**4.1 BLACK BODY RADIATION:**

Definition:

Radiation is the energy transfer across a system boundary due to a  $\Delta T$ , by the mechanism of photon emission or electromagnetic wave emission.

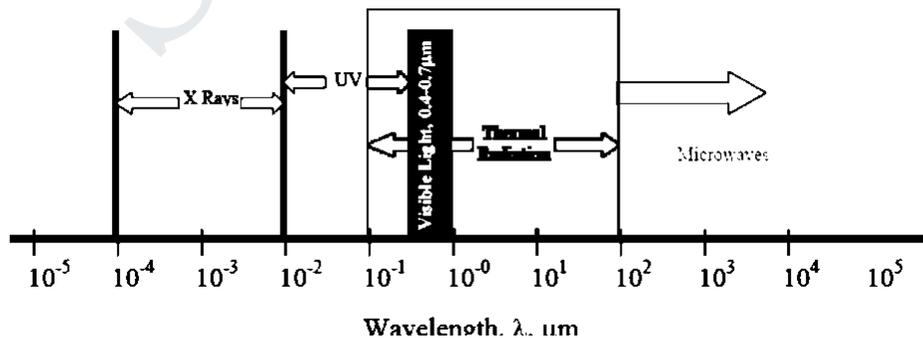
Because the mechanism of transmission is photon emission, unlike conduction and convection, there need be no intermediate matter to enable transmission.



The significance of this is that radiation will be the only mechanism for heat transfer whenever a vacuum is present.

**4.2 Electromagnetic Phenomena.**

We are well acquainted with a wide range of electromagnetic phenomena in modern life. These phenomena are sometimes thought of as wave phenomena and are, consequently, often described in terms of electromagnetic wave length,  $\lambda$ . Examples are given in terms of the wave distribution shown below:



One aspect of electromagnetic radiation is that the related topics are more closely associated with optics and electronics than with those normally found in mechanical engineering courses. Nevertheless, these are widely encountered topics and the student is familiar with them through every day life experiences.

From a viewpoint of previously studied topics students, particularly those with a background in mechanical or chemical engineering, will find the subject of Radiation Heat Transfer a little unusual. The physics background differs fundamentally from that found in the areas of continuum mechanics. Much of the related material is found in courses more closely identified with quantum physics or electrical engineering, i.e. Fields and Waves. At this point, it is important for us to recognize that since the subject arises from a different area of physics, it will be important that we study these concepts with extra care.

#### 4.3 Stefan-Boltzman Law

Both Stefan and Boltzman were physicists; any student taking a course in quantum physics will become well acquainted with Boltzman's work as he made a number of important contributions to the field. Both were contemporaries of Einstein so we see that the subject is of fairly recent vintage. (Recall that the basic equation for convection heat transfer is attributed to Newton)

$$E_b = \sigma \cdot T_{abs}^4$$

where:  $E_b$  = Emissive Power, the gross energy emitted from an ideal surface per unit area, time.

$\sigma$  = Stefan Boltzman constant,  $5.67 \cdot 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$

$T_{abs}$  = Absolute temperature of the emitting surface, K.

Take particular note of the fact that absolute temperatures are used in Radiation. It is suggested, as a matter of good practice, to convert all temperatures to the absolute scale as an initial step in all radiation problems.

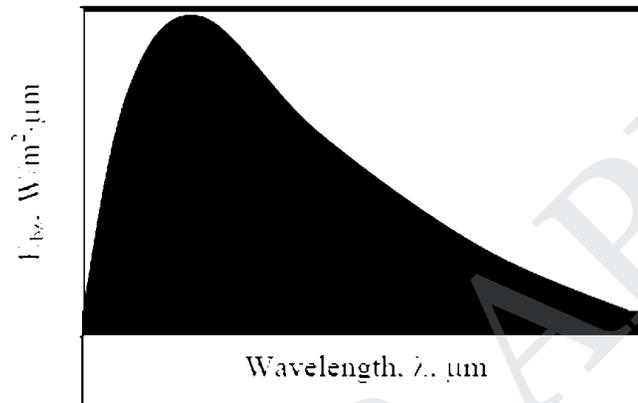
You will notice that the equation does not include any heat flux term,  $q''$ . Instead we have a term the emissive power. The relationship between these terms is as follows. Consider two infinite plane surfaces, both facing one another. Both surfaces are ideal surfaces. One surface is found to be at temperature,  $T_1$ , the other at temperature,  $T_2$ . Since both temperatures are at temperatures above absolute zero, both will radiate energy as described by the Stefan-Boltzman law. The heat flux will be the net radiant flow as given by:

$$q'' = E_{b1} - E_{b2} = \sigma \cdot T_1^4 - \sigma \cdot T_2^4$$

#### 4.4 Plank's Law

While the Stefan-Boltzman law is useful for studying overall energy emissions, it does not allow us to treat those interactions, which deal specifically with wavelength,  $\lambda$ . This problem was overcome by another of the modern physicists, Max Plank, who developed a relationship for wave-based emissions.

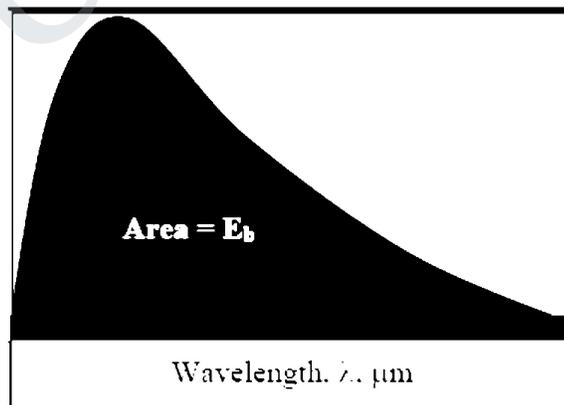
$$E_{b\lambda} = f(\lambda)$$



We haven't yet defined the Monochromatic Emissive Power,  $E_{b\lambda}$ . An implicit definition is provided by the following equation:

$$E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda$$

We may view this equation graphically as follows:



A definition of monochromatic Emissive Power would be obtained by differentiating the integral equation:

$$E_{b\lambda} \equiv \frac{dE_b}{d\lambda}$$

The actual form of Plank's law is:

$$E_{b\lambda} = \frac{C_1}{\lambda^5 \cdot \left[ e^{C_2/\lambda T} - 1 \right]}$$

$$C_1 = 2 \cdot \pi \cdot h \cdot c_0^2 = 3.742 \cdot 10^8 \text{ W} \cdot \mu\text{m}^4/\text{m}^2$$

$$C_2 = h \cdot c_0/k = 1.439 \cdot 10^4 \mu\text{m} \cdot \text{K}$$

Where:  $h$ ,  $c_0$ ,  $k$  are all parameters from quantum physics. We need not worry about their precise definition here.

This equation may be solved at any  $T$ ,  $\lambda$  to give the value of the monochromatic emissivity at that condition. Alternatively, the function may be substituted into the integral  $E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda$  to find the Emissive power for any temperature. While performing this integral by hand is difficult, students may readily evaluate the integral through one of several computer programs, i.e. MathCad, Maple, Mathematica, etc.

$$E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda = \sigma \cdot T^4$$

#### 4.5 Emission Over Specific Wave Length Bands

Consider the problem of designing a tanning machine. As a part of the machine, we will need to design a very powerful incandescent light source. We may wish to know how much energy is being emitted over the

ultraviolet band ( $10^{-4}$  to  $0.4 \mu\text{m}$ ), known to be particularly dangerous.

$$E_b(0.0001 \rightarrow 0.4) = \int_{0.0001 \mu\text{m}}^{0.4 \mu\text{m}} E_{b\lambda} \cdot d\lambda$$

With a computer available, evaluation of this integral is rather trivial. Alternatively, the text books provide a table of integrals. The format used is as follows:

$$\frac{E_b(0.001 \rightarrow 0.4)}{E_b} = \frac{\int_{0.001 \mu m}^{0.4 \mu m} E_{b\lambda} \cdot d\lambda}{\int_0^{\infty} E_{b\lambda} \cdot d\lambda} = \frac{\int_0^{0.4 \mu m} E_{b\lambda} \cdot d\lambda}{\int_0^{\infty} E_{b\lambda} \cdot d\lambda} - \frac{\int_0^{0.0001 \mu m} E_{b\lambda} \cdot d\lambda}{\int_0^{\infty} E_{b\lambda} \cdot d\lambda} = F(0 \rightarrow 0.4) - F(0 \rightarrow 0.0001)$$

Referring to such tables, we see the last two functions listed in the second column. In the first column is a parameter,  $\lambda \cdot T$ . This is found by taking the product of the absolute temperature of the emitting surface,  $T$ , and the upper limit wave length,  $\lambda$ . In our example, suppose that the incandescent bulb is designed to operate at a temperature of 2000K. Reading from the table:

$\lambda,$

| $\lambda, \mu m$   | $T, K$ | $\lambda \cdot T, \mu m \cdot K$ | $F(0 \rightarrow \lambda)$ |
|--|--------|----------------------------------|----------------------------|
| 0.0001   | 2000   | 0.2                              | 0                          |
| 0.4  | 2000   | 600                              | 0.000014                   |
| <b><math>F(0.4 \rightarrow 0.0001 \mu m) = F(0 \rightarrow 0.4 \mu m) - F(0 \rightarrow 0.0001 \mu m)</math></b> |        |                                  | <b>0.000014</b>            |

This is the fraction of the total energy emitted which falls within the IR band. To find the absolute energy emitted multiply this value times the total energy emitted:

$$E_{bIR} = F(0.4 \rightarrow 0.0001 \mu m) \cdot \sigma \cdot T^4 = 0.000014 \cdot 5.67 \cdot 10^{-8} \cdot 2000^4 = 12.7 \text{ W/m}^2$$

#### 4.6 Solar Radiation

The magnitude of the energy leaving the Sun varies with time and is closely associated with such factors as solar flares and sunspots. Nevertheless, we often choose to work with an average value. The energy leaving the sun is emitted outward in all directions so that at any particular distance from the Sun we may imagine the energy being dispersed over an imaginary spherical area. Because this area increases with the distance squared, the solar flux also decreases with the distance squared. At the average distance between Earth and Sun this heat flux is 1353 W/m<sup>2</sup>, so that the average heat flux on any object in Earth orbit is found as:

$$G_{s.o} = S_c \cdot f \cdot \cos \theta$$

Where  $S_c$  = Solar Constant, 1353 W/m<sup>2</sup>

$f$  = correction factor for eccentricity in Earth Orbit, (0.97 <  $f$  < 1.03)

$\theta$  = Angle of surface from normal to Sun.

Because of reflection and absorption in the Earth's atmosphere, this number is significantly reduced at ground level. Nevertheless, this value gives us some opportunity to estimate the potential for using solar energy, such as in photovoltaic cells.

### Some Definitions

In the previous section we introduced the Stefan-Boltzman Equation to describe radiation from an ideal surface.

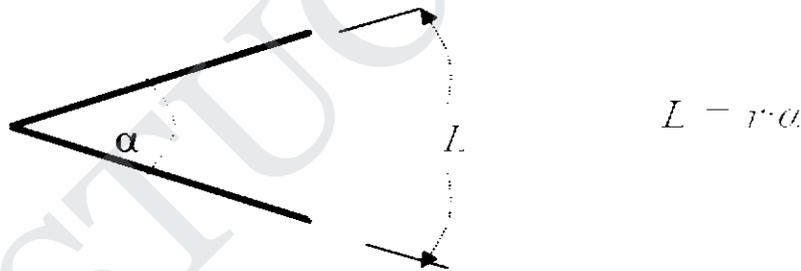
$$E_b = \sigma \cdot T_{abs}^4$$

This equation provides a method of determining the total energy leaving a surface, but gives no indication of the direction in which it travels. As we continue our study, we will want to be able to calculate how heat is distributed among various objects.

For this purpose, we will introduce the radiation intensity,  $I$ , defined as the energy emitted per unit area, per unit time, per unit solid angle. Before writing an equation for this new property, we will need to define some of the terms we will be using.

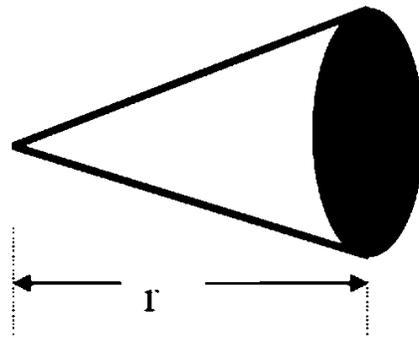
#### 4.7 Angles and Arc Length

We are well accustomed to thinking of an angle as a two dimensional object. It may be used to find an arc length:



#### Solid Angle

We generalize the idea of an angle and an arc length to three dimensions and define a solid angle,  $\Omega$ , which like the standard angle has no dimensions. The solid angle, when multiplied by the radius squared will have dimensions of length squared, or area, and will have the magnitude of the encompassed area.

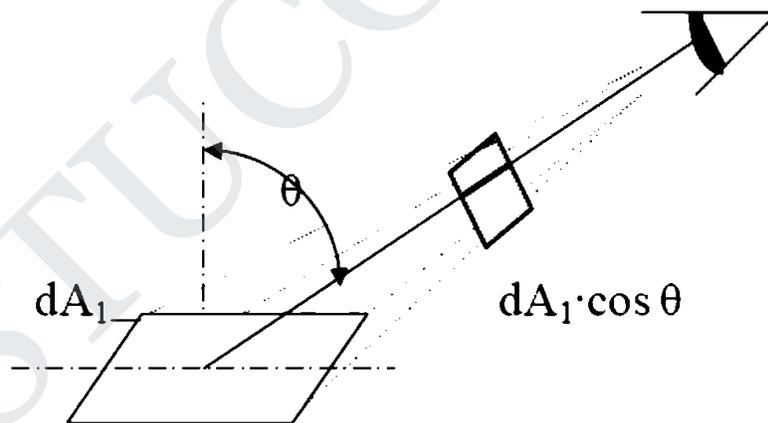


$$A = r^2 \cdot d\Omega$$

#### 4.8 Projected Area

The area,  $dA_1$ , as seen from the perspective of a viewer, situated at an angle  $\theta$  from the normal to the surface, will appear somewhat smaller, as  $\cos \theta \cdot dA_1$ . This smaller area is termed the projected area.

$$A_{\text{projected}} = \cos \theta \cdot A_{\text{normal}}$$



#### 4.9 Intensity

The ideal intensity,  $I_b$ , may now be defined as the energy emitted from an ideal body, per unit projected area, per unit time, per unit solid angle.

$$I = \frac{dq}{\cos \theta \cdot dA_1 \cdot d\Omega}$$

#### 4.10 Spherical Geometry

Since any surface will emit radiation outward in all directions above the surface, the spherical coordinate system provides a convenient tool for analysis. The three basic coordinates shown are  $R$ ,  $\phi$ , and  $\theta$ , representing the radial, azimuthal and zenith directions.

In general  $dA_1$  will correspond to the emitting surface or the source. The surface  $dA_2$  will correspond to the receiving surface or the target. Note that the area proscribed on the hemisphere,  $dA_2$ , may be written as:

$$dA_2 = [(R \cdot \sin \theta) \cdot d\phi] \cdot [R \cdot d\theta]$$

or, more simply as:

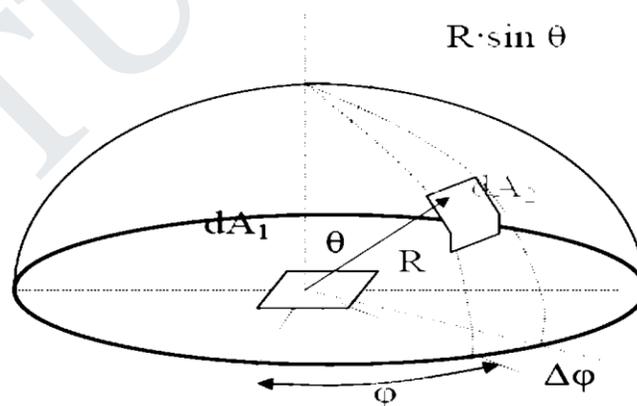
$$dA_2 = R^2 \cdot \sin \theta \cdot d\phi \cdot d\theta$$

Recalling the definition of the solid angle,

$$dA = R^2 \cdot d\Omega$$

we find that:

$$d\Omega = R^{-2} \sin \theta \cdot d\theta \cdot d\phi$$



#### 4.11 Real Surfaces

Thus far we have spoken of ideal surfaces, i.e. those that emit energy according to the Stefan-Boltzman law:

$$E_b = \sigma \cdot T_{abs}^4$$

Real surfaces have emissive powers,  $E$ , which are somewhat less than that obtained theoretically by Boltzman. To account for this reduction, we introduce the emissivity,  $\varepsilon$ .

$$\varepsilon \equiv \frac{E}{E_b}$$

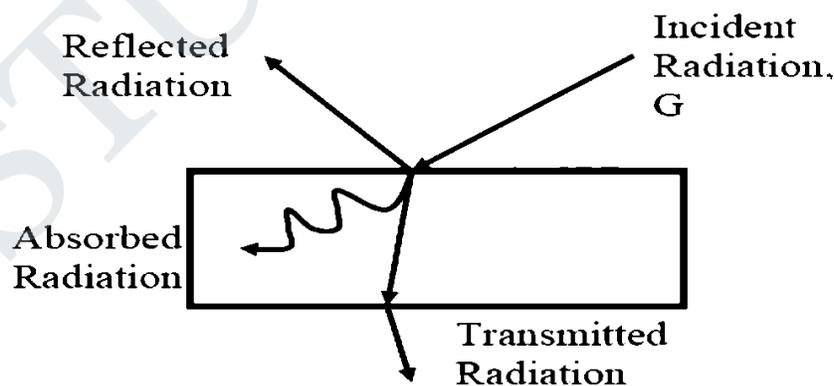
so that the emissive power from any real surface is given by:

$$E = \varepsilon \cdot \sigma \cdot T_{abs}^4$$

### Receiving Properties

Targets receive radiation in one of three ways; they absorption, reflection or transmission. To account for these characteristics, we introduce three additional properties:

- Absorptivity,  $\alpha$ , the fraction of incident radiation absorbed.
- Reflectivity,  $\rho$ , the fraction of incident radiation reflected.
- Transmissivity,  $\tau$ , the fraction of incident radiation transmitted.



We see, from Conservation of Energy, that:

$$\alpha + \rho + \tau = 1$$

In this course, we will deal with only opaque surfaces,  $\tau = 0$ , so that:

$\alpha + \rho = 1$

Opaque Surfaces

**4.12 SHAPE FACTOR:**

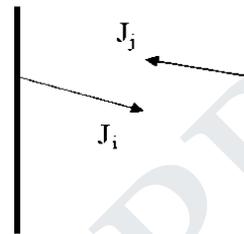
**Net Exchange Between Surfaces**

Consider the two surfaces shown. Radiation will travel from surface i to surface j and will also travel from j to i.

$$q_{i \rightarrow j} = J_i \cdot A_i \cdot F_{i \rightarrow j}$$

likewise,

$$q_{j \rightarrow i} = J_j \cdot A_j \cdot F_{j \rightarrow i}$$



The net heat transfer is then:

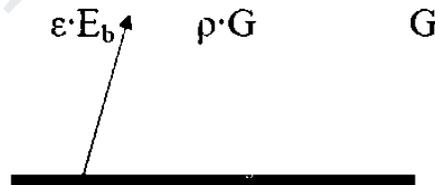
$$q_{j \rightarrow i \text{ (net)}} = J_i \cdot A_i \cdot F_{i \rightarrow j} - J_j \cdot A_j \cdot F_{j \rightarrow i}$$

From reciprocity we note that  $F_{1 \rightarrow 2} \cdot A_1 = F_{2 \rightarrow 1} \cdot A_2$  so that

$$q_{j \rightarrow i \text{ (net)}} = J_i \cdot A_i \cdot F_{i \rightarrow j} - J_j \cdot A_i \cdot F_{i \rightarrow j} = A_i \cdot F_{i \rightarrow j} \cdot (J_i - J_j)$$

**Net Energy Leaving a Surface**

The net energy leaving a surface will be the difference between the energy leaving a surface and the energy received by a surface:



$$q_{1 \rightarrow} = [\epsilon \cdot E_b - \alpha \cdot G] \cdot A_1$$

Combine this relationship with the definition of Radiosity to eliminate G.

$$J \equiv \epsilon \cdot E_b + \rho \cdot G \rightarrow G = [J - \epsilon \cdot E_b] / \rho$$

$$q_{1\rightarrow} = \{\epsilon \cdot E_b - \alpha \cdot [J - \epsilon \cdot E_b] / \rho\} \cdot A_1$$

Assume opaque surfaces so that  $\alpha + \rho = 1 \rightarrow \rho = 1 - \alpha$ , and substitute for  $\rho$ .

$$q_{1\rightarrow} = \{\epsilon \cdot E_b - \alpha \cdot [J - \epsilon \cdot E_b] / (1 - \alpha)\} \cdot A_1$$

Put the equation over a common denominator:

$$q_{1\rightarrow} = \left[ \frac{(1 - \alpha) \cdot \epsilon \cdot E_b - \alpha \cdot J + \alpha \cdot \epsilon \cdot E_b}{1 - \alpha} \right] \cdot A_1 = \left[ \frac{\epsilon \cdot E_b - \alpha \cdot J}{1 - \alpha} \right] \cdot A_1$$

If we assume that  $\alpha = \epsilon$  then the equation reduces to:

$$q_{1\rightarrow} = \left[ \frac{\epsilon \cdot E_b - \epsilon \cdot J}{1 - \epsilon} \right] \cdot A_1 = \left[ \frac{\epsilon \cdot A_1}{1 - \epsilon} \right] \cdot (E_b - J)$$

#### 4.13 ELECTRICAL ANALOGY :

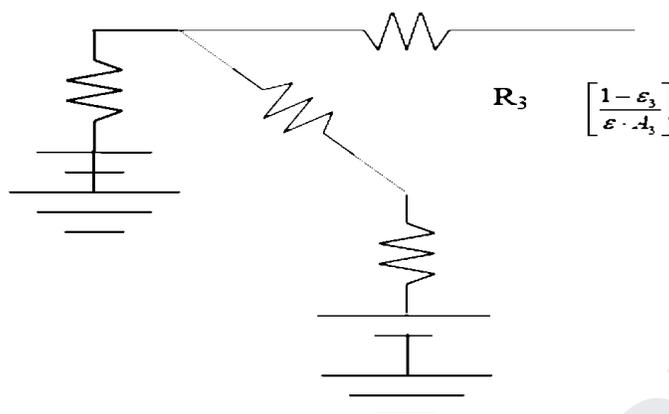
##### Electrical Analogy for Radiation

We may develop an electrical analogy for radiation, similar to that produced for conduction. The two analogies should not be mixed: they have different dimensions on the potential differences, resistance and current flows.

|                                      | <b>Equivalent Current</b> | <b>Equivalent Resistance</b>                           | <b>Potential Difference</b> |
|--------------------------------------|---------------------------|--|-----------------------------|
| <b>Ohms Law</b>                      | I                         | R  | $\Delta V$                  |
| <b>Net Energy Leaving Surface</b>    | $q_{1\rightarrow}$        | $\left[ \frac{1 - \epsilon}{\epsilon \cdot A} \right]$ | $E_b - J$                   |
| <b>Net Exchange Between Surfaces</b> | $q_{i\rightarrow j}$      | $\frac{1}{A_1 \cdot F_{1\rightarrow 2}}$               | $J_1 - J_2$                 |

#### 4.14 RADIATION SHIELDS ;

Insulated surfaces. In steady state heat transfer, a surface cannot receive net energy if it is insulated. Because the energy cannot be stored by a surface in steady state, all energy must be re-radiated back into the enclosure. *Insulated surfaces are often termed as re-radiating surfaces.*



Electrically cannot flow through a battery if it is not grounded.

Surface 3 is not grounded so that the battery and surface resistance serve no purpose and are removed from the drawing.

- Black surfaces: A black, or ideal surface, will have no surface resistance:

$$\left[ \frac{1 - \epsilon}{\epsilon \cdot A} \right] = \left[ \frac{1 - 1}{1 \cdot A} \right] = 0$$

In this case the nodal Radiosity and emissive power will be equal.

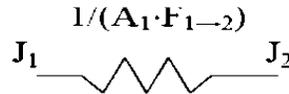
This result gives some insight into the physical meaning of a black surface. Ideal surfaces radiate at the maximum possible level. Non-black surfaces will have a reduced potential, somewhat like a battery with a corroded terminal. They therefore have a reduced potential to cause heat/current flow.

- Large surfaces: Surfaces having a large surface area will behave as black surfaces, irrespective of the actual surface properties:

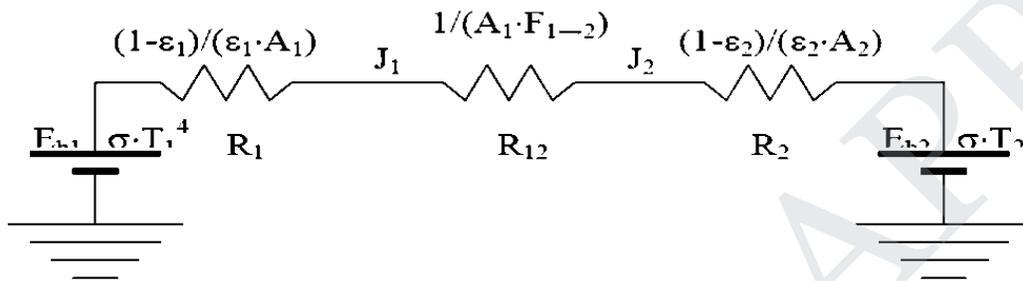
$$\left[ \frac{1 - \epsilon}{\epsilon \cdot A} \right] = \left[ \frac{1 - \epsilon}{\epsilon \cdot \infty} \right] = 0$$

Physically, this corresponds to the characteristic of large surfaces that as they reflect energy, there is very little chance that energy will strike the smaller surfaces; most of the energy is reflected back to another part of the same large surface. After several partial absorptions most of the energy received is absorbed.

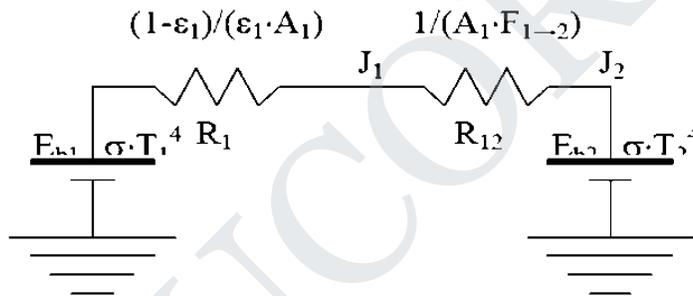
Consider the case of an object, 1, placed inside a large enclosure, 2. The system will consist of two objects, so we proceed to construct a circuit with two radiosity nodes



Now we ground both Radiosity nodes through a surface resistance.



Since  $A_2$  is large,  $R_2 = 0$ . The view factor,  $F_{1-2} = 1$



Sum the series resistances:

$$R_{\text{Series}} = (1-\epsilon_1)/(\epsilon_1 \cdot A_1) + 1/A_1 = 1/(\epsilon_1 \cdot A_1)$$

Ohm's law:

$$i = \Delta V/R$$

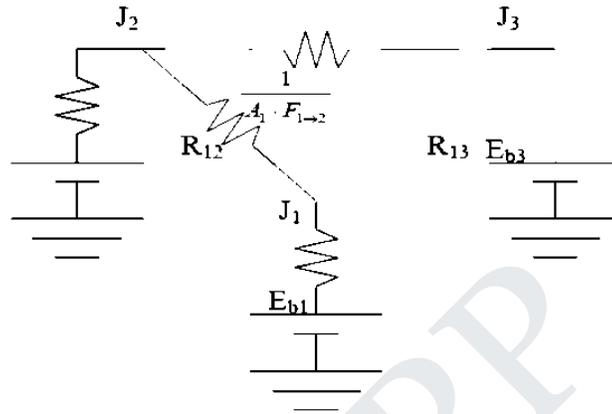
or by analogy:

$$q = \Delta E_b/R_{\text{Series}} = \epsilon_1 \cdot A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$$

You may recall this result from Thermo I, where it was introduced to solve this type of radiation problem.

• Networks with Multiple Potentials

Systems with 3 or more grounded potentials will require a slightly different solution, but one which students have previously encountered in the Circuits course.



The procedure will be to apply Kirchoff's law to each of the Radiosity junctions.

$$\sum_{i=1}^3 q_i = 0$$

In this example there are three junctions, so we will obtain three equations. This will allow us to solve for three unknowns.

Radiation problems will generally be presented on one of two ways:

1. The surface net heat flow is given and the surface temperature is to be found.
2. The surface temperature is given and the net heat flow is to be found.

Returning for a moment to the coal grate furnace, let us assume that we know (a) the total heat being produced by the coal bed, (b) the temperatures of the water walls and (c) the temperature of the super heater sections.

Apply Kirchoff's law about node 1, for the coal bed:

$$q_1 + q_{2 \rightarrow 1} + q_{3 \rightarrow 1} = q_1 + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$

Similarly, for node 2:

$$q_2 + q_{1 \rightarrow 2} + q_{3 \rightarrow 2} = \frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0$$

(Note how node 1, with a specified heat input, is handled differently than node 2, with a specified temperature.

And for node 3:

$$q_3 + q_{1 \rightarrow 3} + q_{2 \rightarrow 3} = \frac{E_{b3} - J_3}{R_3} - \frac{J_1 - J_3}{R_{13}} - \frac{J_2 - J_3}{R_{23}} = 0$$

The three equations must be solved simultaneously. Since they are each linear in J, matrix methods may be used:

$$\begin{bmatrix} \frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{12}} & \frac{1}{R_{13}} \\ \frac{1}{R_{12}} & -\frac{1}{R_2} - \frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{23}} \\ \frac{1}{R_{13}} & \frac{1}{R_{23}} & -\frac{1}{R_3} - \frac{1}{R_{13}} - \frac{1}{R_{23}} \end{bmatrix} \cdot \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} -q_1 \\ -\frac{E_{b2}}{R_2} \\ -\frac{E_{b3}}{R_3} \end{bmatrix}$$

The matrix may be solved for the individual Radiosity. Once these are known, we return to the electrical analogy to find the temperature of surface 1, and the heat flows to surfaces 2 and 3.

Surface 1: Find the coal bed temperature, given the heat flow:

$$q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{\sigma \cdot T_1^4 - J_1}{R_1} \Rightarrow T_1 = \left[ \frac{q_1 \cdot R_1 + J_1}{\sigma} \right]^{0.25}$$

Surface 2: Find the water wall heat input, given the water wall temperature:

$$q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{\sigma \cdot T_2^4 - J_2}{R_2}$$

Surface 3: (Similar to surface 2) Find the water wall heat input, given the water wall temperature:

$$q_3 = \frac{E_{b3} - J_3}{R_3} = \frac{\sigma \cdot T_3^4 - J_3}{R_3}$$

**UNIT 4: RADIATION****PART A****ANSWER THE FOLLOWING:****1. Define Radiation.****(AU 2013)**

The heat transfer from one body to another without any transmitting medium is known as radiation. It is an electromagnetic wave phenomenon.

**2. What is black body?****(AU 2012)**

Black body is an ideal surface having the following properties.

1. A black body absorbs all incident radiation, regardless of wave length and direction.
2. For a prescribed temperature and wave length, no surface can emit more energy than black body.

**3. State Wien's displacement law.****(AU2010)**

The Wien's law gives the relationship between temperature and wave length corresponding to the maximum spectral emissive power of the black body at that temperature.

$$\lambda_{\text{mas}} T = c_3$$

Where  $c_3 = 2.9 \times 10^{-3}$  [Radiation constant]

$$\Rightarrow \lambda_{\text{mas}} T = 2.9 \times 10^{-3} \text{ mK}$$

**4. What is meant by gray body?****(AU2011)**

If a body absorbs a definite percentage of incident radiation irrespective of their wave length, the body is known as gray body. The emissive power of a gray body is always less than that of the black body.

**5. State Kirchhoff's law of radiation.****(AU2010)**

This law states that the ratio of total emissive power to the absorptive is constant for all surfaces which are in thermal equilibrium with the surroundings. This can be written as

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3}$$

It also states that the emissivity of the body is always equal to its absorptive when the body remains in thermal equilibrium with its surroundings.

$\alpha_1 = E_1$ ;  $\alpha_2 = E_2$  and so on.

**6. Define irradiation (G)**

**(AU 2011)**

It is defined as the total radiation incident upon a surface per unit time per unit area. It is expressed in  $W/m^2$ .

**7. What is the purpose of radiation shield?**

Radiation shields constructed from low emissivity (high reflective) materials. It is used to reduce the net radiation transfer between two surfaces.

**8. Define intensity of radiation ( $I_b$ )?**

**(AU2010)**

It is defined as the rate of energy leaving a space in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

$$I_n = \frac{E_b}{\pi}$$

**9. What is radiosity (J)?**

It is used to indicate the total radiation leaving a surface per unit time per unit area. It is expressed in  $W/m^2$ .

**10. What is meant by shape factor?**

**(AU 2009)**

The shape factor is defined as the fraction of the radioactive energy that is diffused from on surface element and strikes the other surface directly with no intervening reflections. It is represented by  $F_{ij}$ . Other names for radiation shape factor are view factor, angle factor and configuration factor.

**11. State Stefan – Boltzmann law.**

**(AU2010)**

The emissive power of a black body is proportional to the fourth power of absolute temperature.

$$E_b \propto T^4$$

$$E_b = \sigma T^4$$

Where  $E_b$  = Emissive power,  $w/m^2$   
 $\sigma$  = Stefan. Boltzmann constant  
 =  $5.67 \times 10^{-8} W/m^2 K^4$   
 $T$  = Temperature, K

**12. What are the assumptions made to calculate radiation exchange between the surfaces?**

1. All surfaces are considered to be either black or gray
2. Radiation and reflection process are assumed to be diffuse.
3. The absorptivity of a surface is taken equal to its emissivity and independent of temperature of the source of the incident radiation.

**PART B****ANSWER THE FOLLOWING:****1. A black body at 3000 K emits radiation. Calculate the following: (AU2013)**

- i) Monochromatic emissive power at 7  $\mu\text{m}$  wave length.
- ii) Wave length at which emission is maximum.
- iii) Maximum emissive power.
- iv) Total emissive power,
- v) Calculate the total emissive of the furnace if it is assumed as a real surface having emissivity equal to 0.85.

**Given:** Surface temperature  $T = 3000\text{K}$

**Solution:****1. Monochromatic Emissive Power:**

From Planck's distribution law, we know

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left(\frac{C_2}{\lambda T}\right) - 1}}$$

[From HMT data book, Page No.71]

Where

$$c_1 = 0.374 \times 10^{-15} \text{ W m}^2$$

$$c_2 = 14.4 \times 10^{-3} \text{ mK}$$

$$\lambda = 1 \times 10^{-6} \text{ m} \quad \text{[Given]}$$

$$\Rightarrow E_{b\lambda} = \frac{0.374 \times 10^{-15} [1 \times 10^{-6}]^{-5}}{\left[ \frac{14.4 \times 10^{-3}}{1 \times 10^{-6} \times 3000} \right]^{-1}}$$

$$E_{b\lambda} = 3.10 \times 10^{12} \text{ W/m}^2$$

**2. Maximum wave length ( $\lambda_{\max}$ );**

From Wien's law, we know

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ mK}$$

$$\Rightarrow \lambda_{\max} = \frac{2.9 \times 10^{-3}}{3000}$$

$$\lambda_{\max} = 0.966 \times 10^{-6} \text{ m}$$

**3. Maximum emissive power ( $E_{b\lambda}$ ) max:**

Maximum emissive power

$$\begin{aligned} (E_{b\lambda})_{\max} &= 1.307 \times 10^{-5} T^5 \\ &= 1.307 \times 10^{-5} \times (3000)^5 \\ (E_{b\lambda})_{\max} &= 3.17 \times 10^{12} \text{ W/m}^2 \end{aligned}$$

**4. Total emissive power ( $E_b$ ):**

From Stefan – Boltzmann law, we know that

$$E_b = \sigma T^4$$

[From HMT data book Page No.71]

Where  $\sigma$  = Stefan – Boltzmann constant  
 $= 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

$$\begin{aligned} \Rightarrow E_b &= (5.67 \times 10^{-8}) (3000)^4 \\ E_b &= 4.59 \times 10^6 \text{ W/m}^2 \end{aligned}$$

**5. Total emissive power of a real surface:**

$$(E_b)_{\text{real}} = \epsilon \sigma T^4$$

Where  $\epsilon$  = Emissivity = 0.85

$$(E_b)_{\text{real}} = 0.85 \times 5.67 \times 10^{-8} \times (3000)^4$$

$$(E_b)_{\text{real}} = 3.90 \times 10^6 \text{ W/m}^2$$

**2. The energy received by a  $2 \times 2$  m solar collector whose normal is inclined at  $45^\circ$  to the sun. The energy loss through the atmosphere is 50% and the diffuse radiation is 20% of direct radiation. (AU 2010)**

**Given:** Surface temperature  $T = 6000 \text{ K}$

Distance between earth and sun  $R = 12 \times 10^{10} \text{ m}$

Diameter on the sun  $D_1 = 1.5 \times 10^9 \text{ m}$

Diameter of the earth  $D_2 = 13.2 \times 10^6$  m

**Solution:**

**1. Energy emitted by sun  $E_b = \sigma T^4$**

$$\Rightarrow E_b = 5.67 \times 10^{-8} \times (6000)^4$$

[ $\because \sigma = \text{Stefan - Boltzmann constant}$   
 $= 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$ ]

$$E_b = 73.4 \times 10^6 \text{ W/m}^2$$

$$\text{Area of sun } A_1 = 4\pi R_1^2$$

$$= 4\pi \times \left( \frac{1.5 \times 10^9}{2} \right)^2$$

$A_1 = 7 \times 10^{18} \text{ m}^2$

$\Rightarrow$  Energy emitted by the sun

$$E_b = 73.4 \times 10^6 \times 7 \times 10^{18}$$

$E_b = 5.14 \times 10^{26} \text{ W}$

**2. The emission received per  $\text{m}^2$  just outside the earth's atmosphere:**

The distance between earth and sun

$$R = 12 \times 10^{10} \text{ m}$$

$$\text{Area, } A = 4\pi R^2$$

$$= 4 \times \pi \times (12 \times 10^{10})^2$$

$A = 1.80 \times 10^{23} \text{ m}^2$

$\Rightarrow$  The radiation received outside the earth atmosphere per  $\text{m}^2$

$$= \frac{E_b}{A}$$

$$= \frac{5.14 \times 10^{26}}{1.80 \times 10^{23}}$$

$$= 2855.5 \text{ W/m}^2$$

**3. Energy received by the earth:**

$$\begin{aligned}\text{Earth area} &= \frac{\pi}{4}(D_2)^2 \\ &= \frac{\pi}{4} \times [13.2 \times 10^6]^2\end{aligned}$$

$$\boxed{\text{Earth area} = 1.36 \times 10^4 \text{m}^2}$$

Energy received by the earth

$$\begin{aligned}&= 2855.5 \times 1.36 \times 10^4 \\ &= 3.88 \times 10^{17} \text{ W}\end{aligned}$$

#### 4. The energy received by a 2 × 2 m solar collector;

Energy loss through the atmosphere is 50%. So energy reaching the earth.

$$\begin{aligned}&= 100 - 50 = 50\% \\ &= 0.50\end{aligned}$$

Energy received by the earth

$$\begin{aligned}&= 0.50 \times 2855.5 \\ &= 1427.7 \text{ W/m}^2 \quad \dots\dots(1)\end{aligned}$$

Diffuse radiation is 20%

$$\Rightarrow 0.20 \times 1427.7 = 285.5 \text{ W/m}^2$$

$$\boxed{\text{Diffuse radiation} = 285.5 \text{ W/m}^2} \quad \dots\dots\dots(2)$$

Total radiation reaching the collection

$$\begin{aligned}&= 1427.7 + 285.5 \\ &= 1713.2 \text{ W/m}^2\end{aligned}$$

$$\begin{aligned}\text{Plate area} &= A \times \cos \theta \\ &= 2 \times 2 \times \cos 45^\circ \\ &= 2.82 \text{ m}^2\end{aligned}$$

Energy received by the collector

$$\begin{aligned}&= 2.82 \times 1713.2 \\ &= 4831.2 \text{ W}\end{aligned}$$

3. A furnace wall emits radiation at 2000 K. Treating it as black body radiation, calculate

1. Monochromatic radiant flux density at 1  $\mu\text{m}$  wave length.
2. Wave length at which emission is maximum and the corresponding emissive power.
3. Total emissive power

**Given:** Temperature  $T = 2000 \text{ K}$ ;  $\lambda = 1 \mu\text{m} = 1 \times 10^{-6}$

**Solution:**

**1. Monochromatic emissive power ( $E_{b\lambda}$ ):**

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left(\frac{C_2}{\lambda T}\right) - 1}}$$

[From HMT data book, Page No.71]

where  $C_1 = 0.374 \times 10^{-15} \text{ W m}^2$   
 $C_2 = 14.4 \times 10^{-3} \text{ mK}$   
 $\lambda = 1 \mu\text{m} = 1 \times 10^{-6} \text{ m}$  [Given]

$$\Rightarrow E_{b\lambda} = \frac{0.374 \times 10^{-15} \times (1 \times 10^{-6})^{-5}}{e^{\left(\frac{14.4 \times 10^{-3}}{1 \times 10^{-6} \times 2000}\right) - 1}}$$

$$E_{b\lambda} = 2.79 \times 10^{11} \text{ W/m}^2$$

**2. Maximum Wave Length ( $\lambda_{\text{max}}$ ):**

From Wien's Law, we know that

$$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ mK}$$

[From HMT data book, Page No.71]

$$\lambda_{\text{max}} = \frac{2.9 \times 10^{-3}}{T}$$

$$= \frac{2.9 \times 10^{-3}}{2000} = 1.45 \times 10^{-6} \text{ m}$$

$$\lambda_{\text{max}} = 1.45 \mu$$

Corresponding emissive power

$$E_{b\lambda} = \frac{C_1 \lambda_{\max}^{-5}}{e^{\left(\frac{C_2}{\lambda_{\max} T}\right) - 1}}$$

$$= \frac{0.374 \times 10^{-15} \times [1.45 \times 10^{-6}]^{-5}}{e^{\left(\frac{14.4 \times 10^{-3}}{1.45 \times 10^{-6} \times 2000}\right) - 1}}$$

$$= 4.09 \times 10^{11} \text{ W/m}^2$$

**3. Total emissive power ( $E_b$ ):**

From Stefan – Boltzmann law, we know

$$E_b = \sigma T^4$$

Where  $\sigma$  - Stefan – Boltzmann constant

$$= 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

$$E_b = 5.67 \times 10^{-8} \times (2000)^4$$

$$\boxed{E_b = 907.2 \times 10^3 \text{ W/m}^2}$$

**4. Calculate the heat exchange by radiation between the surfaces of two long cylinders having radii 120mm and 60mm respectively. The axis of the cylinder is parallel to each other. The inner cylinder is maintained at a temperature of 130°C and emissivity of 0.6. Outer cylinder is maintained at a temperature of 30°C and emissivity of 0.5. (AU 2012)**

**Given:**  $r_1 = 60 \text{ mm}$

$$= 0.060 \text{ m}$$

$$r_2 = 120 \text{ mm}$$

$$= 0.12$$

$$T_1 = 130^\circ\text{C} + 273$$

$$= 403$$

$$\epsilon_1 = 0.6$$

$$T_2 = 30^\circ\text{C} + 273 = 303 \text{ K}$$

$$\epsilon_2 = 0.5$$

**To find:** Heat exchange (Q)

**Solution:** Heat exchange between two large concentric cylinders is given by

$$Q = \bar{\varepsilon} \sigma A [T_1^4 - T_2^4] \quad \dots(1)$$

[From equation No.27]

$$\text{where } \bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \left( \frac{1}{\varepsilon_2} - 1 \right)}$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{0.6} + \frac{\pi D_1 L_2}{\pi D_2 L_2} \left( \frac{1}{0.5} - 1 \right)} \quad [\because A = \pi DL]$$

$$= \frac{1}{\frac{1}{0.6} + \frac{0.12}{0.24} \left( \frac{1}{0.5} - 1 \right)} \quad [\because L_1 = L_2 = 1]$$

$$\boxed{\bar{\varepsilon} = 0.46}$$

(1)  $\Rightarrow$

$$Q_{12} = 0.46 \times 5.67 \times 10^{-8} \times \pi \times D_1 \times L \times [(403)^4 - (303)^4]$$

$$= 0.46 \times 5.67 \times 10^{-8} \times \pi \times 0.12 \times 1 \times [(403)^4 - (303)^4]$$

$$\boxed{Q_{12} = 176.47 \text{ W}}$$

**5. Emissivity's of two large parallel plates maintained at 800°C and 300°C are 0.5 respectively. Find net radiant hat exchange per square meter for these plates. Find the percentage reduction in heat transfer when a polished aluminums radiation shield of emissivity 0.06 is placed between them. Also find the temperature of the shield. (AU 2010)**

**Given:**  $T_1 = 800^\circ\text{C} + 273$

$$= 1073 \text{ K}$$

$$T_2 = 300^\circ\text{C} + 273$$

$$= 573 \text{ K}$$

$$\varepsilon_1 = 0.3 \quad \varepsilon_2 = 0.5$$

Shield emissivity  $\varepsilon_3 = 0.06$

**To find:**

1. Net radiant heat exchange per square meter. (Q/A)

2. Percentage reduction in heat loss due to radiation shield.
3. Temperature of the shield ( $T_3$ ).

**Solution:** Heat exchange between two large parallel plates without radiation shield is given by

$$Q_{12} = \bar{\varepsilon} \sigma A [T_1^4 - T_2^4]$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{1}{\frac{1}{0.3} + \frac{1}{0.5} - 1}$$

$$\boxed{\bar{\varepsilon} = 0.230}$$

$$(1) \Rightarrow Q_{12} = 0.230 \times \sigma \times A \times [T_1^4 - T_2^4]$$

$$= 0.230 \times 5.67 \times 10^{-8} \times A \times [(1073)^4 - (573)^4]$$

$$\frac{Q_{12}}{A} = 15,879.9 \text{ W/m}^2$$

Heat transfer square metre without radiation shield

$$\boxed{\frac{Q_{12}}{A} = 15.87 \text{ k W/m}^2} \dots\dots(1)$$

Heat exchange between plate 1 and radiation shield 3 is given by

$$(1) \Rightarrow Q_{13} = \bar{\varepsilon} \sigma A [T_1^4 - T_3^4]$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

$$\Rightarrow Q_{13} = \frac{\sigma \times A [T_1^4 - T_3^4]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \dots\dots(A)$$

Heat exchange between radiation shield 3 and plate 2 is given by

$$Q_{32} = \bar{\varepsilon} \sigma A [T_3^4 - T_2^4]$$

$$\text{Where } \bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$\Rightarrow Q_{32} = \frac{\sigma \times A [T_3^4 - T_2^4]}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \dots (B)$$

$$\text{We know } Q_{13} = Q_{32}$$

$$\Rightarrow = \frac{\sigma A [T_1^4 - T_3^4]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma A [T_3^4 - T_2^4]}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$\Rightarrow = \frac{[T_1^4 - T_3^4]}{\frac{1}{0.3} + \frac{1}{0.06} - 1} = \frac{[T_3^4 - T_2^4]}{\frac{1}{0.06} + \frac{1}{0.5} - 1}$$

$$\Rightarrow \frac{(1073)^4 - (T_3^4)}{19} = \frac{(T_3^4) - (573)^4}{17.6}$$

$$\Rightarrow T_3^4 = \frac{17.6 [(1073)^4 - (T_3^4)]}{19} + (573)^4$$

$$\Rightarrow T_3^4 = 0.926 [(1073)^4 - (T_3^4)] + (573)^4$$

$$\Rightarrow T_3^4 = 0.926 [(1073)^4 - 0.926 \times (T_3^4)] + (573)^4$$

$$(T_3^4) + 0.926 (T_3^4) = 1.33 \times 10^{12}$$

$$(1.926) (T_3^4) = 1.33 \times 10^{12}$$

$$(T_3^4) = 6.90 \times 10^{11}$$

$$\boxed{T_3 = 911.5 \text{ K}}$$

**Radiation shield temperature  $T_3 = 911.5 \text{ K}$**

Substituting  $T_3$  value in equation (A) (or) equation (B), Heat transfer with radiation shield

$$\Rightarrow Q_{13} = \frac{5.67 \times 10^{-8} \times A \times [(1073)^4 - (911.5)^4]}{\frac{1}{0.3} + \frac{1}{0.06} - 1}$$

$$\frac{Q_{13}}{A} = 1895.76 \text{ W/m}^2$$

Heat transfer with radiation shield

$$\Rightarrow \boxed{\frac{Q_{13}}{A} = 1.89 \text{ kW/m}^2} \dots\dots (2)$$

Reduction in heat loss due to radiation shield

$$\begin{aligned} &= \frac{Q_{\text{without shield}} - Q_{\text{with shield}}}{Q_{\text{without shield}}} = \frac{Q_{12} - Q_{13}}{Q_{12}} \\ &= \frac{15.87 - 1.89}{15.87} \\ &= 0.88 = 88\% \end{aligned}$$

6. Two circular discs of diameter 20 cm each are placed 2 m apart. Calculate the radiant heat exchange for these discs if there are maintained at 800°C and 300°C respectively and the corresponding emissivity are 0.3 and 0.5. (AU 2010)

**Given:**  $D_1 = 20 \text{ cm} = 0.2 \text{ m}$   $D_2 = 0.2 \text{ m}$

$$\begin{aligned} T_1 &= 800^\circ\text{C} + 273 \\ &= 1073 \text{ K} \end{aligned}$$

$$\begin{aligned} T_2 &= 300^\circ\text{C} + 273 \\ &= 573 \text{ K} \end{aligned}$$

$$\varepsilon_1 = 0.3 \quad \varepsilon_2 = 0.5$$

**To find:** Heat exchange (Q)

$$A_1 = \frac{\pi}{4} (D_1^2)$$

**Solution:** Area =  $\frac{\pi}{4} (0.2)^2 = 0.031 \text{ m}^2$

$$A_1 = 0.031 \text{ m}^2$$

$$A_2 = 0.031 \text{ m}^2 [\because D_1 = D_2]$$

Heat transfer by radiation generation equation is

$$Q_{12} = \frac{\sigma [T_1^4 - T_2^4]}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

$$= \frac{5.67 \times 10^{-8} [(1073)^4 - (573)^4]}{\frac{1-0.3}{0.031 \times 0.3} + \frac{1}{0.31 \times F_{12}} + \frac{1-0.5}{0.031 \times 0.5}}$$

$$Q_{12} = \frac{69 \times 10^3}{107.45 + \frac{1}{0.031 F_{12}}} \dots\dots(1)$$

Where  $F_{12}$  = Shape factor for disc.

In order to find shape factor,  $F_{12}$  refer HMT data book, Page No.76.

$$X \text{ axis} = \frac{\text{Diameter}}{\text{Distance between disc}}$$

$$= \frac{0.2}{2}$$

$$X \text{ axis} = 0.1$$

Curve  $\rightarrow 1$  [since given is disc]

X axis value is 0.1, curve is 1, so corresponding Y axis value is 0.01.

$\rightarrow F_{12} = 0.01$

$F_{12} = 0.01$

(1)  $\rightarrow Q_{12} = \frac{69 \times 10^3}{107.45 + \frac{1}{0.031 \times 0.01}}$

$Q_{12} = 20.7 \text{ Watts.}$

**7. A long cylindrical heater 30 in diameter is maintained at 700°C. It has surface emissivity of 0.8. The heater is located in a large room whose wall is 35°C. Find the radiant heat transfer. Find the percentage of reduction in heat transfer if the heater is completely covered by radiation shield ( $\epsilon = 0.05$ ) and diameter 40 mm.**

**Given:** Diameter of cylinder  $D_1=30\text{mm}=0.030 \text{ mm}$

Temperature  $T_1 = 700^\circ\text{C} + 273 = 973 \text{ K}$

Emissivity  $\varepsilon_1 = 0.8$

Room temperature  $T_2 = 35^\circ\text{C} + 273 = 308 \text{ K}$

**Radiation Shield:**

Emissivity  $\varepsilon_3 = 0.05$

Diameter  $D_3 = 40 \text{ mm} = 0.040 \text{ m}$

**Solution:****Case 1: Heat transfer without shield:**

Heat transfer by radiation general equation is

$$Q_{12} = \frac{\sigma [T_1^4 - T_2^4]}{\frac{1 - \varepsilon_1}{A_1 \varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \varepsilon_2}{A_2 \varepsilon_2}}$$

Where  $A_1 = \pi DL = \pi \times 0.030 \times 1 = 0.094 \text{ m}$

$$A_1 = 0.094 \text{ m}^2$$

Since room is large  $A_2 = \infty$

$F_{12} =$  Shape factor

Small body enclosed by large body  $\rightarrow F_{12} = 1$

[Refer HMT data book, Page No.73]

$$(1) \Rightarrow Q_{12} = \frac{5.67 \times 10^{-8} [(973)^4 - (308)^4]}{\frac{1 - 0.8}{0.094 \times 0.8} + \frac{1 - 0.5}{0.094 \times 1} + 0}$$

$$\left[ \text{Since } A_{12} = \infty, \frac{1 - \varepsilon_2}{A_2 \varepsilon_2} = 0 \right]$$

Heat transfer without shield

$$Q_{12} = 3783.2 \text{ W} \quad \dots\dots(2)$$

**Case 2: Heat transfer with shield:**

Heat transfer between heater (1) and radiation shield (3) is given by

$$Q_{13} = \frac{\sigma [T_1^4 - T_3^4]}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{13}} + \frac{1-\epsilon_3}{A_3\epsilon_3}}$$

$$Q_{12} = \frac{\sigma [T_1^4 - T_2^4]}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

Where  $A_3 = \pi D_3 L = \pi \times 0.040 \times 1$

$$A_3 = 0.094 \text{ m}^2$$

Shape factor for concentric long cylinder  $F_{13} = 1$

[Refer HMT data book, Page No. 73]

$$(1) \Rightarrow Q_{13} = \frac{5.67 \times 10^{-8} [(973)^4 - T_3^4]}{\frac{1-0.8}{0.094 \times 0.8} + \frac{1}{0.094 \times 1} + \frac{1-0.5}{0.125 \times 0.05}}$$

$$Q_{13} = 3.43 \times 10^{-10} [(973)^4 - T_3^4] \dots\dots(3)$$

Heat exchange between radiation shield (3) and Room (2) is given by

$$Q_{32} = \frac{\sigma [T_3^4 - T_2^4]}{\frac{1-\epsilon_3}{A_3\epsilon_3} + \frac{1}{A_3F_{32}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

Since room is large,  $A_2 = \infty$

$$\Rightarrow \frac{1-\epsilon_2}{A_2\epsilon_2} = 0$$

Shape factor for small body enclosed by large body

$$F_{32} = 1$$

[Refer HMT data book, Page No.73]

$$\Rightarrow Q_{32} = \frac{5.67 \times 10^{-8} [T_3^4 - (308)^4]}{\frac{1-0.05}{0.125 \times 0.5} + \frac{1}{0.125 \times 1}} + 0$$

$$\boxed{Q_{32} = 3.54 \times 10^{-10} [T_3^4 - (308)^4]} \quad \dots\dots(4)$$

$$\Rightarrow 3.43 \times 10^{-10} [(973)^4 - T_3^4] = 3.54 \times 10^{-10} [T_3^4 - (308)^4]$$

$$\Rightarrow 307.4 - 3.43 \times 10^{-10} T_3^4 = 3.54 \times 10^{-10} T_3^4 - 3.18$$

$$310.58 = 6.97 \times 10^{-10} T_3^4$$

$$\Rightarrow \boxed{T_3 = 817 \text{ K}}$$

Substitute  $T_3$  value in (3) or (4).

Heat transfer with radiation shield

$$Q_{13} = 3.43 \times 10^{-10} [(973)^4 - (817)^4]$$

$$\boxed{Q_{13} = 154.6 \text{ W}}$$

$$\left. \begin{array}{l} \text{Reduction heat} \\ \text{loss due to} \\ \text{radiation shield} \end{array} \right\} = \frac{Q_{\text{without shield}} - Q_{\text{with shield}}}{Q_{\text{without shield}}}$$

$$= \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= \frac{3783.2 - 154.6}{3783.2} = 95.9\%$$

## UNIT 5 MASS TRANSFER

### 5.1 BASIC CONCEPTS

Air is a mixture of various gases. Whenever we have a multicomponent system with a concentration gradient, one constituent of the mixture gets transported from the region of higher concentration to the region of lower concentration till the concentration gradient reduces to zero. This phenomenon of the transport of mass as a result of concentration gradient is called 'Mass Transfer'.

The mass transfer phenomenon is analogous to heat transfer phenomenon. In heat transfer - heat energy flows in a direction of decreasing temperature gradient and ceases when the temperature gradient reduces to zero. In mass transfer - the transfer of mass takes place in the

direction of decreasing concentration gradient and ceases when the concentration gradient is zero.

The common examples of mass transfer in our everyday life and in many industries are:

- diffusion of smoke discharged by tall chimney into the atmosphere,
- a drop of ink diffusing in a glass of still water,
- evaporation of a drop of perfume in a room,
- humidification of air flowing over a spray pond or cooling tower,
- mixing of diesel or petrol with air inside an internal combustion engine,
- diffusion welding of metals,
- diffusion of neutron in a nuclear reactor.

## 5.2. Different Modes of Mass Transfer

There are basically two modes of mass transfer:

(i) Mass Transfer by Diffusion - the transport of mass by random molecular motion in quiescent or laminar flowing fluids is known as mass transfer by 'diffusion' and is analogous to heat transfer by conduction. Mass transfer by diffusion occurs due to (a) concentration gradient, (b) temperature gradient, and (c) hydrostatic pressure difference.

(ii) Convective Mass Transfer - the rate of molecular diffusion of mass can be accelerated by the bulk motion of the fluid. Mass can be transported between the boundary of a surface and a moving fluid (drying of clothes, molecular diffusion of a sugar cube in a cup of coffee by stirring, moist air flowing over the surface of an ocean and precipitation on a dry land etc.), or between two moving fluids which are relatively immiscible (formation of clouds, vapourisation of water in a tea kettle). This mechanism of mass transfer is called 'convective mass transfer' and is analogous to heat transfer by convection (free or forced).

## 5.3. Dalton's Law of Partial Pressure

Each constituent of a multicomponent system contributes to the total pressure by an amount which is known as the 'partial pressure' of the constituent. The relationship between the

partial pressures of the constituents is expressed by Dalton's Law:

The pressure of a mixture of gases is equal to the sum of the partial pressure of the constituents. The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume occupied by the mixture at the same temperature.

For a mixture of ideal gases, we have

$$P = P_A + P_B + \dots + P_K;$$

where  $P_A$  is the partial pressure of the species A and so on.

$$= \sum_i P_i \quad (5.1)$$

Dalton's law was reformulated by Gibbs to include a second statement on the properties of mixtures. The combined statement is Gibbs-Dalton law:

The internal energy, enthalpy and entropy of a gaseous mixture are respectively equal to the sum of the internal energies, enthalpies, and entropies of the constituents. The internal energy, enthalpy and entropy which a constituent would have if it occupied alone that volume occupied by the mixture at the temperature of the mixture.

#### 5.4. Molar Density, Mass Density, Mass Fraction and Mole Fraction

There are a number of ways by which the concentration for a species in a multicomponent mixture can be defined:

(i) Molar Density or Molar Concentration,  $C_A$  = number of moles of the species A per unit volume of mixture,  $\text{kg-mol/m}^3$

(ii) Mass Density or Mass Concentration,  $\rho_A$  = mass of the species A per unit volume of the mixture,  $\text{kg/m}^3$ .

(iii) Mass Fraction,  $m_A$  = mass concentration of component A / total mass density of the mixture.

(iv) Mole Fraction,  $X_A$  = number of moles of species A / total number of moles of the mixture. =  $C_A/C$

Therefore, the following summation rules hold true:

$$C_A + C_B + \dots + C_K = C$$

$$\rho_A + \rho_B + \dots + \rho_K = \rho$$

$$X_A + X_B + \dots + X_K = 1$$

$$m_A + m_B + \dots + m_K = I \quad (5.2)$$

Since the number of moles = mass of species/molecular weight, we have

$$C_A = \rho_A / M_A$$

For a perfect gas, we have:

$$P_A V = n_A R_o T, \text{ where } R_o \text{ is the universal gas constant,}$$

$$\text{and, } C_A = n_A / V = P_A / R_o T$$

$$X_A = C_A / C = P_A / P \text{ and } C = p / R_o T$$

#### 5.4 Mass Average and Molar Average Velocities and Different Types of Fluxes Velocity

ies: In a multicomponent mixture, the bulk velocity of the mixture can be defined on the basis of mass average or molar average velocity. Let  $V_A$  be the velocity of the species A and  $\rho_A$  is the mass density of the species A, then the mass average velocity would be:

$$\begin{aligned} V &= \frac{\rho_A V_A + \rho_B V_B + \dots}{\rho_A + \rho_B} = \frac{\rho_A V_A + \rho_B V_B + \dots}{\rho} \\ &= m_A V_A + m_B V_B + \dots \end{aligned} \quad (5.3)$$

Similarly, the molar average velocity would be:

$$U = \frac{C_A V_A + C_B V_B + \dots}{C_A + C_B} = \frac{C_A V_A + C_B V_B + \dots}{C} = X_A V_A + X_B V_B + \dots$$

Since mass transfer requires the diffusion of a species with respect to a plane moving with an average velocity, diffusion will take place when the diffusion velocity is in excess of the average velocity. Thus

$$\text{Mass diffusion velocity of the species A : } V_A - V \quad (5.4)$$

$$\text{Molar diffusion velocity of the species A : } V_A - U \quad (5.5)$$

Fluxes: The mass flux of species A can be expressed relative to either a fixed observer or an observer moving with the bulk velocity. For a stationary observer, the absolute flux of any species A will be equal to the sum of the flux due to the molecular diffusion and that due to the bulk motion.

Thus, Absolute flux:  $\rho_A V_A$  and, Diffusion flux:  $\dot{m}/A$

Flux due to bulk motion:  $\rho_A V$

$$\rho_A V_A = \dot{m}/A + \rho_A V, \quad \text{or} \quad \dot{m}/A = \rho_A (V_A - V) \quad (5.6)$$

Similarly, molar diffusion flux =  $C_A(V_A - V)$

### 5.5 Fick's Law of Diffusion\*

The fundamental equation (one-dimensional) of molecular diffusion is known as Fick's law. It has been derived from the kinetic theory of gases, and can be written for a binary mixture as

$$J_A = -D_{AB} (d C_A/dx) \quad (5.8)$$

where  $D_{AB}$  = diffusion coefficient of species A with respect to species B,  
 $J_A$  = molar flux in the X-direction relative to the molar average velocity,

$dC_A/dx$  = Concentration gradient in X-direction.

Let us consider a two compartment tank as shown in Fig. 5.1. One compartment contains gas A and the other compartment contains gas B and both the compartments are initially at a uniform pressure and temperature throughout. When the partition between the compartments is removed, the two gases will diffuse through each other until equilibrium is established and the concentration of the gases is uniform throughout the tank.

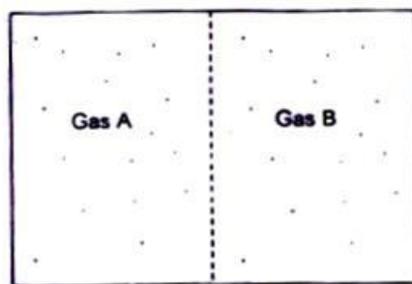


Fig. 5.1 Diffusion of species A in to species B

Fig 5.2 illustrates the dependence of diffusion on the concentration profile. The concentration of the species A on the left side of the imaginary plane is greater than that on the right side. As such, more molecules will cross the plane per unit time from left to right. This would lead to a net transfer of mass from the region of higher concentration to the region of lower concentration.

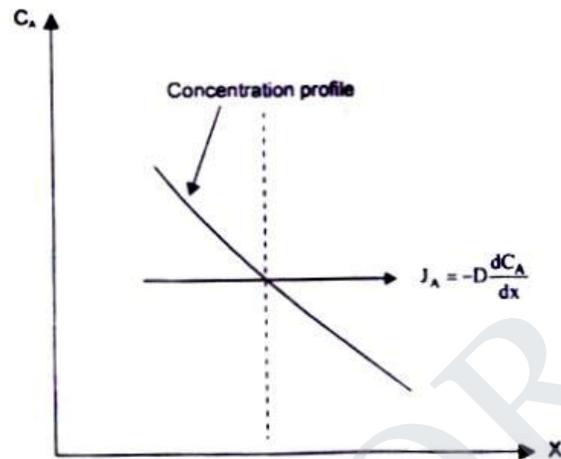


Fig. 5.2 Dependence of diffusion on concentration profile

\* This law assumes that fluxes are measured relative to the coordinates that move with the average velocity of the mixture.

### 5.6.CONVECTIVE MASS TRANSFER :

(i) Diffusion in Gases - the diffusion rates in gases are dependent on the molecular speed which is a function of temperature and therefore, the diffusion coefficient depends upon the temperature of gases.

Gilliland has proposed a semi-empirical equation for diffusion coefficient in a binary gas mixture –

$$D = 435.7 \frac{T^{3/2}}{p(V_A^{1/3} + V_B^{1/3})^2} \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} \quad (5.9)$$

where D is in square centimeters per second, T is in Kelvin, p is the total pressure of the

system in pascals,  $V_A$  and  $V_B$  are the molecular volumes of the species A and R as calculated from the atomic volumes in Table 12.1,  $M_A$  and  $M_B$  are the molecular weights of species A and B.

Diffusion coefficients for gases depend upon pressure, temperature and other molecular properties of diffusing gases. At two different pressure and temperature, we have

$$D_2/D_1 = (p_1/p_2) \cdot (T_2/T_1)^{3/2} \quad (5.10a)$$

**Table 5.1 Atomic volumes\***

|                                      |      |                                    |      |
|--------------------------------------|------|------------------------------------|------|
| Air                                  | 29.9 | In secondary amines                | 1.2  |
| Bromine                              | 27.0 | Oxygen, molecule (O <sub>2</sub> ) | 7.4  |
| Carbon                               | 14.8 | Coupled to two other elements:     |      |
| Carbon dioxide                       | 34.0 | In aldehydes and ketones           | 7.4  |
| Chlorine                             |      | In methyl esters                   | 9.1  |
| Terminal as in R-Cl                  | 21.6 | In ethyl esters                    | 9.9  |
| Medial as in R-CHCl-R                | 24.6 | In higher esters & ethers          | 11.0 |
| Flourine                             | 8.7  | In acids                           | 12.0 |
| Hydrogen, molecule (H <sub>2</sub> ) | 14.3 | In union with S, P, N              | 8.3  |
| in compounds                         | 3.7  | Phosphorous                        | 27.0 |
| Iodine                               | 37.0 | Sulphur                            | 25.6 |
| Nitrogen, molecule (N <sub>2</sub> ) | 15.6 | Water                              | 18.8 |
| in primary amines                    | 10.5 |                                    |      |

\*(For three numbered ring like ethylene oxide, deduct 6.0, for four numbered ring like cyclobutane, deduct 8.5, for six numbered ring like benzene, deduct 15.6, for naphthelene ring, deduct 30.0.)

(ii) Diffusion in Liquids and Solids - Diffusion in liquids occurs at much slower rate than in gases. Since kinetic theory of liquids is not as much developed as that of gases, it is usually assumed as a first approximation that equations of the same general form are applicable to the diffusion of a solute in a solvent as to the diffusion in gases, i.e., Fick's law is assumed valid for liquids.

Diffusion coefficient for most of the common organic and inorganic materials in the

usual solvents such as water, alcohol and benzene at room temperature lie in the range of  $1.79 \times 10^{-3}$  to  $1.075 \times 10^{-7}$  cm<sup>2</sup>/s.

Diffusion in solids is much slower than in liquids. Diffusion of solids in solid has limited engineering applications but diffusion of fluids in solids have extensive applications. Fick's law is sometimes used, with an empirically determined effective diffusivity which takes care of the structure of solid. A typical problem of liquid transfer in a solid, of interest, is drying of solids.

### 5.7. HEAT AND MASS TRANSFER ANALOGY:

Fick's law (Eq. 5.8) can also be expressed in terms of mass flux per unit area or mass concentration or in terms of molal concentrations and fluxes. For gases, the law may be expressed in terms of partial pressures by making use of the perfect gas equation of state:

$$p = p_A + p_B + p_C + \dots + p_D$$

Since the characteristic gas constant of a gas is:  $R_A = R_o/M_A$

We have  $p_A = p_A M_A/R_o T$

and  $m_A/A = -D_{AB} (M_A/R_o T) dp_A/dx$  for isothermal diffusion. (5.10b)

Similarly, the diffusion of the component B, for the system shown in Fig. 5.1, we can write

$$\frac{\dot{m}_B}{A} = D_{BA} \frac{M_B}{R_o T} \frac{dp_B}{dx} \quad (5.11)$$

When we have equimolar counter diffusion, shown in Fig. 12.3 (a, b), the steady state molal diffusion rates of the species A and B, represented by  $N_A$  and  $N_B$  will be given by

$$N_A = \frac{\dot{m}_A}{M_A} = -D_{AB} \left( \frac{A}{R_o T} \right) \left( \frac{dp_A}{dx} \right) \quad (5.12)$$

$$\text{and } N_B = \frac{\dot{m}_B}{M_B} = +D_{BA} \left( \frac{A}{R_o T} \right) \left( \frac{dp_B}{dx} \right) \quad (5.13)$$

The total pressure of the system remains constant at steady state,

or,  $p = p_A + p_B$  ; and  $dp_A/dx + dp_B/dx = 0$

as  $dp_A/dx = -dp_B/dx$

Since each molecule of A is replaced by a molecule of B, the molal diffusion rates must be equal. Thus:  $N_A = -N_B$ , and

$$-D_{AB} \left( \frac{A}{R_0 T} \right) \left( \frac{dp_A}{dx} \right) = -D_{BA} \left( \frac{A}{R_0 T} \right) \left( \frac{dp_A}{dx} \right) \tag{5.14}$$

or  $D_{AB} = D_{BA} = D$

This fact is known as the equivalence of diffusion coefficients or diffusivities in binary mixtures, and is a property of the binary mixture.

By integrating Eq. (12.10), we can obtain the mass flux of the species A as;

$$\frac{\dot{m}_A}{A} = -\frac{DM_A}{R_0 T} \times (p_{A_2} - p_{A_1}) / \Delta x \tag{5.15}$$

corresponding to the nomenclature used in Fig. 5.3 (a, b). Table 5.2 gives the values of the binary diffusion coefficients.

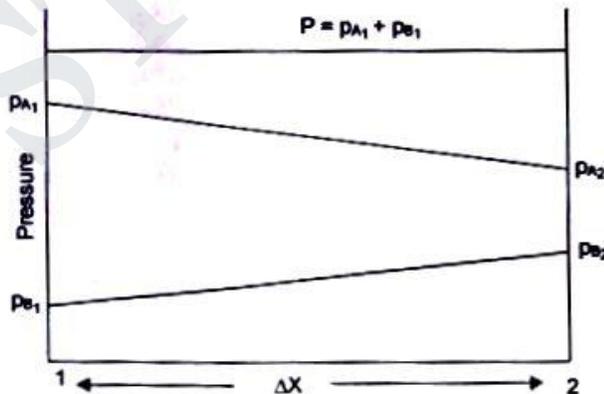
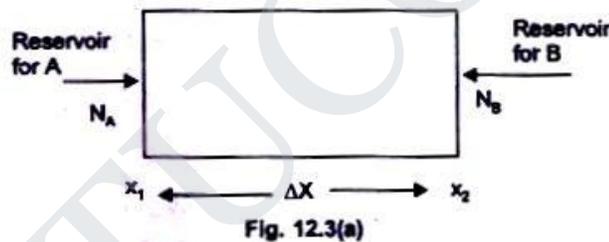


Fig. 5.3(b) Equimolar counter-diffusion (partial pressure profile)

(iv) It does not tell about diffusion due to temperature or pressure gradient or due to external forces.

### 5.8 CONVECTIVE MASS TRANSFER CORRELATIONS :

Let us consider a tank containing water which is exposed to air in the tank as shown in Fig. 5.4. We assume that:

(i) the system is isothermal,

(ii) the total pressure remains constant,

(iii) the system is in steady state. Since there has to be a little movement of air over the top of the tank to remove the water vapour that diffuses to that point, the air movement does not create any turbulence to alter the concentration profile in the tank, and

(iv) air and water vapour both behave like ideal gases.

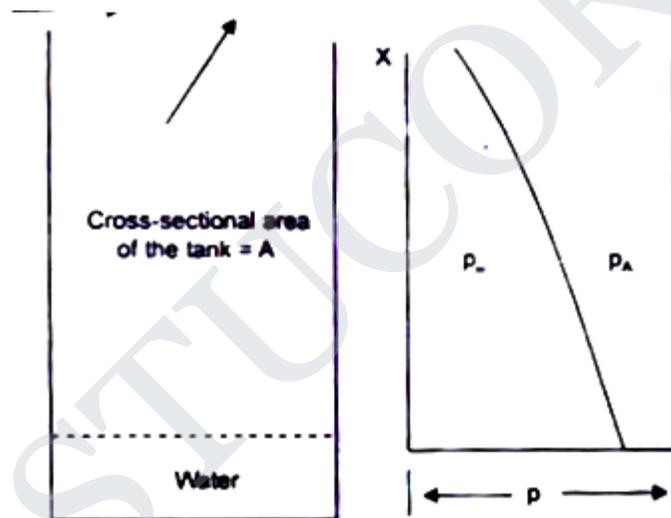


Fig. 5.4 Diffusion of water vapour in air

From Eq. (5.10), the downward diffusion of air can be written as

$$\dot{m}_A = -(DAM_A / R_o T)(dp_A / dx) \quad (5.16)$$

and this has to be balanced by the bulk mass transfer upward. Therefore,

$$-p_A AV = -\frac{p_A M_{\text{air}}}{R_o T} AV ; \text{ where } V \text{ is the upward bulk mass velocity.}$$

$$\therefore V = \frac{D}{p_A} \frac{dp_A}{dx} \quad (5.17)$$

The mass diffusion of water vapour upward is

$$\dot{m}_w = -DA \frac{M_w}{R_o T} \frac{dp_w}{dx} \quad (5.18)$$

and the bulk transport of water vapour would be

$$p_w AV = \frac{p_w M_w}{R_o T} AV \quad (5.19)$$

And, the total mass transport is then,

$$\dot{m}_{w\text{total}} = -\frac{DAM_w}{R_o T} \frac{dp_w}{dx} + \frac{p_w M_w}{R_o T} A \frac{D}{p_A} \frac{dp_A}{dx}$$

Since the total pressure remains constant, by Dalton's law we get

$$p = p_A + p_w \text{ or, } dp_A / dx = -dp_w / dx$$

$$\dot{m}_{w\text{total}} = -\frac{DAM_w}{R_o T} \frac{dp_w}{dx} \left[ 1 + \frac{p_w}{p_A} \right]$$

$$\square \quad \dot{m}_{\text{total}} = -\frac{DM_w A}{R_o T} \frac{p}{p - p_w} \frac{dp_w}{dx} \quad (5.20)$$

This relation is called the Stefan's law. Upon integration,

$$\dot{m}_{w\text{total}} = +\frac{DpM_w A}{R_o T(x_2 - x_1)} \log_e \frac{p - p_{w2}}{p - p_{w1}} = \frac{DpM_w A}{R_o T(x_2 - x_1)} \log_e \frac{p_{A2}}{p_{A1}} \quad (5.21)$$

**UNIT 5: MASS TRANSFER****PART A****TWO MARK QUESTIONS:****1. WHAT IS MASS TRANSFER? (AU 2010)**

The process of transfer of mass as a result of the species concentration difference in a mixture is known as mass transfer

**2. GIVE THE EXAMPLES OF MASS TRANSFER.**

1. Humidification of air in cooling tower.
2. The transfer of water vapor into dry air.

**3. WHAT ARE THE MODES OF MASS TRANSFER? (AU2013)**

There are basically two modes of mass transfer.

1. Diffusion mass transfer.
2. Convective mass transfer.

**4. WHAT IS MOLECULAR DIFFUSION? (AU2012)**

The transport of water on a microscopic level as a result of diffusion from a region of higher concentration to a region of lower concentration in a mixture of liquids or gases is known as molecular diffusion.

#### 5. STATE FICK'S LAW OF DIFFUSION?

(AU2014)

The diffusion rate is given by the Fick's law, which states that molar flux of an element per unit area is directly proportional to concentration gradient.

$$J_A = -D_{AB} \left( \frac{dC_A}{dx} \right)$$

#### 6. DEFINE SCHMIDT NUMBER.

(AU 2012)

It is defined as the ratio of the molecular diffusivity of momentum to the molecular diffusivity of mass.

$$Sc = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of mass}}$$

#### 7. DEFINE SCHERWOOD NUMBER.

(AU 2010)

It is defined as the ratio of concentration gradients at the boundary.

$$Sh = h_M \frac{X}{D_{ab}}$$

#### 8. DEFINE MOLAR CONCENTRATION:

Number of molecules of a component per unit volume of the mixture. It is expressed in  $\text{kg} - \text{mole}/\text{m}^3$ .

$$\text{MOLAR CONCENTRATION} = \frac{\text{Number of moles of component}}{\text{Unit Volume of mixture}}$$

#### 9. DEFINE MASS FRACTION.

The mass fraction is defined as the ratio of mass concentration species to the total mass density of the mixture.

#### 10. DEFINE MOLE FRACTION.

(AU 2014)

The mole fraction is defined as the ratio of mole concentration of a species to the total molar concentration.

#### Part B

#### ANSWER THE FOLLOWING:

1. A vessel contains a mixture of 2 kmol of  $\text{CO}_2$  and 4.5 kmol of air at 1 bar and  $25^\circ\text{C}$ . If air

contains 21 % oxygen and 79% nitrogen by volume, calculate for the mixture:

- (i) The mass of CO<sub>2</sub>, O<sub>2</sub> and N<sub>2</sub>, and the total mass;
- (ii) The percentage carbon content by mass;
- (iii) The molar mass and the gas constant for the mixture;
- (iv) The specific volume of the mixture.

**Solution:** (1) Number of moles of O<sub>2</sub> = 0.21 × 4.5 = 0.945 kmol,

Number of moles of N<sub>2</sub> = 0.79 × 4.5 = 3.55 kmol

Mass of CO<sub>2</sub> = 2 × 44 = 88kg; Mass of O<sub>2</sub> = 0.945 × 32 = 30.24 kg

Mass of N<sub>2</sub> = 3.55 × 28 = 99.54 kg

The total mass = 88 + 30.24 + 99.54 = 217.48 kg

(ii) Percentage of carbon in the mixture;  $(24/217.48) \times 100 = 11.035\%$  by mass.

(iii) Total number of moles =  $n_{\text{CO}_2} + n_{\text{N}_2} = 2 + 0.945 + 3.555 = 6.5$  kmol

$$\text{Molar mass} = \sum \frac{n_i}{n} m_i$$

$$= (2/6.5) \times 44 + (0.945/6.5) \times 32 + (3.555/6.5) \times 28$$

$$= 33.5 \text{ kg/kmol}$$

And the gas constant of the mixture;  $8314/33.5 = 248.18 \text{ J/kgK}$

(iv) Specific volume of the mixture,  $v = RT/p = 248.18 \times 298 / (1 \times 10^5)$

$$= 0.7395 \text{ m}^3/\text{kg}.$$

**2. The air pressure inside a synthetic rubber ball (400 mm inside diameter and 15 mm thick) decreases from 3.5 bar to 3.45 bar in seven days. Estimate the coefficient of diffusion of air in synthetic rubber if the temperature is 25°C and the solubility of air in the rubber is  $1.8 \times 10^{-3} \text{ kmol/m}^3 \text{ bar}$ . (AU 2012)**

**Solution:** Since the pressure change is very small during a period of seven days, the problem can be treated as quasi-steady. The initial mass of air inside the ball

$$m_1 = p_1 V / RT = \frac{3.5 \times 10^5 \times (4/3) \pi (0.2)^3}{287 \times 298} = 0.137 \text{ kg}$$

$$\text{The final mass, } m_2 = \frac{3.45 \times 10^5 \times (4/3) \pi (0.2)^3}{287 \times 298} = 0.1352 \text{ kg}$$

$$\text{The rate of leakage} = \frac{0.137 - 0.1352}{7 \times 24 \times 3600} = 2.976 \times 10^{-9} \text{ kg/s}$$

The average pressure inside the ball =  $(3.45 + 3.5)/2 = 3.475 \text{ bar}$

Concentration inside the ball =  $p_1 \times S = 3.475 \times 1.8 \times 10^{-3} \times 29, \text{ kg/m}^3$ .

$$= 0.1814 \text{ kg/m}^3$$

Concentration at the outside surface =  $p_2 \times S = 1 \times 1.8 \times 10^{-3} \times 29$

$$= 0.0522 \text{ kg/m}^3$$

Since conduction heat transfer is analogous to diffusion mass transfer, the diffusive resistance for the spherical shell can be written as

$$R_D = (r_2 - r_1) / (4\pi D r_1 r_2), \text{ and}$$

$$\dot{m}_A = (C_{A_1} - C_{A_2}) / R_D = 4\pi D r_1 r_2 (C_{A_1} - C_{A_2}) / (r_2 - r_1)$$

$$\text{Or, } D = \dot{m}_A (r_2 - r_1) / [4\pi r_1 r_2 (C_{A_1} - C_{A_2})]$$

$$= \frac{2.976 \times 10^{-9} \times 0.015}{4 \times 3.142 \times 0.2 \times 0.215 \times (0.1814 - 0.0522)} = 6.4 \times 10^{-10} \text{ m}^2/\text{s}$$

**3. Estimate the rate of burning of a pulverized carbon particle in a furnace if the diameter of the particle is 4 mm, pressure 1 bar. The oxygen is available at 1100 K. Assume that fairly large layer of CO<sub>2</sub> surrounds the carbon particle. Take D = 1 cm<sup>2</sup>/s. (AU2011)**

**Solution:** The combustion equation is  $C + O_2 \rightarrow CO_2$ , i.e., there will be an Equimolal counter-diffusion between O<sub>2</sub> and CO<sub>2</sub>,

Since a fairly large blanket of carbon dioxide surrounds the carbon particle, the partial pressure

of carbon dioxide at the surface of the carbon particle will be 1 bar and the partial pressure of oxygen will be zero. Similarly, the partial pressure of carbon dioxide far outside will be zero and the partial pressure of oxygen will be 1 bar.

From Eq. (12.12), we have:  $\frac{N_A}{A} = -D \frac{1}{R_o T} \frac{dp_A}{dx}$

$$\text{Or, } \frac{N_A}{4\pi r^2} = -\frac{D}{R_o T} \frac{dp_A}{dr}$$

Separating the variables and integrating, we get

$$\frac{N_A R_o T}{4\pi D} \int_{r=r_1}^{\infty} \frac{dr}{r^2} = -\int_1^0 dp_A$$

$$\text{Or, } p_{A_1} = \frac{N_A R_o T}{4\pi D} \cdot \frac{1}{r_1} \quad \text{and} \quad N_{CO_2} = \frac{4\pi \times 1 \times 10^{-4} \times 10^5 \times 2 \times 10^{-3}}{8314 \times 1100}$$

$$= 2.748 \times 10^{-8} \text{ kgmol/s}$$

Since 1 mol of carbon will produce 1 mol of CO<sub>2</sub>, the rate of burning of carbon will be

$$= 2.748 \times 10^{-8} \times 12 = 3.298 \times 10^{-7} \text{ kg/so}$$

**4. Carbon dioxide at 30°C and at a pressure of 2.5bar is flowing through a rubber pipe, inside diameter 25 mm and thickness 5 mm. The coefficient of diffusion of carbon dioxide-rubber combination is  $0.11 \times 10^{-9} \text{ m}^2/\text{s}$  and the solubility of carbondioxide  $4 \times 10^{-2} \text{ k mol/m}^3 \text{ bar}$ . Calculate the loss of CO<sub>2</sub> by diffusion per unit length of the pipe.**

**Solution:** The species concentration at the gas-solid interface is obtained in terms of the partial pressure of the gas adjacent to the solid surface and a solubility factor, S.

Or, Concentration = Partial pressure  $\times$  S

The carbon dioxide is flowing through the rubber tube at 2.5bar. As such, the partial pressure of CO, at the rubber-gas interface is 2.5bar and the concentration is then,

$$C_{A_1} = 2.5 \times 4 \times 10^{-2} \text{ kmol/m}^3 = 4.4 \text{ kg/m}^3$$

Assuming that at the outer surface of the rubber pipe, the partial pressure of carbon dioxide is zero, the concentration at the outer surface is zero. Further, the diffusion mass transfer is analogous to conduction heat transfer, the diffusion resistance in cylindrical system can be written as,

$$R_D = \ln(r_2/r_1)/(2\pi LD)$$

$$= \ln(17.5/12.5)/(2 \times 3.142 \times 1 \times 0.11 \times 10^{-9}) = 4.867 \times 10^8$$

$$\text{And } \dot{m}_{\text{CO}_2} = (C_{A_1} - C_{A_2})/R_D = (4.4 - 0.0)/4.867 \times 10^8$$

$$= 9.04 \times 10^{-4} \text{ kg/s, or } 3.25 \times 10^{-5} \text{ kg/hr}$$

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